IV 3-D Image Reconstruction for Synthetic Aperture Radar (SAR)

PART 1

Generalised signal model for FMCW, SF & Pulsed radars.

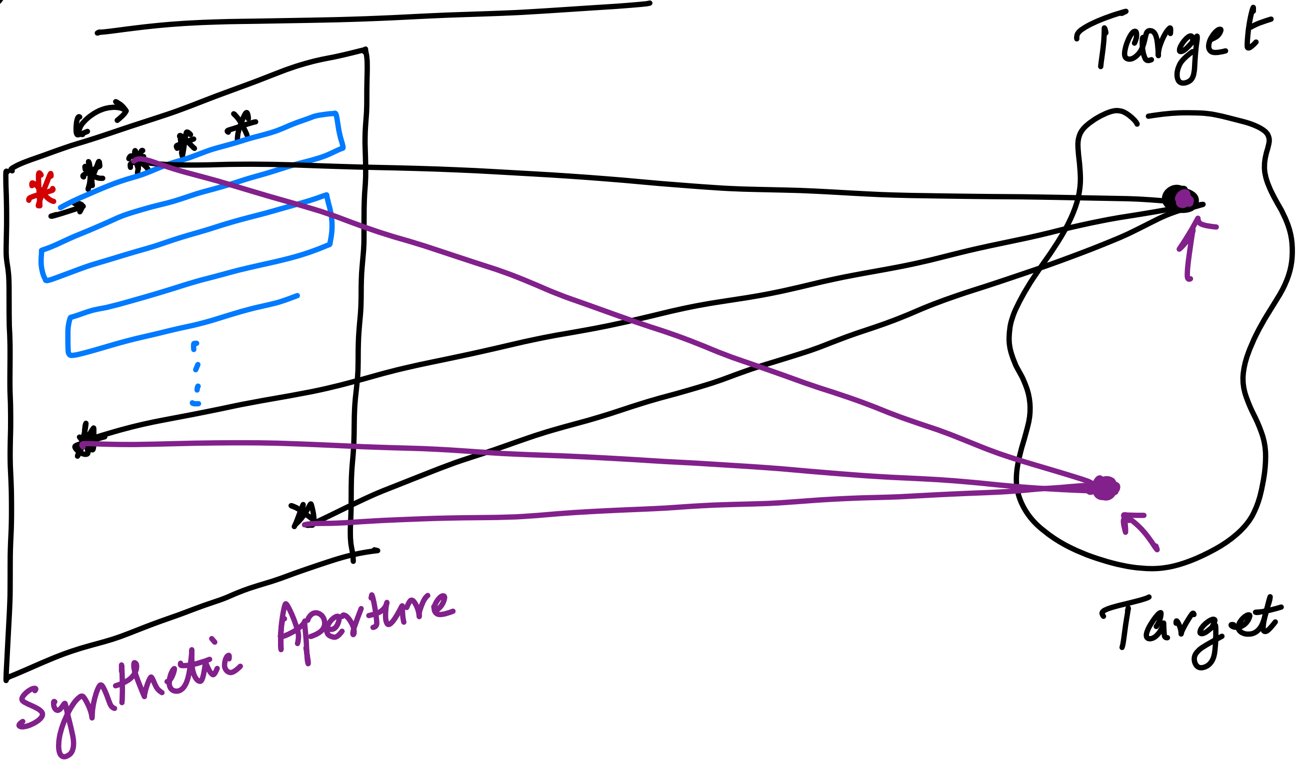
PART 2 (They are all the same!)

> Back projection or Back propagation (BP)
> Delay And Sum (DAS)
> Time Domain Correlation (TDC)
> 3-D Matched Filtering. (MF)

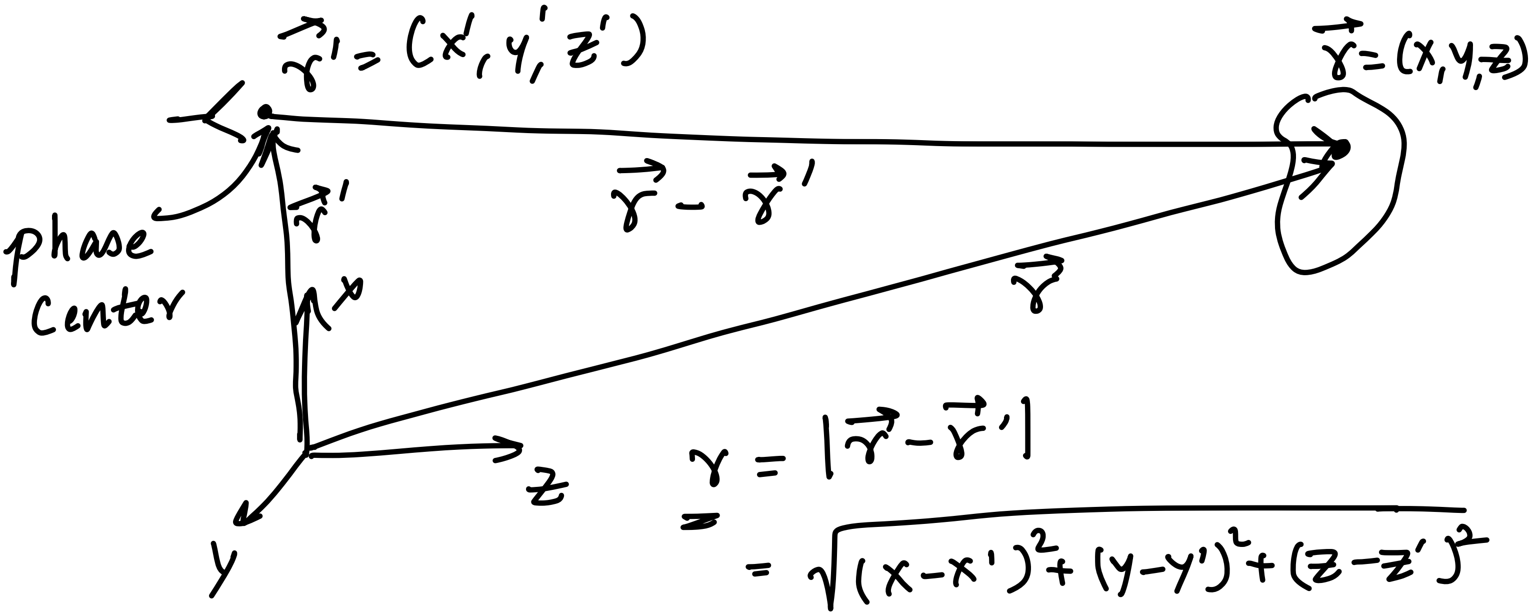
PART 3

> MATLAB code for reconstruction.
> HFSS $\longrightarrow$ MATLAB. {already done}
> Experimental data $\longrightarrow$ MATLAB

4.1 Intuition (SAR)



4.2 Generalized Signal model. (3-D)



$$r = |\vec{r} - \vec{r}'|$$

$$= \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

4.2.1 FMCW

Center freq

Chirp rate

$$S_{IF}(r) = \cos\left(\frac{2 w_c r}{c} + \frac{2\beta r t}{c} - \underbrace{\frac{2\beta r^2}{c^2}}_{\text{RVP}}\right)$$

$$t \in \left[-\frac{T_s}{2}, \frac{T_s}{2}\right]$$

> Ignore RVP or RVP can be removed!
(Carrara & Goodman)

> Cos → $\boxed{\exp(j)}$ → Exercise
What does this do?

$$S_b(r) = \exp\left\{-j\left(\frac{2\omega_c}{c} + \frac{2\beta t}{c}\right) r\right\}$$

$$K_t = \frac{2\omega_c}{c} + \frac{2\beta t}{c}$$

$$\Rightarrow \boxed{S_b(r) = e^{-j2K_t r}} \rightarrow \text{IF signal or beat signal.}$$

4.2.2
Stepped Frequency

TX: $e^{j\omega t}$ RX: $e^{j(\omega t - 2kr)}$

$k \hookrightarrow \frac{\omega}{c}$

$$S_b(r) = \frac{RX}{TX} = S_{11} \text{ or } S_{21} = e^{-j2kr}$$

$$\boxed{S_b(r) = e^{-j2kr}}$$

4.2.3 Pulsed radar

TX: $p(t)$ **RX:** $p\left(t - \frac{2r}{c}\right)$

$$S_{MF}(r) = |P(\omega)|^2 e^{-2jkr}$$

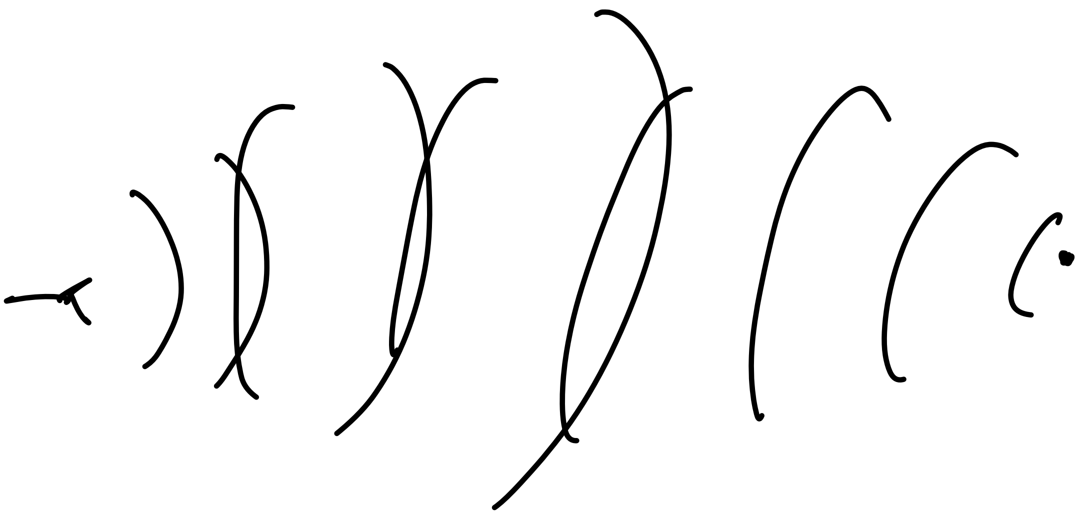
$$\boxed{S_b(r) = e^{-2jkr}}$$

Therefore, the generalized signal model can be written as

$$S_b(\vec{r}, \vec{r}', k) = e^{-2jk|\vec{r} - \vec{r}'|}$$

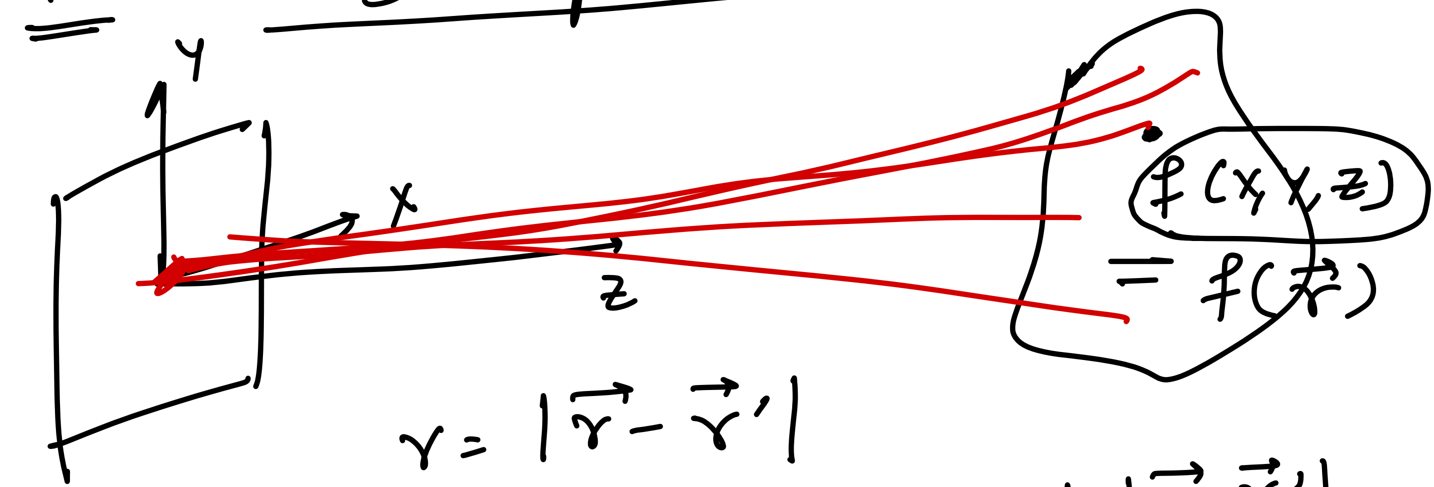
$\rightarrow k_t$ for FMCW $= \dfrac{\omega_c + \beta t}{c}$

$\rightarrow k = \dfrac{\omega}{c}$ for P & SF.



$$\boxed{S_b(r) = \frac{e^{-2jkr}}{r^2}} = \underbrace{\left(\frac{e^{-jkr}}{r}\right)}_{h_0^{(1)}(r)}\left(\frac{e^{-jkr}}{r}\right)$$

4.3 3-D Signal model



$$r = |\vec{r} - \vec{r}\,'|$$

$$S_b(\vec{r}, \vec{r}\,', k) = \underbrace{f(\vec{r})}\ \underbrace{\frac{e^{-2jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|^2}}$$

$$\tilde{S}_b(\vec{r}\,', k) \simeq \int_{\vec{r}} f(\vec{r})\, \frac{e^{-2jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|^2}\, d\vec{r}$$

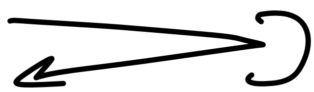
$$= \iiint_{x\,y\,z} \overbrace{f(x,y,z)}\ \frac{e^{-2jk\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}}{(x-x')^2+(y-y')^2+(z-z')^2}\, dx\, dy\, dz$$

Assumptions

1) Antenna is omnidirectional.
2) Target is a contiguous collection of point scatterers. Ignored multiple scattering.
3) Ignored dispersion of channel (system, RVP, channel).

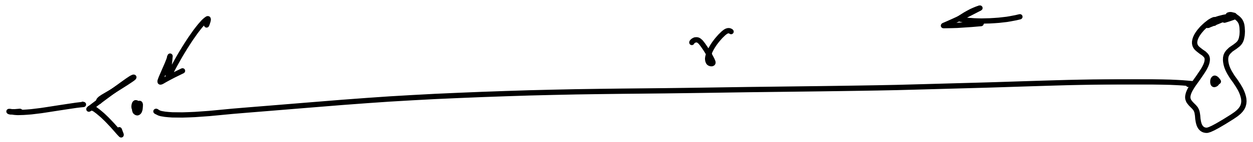
? $\phi \neq \omega t + kr$

4) Ignored target scattering as a fn. of frequency.



5) Ignored wavelength dependance on received power. Mie

4.4 A more accurate signal model



In the far-field of the antenna, the power density is $P_b = \dfrac{P_t G_t}{4\pi r^2}$

power reflected at target : $\dfrac{P_t G_t}{4\pi r^2}\sigma$

power density at receiver : $\dfrac{P_t G_t \sigma}{4\pi r} \cdot \dfrac{1}{4\pi r^2}$

power received by RX : $\dfrac{P_t G_t \sigma A_e}{(4\pi r^2)^2}$; $A_e = \dfrac{\lambda^2 G_t}{4\pi}$

$$\frac{P_r}{P_t} = \frac{G_t^2 \lambda^2 \sigma}{(4\pi)^3 r^4}$$

$\sigma = 4\pi |S|^2$ where $S = \dfrac{E_s}{E_i}$

↳ Scattering amp.

$$S_b = \frac{E_r}{E_t} = \sqrt{\frac{P_r}{P_t}}\, e^{-2jkr} = \frac{\lambda G(\hat{r}) S(\vec{r})}{4\pi r^2} e^{-2jkr}$$

$$\tilde{S}_b(\vec{r}',k) = \iiint_{xyz} \underbrace{\frac{\lambda\, G(\hat{r})\, S(\vec{r})\, L}{4\pi}}_{f(x,y,z)} \cdot \frac{e^{-2jkr}}{r^2}\, dx\,dy\,dz$$

Loss in system.

$$\underbrace{\tilde{S}_b(x',y',k)}_{\text{measured}} = \iiint_{xyz} f(x,y,z)\, \frac{e^{-2jkr}}{r^2}\, dx\,dy\,dz$$

The ! Goal ! of radar imaging / reconstruction algorithms is to reproduce $f(x,y,z)$ from $\tilde{S}_b(x',y',k)$ as accurately & quickly / efficiently as possible!

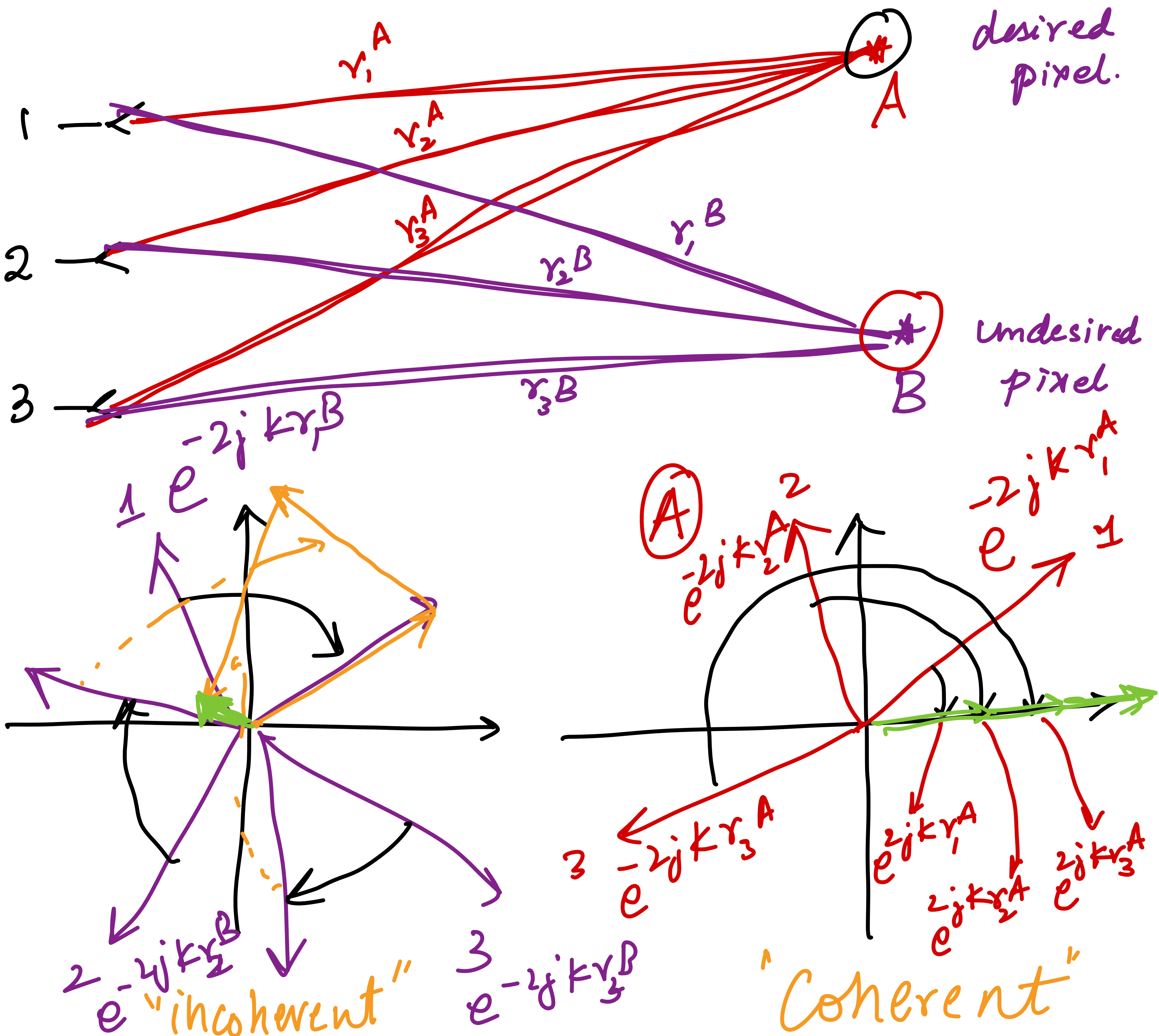
4.5 Reconstruction.

Back projection, DAS } Pulsed / SF frequency domain

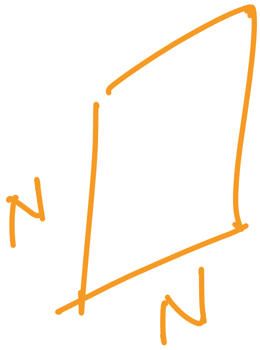
Time domain correlation / MF } FMCW radars Time domain.

$\underbrace{\hphantom{Time\ domain.}}_{e^{-2jkr}}$

4.5.1 Frequency domain (P & SF)

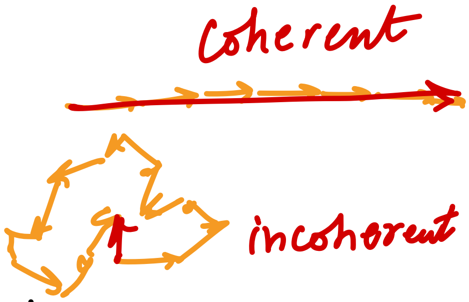


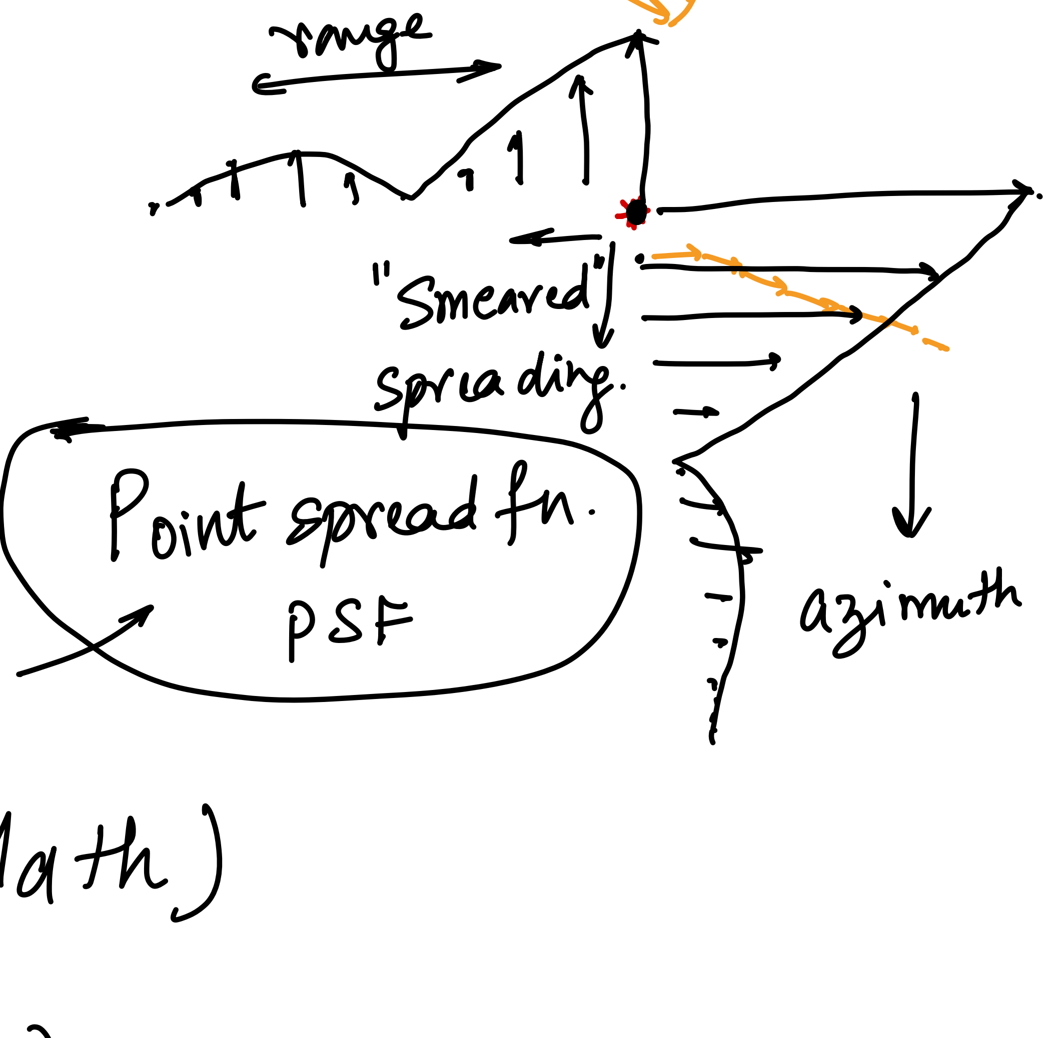
For desired pixel when phase is unwrapped, coherent addition of $\underbrace{O(N^3)}_{\text{processing gain of SAR.}}$ phasors occurs!



& N samples in time/freq.

$$\tilde{S}_b(\underset{N}{x'}, \underset{N}{y'}, \underset{N}{k})$$





4.5.2 (Math)

$$\tilde{S}_b(x', y', k)$$

$\downarrow$ for a pixel at $\vec{r} = (x, y, z)$

$$\tilde{S}_b(x', y', k)\, e^{j2k|\vec{r} - \vec{r}'|}$$

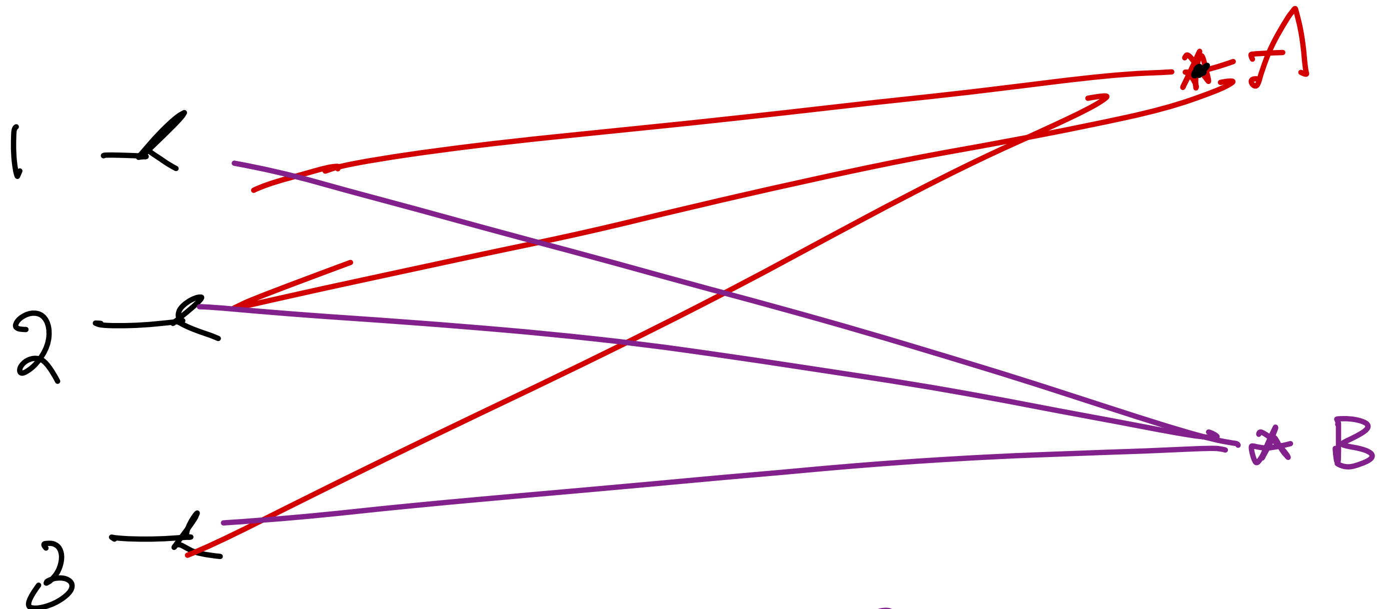
$\downarrow$ Integrate/sum over $\underset{N}{x'}, \underset{N}{y'}$ & $\underset{N}{k}$

$$\tilde{f}(x,y,z) = \sum_{k} \sum_{x'} \sum_{y'} \underbrace{S_b(x',y',k)^*} \underbrace{\exp\left(j2k\sqrt{(x-x')^2+(y-y')^2+z^2}\right)}_{\text{Steering matrix}}$$

$\downarrow$ complex image

sum(F(:))

4.5.3 Correlation (time)



$$S_b^A(r) \neq S_b^B(r)$$

How similar is $\tilde{S}_b$ to S_b^A.

$\Rightarrow$ Correlation of $\tilde{S}_b$ with S_b^A ↳ pixel of interest.

$$S_b^A(r) = \frac{f(\vec{r})}{\cancel{r^2}} e^{-2jkr}$$

$\cancel{r^2} \to 1$

$$C(\tau) = \int_{t=-\infty}^{\infty} x(t)\, y^*(t-\tau)\, dt$$

$$\tilde{f}(r) = \tilde{S}_b(\vec{r}', k) \circledast \underbrace{S_b^{A*}(\vec{r}, \vec{r}', k)}$$

$$= \iiint_{x'y'k} \tilde{S}_b(\vec{r}', k)\, e^{2jkr}\, dx'dy'dk$$

$$\tilde{f}(x, y, z) \simeq \sum_k \sum_{x'} \sum_{y'} \tilde{S}_b(x', y', k)$$

$$\cdot \exp\left(2jk\sqrt{(x-x')^2 + (y-y')^2 + z^2}\right)$$

Therefore with the generalized signal model, BP, DAS, TDC, MF are all mathemati-cally identical!

Computation Time of BP/TDC

Assume $N \times N \times N$ pixels & $N \times N$ array & N freq/time samples. For each pixel we have N^3 multiplications (FLOPS).

$\Rightarrow$ Computational burden $\underbrace{O(N^6)}_{\text{SLOW}}$.

$N = 10^2 \Rightarrow 10^{12}$ FLOPS.

4.? Advantages & Limitations

? 1) Highest SNR!

X 2) Arbitrary geometry of array

X 3) Arbitrary domain of image

4) Slow!!!

Can we make it faster? YES!!

$$\sum^3 S_b(\vec{r}\,')\, \underbrace{e^{2jkr}}\Big\}$$

$$\updownarrow$$

$$\sum^3 S_b(\vec{r}\,')\, e^{\underbrace{j(k_x x + k_y y + k_z z)}_{\vec{k}\cdot\vec{r} \,=\, kr\cos\theta}}$$

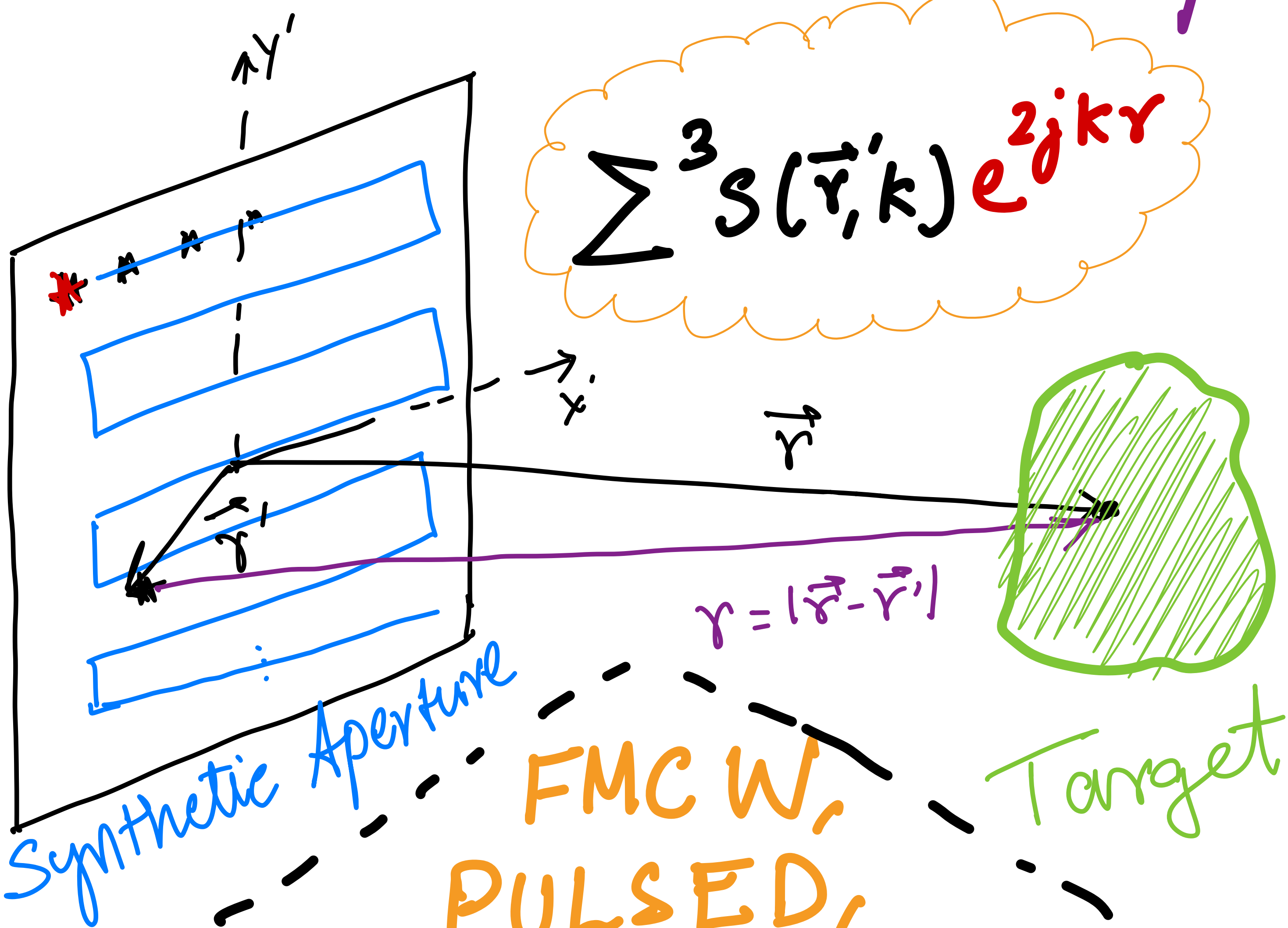
$$\tilde{f} = \sum^3 \underbrace{S_b(\vec{r}\,')}\, e^{j\vec{k}\cdot\vec{r}}\Big\}$$

$$O(N^6) \longrightarrow O(N^3 (\log N)^3)$$

RMA, ω-K, Holographic etc....

- MATLAB
- HFSS
- Experiment

3D SAR IMAGING



FMCW, PULSED, & STEPPED FREQUENCY

RECONSTRUCTION ALGORITHMS

BACK PROJECTION

TIME DOMAIN CORRELATION

DELAY AND SUM

3D MATCHED FILTERING