EE 210 - Applied Electromagnetic Theory

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Contents

Chapter 1

Introduction and Course Logistics

1.1 Course Information

- **Instructor:** Aditya Varma Muppala
- **Lectures:** Wed/Fri, 12:30–2:00 PM at Cory 293.
- **Office Hours:** Wed/Fri, 4-6 PM at Cory 510 (Cory 400 if my office is overflowing).

1.2 Course Objectives

To develop a strong foundation in electromagnetic theory for applications in antenna design, RF/microwave circuits, waveguides and resonators, computational electromagnetics, wave propagation, optics, and scattering. Emphasis will be placed on understanding the theory and applying it in a wide range of CAD experiments using ANSYS HFSS.

1.3 Syllabus

1. **Maxwell's equations:** Differential and integral form, boundary conditions.
2. **Wave–matter interactions:** Constitutive relations, Lorentz model, Drude model, dispersion relations.
3. **Electromagnetic theorems:** Image theory, Poynting theorem, uniqueness, field equivalence, reciprocity, and extinction theorems.
4. **Electromagnetic potentials:** Wave equation, Green's functions, radiation boundary condition, and antenna concepts.

5. **Computational EM:** Integral equations, method of moments, physical optics, and geometrical optics.

6. **Plane waves:** Plane wave expansion and angular spectrum, TE/TM field solutions, reflections from boundaries, wave propagation in inhomogeneous media, and negative index media.

7. **Waveguides and resonators:** Rectangular, cylindrical, and arbitrary cross-section waveguides of metal and dielectric materials; calculus of variations.

8. **Scattering:** Multipole expansions, plane wave scattering from spheres, creeping waves.

9. **Advanced topics in antennas:** Electrically small antennas, characteristic mode analysis (CMA), UWB antennas, and metamaterials.

1.4 Textbook

Foundations of Applied Electromagnetics by Kamal Sarabandi (free link).

1.5 Reference books

1. Kong, J. A., *Electromagnetic Wave Theory* (free link).

2. Harrington, R. F., *Time-Harmonic Electromagnetic Fields.*

3. Balanis, C. A., *Advanced Electromagnetics.*

4. Jin, J. M., *Theory and Computation of Electromagnetic Fields.*

5. Jackson, J. D., *Classical Electrodynamics.*

6. Born, M., and Wolf, E., *Principles of Optics.*

7. Chew, W. C., *Lectures on Electromagnetic Field Theory* (free link).

1.6 Lecture Recordings and Notes

I will record lectures separately and post them to my YouTube channel. I prefer this since web capture on a whiteboard does not do a good job. Also, I will post the lecture notes here and to Canvas as the course progresses.

1.7 Homework and Exams

One homework per week will be assigned on Friday and due the following Friday at midnight (11:59 PM).

The first part of every homework will be to upload your notes from that week's lectures. This is to encourage you to take notes, which has been shown to improve learning and long-term retention, especially in mathematics. It is also an easy way to get points in the homework. You need not take notes in class during lecture. You can take them any time from the uploaded lecture notes or the lecture recordings. They can be handwritten or typed. If you do not want to take notes, that is fine, see grading policy below.

Additionally, there will be two to three problems and/or one CAD assignment in HFSS per week. I encourage collaboration, but every student must turn in individual homework and CAD solutions. I don't oppose the use of AI tools in solving homework problems. If you did use AI to solve part of a problem, say to solve an integral, be sure to cite it in your solution. There is no penalty for using AI, but remember that deep understanding comes from the confusion and frustration you go through when wrapping your head around a new problem or concept. If you don't put yourself through this, you will have learned nothing more than how to be a good prompt engineer, which is not the goal of this course.

We will have one mid-term and one final. Exam problems will be equivalent in difficulty to the homeworks. The main goal of exams is to force you to review the class material and test your conceptual understanding of the subject. They are not representative of research in real life so I don't take them too seriously and neither should you. As long as you do all the homeworks and have reviewed the material in class well, you will do well on the exams.

Lastly, I highly recommend you attend office hours because this is where the magic happens in this class as we get to discuss problems on the homework, CAD and other cool stuff.

1.8 Grading

Your lowest homework grade will be dropped.
Score 1 = 20% Notes + 40% HW (including CAD) + 20% Mid-term + 20% Final.
Score 2 = 40% HW (including CAD) + 30% Mid-term + 30% Final.
Final grade = max{Score 1, Score 2}

Chapter 2

Mathematical Pre-requisites: Vector Calculus and Linear Algebra

2.1 Vector Calculus

Electromagnetics and wave propagation is written in the language of vector calculus. Therefore, to make sure we are all on the same page before the beginning of classes it would be nice to do a quick review. Here are some great videos resources that serve as an excellent review for those who are rusty on their vector calculus:

1. 3Blue1Brown's video on Vector Calculus. Amazing for intuition into curl, gradient and divergence. Very succinct video and a good refresher for most people. Grant' channel is the best at developing intuition from animations.

2. Steve Brunton's course on Vector Calculus and PDEs. Great solid intro to vector calculus. The first 8 lectures are a good enough background for our course.

3. Trefor Bazett's course on Vector Calculus. Another great intro to vector calculus. Maybe good for those who have not seen this material in a long time.

2.2 Linear Algebra

A strong background in an undergraduate Linear Algebra course is a pre-requisite for a lot of the material covered in this class. You should be familiar with concepts like orthogonal basis, inner products, matrix methods, eigenvalues and eigenvectors, Gram-Schmidt orthogonalization. For a refresher I highly recommend these video resources.

1. 3Blue1Brown's video series on Linear Algebra. This is where linear algebra *clicked* for me. I cannot recommend this series highly enough, it is perfect.

2. Gil Strang's timeless course on Linear Algebra. A great complete course for those who don't have a good linear algebra background.

2.3 Useful Results in Vector Calculus

The following tables (from the Wikipedia page: Del in cylindrical and spherical coordinates) summarize coordinate transformations and elementary vector calculus identities. Having these handy is super useful so I have compiled them here for you (and for me).

2.3.1 Coordinate Conversions

		From		
		Cartesian	**Cylindrical**	**Spherical**
To	**Cartesian**	$x = x$ $y = y$ $z = z$	$x = \rho\cos\varphi$ $y = \rho\sin\varphi$ $z = z$	$x = r\sin\theta\cos\varphi$ $y = r\sin\theta\sin\varphi$ $z = r\cos\theta$
	Cylindrical	$\rho = \sqrt{x^2+y^2}$ $\varphi = \arctan\left(\frac{y}{x}\right)$ $z = z$	$\rho = \rho$ $\varphi = \varphi$ $z = z$	$\rho = r\sin\theta$ $\varphi = \varphi$ $z = r\cos\theta$
	Spherical	$r = \sqrt{x^2+y^2+z^2}$ $\theta = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right)$ $\varphi = \arctan\left(\frac{y}{x}\right)$	$r = \sqrt{\rho^2+z^2}$ $\theta = \arctan\left(\frac{\rho}{z}\right)$ $\varphi = \varphi$	$r = r$ $\theta = \theta$ $\varphi = \varphi$

Figure 2.1: Coordinate conversion formulas between Cartesian, cylindrical, and spherical coordinates.

2.3.2 Unit Vector Conversions

	Cartesian	Cylindrical	Spherical
Cartesian	$\hat{\mathbf{x}} = \hat{\mathbf{x}}$ $\hat{\mathbf{y}} = \hat{\mathbf{y}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \cos\varphi\hat{\boldsymbol{\rho}} - \sin\varphi\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{y}} = \sin\varphi\hat{\boldsymbol{\rho}} + \cos\varphi\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \sin\theta\cos\varphi\hat{\mathbf{r}} + \cos\theta\cos\varphi\hat{\boldsymbol{\theta}} - \sin\varphi\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{y}} = \sin\theta\sin\varphi\hat{\mathbf{r}} + \cos\theta\sin\varphi\hat{\boldsymbol{\theta}} + \cos\varphi\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$
Cylindrical	$\hat{\boldsymbol{\rho}} = \frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}}}{\sqrt{x^2+y^2}}$ $\hat{\boldsymbol{\varphi}} = \frac{-y\hat{\mathbf{x}} + x\hat{\mathbf{y}}}{\sqrt{x^2+y^2}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\rho}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\boldsymbol{\rho}} = \sin\theta\hat{\mathbf{r}} + \cos\theta\hat{\boldsymbol{\theta}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$
Spherical	$\hat{\mathbf{r}} = \frac{x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}}{\sqrt{x^2+y^2+z^2}}$ $\hat{\boldsymbol{\theta}} = \frac{(x\hat{\mathbf{x}} + y\hat{\mathbf{y}})\,z - (x^2+y^2)\,\hat{\mathbf{z}}}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}$ $\hat{\boldsymbol{\varphi}} = \frac{-y\hat{\mathbf{x}} + x\hat{\mathbf{y}}}{\sqrt{x^2+y^2}}$	$\hat{\mathbf{r}} = \frac{\rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}}{\sqrt{\rho^2+z^2}}$ $\hat{\boldsymbol{\theta}} = \frac{z\hat{\boldsymbol{\rho}} - \rho\hat{\mathbf{z}}}{\sqrt{\rho^2+z^2}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{r}} = \hat{\mathbf{r}}$ $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$

Figure 2.2: Conversion between unit vectors in Cartesian, cylindrical, and spherical coordinate systems in terms of **destination** coordinates.

	Cartesian	Cylindrical	Spherical
Cartesian	$\hat{\mathbf{x}} = \hat{\mathbf{x}}$ $\hat{\mathbf{y}} = \hat{\mathbf{y}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \frac{x\hat{\boldsymbol{\rho}} - y\hat{\boldsymbol{\varphi}}}{\sqrt{x^2+y^2}}$ $\hat{\mathbf{y}} = \frac{y\hat{\boldsymbol{\rho}} + x\hat{\boldsymbol{\varphi}}}{\sqrt{x^2+y^2}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\mathbf{x}} = \frac{x\left(\sqrt{x^2+y^2}\hat{\mathbf{r}} + z\hat{\boldsymbol{\theta}}\right) - y\sqrt{x^2+y^2+z^2}\hat{\boldsymbol{\varphi}}}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}}$ $\hat{\mathbf{y}} = \frac{y\left(\sqrt{x^2+y^2}\hat{\mathbf{r}} + z\hat{\boldsymbol{\theta}}\right) + x\sqrt{x^2+y^2+z^2}\hat{\boldsymbol{\varphi}}}{\sqrt{x^2+y^2}\sqrt{x^2+y^2+z^2}}$ $\hat{\mathbf{z}} = \frac{z\hat{\mathbf{r}} - \sqrt{x^2+y^2}\hat{\boldsymbol{\theta}}}{\sqrt{x^2+y^2+z^2}}$
Cylindrical	$\hat{\boldsymbol{\rho}} = \cos\varphi\hat{\mathbf{x}} + \sin\varphi\hat{\mathbf{y}}$ $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\rho}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$\hat{\boldsymbol{\rho}} = \frac{\rho\hat{\mathbf{r}} + z\hat{\boldsymbol{\theta}}}{\sqrt{\rho^2+z^2}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \frac{z\hat{\mathbf{r}} - \rho\hat{\boldsymbol{\theta}}}{\sqrt{\rho^2+z^2}}$
Spherical	$\hat{\mathbf{r}} = \sin\theta\,(\cos\varphi\hat{\mathbf{x}} + \sin\varphi\hat{\mathbf{y}}) + \cos\theta\hat{\mathbf{z}}$ $\hat{\boldsymbol{\theta}} = \cos\theta\,(\cos\varphi\hat{\mathbf{x}} + \sin\varphi\hat{\mathbf{y}}) - \sin\theta\hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}} = -\sin\varphi\hat{\mathbf{x}} + \cos\varphi\hat{\mathbf{y}}$	$\hat{\mathbf{r}} = \sin\theta\hat{\boldsymbol{\rho}} + \cos\theta\hat{\mathbf{z}}$ $\hat{\boldsymbol{\theta}} = \cos\theta\hat{\boldsymbol{\rho}} - \sin\theta\hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{r}} = \hat{\mathbf{r}}$ $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}$ $\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}$

Figure 2.3: Conversion between unit vectors in Cartesian, cylindrical, and spherical coordinate systems in terms of **source** coordinates.

2.3.3 Computing Divergence, Curl, Gradient, and Laplacian

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, φ, z)
Vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$
Gradient ∇f[1]	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$
Divergence $\nabla \cdot \mathbf{A}$[1]	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$
Curl $\nabla \times \mathbf{A}$[1]	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$
Laplace operator $\nabla^2 f \equiv \Delta f$[1]	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$

Figure 2.4: Gradient, Divergence, Curl and Laplacian in cartesian and cylindrical coordinates.

Operation	Spherical coordinates (r, θ, φ), where θ is the polar angle and φ is the azimuthal angle[α]
Vector field $\mathbf{A}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Gradient ∇f[1]	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla \cdot \mathbf{A}$[1]	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$
Curl $\nabla \times \mathbf{A}$[1]	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(A_\varphi \sin\theta) - \frac{\partial A_\theta}{\partial \varphi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\varphi}}$
Laplace operator $\nabla^2 f \equiv \Delta f$[1]	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$

Figure 2.5: Gradient, Divergence, Curl and Laplacian in spherical polar coordinates.

	Cartesian	Cylindrical	Spherical
Differential displacement $d\ell$[1]	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$d\rho\,\hat{\boldsymbol{\rho}} + \rho\,d\varphi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\,\sin\theta\,d\varphi\,\hat{\boldsymbol{\varphi}}$
Differential normal area $d\mathbf{S}$	$dy\,dz\,\hat{\mathbf{x}} + dx\,dz\,\hat{\mathbf{y}} + dx\,dy\,\hat{\mathbf{z}}$	$\rho\,d\varphi\,dz\,\hat{\boldsymbol{\rho}} + d\rho\,dz\,\hat{\boldsymbol{\varphi}} + \rho\,d\rho\,d\varphi\,\hat{\mathbf{z}}$	$r^2\sin\theta\,d\theta\,d\varphi\,\hat{\mathbf{r}} + r\sin\theta\,dr\,d\varphi\,\hat{\boldsymbol{\theta}} + r\,dr\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume dV[1]	$dx\,dy\,dz$	$\rho\,d\rho\,d\varphi\,dz$	$r^2\sin\theta\,dr\,d\theta\,d\varphi$

Figure 2.6: Differential length, area and volume.

2.3.4 Useful Vector Calculus Identities

Product Rules

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\phi\mathbf{A}) = \phi(\nabla \cdot \mathbf{A}) + \nabla\phi \cdot \mathbf{A}$

$\nabla \times (\phi\mathbf{A}) = \phi(\nabla \times \mathbf{A}) + \nabla\phi \times \mathbf{A}$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times (\nabla\phi) = \mathbf{0}$

$\nabla \cdot (\nabla\phi) = \nabla^2\phi$

$\nabla^2\mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A})$

Vector Triple Products

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = -\mathbf{C} \times (\mathbf{A} \times \mathbf{B})$

Integral Theorems

Useful integral theorems that we will use throughout the course:

Gradient Theorem (Fundamental Theorem for Line Integrals)

$$\int_C \nabla\phi \cdot d\mathbf{r} = \phi(B) - \phi(A)$$

Divergence Theorem (Gauss)

$$\iiint_V (\nabla \cdot \mathbf{A})\, dV = \oiint_{\partial V} \mathbf{A} \cdot d\mathbf{S}$$

Stokes' Theorem

$$\iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{A} \cdot d\mathbf{r}$$

Stokes' Theorem (Volume form)

$$\iiint_V (\nabla \times \mathbf{A})\, dV = \oiint_{\partial V} \hat{\mathbf{n}} \times \mathbf{A}\, dS$$

Green's First Identity

$$\iiint_V \left(\nabla u \cdot \nabla v + u\, \nabla^2 v\right)\, dV = \oiint_{\partial V} u\, \frac{\partial v}{\partial n}\, dS$$

Green's Second Identity

$$\iiint_V \left(u\, \nabla^2 v - v\, \nabla^2 u\right) dV = \oiint_{\partial V} \left(u\, \frac{\partial v}{\partial n} - v\, \frac{\partial u}{\partial n}\right) dS$$