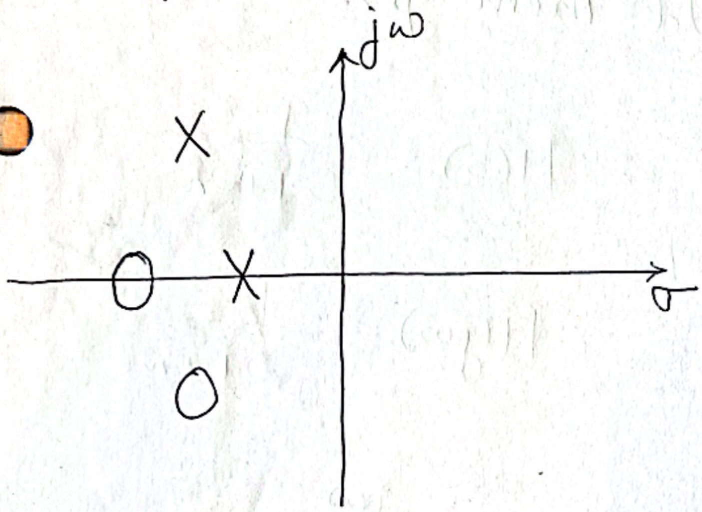
Lec 6 - Poles & Zeroes, Bode Plots, ~~[illegible]~~ Root Locus

Become trivial from Laplace domain picture.

$$H(s) = \frac{b_m s^m + \ldots + b_0}{a_n s^n + \ldots + a_0} \overset{\text{Fund Thm Alg}}{=} G \cdot \frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

p_k, z_i are real or complex conjugate pairs.

S-plane



$\left.\begin{array}{l} X \rightarrow \text{tent} \\ O \rightarrow \text{stake} \end{array}\right\}$ Just for intuition.

> We are interested in $H(j\omega) \Rightarrow s = j\omega.$ \] Eigenfunction excitations.

$$|H(j\omega)| = G \frac{\prod_{i=1}^{m} |(j\omega - z_i)|}{\prod_{k=1}^{n} |(j\omega - p_k)|}$$

since $j\omega - z_i = R_{z_i} e^{j\theta_{z_i}}$ & $j\omega - p_k = R_{p_k} e^{j\theta_{p_k}}$

$$\angle H(j\omega) = \sum_{i=1}^{m} \angle(j\omega - z_i) - \sum_{k=1}^{n} \angle(j\omega - p_k).$$

We want to convert from $\prod \longrightarrow \sum$ in $|H(j\omega)|$

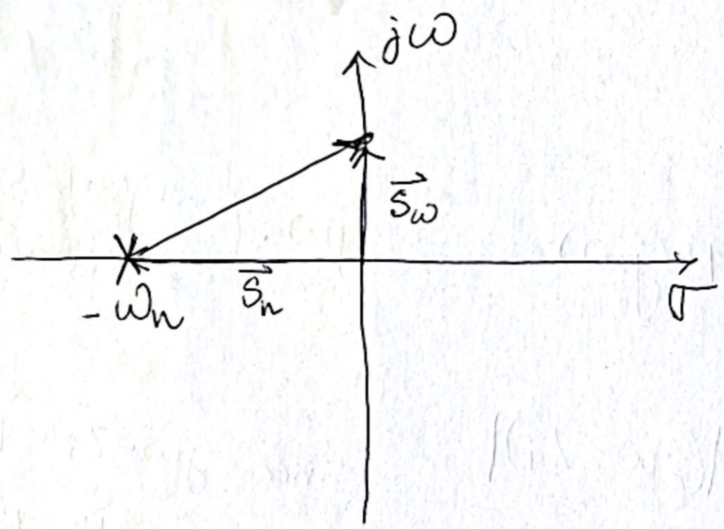
$\Rightarrow$ Take log on both sides.

$$20\log|H(j\omega)| = 20\log G + \sum_{i=1}^{m} 20\log|j\omega - z_i|$$
$$- \sum_{j=1}^{n} 20\log|j\omega - p_j|$$

$\Rightarrow$ It is sufficient to study the effect of a single pole or zero (& then add them up).

Single Pole at $s = -\omega_n \Rightarrow H(s) = \dfrac{1}{s+\omega_n}$

$$\Rightarrow H(j\omega) = \frac{1}{j\omega + \omega_n}$$



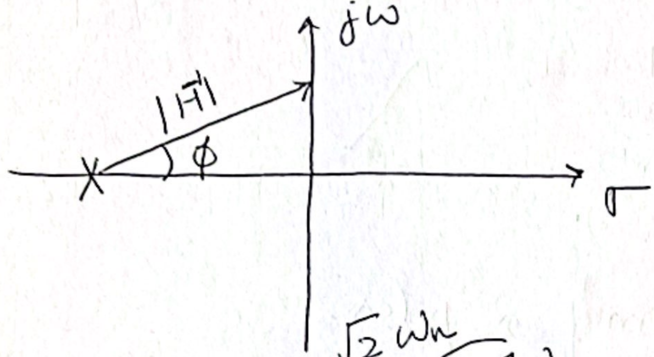
$\vec{S}_\omega = j\omega$

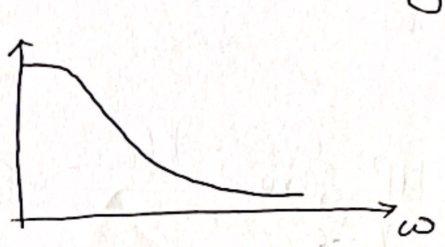
$\vec{S}_n = -\omega_n$

$\vec{S}_\omega - \vec{S}_n = j\omega + \omega_n$

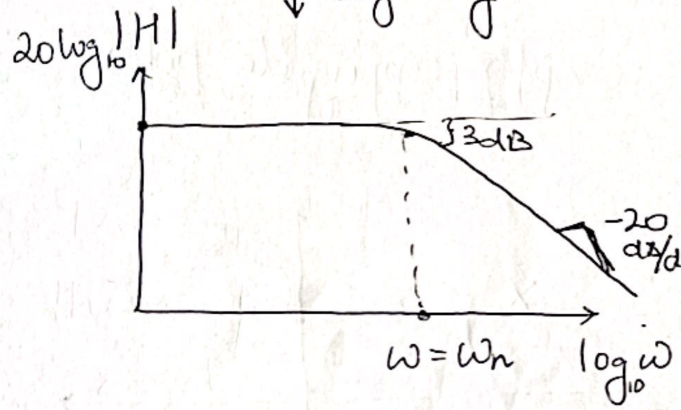
$$|H(j\omega)| = \frac{1}{|j\omega + \omega_n|} = \frac{1}{\sqrt{\omega_n^2 + \omega^2}} \quad \& \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_n}\right)$$

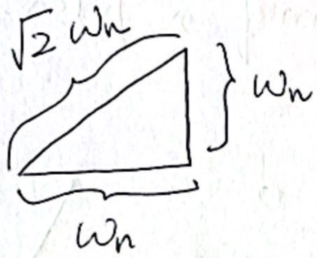
- From tenting picture it can be obtained intuitively.





↓ log-log





> At $\omega = \omega_n$

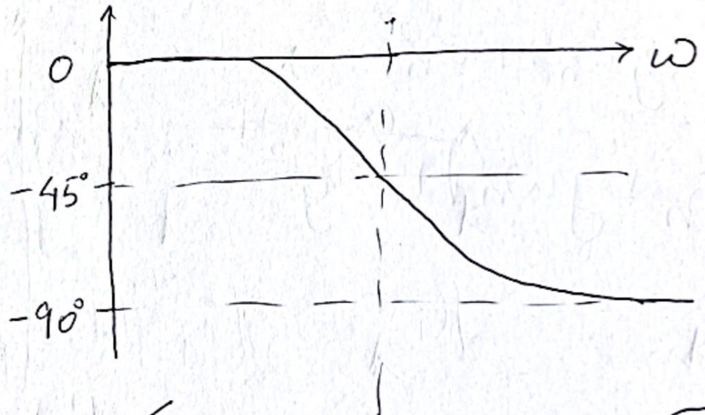
$\Rightarrow$ $|H|$ drops by 3dB

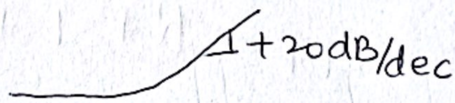
- $20\log\frac{1}{\sqrt{2}} \simeq -3.01\,\text{dB}$

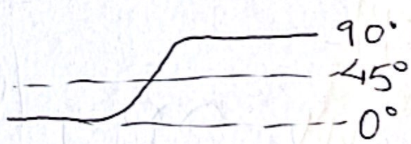
When $\omega \to \infty$, $|H| \simeq \frac{1}{\omega} \Rightarrow -20\,\text{dB/dec}$.

Also $\phi = 45°$. When $\omega = 0$, $\phi = 0°$ & $\omega \to \infty \Rightarrow \phi = 90°$

$\angle H = -\phi$ $\Rightarrow$

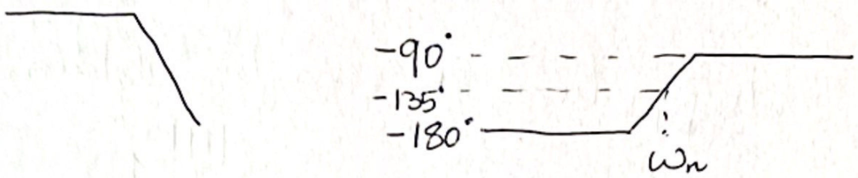


- > Zero $\Rightarrow$ 

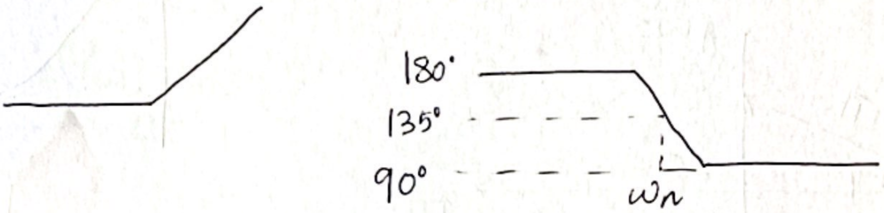
&


RHP pole $\Rightarrow$ angle is ~~flipped~~. $180 - \cancel{\phi}$ ~~...~~ ϕ
& Zero

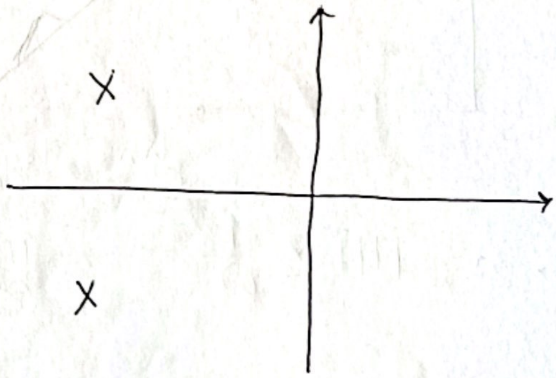
⇒ RHP pole

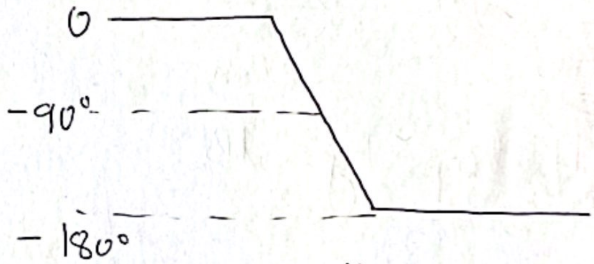


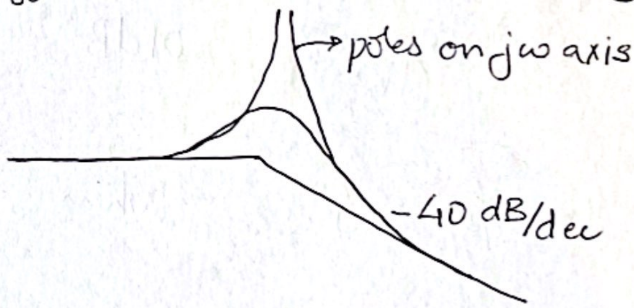
RHP zero



Complex conjugate poles → Sum of 2 poles.







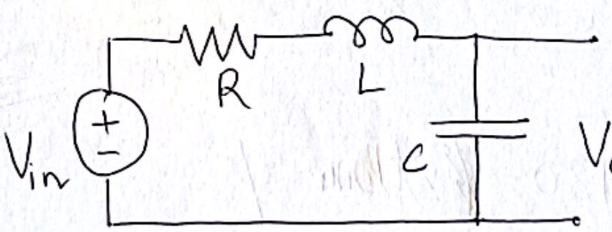
$$H(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Rightarrow s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

ω_n = natural freq

ζ = damping ratio.

Example



$$Z_{tot} = R + sL + \frac{1}{sC}$$

$$H(s) = \frac{V_c(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$\Rightarrow H(s) = \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{LC}} \quad \& \quad 2\zeta\omega_n = \frac{R}{L} \Rightarrow \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

natural ~~resonance~~ freq.

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \Rightarrow \text{LHP since } \zeta, \omega_n > 0.$$

When $\zeta \in (0,1)$ s is complex conjugate.

$\zeta = 0 \Rightarrow R = 0 \;\&\; \omega_0 = \omega_n$

resonance freq — natural freq

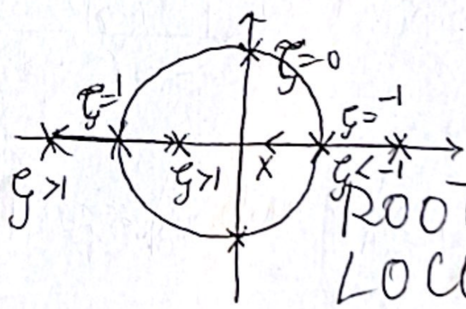
excitation at ω_n leads to ∞ amplitude at steady state $\rightarrow$ RESONANCE (no loss to damp out the energy put in)

$\zeta < 1 \Rightarrow$ oscillations $\Rightarrow$ underdamped.

$\zeta = 1 \Rightarrow$ 2 real roots that are equal $\Rightarrow$ critically damped

$\zeta > 1 \Rightarrow$ over damped $\Rightarrow$ no oscillations. $\left(R >> 2\sqrt{\frac{L}{C}}\right)$

2 distinct real roots.



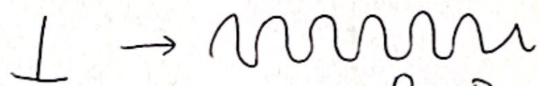
ROOT LOCUS.

Connecting to Impulse Response

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

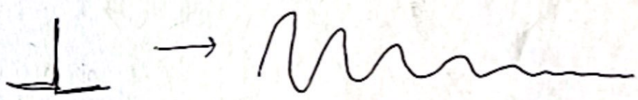
$$\Rightarrow h(t) = \frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t). \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\zeta = 0 \Rightarrow h(t) = \sin(\omega_n t)$$



$R = 0$

$$\zeta < 1 \Rightarrow h(t) = \frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

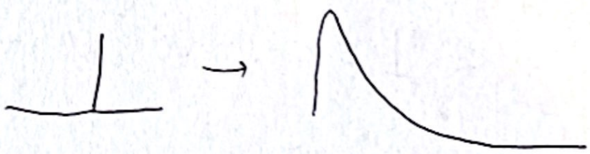


$R > 0$

$$\zeta = 1 \Rightarrow h(t) = \omega_n e^{-\zeta\omega_n t} \frac{\sin(\omega_d t)}{\omega_d}$$

$$\lim_{\omega_d \to 0} \frac{\sin(\omega_d t)}{\omega_d} = t$$

$$h(t) = \omega_n t e^{-\omega_n t}$$



$\zeta > 0 \Rightarrow$ $H(s)$ has 2 distinct real negative poles

$$H(s) = \frac{\omega_n^2}{(s-s_1)(s-s_2)} \Rightarrow h(t) = \frac{\omega_n^2}{s_1 - s_2}\left(e^{s_1 t} - e^{s_2 t}\right)$$

$t > 0$

where $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

$\Rightarrow$ Sum of two decaying exponentials