Predicate Logic

OVERVIEW AND PARSING

PL – Restrictions

Restrictions with PL

- Facts (that are either T/F) only can be represented
- NOT sufficient to represent the complex / (NL) natural language statements
- Having limited expressive power
- Examples:

Some humans are intelligent

All children are naughty

- So, some more powerful logic required

Predicate logic / First-order logic

Predicate Logic

- Another name: ***First Order Logic (FOL)***
- One more way of knowledge representation in AI
- Extension to PL
- Sufficiently expressive to *represent the natural language statements* in a concise way
- Powerful language that
  - Develops information about the objects in a more easy way &
  - Can also express the relationship between those objects.

Predicate Logic

- Not like PL in way that
  - Does not only assume that the world contains facts
  - but also assumes the following things in the world:
    - **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
    - **Relations:**
      - **Unary relation (**red, round, is adjacent,…) /
      - **n-any relation (**the sister of, brother of, has color, comes between)
    - **Function:** Father of, best friend, third inning of, end of, ...
- FOL **(**as a natural language**)** also has two main parts:
  - **Syntax**
  - **Semantics**

Predicate Logic
Syntax

- Basic syntactic elements: ***Symbols***
  - Statements written in short-hand notation
- Basic elements of the FOL:

Constant	$1,2,A,John,Mumbai,cat,\ldots$
Variables	$x,y,z,a,b,\ldots$
Predicates	$Brother,Father,>,\ldots$
Function	$sqrt,LeftLegOf,\ldots$
Connectives	$\neg,\wedge,\vee,\rightarrow,\Leftrightarrow$
Equality	$==$
Quantifier	$\forall,\exists$

Atomic Sentences & Compound Sentences

Atomic sentences:

- The most basic sentences of FOL
  - Formed from a predicate symbol followed by a parenthesis with a sequence of terms
  - Can be represented as

$$Predicate\ (term1, term2, \dots \dots, term\ n)$$

  - **Examples:**
    - **Ravi and Ajay are brothers:**

$$brothers(Ravi, Ajay)$$

    - **Chinky is a cat:**

$$cat\ (Chinky)$$

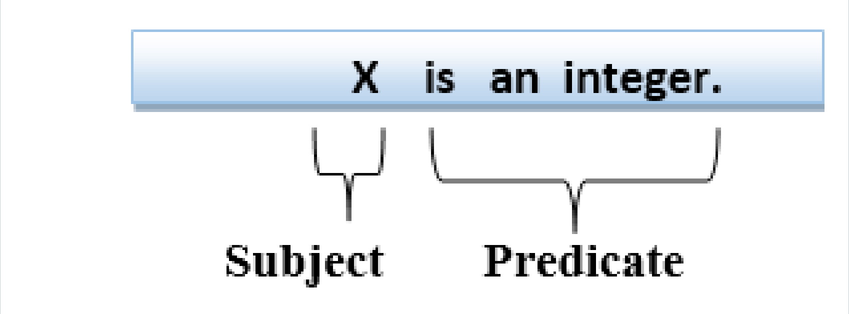
Complex Sentences:

- Made by combining atomic sentences
  - Using connectives

Predicate Logic
Basic Idea

FOL statements divided into two parts:

- **Subject:**
  - Main part of the statement
- **Predicate:**
  - Defined as a relation - Binds two atoms together in a statement
- **Example:**



<u>Quantifier:</u>

- Language element which generates quantification
  - Quantification specifies the quantity of specimen in the universe of discourse
- Basically symbols
  - Determine / Identify the range and scope of the variable in the logical expression
- There are two types of quantifier:
  - ***Universal Quantifier, (for all, everyone, everything)***
  - ***Existential quantifier, (for some, at least one).***

Universal Quantifier:

- Symbol of logical representation:
  - Specifies that the statement within its range is true for everything /
  - Every instance of a particular thing
- Represented by a symbol ∀,
  - Resembles an inverted *A*
- If x is a variable, then $\forall x$ is read as:
  - **For all *x***
  - **For each *x***
  - **For every *x***

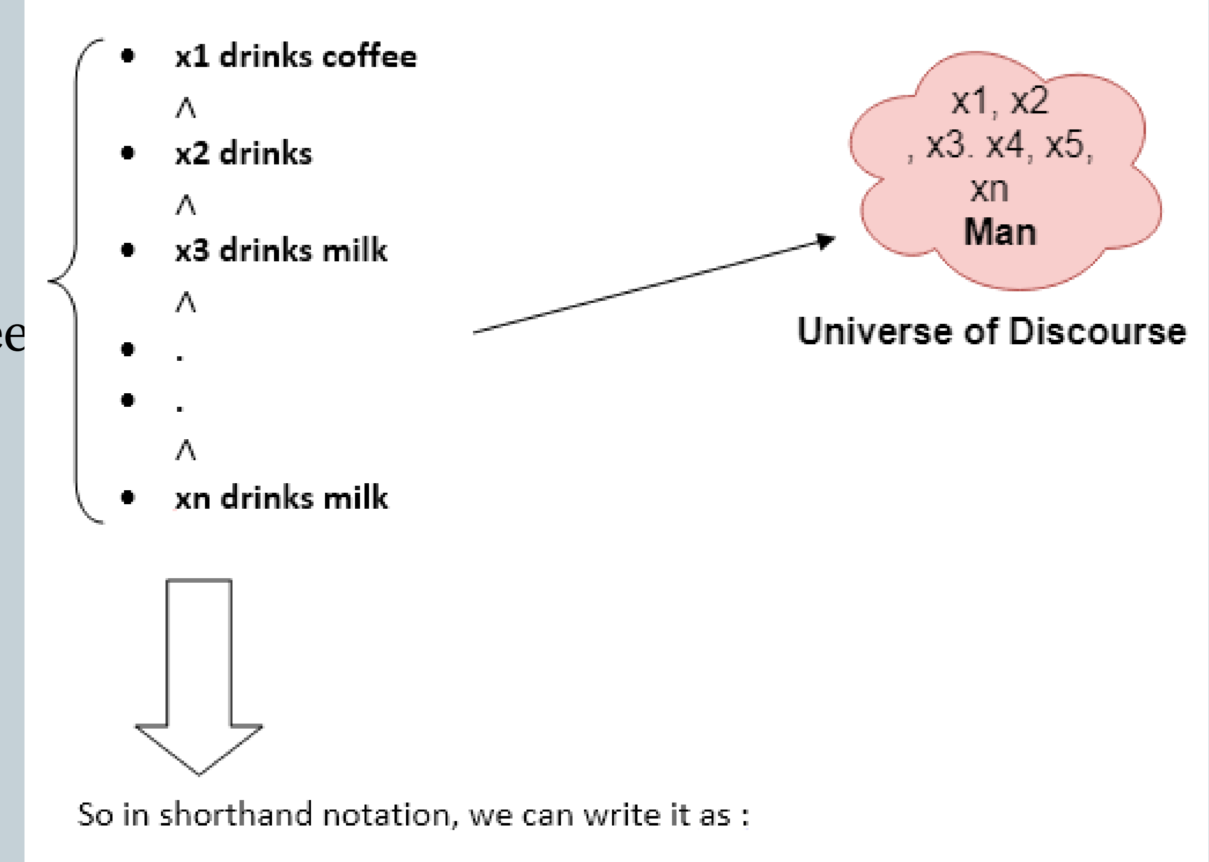
In universal quantifier we use implication →

FOL - Universal Quantifier Example

All man drink coffee

$$\forall x\; man(x) \rightarrow drink(x, cofee)$$

- There are all x where x is a man who drink coffee



Existential Quantifier:

- Type of quantifiers
  - Which express that the statement within its scope is true for at least one instance of something
- Denoted by ∃
  - Resembles as inverted E
  - When used with a predicate variable, it is called as an existential quantifier
- If x is a variable, then $\exists x/\exists(x)$is read as:
  - **There exists a x**
  - **For some x**
  - **For at least one x**

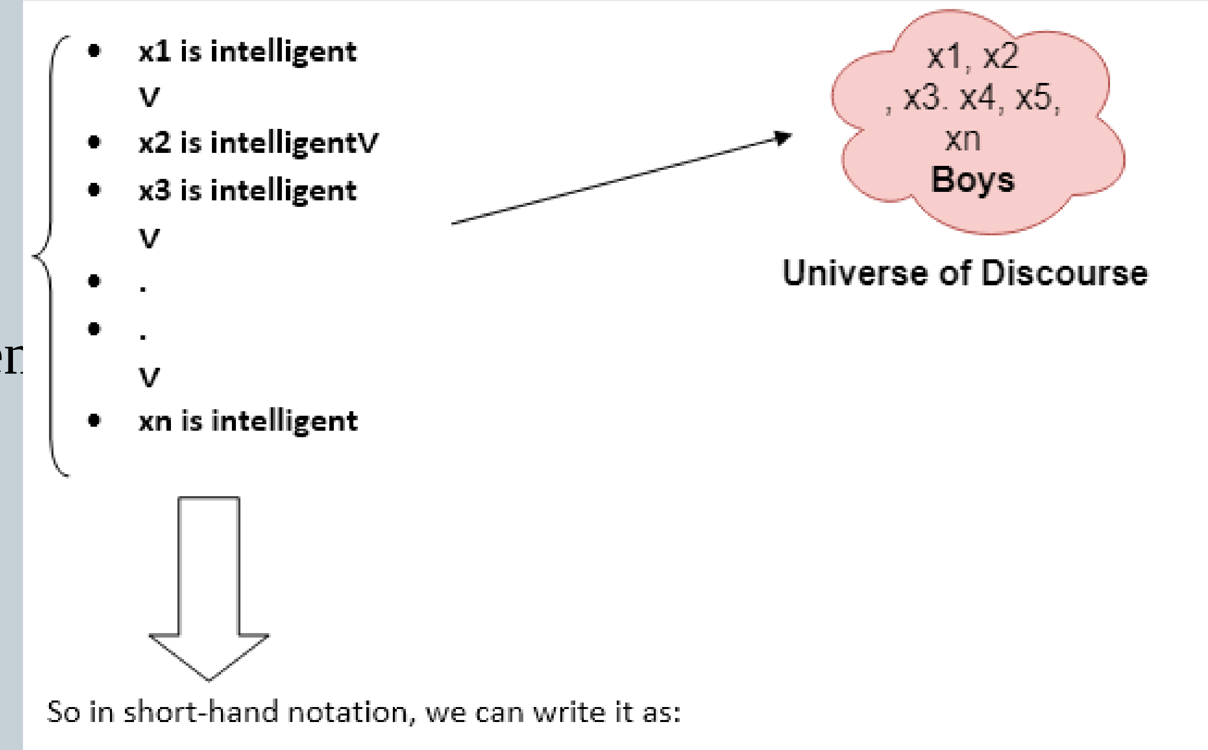
In Existential quantifier we always use AND or Conjunction symbol $\wedge$

FOL - Existential Quantifier Example

Some boys are intelligent

$\exists x\ boys(x) \wedge intelligent(x)$

There are some x where x is a boy who is intelligen



FOL - Quantifier Properties

- Universal quantifier:

$$\forall \boldsymbol{x} \forall \boldsymbol{y} \text{ is similar to } \forall \boldsymbol{y} \forall \boldsymbol{x}$$

- Existential quantifier

$$\exists \boldsymbol{x} \exists \boldsymbol{y} \text{ is similar to } \exists \boldsymbol{y} \exists \boldsymbol{x}$$

- Universal & Universal quantifier:

$$\exists \boldsymbol{x} \forall \boldsymbol{y} \text{ is NOT similar to } \forall \boldsymbol{y} \exists \boldsymbol{x}$$

FOL
Free & Bound Variables

Free Variable:

- A variable is a free ***variable*** in a formula if it ***occurs outside the scope of the quantifier***
- **Example:**

$$\forall \boldsymbol{x}\ \exists(\boldsymbol{y})[\boldsymbol{P}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})]$$

  - z is a free variable

Bound Variable:

- A variable is a bound ***variable*** in a formula if it ***occurs within the scope of the quantifier***
- **Example:**

$$\forall \boldsymbol{x}\ \exists(\boldsymbol{y})[\boldsymbol{P}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})]$$

  - x&y are bound variable

Rules of Inference Predicate Logic

OVERVIEW AND PARSING

FOL – Inference

- Inference in First-Order Logic
  - Used to deduce new facts or sentences from existing sentences

- Let’s go thru’ some basic terminologies used in FOL

Substitution:

- Fundamental operation performed on terms and formulas
- It occurs in all inference systems in FOL
- Becomes complex in the presence of quantifiers in FOL

$F[a/x]$ refers to substitute a constant $\underline{a}$ in place of variable $\underline{x}$

- Note: FOL is capable of expressing facts about some or all objects in the universe

Equality:

- *Equality symbols* used to specify that the two terms refer to the same object

Example: Brother (John) = Smith

- Object referred by the **Brother (John)** is similar to the object referred by ***Smith***

- Equality symbol also used with negation:
  - To represent that two terms are not the same objects

Example: $\neg(x = y)$ ***which is equivalent to*** $x \neq y$

Basic Inference Rules in FOL:

- **Universal Generalization**
- **Universal Instantiation**
- **Existential Instantiation**
- **Existential introduction**

Universal Generalization

- A valid inference rule
- If *premise* ***P(c) is true for any arbitrary element c*** in the universe of discourse, then we can have a *conclusion* as ***∀x P(x)***

$$\frac{P(c)}{\forall x P(x)}$$

  - Used if we want to show that every element has a similar property
  - [x must not appear as a free variable]
- **Example:**

$P(c)$: **A byte contains 8 bits**

$\forall x P(x)$: **All bytes contain 8 bits**

Universal Instantiation (UI):

- Also called ***universal elimination***
  - Can be applied multiple times to add new sentences
  - The new KB is logically equivalent to the previous KB
- **we can infer any sentence obtained by substituting a ground term for the variable**
- Any sentence $\boldsymbol{P(c)}$ **can be inferred by substituting a ground term $\boldsymbol{c}$** (constant within domain x) from $\boldsymbol{\forall x P(x)}$ for any object in the universe of discourse
  - Premise: $\forall x P(x)$ & Conclusion: $P(c)$

$$\frac{\forall x P(x)}{P(c)}$$

- **Example:**

$\forall x P(x)$: **Every person likes ice-cream**

$P(c)$: **Arun likes ice-cream**

FOL
Rules of Inference

- **Example: 2.**
- "All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

$$\forall x\, king(x) \land greedy\,(x) \rightarrow Evil\,(x)$$

- So from this information, we can infer any of the following statements using Universal Instantiation:

$$King(John) \land Greedy\,(John) \rightarrow Evil\,(John),$$
$$King(Richard) \land Greedy\,(Richard) \rightarrow Evil\,(Richard),$$
$$King(Father(John)) \land Greedy\,(Father(John)) \rightarrow Evil\,(Father(John)),$$

Existential Instantiation

- Also called ***existential elimination***
  - It can be applied only once to replace the existential sentence
  - The new KB is not logically equivalent to old KB
    - But it will be satisfiable if old KB was satisfiable
- This rule states that one can infer P(c) from the formula given in the form of ∃x P(x) for a new constant symbol c.
- Any sentence $\boldsymbol{P(c)}$ **can be inferred from** $\boldsymbol{\exists x P(x)}$ for a new constant symbol $\boldsymbol{c}$
- The restriction with this rule is that c used in the rule must be a new term for which P(c) is true
  - Premise: $\exists x P(x)$ & Conclusion: $P(c)$

$$\frac{\exists x P(x)}{P(c)}$$

- **Example:**

Given: $\boldsymbol{\exists x Crown(x) \land OnHead(x, Arun)}$
So, it can be inferred that $\boldsymbol{Crown(K) \land OnHead(K, Arun)}$

K: **Skolem constant**

The Existential instantiation is a special case of **Skolemization process**

Existential Introduction

- Also known as an ***existential generalization***
- If there is ***some element c in the universe of discourse having a property P***, then we can infer that ***there exists something in the universe which has the property P***.
  - Premise: $\exists x P(x)$ & Conclusion: $P(c)$

$$\frac{P(c)}{\exists x P(x)}$$

- **Example:**

Given: *Priyanka got good marks in English*

So, it can be inferred that *Someone got good marks in English*

Generalized Modus Ponens Rule

- Lifted version of Modus ponens
- Summarized as: ***P implies Q and P is asserted to be true, therefore Q must be True***
- According to Modus Ponens, for atomic sentences **pi, pi', q**. Where there is a substitution θ such that SUBST **(θ, pi',) = SUBST(θ, pi)**

$$\frac{\boldsymbol{p1', p2', \dots, pn', (p1 \land p2 \land \cdots \land pn \to q)}}{\boldsymbol{SUBST(\theta, q)}}$$

- **Example:**
- **We will use this rule for Kings are evil, so we will find some x such that x is king, and x is greedy so we can infer that x is evil.**

- **Example:**

Given: $\boldsymbol{Kings\ are\ evil\ \&\ \exists x\ king(x), greedy(x)}$

So, it can be inferred that ***x is evil***