

Let's go to work

# Find the majority element in an array

A **majority element** in an array  $A[]$  of size  $n$  is an element that appears more than  $n/2$  times (and hence there is at most one such element).

Optimum Run time :  $O(N)$

Optimum Aux. Space Complexity:  $O(1)$

**Examples :**

**Input :** [3, 3, 4, 2, 4, 4, 2, 4, 4]

**Output :** 4

**Explanation:** The frequency of 4 is 5 which is greater than the half of the size of the array size.

**Input :** [1, 2, 3, 5, 1, 1, 1, 1, 1]

**Output :** 1

**Explanation:** The frequency of 1 is 6 which is greater than the half of the size of the array size.



# **Approach 1 : Brute Force**



# Approach 1 : Brute Force



# Let's solve it with Brute Force First

- We can exhaust the search space in quadratic time by checking whether each element is the majority element.
- Algorithm
- Brute force:
  - This approach iterates over the array, and then iterates again for each number to count its occurrences.
  - As soon as a number is found to have appeared more than any other can possibly have appeared, return it.

SOLUTION 1:

Run Time  $O(N^2)$

Space  $O(1)$



```
# Brute Force
def find_majority_element_bf(nums: List[int]) -> int:
    """
    param nums: int container to look up
    :return: frequent value in array
    :pre-cond: there will be at least one elem in list
    """

    pass
```





# Let's solve it with Sorting in mind

---

Use Sorted to sort the list and a nested loop!

SOLUTION 1:

Run Time  $O(N \log N)$

Space  $O(1)$



```
# Brute Force
def find_majority_element_bf(nums: List[int]) -> int:
    """
    param nums: int container to look up
    :return: frequent value in array
    :pre-cond: there will be at least one elem in list
    """

    pass
```





Lets try another  
approach

## Approach 2 : Sorting

# Sorting

- If the elements are sorted in monotonically increasing (or decreasing) order, the majority element can be found at index  $\lfloor n/2 \rfloor$  (and also at  $\lfloor n/2 \rfloor - 1$ , if  $n$  is even).

# How to implement ?

Sort nums, and return the element in question.

Array with even number of elements:

[4, 4, 2, 4, 3, 5, 4]

After sorting, see the mid point

[2, 3, 4, 4, 4, 5]

Array with odd number of elements

[1, 2, 3, 5, 1, 1, 1, 1, 1]

[1, 1, 1, 1, 1, 1, 2, 3, 5]



Let's complete this:

```
def find_majority_element_v1(self, nums: List[int]) -> int:  
    pass
```



Let's try another  
approach

## Approach 3 : Using a Hash Map



# Using a Hash Map

---

- We know that the majority element occurs more than  $\lfloor n/2 \rfloor$  times, and a HashMap allows us to count element occurrences efficiently.





# How do we implement this?

---

- We can use a HashMap that maps elements to counts in order to count occurrences in linear time by looping over nums.
- Then, we simply return the key with maximum value.



# Let's complete this

```
def find_majority_element_v2(self, nums: List[int]) -> int:  
    pass
```



# Analysis

---

- We can use a HashMap that maps elements to counts in order to count occurrences in linear time by looping over nums.
- Then, we simply return the key with maximum value.

SOLUTION 3:

Using a dictionary

Run Time  $O(N)$

Space  $O(N)$

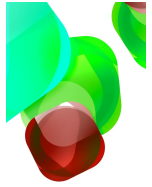


Let's try another  
approach

## Approach 4 : Using a Hash Map with a twist

The background features a collection of abstract, overlapping geometric shapes. On the left, a large cyan shape with a white diagonal line points towards the center. To its right are several green and cyan circular and semi-circular shapes, some with internal gradients. In the bottom right, there are several yellow and orange diamond and square shapes. A small red and brown shape is located near the bottom center. The overall aesthetic is modern and minimalist.

# Using Collections



# Let's complete this



```
# using collections module  
def find_majority_element_v3(self, nums: List[int]) -> int:  
    pass
```





Let's try another  
approach

Approach 5 : We will try to  
improve our space  
complexity

# BOYER-MOORE MAJORITY VOTING ALGORITHM

Run Time  $O(N)$   
Space  $O(1)$

# BOYER-MOORE MAJORITY VOTING ALGO

---

- The algorithm maintains in its [local variables](#)
  - a sequence element(candidate)
  - and a counter, with the counter initially zero.
- It then processes the elements of the sequence, one at a time.
  - Initialize an element  $m$  and a counter  $i$  with  $i = 0$
  - For each element  $x$  of the input sequence:
    - If  $i = 0$ , then assign  $m = x$  and  $i = 1$
    - else if  $m = x$ , then assign  $i = i + 1$
    - else assign  $i = i - 1$
  - Return  $m$





Initialize an element  $m$  and a counter  $i$  with  $i = 0$

For each element  $x$  of the input sequence:

- If  $i = 0$ , then assign  $m = x$  and  $i = 1$
- else if  $m = x$ , then assign  $i = i + 1$
- else assign  $i = i - 1$

Return  $m$

# BOYER-MOORE MAJORITY VOTING ALGO

---

- When processing an element  $x$ , if the counter is zero, the algorithm stores  $x$  as its remembered sequence element(candidate) and sets the counter to one.
- Otherwise, it compares  $x$  to the stored element(candidate) and either increments the counter (if they are equal) or decrements the counter (otherwise).
- At the end of this process, if the sequence has a majority, it will be the element stored by the algorithm.



# BOYER-MOORE MAJORITY VOTING ALGORITHM

- Initialize an element  $m$  and a counter  $i$  with  $i = 0$
- For each element  $x$  of the input sequence:
  - If  $i = 0$ , then assign  $m = x$  and  $i = 1$
  - else if  $m = x$ , then assign  $i = i + 1$
  - else assign  $i = i - 1$
- Return  $m$

Run Time  $O(N)$   
Space  $O(1)$

# Boyer-Moore



$N=9$

There are total of 5 2's

$9//2 = 4$

So: 2 is our majority element.

We will initialize 2 variables.

Candidate is the majority element we are looking for

Count is the number of times we have seen this element

**candidate = 0**

**count = 0**

**element = 2**

- Initialize an element  $m$  and a counter  $i$  with  $i = 0$
- For each element  $x$  of the input sequence:
  - If  $i = 0$ , then assign  $m = x$  and  $i = 1$
  - else if  $m = x$ , then assign  $i = i + 1$
  - else assign  $i = i - 1$
- Return  $m$



# Boyer-Moore



Starting from index 0 , count =0  
candidate becomes 2  
Increment count by 1

**candidate = 2**

**count = 1**

**element = 2**

# Boyer-Moore



At index 1:  
current candidate is 2  
Compare it with the value in index 1 ,  
if they are different , decrement count

**candidate = 2**

**count = 0**

**element = 1**

# Boyer-Moore



candidate = 2

count = 1

element = 2

# Boyer-Moore



candidate = 2

count = 2

element = 2

# Boyer-Moore

2	1	2	2	2	1	1	3	2
---	---	---	---	---	---	---	---	---



candidate = 2

count = 3

element = 2

# Boyer-Moore

2	1	2	2	2	1	1	3	2
---	---	---	---	---	---	---	---	---



candidate = 2

count = 2

element = 1



# Boyer-Moore

2	1	2	2	2	1	1	3	2
---	---	---	---	---	---	---	---	---



candidate = 2

count = 1

element = 1

# Boyer-Moore

2	1	2	2	2	1	1	3	2
---	---	---	---	---	---	---	---	---



candidate = 2

count = 0

element = 3

# Boyer-Moore

2	1	2	2	2	1	1	3	2
---	---	---	---	---	---	---	---	---



candidate = 2

count = 1

element = 2



# Let's complete this

---

```
def find_majority_element_Boyer_Moore(self, nums: List[int]) -> int:  
    pass
```

# Boyer-Moore Analysis

Majority element means it is an element that appears more than  $n/2$  time,  $n$  being the length of the array) in an array in  **$O(N)$  run time and  $O(1)$  Space complexity.**



**If time permits....**



**One more problem to work on...**

# Logger Rate Limiter

Design a logger system that receives a stream of messages along with their timestamps.

Each **unique** message should only be printed **at most every 10 seconds** (i.e. a message printed at timestamp  $t$  will prevent other identical messages from being printed until timestamp  $t + 10$ ).

All messages will come in chronological order.

Several messages may arrive at the same timestamp.



# Logger Rate Limiter

Implement the Logger class:

**Logger()** Initializes the logger object.

**bool shouldPrintMessage(int timestamp, string message)** Returns true if the message should be printed in the given timestamp, otherwise returns false.

Optimum Run Time  $O(1)$

Space Complexity  $O(M)$   $\rightarrow$  M being the number of incoming messages

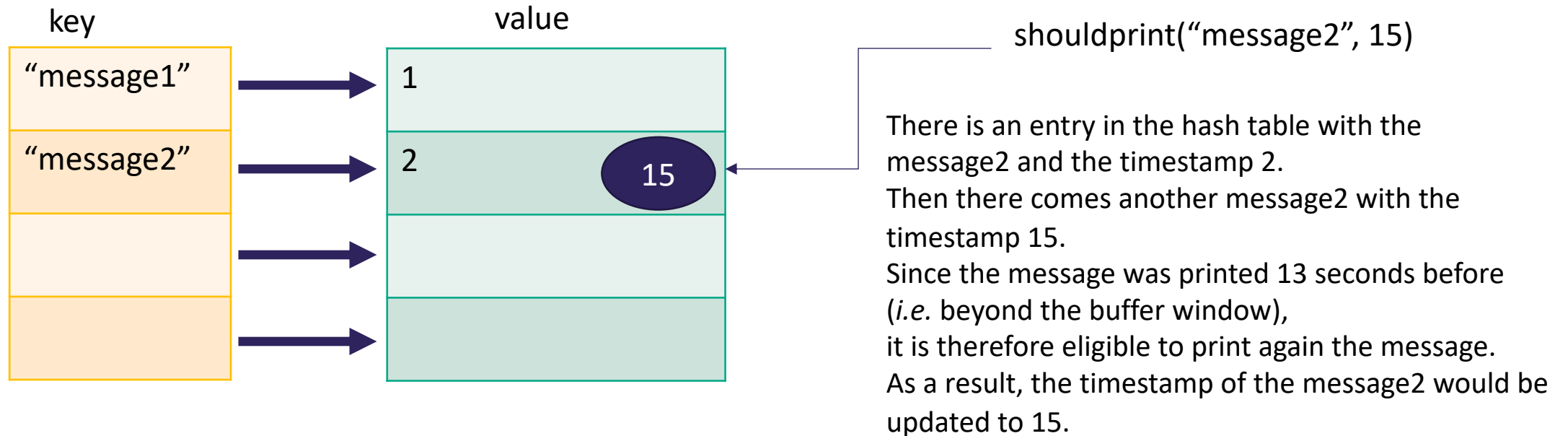
# Practicing Hash Table

```
Logger logger = new Logger();  
logger.shouldPrintMessage(1, "foo"); // return true, next allowed timestamp for "foo" is  $1 + 10 = 11$   
logger.shouldPrintMessage(2, "bar"); // return true, next allowed timestamp for "bar" is  $2 + 10 = 12$   
logger.shouldPrintMessage(3, "foo"); //  $3 < 11$ , return false  
logger.shouldPrintMessage(8, "bar"); //  $8 < 12$ , return false  
logger.shouldPrintMessage(10, "foo"); //  $10 < 11$ , return false  
logger.shouldPrintMessage(11, "foo"); //  $11 \geq 11$ , return true, next allowed timestamp for "foo" is  $11 + 10 = 21$ 
```

# Practicing with Hash Maps

We can have a hash table/dictionary with the **message as key**, and **its timestamp as the value**.

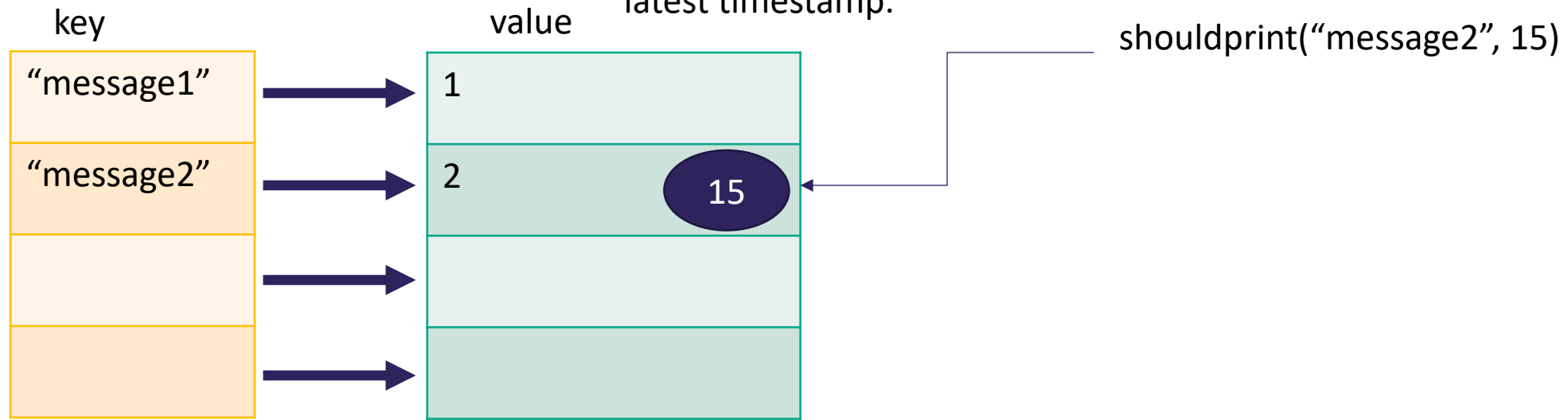
The hash table keeps all the unique messages along with the latest timestamp that the message was printed.



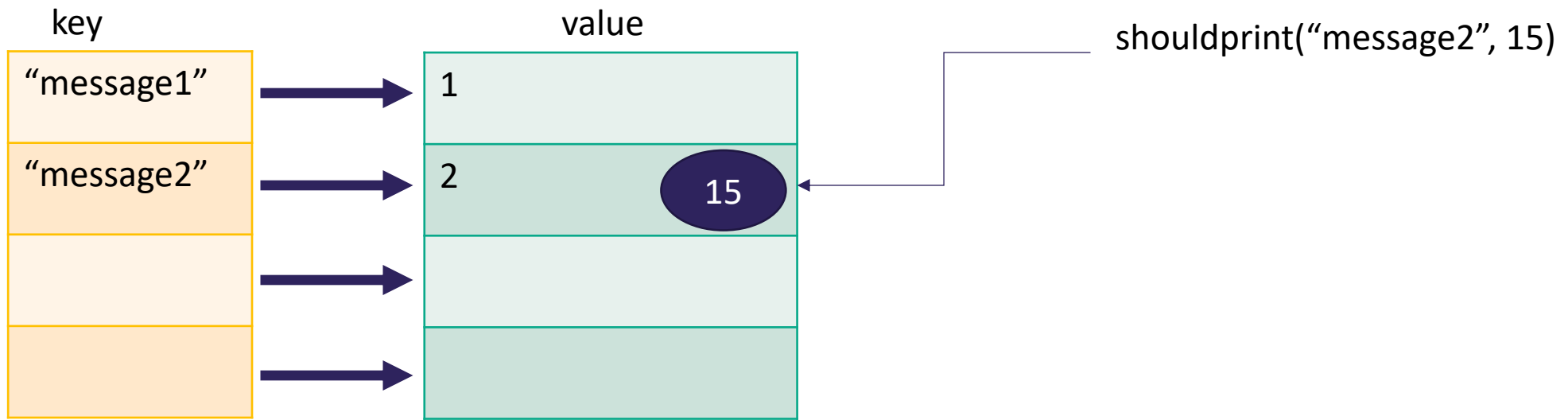
# Algorithm

## Algorithm

- We initialize a hash table/dictionary to keep the messages along with the timestamp.
- At the arrival of a new message, the message is eligible to be printed with either of the two conditions as follows:
  - case 1). we have never seen the message before.
  - case 2). we have seen the message before, and it was printed more than 10 seconds ago.
- In both of the above cases, we would then update the entry that is associated with the message in the hash table, with the latest timestamp.



# Algorithm





# Let's complete this

---

```
def shouldPrintMessage(self, timestamp, message) -> bool:
    """
    :return true if the message should be printed in the given timestamp, otherwise returns false.
    """
    pass
```