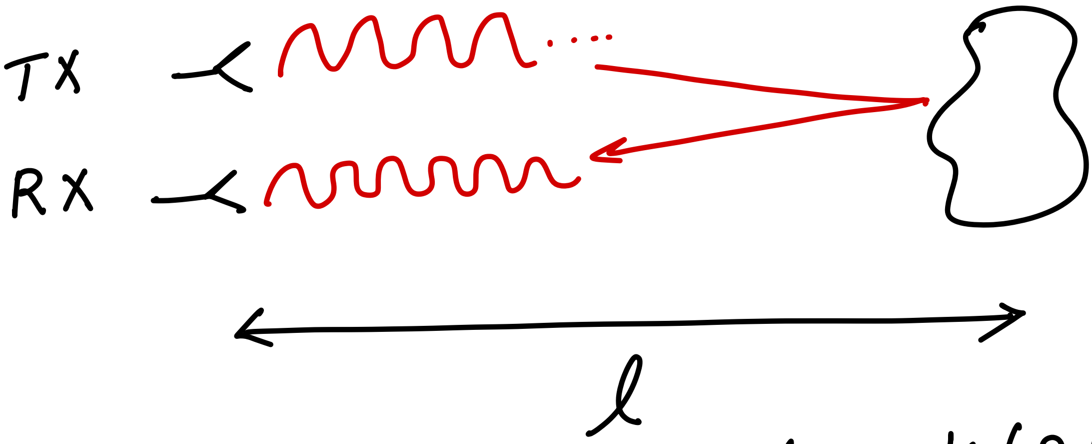
III Stepped Frequency Radars.

3.1 Intuition



$$\underbrace{\phi_0 = K(2l)} = \frac{\overset{2\pi f}{\omega}}{c}(2l)$$

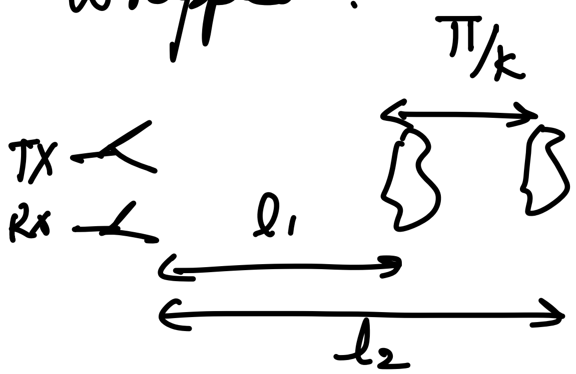
TX: $\cos(\omega t)$

RX: $A_0 \cos(\omega t + \phi_0)$

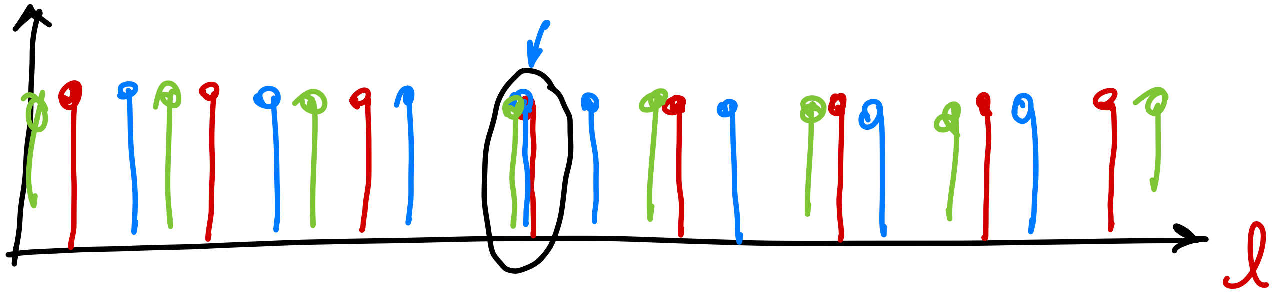
$$\boxed{l = \frac{\phi_0}{2K}}$$

ϕ_0 is an angle which is "wrapped".

$$l_2 = l_1 + \frac{\pi}{K} = \frac{\phi_1}{2K} + \frac{2\pi}{2k}$$



$$= \frac{\phi_1 + 2\pi}{2K} = \frac{\phi_1}{2K} = l_1 = \frac{\phi_2}{2K_2}$$



3.2 Single target Signal model

"phasors"

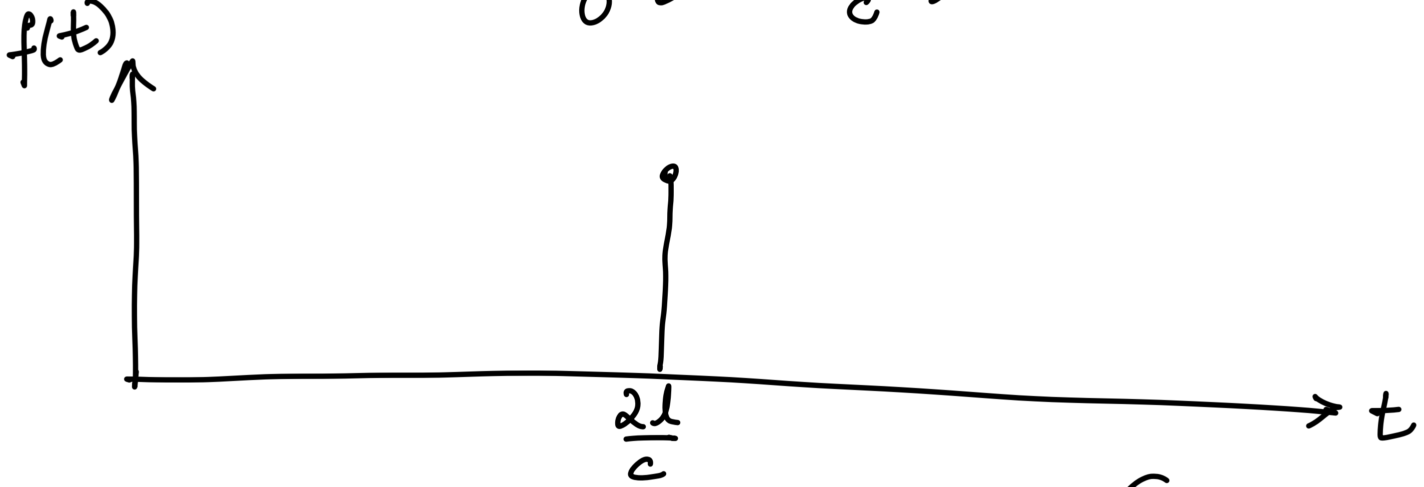
$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

Tx: $e^{j\omega t}$

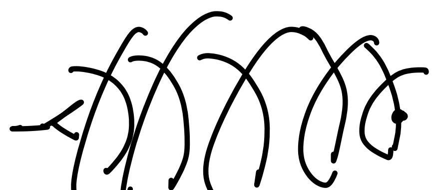
Rx: $$e^{j(\omega t - 2kl)} = e^{j\omega t}\, e^{-j2\overset{\omega/c}{k}l} = F(\omega)$$

$$\mathcal{F}^{-1}\{F(\omega)\} = \mathcal{F}^{-1}\{e^{j\omega t}\, e^{-j2kl}\}$$

$$= \delta\left(t - \frac{2l}{c}\right) = f(t)$$



3.3 Multiple targets

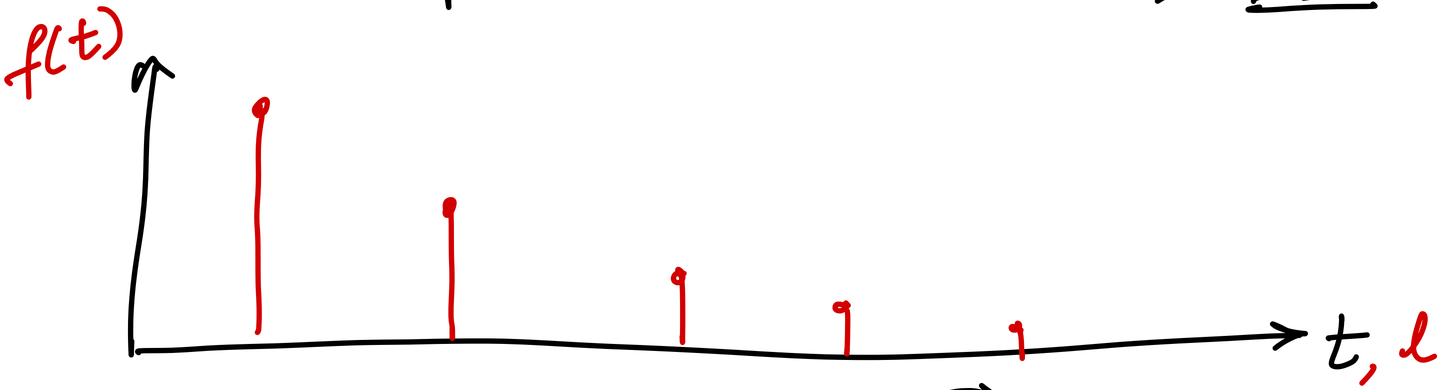


Tx: $e^{j\omega t}$

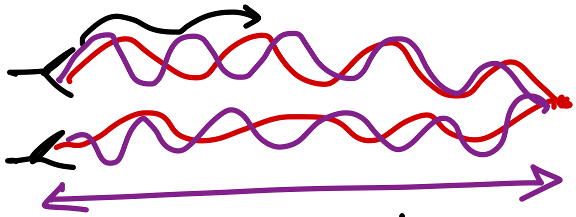
Rx: $$\sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2} e^{j(\omega t - 2k l_n)} = F(\omega)$$

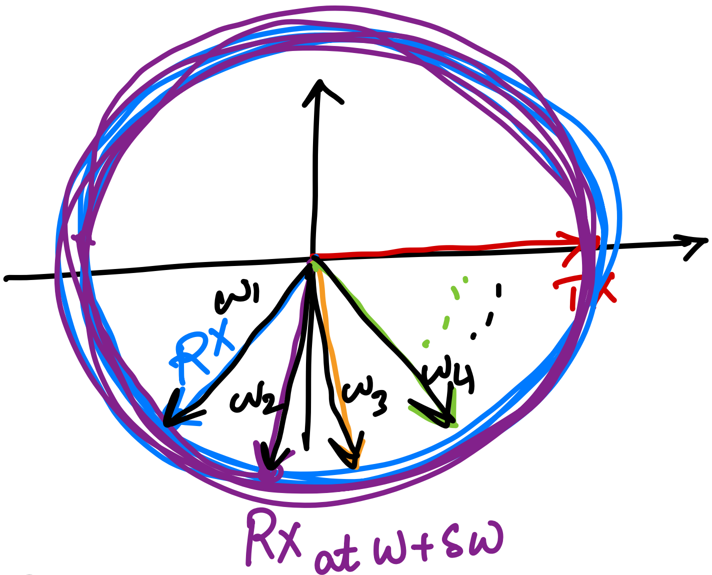
$$\mathcal{F}^{-1}\{F(\omega)\} = \sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2}\left[\underbrace{\mathcal{F}^{-1}\left(e^{-j(\omega t - 2k l_n)}\right)}\right]$$

$$f(t) = \sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2}\left[\delta\left(t - \frac{2 l_n}{c}\right)\right]$$



3.4 Phasor model.





Tx: $e^{j\omega t}$

RX: $\underbrace{A_0\, e^{j(\omega t - k2l)}}$

$\cancel{A_0} \angle -2kl$

$2kl = \frac{2\omega}{c} l$

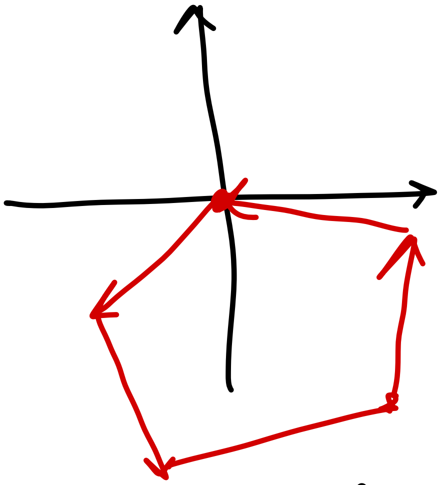
-ve ⇒ counter Clockwise

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

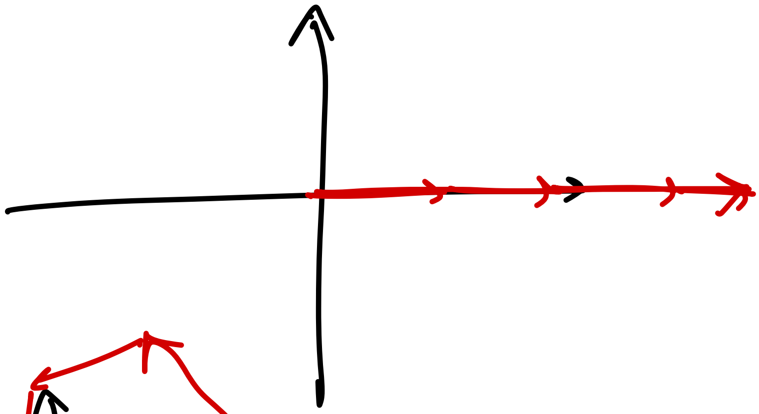
$\omega t = 2kl$

$\Rightarrow t = \frac{2kl}{\omega} = \frac{2l}{c}$

At $t=0$, no unwrapping.



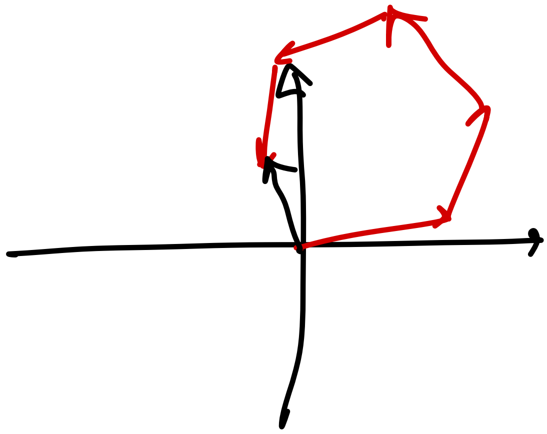
At $t = \frac{2l}{c}$, $e^{j\omega t} = e^{j\frac{2kl}{c}}$



$\Rightarrow \int F(\omega)\, e^{j\omega t}$

e^{j2kl}

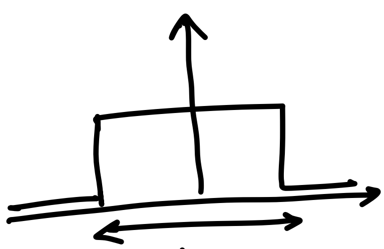
At $t \neq \frac{2l}{c}$



(3.5) Bandwidth limited signal.

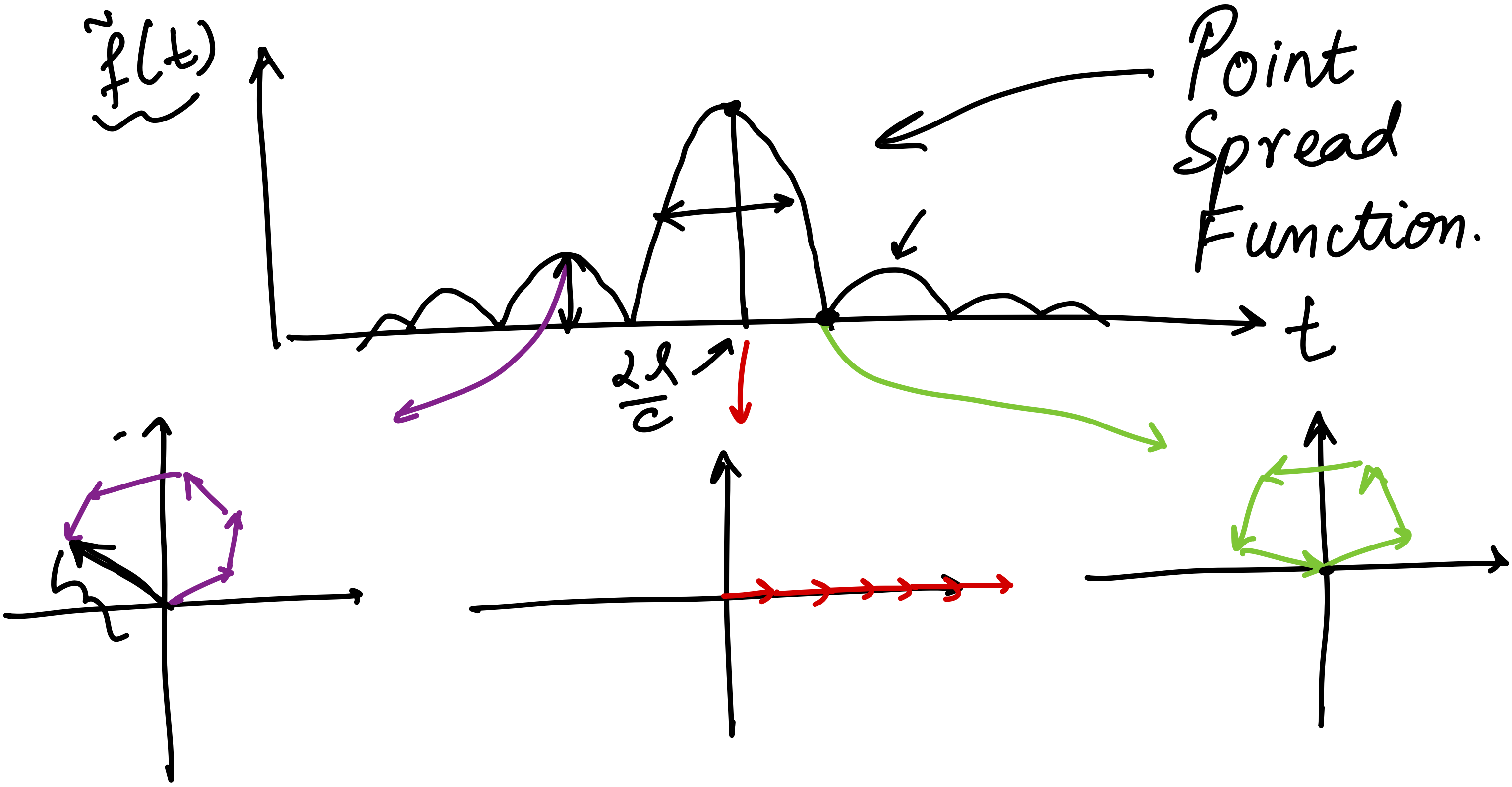
$\omega \in [\omega_l, \omega_h]$ & $BW = \Delta\omega = \omega_h - \omega_l$.

$F(\omega) \to \text{rect}(\Delta\omega)\, F(\omega)$.



$\mathcal{F}^{-1}\{\text{rect}(\Delta\omega)\ F(\omega)\}$

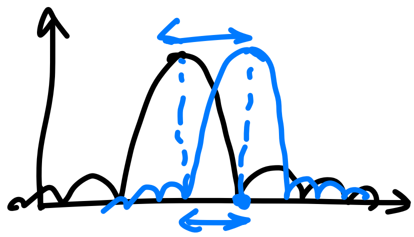
$= \Delta\omega \,\text{sinc}\left(\frac{\Delta\omega t}{2}\right) \circledast f(t) = \tilde{f}(t)$



3.6 Range Resolution.

$$\mathcal{F}^{-1}\{\mathrm{rect}(\Delta\omega)\, F(\omega)\} = \Delta\omega\, \mathrm{sinc}\left(\frac{\Delta\omega}{2}\left(t - \frac{2l}{c}\right)\right)$$

$$\frac{\Delta\omega}{2}\Delta t = \pi \Rightarrow \frac{\Delta\omega\, \Delta l}{2c} = \frac{\pi}{2}$$



$$\Rightarrow \Delta l = \frac{\cancel{2}\, c\, \cancel{\pi}}{\cancel{2}\,(2\cancel{\pi}\Delta f)} = \frac{c}{2\Delta f}$$

$$\Rightarrow \boxed{RR = \frac{c}{2\Delta f}}$$

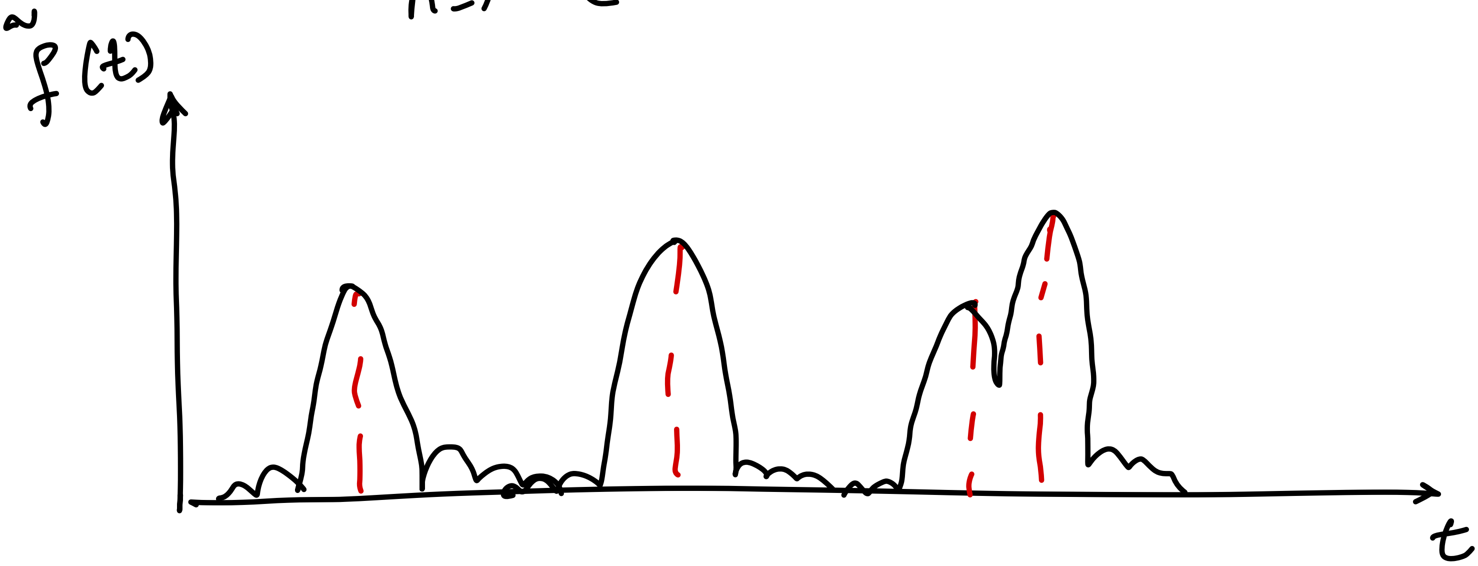
3.7 Multi target with finite bandwidth.

$$F(\omega) = \sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2} e^{j(\omega t - 2k l_n)}$$

$$f(t) = \sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2}\, \delta\left(t - \frac{2l_n}{c}\right)$$

$$\tilde{f}(t) = f(t) * \operatorname{Sinc}\left(\frac{\Delta\omega t}{2}\right)$$

$$= \sum_{n=1}^{N} \frac{\sigma_n}{(4\pi l_n)^2}\, \operatorname{sinc}\left(\frac{\Delta\omega}{2}\left(t - \frac{2l_n}{c}\right)\right)$$



3.8 Features

> **Practical:** Use a network analyzer (VNA).

$Tx: e^{j(\omega t + \phi)}$

$Rx: e^{j(\omega t + \phi + 2kl)}$

> **Advantages:** Simple !

$A \cdot e^{j2kl}$

> **Disadvantage:** Slow !
