Linear Active Networks

- Spence

Ch: 1

Notation

v_{AC} → instantaneous value.

V_{AC} → mean or dc value.

v_{ac} → Signal component.

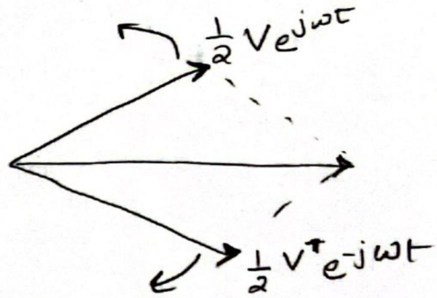
V_{ac} → Complex Voltage.

$\hat{V}_{ac}$ → peak voltage.

→ Let $v = \hat{V} \cos \omega t$

$\Rightarrow v = Re[\hat{V} e^{j\omega t}]$

$= \frac{1}{2}[V e^{j\omega t} + V^* e^{-j\omega t}]$



Sum of these two contrarotating vectors is a sinusoid $\hat{V} \cos \omega t$.

In the most general case.

$$v = \frac{1}{2}[V e^{j\omega t} + V^* e^{-j\omega t}]$$

$$i = \frac{1}{2}[I e^{j\omega t} + I^* e^{-j\omega t}]$$

where V and I are complex quantities since their phases maybe nonzero.

$\Rightarrow V = \hat{V} e^{j(\theta + \phi)}$
$I = \hat{I} e^{j\theta}$

ϕ is relative phase
θ is initial phase. at $t = 0$.

> Therefore the voltage & current in a network (although scalars) can be fully described by V & I (complex).

> θ is usually ignored since it is irrelevant for steady state analysis.

$$V_{rms} = \frac{|V|}{\sqrt{2}} = \hat{V}/\sqrt{2}$$

$$i_{rms} = \frac{|I|}{\sqrt{2}} = \hat{I}/\sqrt{2}$$

Generalising even further ...

Let,

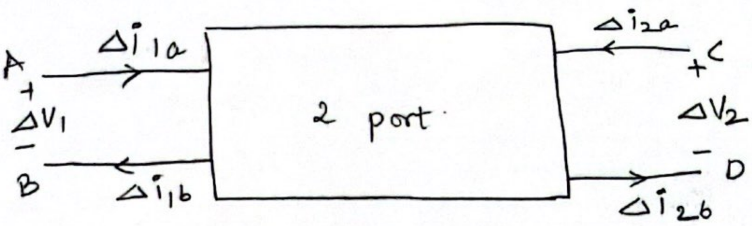
$$i = \frac{1}{2}\left[I e^{j\omega t} + I^{*} e^{-j\omega t}\right]$$

The exponent need not be purely imaginary.

$$\Rightarrow i = \frac{1}{2}\left[I e^{(\sigma + j\omega)t} + I^{*} e^{(\sigma - j\omega)t}\right]$$

$\Rightarrow$ sinusoids whose amplitudes exponentially.

Two port description.



Assumptions.

> The source & load are such that $\Delta i_{1a} = \Delta i_{1b}$ & $\Delta i_{2a} = \Delta i_{2b}$.

> We ignore the voltage V_{AC} & V_{BD}.

At any given frequency

$$\left.\begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}\right\}$$

Y parameters are a function of frequency and also on the bias point since the network is only linear for small signals.

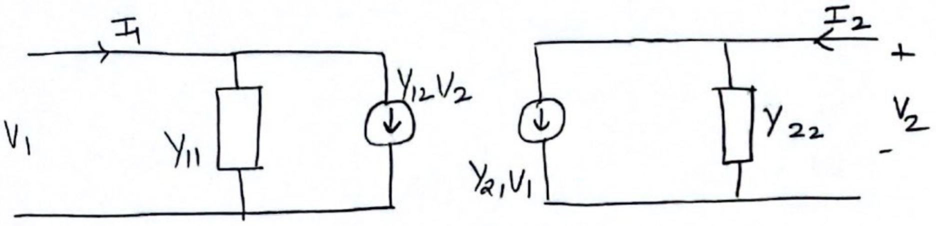
$Y_{11} = g_{11} + j b_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$ → short circuit input admittance (y_i)

$Y_{12} = g_{12} + j b_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$ → reverse transfer " (y_r).

$Y_{21} = g_{21} + j b_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0}$ → forward " " (y_f)

$Y_{22} = g_{22} + j b_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$ → s.c o/p admittance. (y_o)

Alternative representation. (visual)

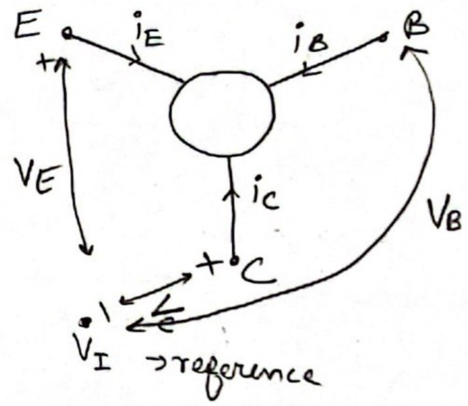


Matrix representation.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Three terminal Networks.

$$I_e = y_{ee} V_e + y_{eb} V_b + y_{ec} V_c$$
$$I_b = y_{be} V_e + y_{bb} V_b + y_{bc} V_c$$
$$I_c = y_{ce} V_e + y_{cb} V_b + y_{cc} V_c.$$



$$\Rightarrow \begin{bmatrix} I_e \\ I_b \\ I_c \end{bmatrix} = \underbrace{\begin{bmatrix} y_{ee} & y_{eb} & y_{ec} \\ y_{be} & y_{bb} & y_{bc} \\ y_{ce} & y_{cb} & y_{cc} \end{bmatrix}}_{\text{Indefinite admittance matrix}} \begin{bmatrix} V_e \\ V_b \\ V_c \end{bmatrix}$$

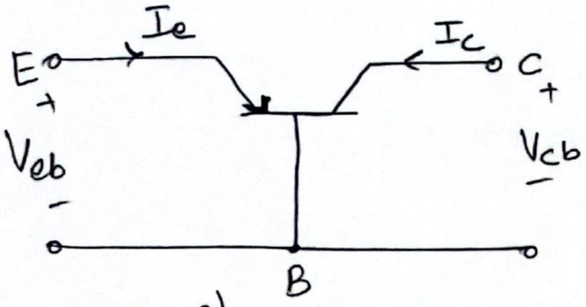
$$y_{bc} = \left.\frac{I_b}{V_c}\right|_{V_e = V_b = 0.}$$

Notice ① KCL ⇒

$$\left.\begin{aligned} y_{ee} + y_{be} + y_{ce} &= 0 \\ y_{eb} + y_{bb} + y_{cb} &= 0 \\ y_{ec} + y_{bc} + y_{cc} &= 0 \end{aligned}\right\} \text{Sum of each column} = 0$$

② If only one node is driven by a voltage all the terminal voltages must be equal since no currents flow. ⇒ This can only be satisfied if the sum of each row = 0.

> Two port from 3 terminal.



$$I_e = y_{ee} V_{eb} + y_{ec} V_{cb}$$
$$I_c = y_{ce} V_{eb} + y_{cc} V_{cb}$$

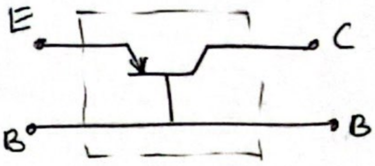
→ Basically remove the B column & row from 3terminal indefinite admittance matrix to arrive at twoport.

$$\begin{array}{c|c} & \begin{matrix} E & B & C \end{matrix} \\ \begin{matrix} E \\ B \\ C \end{matrix} & \begin{bmatrix} y_{ee} & y_{eb} & y_{ec} \\ \cancel{y_{be}} & \cancel{y_{bb}} & \cancel{y_{bc}} \\ y_{ce} & y_{cb} & y_{cc} \end{bmatrix} \end{array}$$

deleted because we are not interested in I_b

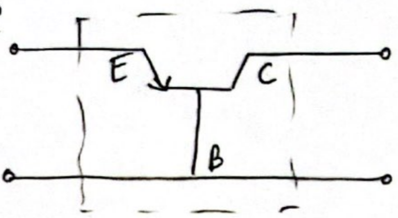
deleted because V_b is reference $\Rightarrow V_b = 0$.

$\Rightarrow$



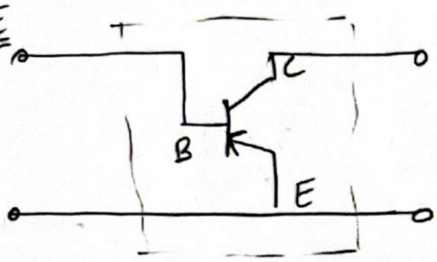
$$\begin{bmatrix} \overset{y_{11}}{y_{ee}} & \overset{y_{12}}{y_{ec}} \\ \underset{y_{21}}{y_{ce}} & \underset{y_{22}}{y_{cc}} \end{bmatrix}$$

CB



> Since the conditions of Σrow=0 & Σcolumn =0 need to be satisfied 4 parameters of 9 in the indefinite admittance matrix are enough to generate the full matrix

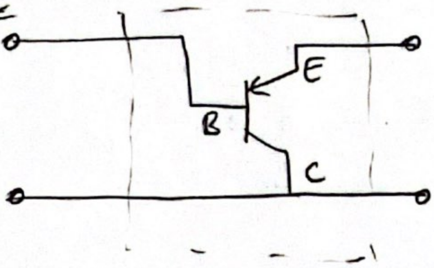
CE



$\Rightarrow$ By characterizing CB behaviour we can derive CC & CE behaviour.

- Fill in CB parameters '4'.
- Fill in remaining 5 elements since Σrow=0 & Σcolumn=0
- Eliminate E row & column to get 2 port parameters for CE.

CC

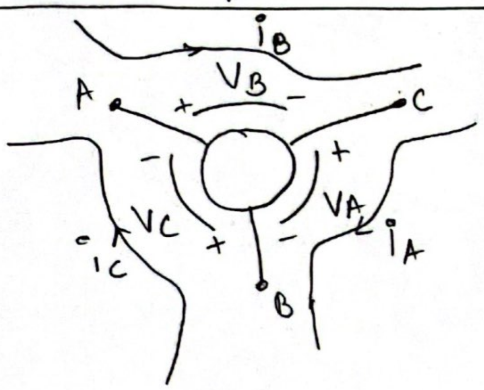


> A four terminal network has a 4X4 indefinite admittance matrix and requires atleast 9 out of the 16 parameters to fully describe it. However redrawing it as a two port only gives 4 parameters. Therefore a 2 port model is a partial description but sufficient for practical situations.

> Impedance matrix

$$Z = Y^{-1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

3 terminal impedance description.



Indefinite impedance matri

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Σ row $= 0$ & Σ column $= 0$

Common base 2 port $\Rightarrow$ $\begin{bmatrix} V_a \\ V_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ac} \\ Z_{ca} & Z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_c \end{bmatrix}$

Let $-V_c = V_{in}$, $-I_c = I_n$, $V_a = V_{out}$, $I_a = I_{out}$.

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} +Z_{cc} & -Z_{ca} \\ -Z_{ac} & +Z_{aa} \end{bmatrix}$$

Hybrid parameters

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$H = K^{-1}$
&
$K = H^{-1}$

> Indefinite matrix description does not exist since units of h & K vary & Σrow & Σcolumn cannot $= 0$.

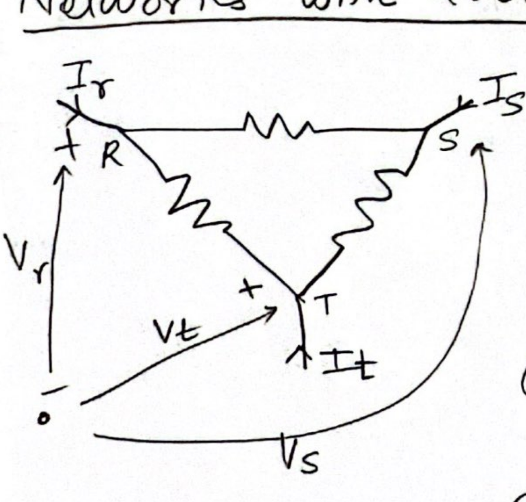
ABCD or Transmission parameters

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

↳ Cascaded 2 port.
> No indefinite matrix.

→ Z parameters use mesh descriptions of V & I and becomes complex for larger networks & therefore Y is preferred.

Networks with 2 terminal elements



$$Y = \begin{array}{c|ccc} & R & S & T \\ \hline R & g_1+g_3 & -g_1 & -g_3 \\ S & -g_1 & g_1+g_2 & -g_2 \\ T & -g_3 & -g_2 & g_2+g_3 \end{array}$$

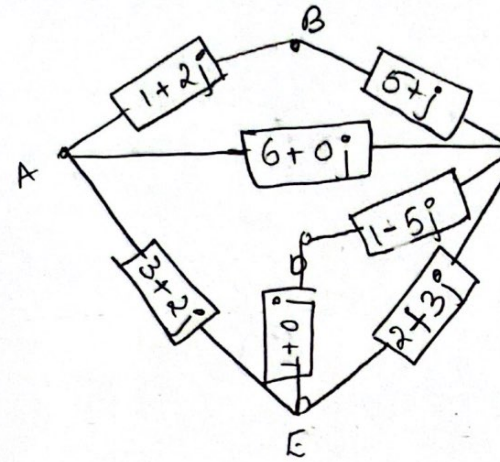
Observations

1. Diagonal elements = sum of conductances incident on the terminal.
2. Off diagonal elements = -ve of conductance between the two terminals involved.

Rules for constructing indefinite admittance matrix

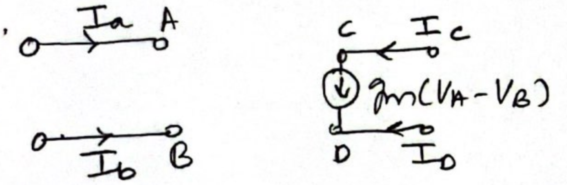
1) No. of rows & columns = no. of network terminals.
2) main diagonal term y_{PP} = sum of all admittances incident on P
3) Off diagonal term y_{PQ} = -ve of all admittances connected b/w P & Q.

Eg:



$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 10+4j & -(1+2j) & -(6+0j) & 0 & -(3+2j) \\ B & -(1+2j) & 6+3j & -(5+j) & 0 & 0 \\ C & -(6+0j) & -(5+j) & 14-j & -(1-5j) & -(2+3j) \\ D & 0 & 0 & -(1-5j) & 2-5j & -(1+0j) \\ E & -(3+2j) & 0 & -(2+3j) & -(1+0j) & 6+5j \end{array}$$

Indefinite admittance matrix for controlled source



KCL ⇒ $I_a = I_b = 0$

$I_c = -I_d = g_m(V_a - V_b)$

$$\Rightarrow Y = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ g_m & -g_m & 0 & 0 \\ -g_m & g_m & 0 & 0 \end{bmatrix} \end{array}$$

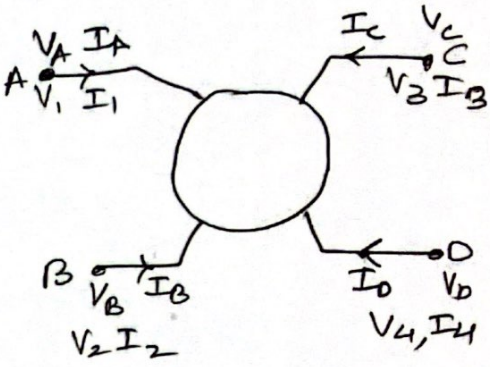
Rules for constructing matrix

1) Non zero elements occur at columns defined by controlling voltages & rows defined by controlled current source.
2) Column of the reference voltage & row of the reference current nodes gives an element with the sign. $+g_m$. Rest of elements' signs are assigned by Σrow=0 & Σcolumn=0 rules.

> For N port network we have 2N terminals. To use an N port description we must ensure the currents entering & leaving each port are equal & that we don't care about voltages b/w 2 port terminals. If these two conditions are not satisfied we must use the 2N terminal description.

Q1.5 Given a 4 terminal network & it's indefinite admittance matrix

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 4 & 7 & -15 & 4 \\ B & -11 & -3 & 19 & -5 \\ C & 8 & -13 & 7 & -2 \\ D & -1 & 9 & -11 & 3 \end{array}$$



Find admittance matrix if port 1 is A B & port 2 is C D

Sol:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 4 & 7 & -15 & 4 \\ -11 & -3 & 19 & -5 \\ 8 & -13 & 7 & -2 \\ -1 & 9 & -11 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$I_1 = -I_2$ and $I_3 = -I_4$ for 2 port to be valid.

$$\Rightarrow 7V_1 - 4V_2 = 4V_3 - V_4 \Rightarrow 7V_{14} = 4V_{24} + 4V_{34}$$
$$\hookrightarrow 7V_{12} = 4V_{32} - V_{42} \quad \Big\} \; (1)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 & 7 & -15 \\ -11 & -3 & 19 \\ 8 & -13 & 7 \end{bmatrix} \begin{bmatrix} V_{14} \\ V_{24} \\ V_{34} \end{bmatrix} \quad \& \quad \begin{bmatrix} I_1 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 4 & -15 & 4 \\ 8 & 7 & -2 \\ -1 & -11 & 3 \end{bmatrix} \begin{bmatrix} V_{12} \\ V_{32} \\ V_{42} \end{bmatrix}$$

We want I_1 & I_3 in terms of V_{12} & V_{34}

$$\Rightarrow I_1 = 4V_{14} + 7V_{24} - 15V_{34} \qquad I_1 = 4V_{12} - 15V_{32} + 4V_{42}$$
$$I_3 = 8V_{14} - 13V_{24} + 7V_{34} \qquad I_3 = 8V_{12} + 7V_{32} - 2V_{42}$$

From ① subs values of V_{32} & V_{14}

$$\Rightarrow I_1 = \frac{4}{7}[4V_{24} + 4V_{34}] + 7V_{24} - 15V_{34}$$

$$I_3 = \frac{8}{7}[4V_{24} + 4V_{34}] - 13V_{24} + 7V_{34}$$

$$I_1 = 4V_{12} - \frac{15}{4}[7V_{12} + V_{42}] + 4V_{42}$$

$$I_3 = 8V_{12} + \frac{7}{4}[7V_{12} + V_{42}] - 2V_{42}$$

$$\Rightarrow 7I_1 = 65V_{24} - 89V_{34}$$

$$7I_3 = -59V_{24} + 81V_{34}$$

$$4I_1 = -89V_{12} - V_{24}$$

$$4I_3 = +81V_{12} + V_{24}$$

$$\Rightarrow 7I_1 = 65[-89V_{12} - 4I_1] - 89V_{34}$$

$$7I_3 = -59[-81V_{12} + 4I_3] + 81V_{34}$$

$$\left.\begin{aligned} \Rightarrow 7I_1 &= -5785V_{12} - 260I_1 - 89V_{34} \\ 7I_3 &= 4779V_{12} - 236I_3 + 81V_{34} \end{aligned}\right\} \Rightarrow \begin{aligned} 267I_1 &= -5785V_{12} - 89V_{34} \\ 243I_3 &= 4779V_{12} + 81V_{34} \end{aligned}$$

$$\Rightarrow I_1 = -21.67V_{12} - 0.33V_{34}$$

$$I_3 = 19.67V_{12} + 0.33V_{34}$$

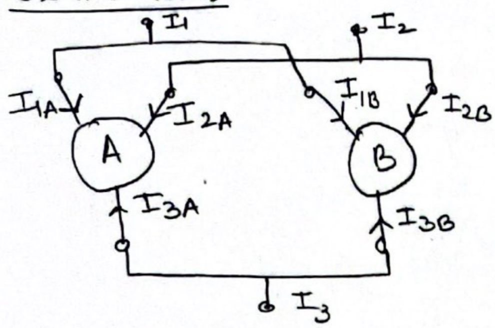
$$\boxed{\Rightarrow \begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} -21.67 & -0.33 \\ 19.67 & +0.33 \end{bmatrix} \begin{bmatrix} V_{12} \\ V_{34} \end{bmatrix}}$$

verified tobe correct

Ch 2 :

Parallel connection of terminals and ports.

① Terminals.

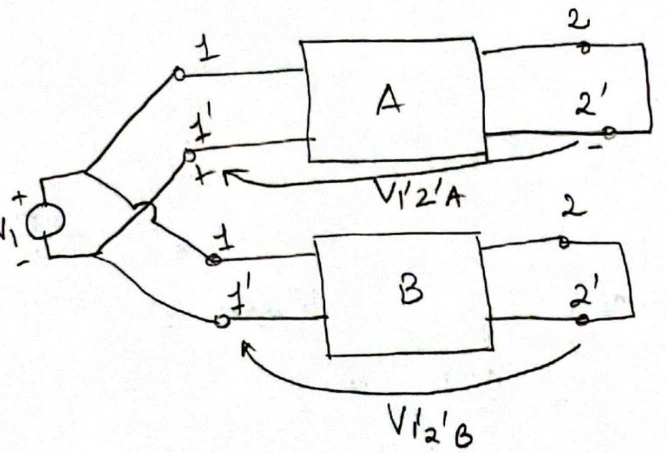


> In this case the indefinite admittance matrices add.

$$Y = Y_A + Y_B$$

② Ports

> In order to add the two matrices the ports must satisfy the "validity test"



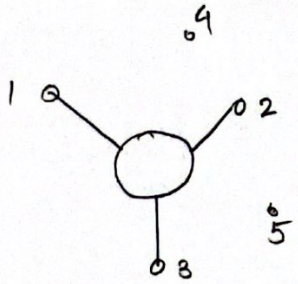
Validity test

$$\boxed{V_{1'2'A} = V_{1'2'B}}$$

This ensures $I_1 = -I_1'$

> If the condition is not satisfied, the two port must be treated as 4 terminal & additional parameters would be needed to fully describe the system.

Unconnected terminals

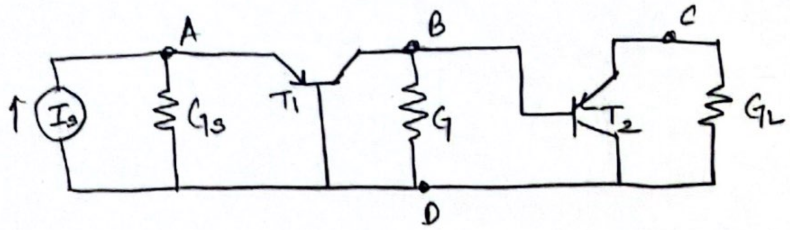


$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & y_{11} & y_{12} & y_{13} & 0 & 0 \\ 2 & y_{21} & y_{22} & y_{23} & 0 & 0 \\ 3 & y_{31} & y_{32} & y_{33} & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

> Add zero row & zero column to the original matrix for each new unconnected terminal.

> Now we have two powerful tools to describe complex networks.
> Parallel combination & unconnected terminals.

Eg:

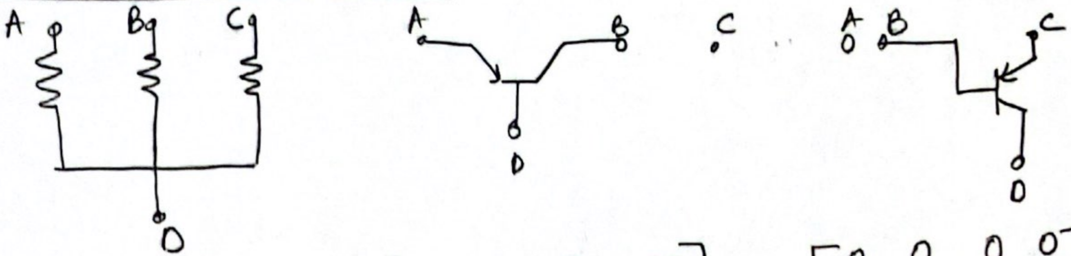


T_1 Common base parameters: $y_{11} = 80$ $y_{12} = -1$ $y_{21} = -180$ $y_{22} = 2$

T_2 ⇒ $y_{ee} = 60$; $y_{bb} = 10$; $y_{cc} = 1$; $y_{bc} = 1$.

$G_s = 2$. ; $G_L = 5$. ; $G = 0.5$

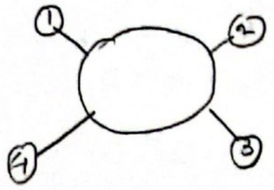
Sol: Divide into 3 circuits. → Notice we can add them all in parallel.



$$\begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 0.5 & 0 & -0.5 \\ 0 & 0 & 5 & -5 \\ -2 & -0.5 & -5 & 7.5 \end{bmatrix} + \begin{bmatrix} 80 & -1 & 0 & -79 \\ -180 & 2 & 0 & 178 \\ 0 & 0 & 0 & 0 \\ 100 & -1 & 0 & -99 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & -11 & 1 \\ 0 & -58 & 60 & -2 \\ 0 & 48 & -49 & 1 \end{bmatrix} = \begin{bmatrix} 82 & -1 & 0 & -8 \\ -180 & 12.5 & -11 & 178 \\ 0 & -58 & 65 & -7 \\ 98 & 46.5 & -54 & -90 \end{bmatrix}$$

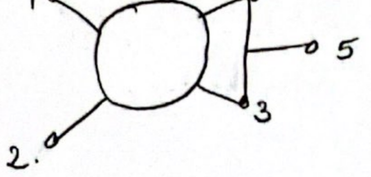
Fill in unknown values using $\Sigma row = \Sigma column = 0$

Short circuit of 2 Terminals.



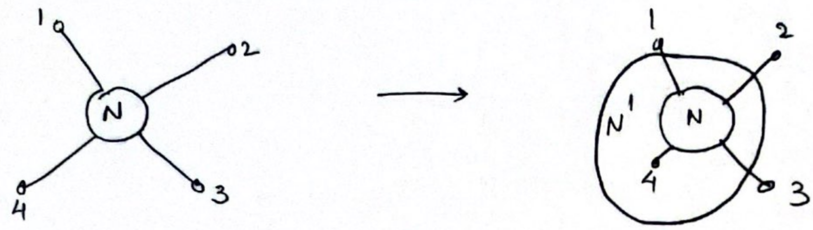
$$\begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix}$$

↓ add rows 2 & 3 and also columns 2 & 3 because new $I_5 = I_2 + I_3$ & $V_2 = V_3 = V_5$



$$\begin{bmatrix} y_{11} & y_{12} + y_{13} & y_{14} \\ y_{21} + y_{31} & y_{22} + y_{23} + y_{32} + y_{33} & y_{24} + y_{34} \\ y_{41} & y_{42} + y_{43} & y_{44} \end{bmatrix}$$

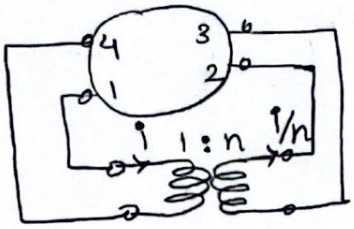
Pivotal Condensation. → Suppressing a terminal that is of no interest.



$$\begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix} \longrightarrow \frac{1}{y_{44}} \begin{bmatrix} y_{11}y_{44} - y_{14}y_{41} & y_{12}y_{44} - y_{14}y_{42} & y_{13}y_{44} - y_{14}y_{43} \\ y_{21}y_{44} - y_{24}y_{41} & y_{22}y_{44} - y_{24}y_{42} & y_{23}y_{44} - y_{24}y_{43} \\ y_{31}y_{44} - y_{34}y_{41} & y_{32}y_{44} - y_{34}y_{42} & y_{33}y_{44} - y_{34}y_{43} \end{bmatrix}$$

- The whole matrix is dividey by y_{mm} if m^{th} element is suppressed.
- Replace $y_{jk}' = y_{jk}y_{mm} - y_{jm}y_{mk}$
- y_{mm} is called the pivot. Main diagonal elements are preferred as pivots because they are rarely $= 0$.
- Same procedure can be followed for port admittance matrix to suppress an open circuited port.
- If a port is short circuit in the port admittance matrix then a reduced port admittance matrix can be obtained by simply eliminating the associated row & column.

Introducing an ideal transformer.



Constraints

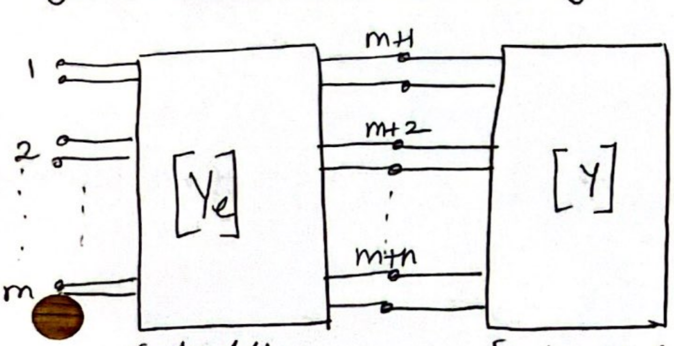
> $(V_2 - V_3) = n\,(V_1 - V_4)$
> i & i/n must be included in KCL.

$$\Rightarrow \begin{bmatrix} I_1 - i \\ I_2 + i/n \\ I_3 - i/n \\ I_4 + i \end{bmatrix} = \begin{bmatrix} y_{11} + n y_{12} & y_{13} + y_{12} & y_{14} - n y_{12} \\ y_{21} + n y_{22} & y_{23} + y_{22} & y_{24} - n y_{22} \\ y_{31} + n y_{32} & y_{33} + y_{32} & y_{34} - n y_{32} \\ y_{41} + n y_{42} & y_{43} + y_{42} & y_{44} - n y_{42} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix}$$

> We have eliminated V_2 (arbitrarily). Since we don't care about i we can eliminate it.

$$\begin{bmatrix} I_1 + n I_2 \\ I_3 + I_2 \\ I_4 - n I_2 \end{bmatrix} = \begin{bmatrix} y_{11} + n y_{12} + n y_{21} + n^2 y_{22} & y_{13} + y_{12} + n y_{23} + n y_{22} & y_{14} - n y_{12} + n y_{24} - n^2 y_{22} \\ y_{31} + n y_{32} + y_{21} + n y_{22} & y_{33} + y_{32} + y_{23} + y_{22} & y_{34} - n y_{32} + y_{24} - n y_{22} \\ y_{41} + n y_{42} - n y_{21} - n^2 y_{22} & y_{43} + y_{42} - n y_{23} - n y_{22} & y_{44} - n y_{42} - n y_{24} + n^2 y_{22} \end{bmatrix}$$

General port embedding.



Embedding network

$(m+n)$ port

$(m+n) \times (m+n)$

Embedded network

(n port)

$(n \times n)$ $\longrightarrow$ $\boxed{\text{gives } m \times m \text{ port}}$

$$Y_e = \left[\begin{array}{ccc|ccc} y_{11} & \cdots & y_{1m} & y_{1(m+1)} & \cdots & y_{1(m+n)} \\ \vdots & & \vdots & \vdots & & \\ y_{m1} & \cdots & y_{mm} & y_{m(m+1)} & \cdots & y_{m(m+n)} \\ \hline y_{(m+1)1} & \cdots & y_{(m+1)m} & y_{(m+1)(m+1)} & \cdots & y_{(m+1)(m+n)} \\ \vdots & & \vdots & \vdots & & \vdots \\ y_{(m+n)1} & \cdots & y_{(m+n)m} & y_{(m+n)(m+1)} & \cdots & y_{(m+n)(m+n)} \end{array}\right]$$

$$\Rightarrow Y_e = \left[\begin{array}{c|c} Y_{mm} & Y_{mn} \\ \hline Y_{nm} & Y_{nn} \end{array}\right]$$

$$\Rightarrow \boxed{Y' = Y_{mm} - Y_{mn}\left[Y_{nn} + Y\right]^{-1} Y_{nm}}$$

→ The form is similar to pivotal condensation since here we are suppressing a matrix and not just a port.

Ch 3

Determinant of a matrix → only exists for square matrices.

① Minors

Given $A = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix}$

Minor $M_{32} = \begin{vmatrix} y_{11} & y_{13} & y_{14} \\ y_{21} & y_{23} & y_{24} \\ y_{41} & y_{43} & y_{44} \end{vmatrix}$ → remove corresponding row & column & take determinant.

$M_{12,13} = \begin{vmatrix} y_{21} & y_{24} \\ y_{31} & y_{34} \end{vmatrix}$

② Second minor ⇒ remove two rows & columns & take det.

③ Principal minor ⇒ remove identical row & column ⇒ M_{11}, M_{22}, M_{33}

④ Cofactor is a signed minor.

cofactor of element y_{32} is $Y_{32} = (-1)^{3+2} \cdot M_{32} = -M_{32}$

⑤ Determinant

Laplace development of determinant: Take any row or column & form corresponding cofactor for each element. Multiply corresponding elements & cofactors & add the products.

Rules

① The algebraic sign is changed for the det if any two rows or columns are interchanged.

② If any 2 rows or columns are identical, det = 0.

③ det is unchanged if to any row, a linear combination of any of its other rows is added. Same applies to columns.

Inverse.

$$A^{-1} = \frac{(\text{Cofactor of } A)^T}{\det|(A)|}$$

$\det(A) = 0 \Rightarrow$ Singular matrix $\Rightarrow$ no inverse!

$$I = YV$$

& $V = ZI$ where $Z = Y^{-1}$.

Cramer's rule.

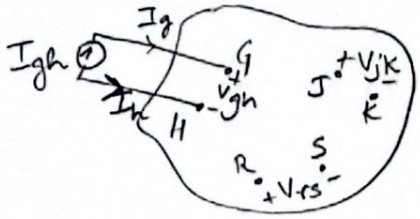
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \overset{A}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\Rightarrow x_1 = \frac{\begin{vmatrix} y_1 & a_{12} & a_{13} & \cdots & a_{1n} \\ y_2 & a_{22} & a_2 & \cdots & a_{2n} \\ \vdots & \vdots & & & \\ y_n & a_{n2} & \cdots & \cdots & a_{nn} \end{vmatrix}}{|A|}$$

$$x_k = \frac{\begin{vmatrix} a_{11} & \cdots & y_1 & \cdots & a_{1n} \\ a_{12} & & y_2 & \cdots & a_{2n} \\ \vdots & & \vdots & & \vdots \\ a_{1n} & \cdots & y_n & \cdots & a_{nn} \end{vmatrix}}{|A|}$$

$\rightarrow$ replace column k by column y & take det.

** General Solutions of N terminal networks.



> Assume an arbitrary network that is driven by current source at terminals G, H.

> If more current sources exist, use Superposition.

> If all terminals except G & H are left open, then the transfer admittance b/w G,H & J,K is $\frac{I_{gh}}{V_{jk}} = \text{sgn}(k-j)\,\text{sgn}(h-g)\,\frac{Y_{uv}}{Y_{gj,hk}}$

> Y_{uv} is any cofactor chosen arbitrarily. A smart choice would be such that row u & column v do not have any zeros

*> The 1st cofactors of all elements of an indefinite admittance matrix are equal.

> $\text{sgn}(k-j)$ = sign of $(k-j)$. If $k=3,\ j=1 \Rightarrow \text{sgn}(k-j) = +ve$.

> $Y_{gj,hk}$ is the cofactor formed by deleting the rows $\underbrace{g,h}_{\text{current excitation}}$ & $\underbrace{\text{columns } j \,\&\, k}_{\text{voltage observed}}$.

> Note here the indefinite admittance matrix is not the same as the port admittance matrix since all other terminals are short & not open → indefinite

port.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I_g \\ I_h \\ \vdots \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & \cdots & y_{1n} \\ \vdots & & & & \\ y_{g1} & y_{g2} & \cdots & & y_{gn} \\ y_{h1} & y_{h2} & \cdots & & y_{hn} \\ y_{n1} & y_{n2} & \cdots & & y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_n \end{bmatrix}$$

> Other network properties are similarly derived.

① Self admittance at input $= \dfrac{I_{gh}}{V_{gh}} = \dfrac{Y_{uv}}{Y_{gg,hh}}$

take $(-1)^{g+g+h+h} \times$ det after yo

↳ delete row g, column g

row h, column h

② Voltage gain b/w J,K & R,S when current drive is at G&H.

$$G^{gh}_{jk,rs} = \left.\frac{V_{jk}}{V_{rs}}\right|_{I_{gh}} = \mathrm{sgn}(s-r)\,\mathrm{sgn}(k-j)\,\frac{Y_{gj,hk}}{Y_{gr,hs}}$$

③ Driving point voltage gain.

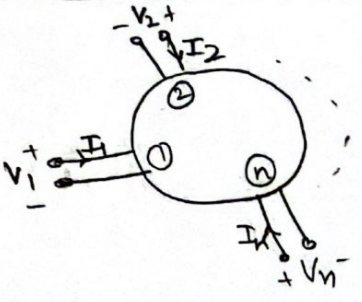
$$G^{gh}_{jk,gh} = \left.\frac{V_{jk}}{V_{gh}}\right|_{I_{gh}} = \mathrm{sgn}(h-g)\,\mathrm{sgn}(k-j)\,\frac{Y_{gj,hk}}{Y_{gg,hh}}$$

→ See examples on Pg 23.

→ All the first cofactors of the indefinite admittance matrix are equal & hence it is aka Equi cofactor matrix. This is because each row & column sum to zero. Proof is complex. Need to manipulate rows & columns s.t det Y remains unchanged.

---

n-Port Network Solution.



$I = YV \longrightarrow \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$

> Remember Y parameters are defined by voltage excitations & short circuit currents measured. Unlike how indefinite admittance matrices are defined (open).

> We want open circuit voltages w.r.t current excitations which comes from Z parameters. $V = ZI$. Where $Z = Y^{-1}$.

> The general sol. of $Z = Y^{-1}$ is what we derived earlier & said that the same could be applied to ports.

> Let I_r be a current excitation at port R & V_s be open.circuit voltage at port S. (all other ports are open).

From Cramer's rule $V_s = \frac{I_r \cdot \Delta_{rs}}{\Delta}$ → cofactor of element y_{rs} . Δ → det

$$\Rightarrow Z_{sr} = \frac{V_s}{I_r} = \frac{\Delta_{rs}}{\Delta} \Rightarrow \boxed{Y_{sr} = \frac{\Delta}{\Delta_{rs}}}$$

Transfer admittance
r→excitation I
s→response V.

defined for open circuit transfer conditions unlike Y parameters.

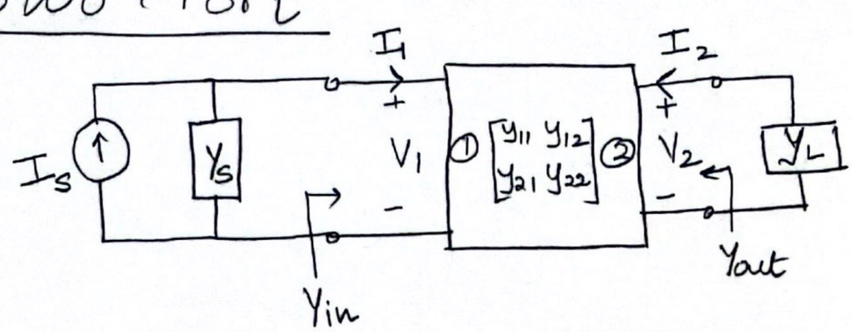
> However Δ_{rs} & Δ are derived from short circuit port admittance matrix Y. Δ is det Y & Δ_{rs} is cofactor of element y_{rs}.

> Y_{sr} is the reciprocal of the open circuited port impedance Z_{sr}.

> Similarly if a network is described by the port impedance matrix Z (which is defined for open circuit configuration), But if the short circuit port current response I_q to an impressed voltage V_p is required when all other ports are shorted, the corresponding $Z_{qp} = \frac{V_p}{I_q} = \frac{\Delta}{\Delta_{pq}}$ $\left.\begin{matrix}\Delta \to \text{det} \\ \Delta_{pq} \to \text{cofactor of } z_{pq}\end{matrix}\right\}$ of the matrix Z!

> $Z_{pq} = \frac{1}{Y_{pq}}$.

Two Port



$$I_1 = I_S - V_1 Y_S \qquad I_2 = y_{21} V_1 + y_{22} V_2$$

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad I_2 = -V_2 Y_L$$

From earlier discussion of self & transfer admittance we can derive.

$$Y_{in} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + y_L} \qquad Y_{out} = y_{22} - \frac{y_{21} y_{12}}{y_{11} + y_s}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + y_L}$$

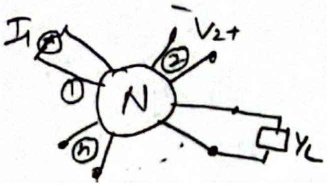
> Immittance ⟶ impedance + admittance.
Transfer immittance ⇒ i/p & o/p are different.
Driving port immittance ⇒ i/p & o/p are same.

In general, input immittance can be expressed with any parameter

$$Y_{IN} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + y_L}; \quad Y_{IN} = K_{11} - \frac{K_{12} K_{21}}{K_{22} + Z_L}$$

$$Z_{IN} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \qquad Z_{IN} = h_{11} - \frac{h_{12} h_{21}}{h_{22} + Y_L}$$

> To get more insight let us see how changing an arbitrary admittance in a network affects the transfer or driving port immittance elsewhere in the network.



$Y_L \rightarrow$ arbitrary impedance.

$$[I] = [Y][V]$$

Let us see how Y_L affects Y_{21}. We know,

$$Y_{21} = \frac{I_1}{V_2} = \frac{\Delta}{\Delta_{12}}$$ from earlier.

Δ → det. of port admittance matrix.

Δ_{12} → cofactor of y_{12} element.

> In terms of y_L:

$$\Delta_{12} = \Delta_{12}^{o} + y_L \cdot \Delta_{3312}$$

$$\Delta = \Delta^{o} + y_L \Delta_{33}$$

$\left\{ \begin{array}{l} \Delta_{12}^{o} \text{ \& } \Delta^{o} \text{ are} \\ \Delta_{12} \text{ \& } \Delta \text{ when } y_L = 0 \end{array} \right.$

$$\Rightarrow \boxed{Y_{21} = \frac{\Delta^{o} + y_L \Delta_{33}}{\Delta_{12}^{o} + y_L \Delta_{3312}}} \quad (3.58)$$

$y = \frac{a+by}{c+dy}$.

→ Any transfer or driving port impedance is related to any single element by the bilinear transformation. $Y = \frac{a+by}{c+dy}$

Let $k_1 + k_2 = b/d$; $k_2 - k_1 = a/c$; $m = c/d$;

$$\Rightarrow \boxed{Y = k_1 \lambda + k_2} \; ; \text{ where } \lambda = \frac{y-m}{y+m}. \quad (3.61)$$

Let $y_N = y/m$

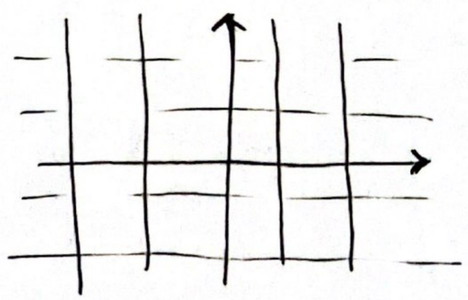
$$\Rightarrow \boxed{\lambda = \frac{y_N - 1}{y_N + 1}}$$

where $a = \Delta^{o}$; $b = \Delta_{33}$

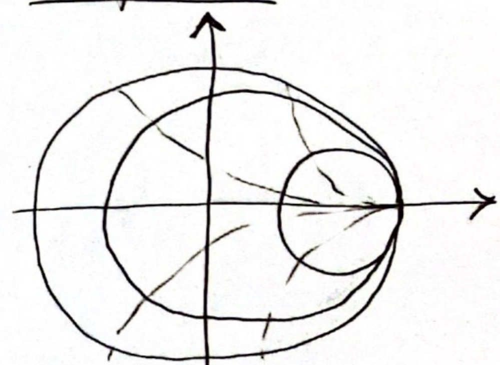
$c = \Delta_{12}^{o}$; $d = \Delta_{3312}$.

$y = y_L$.

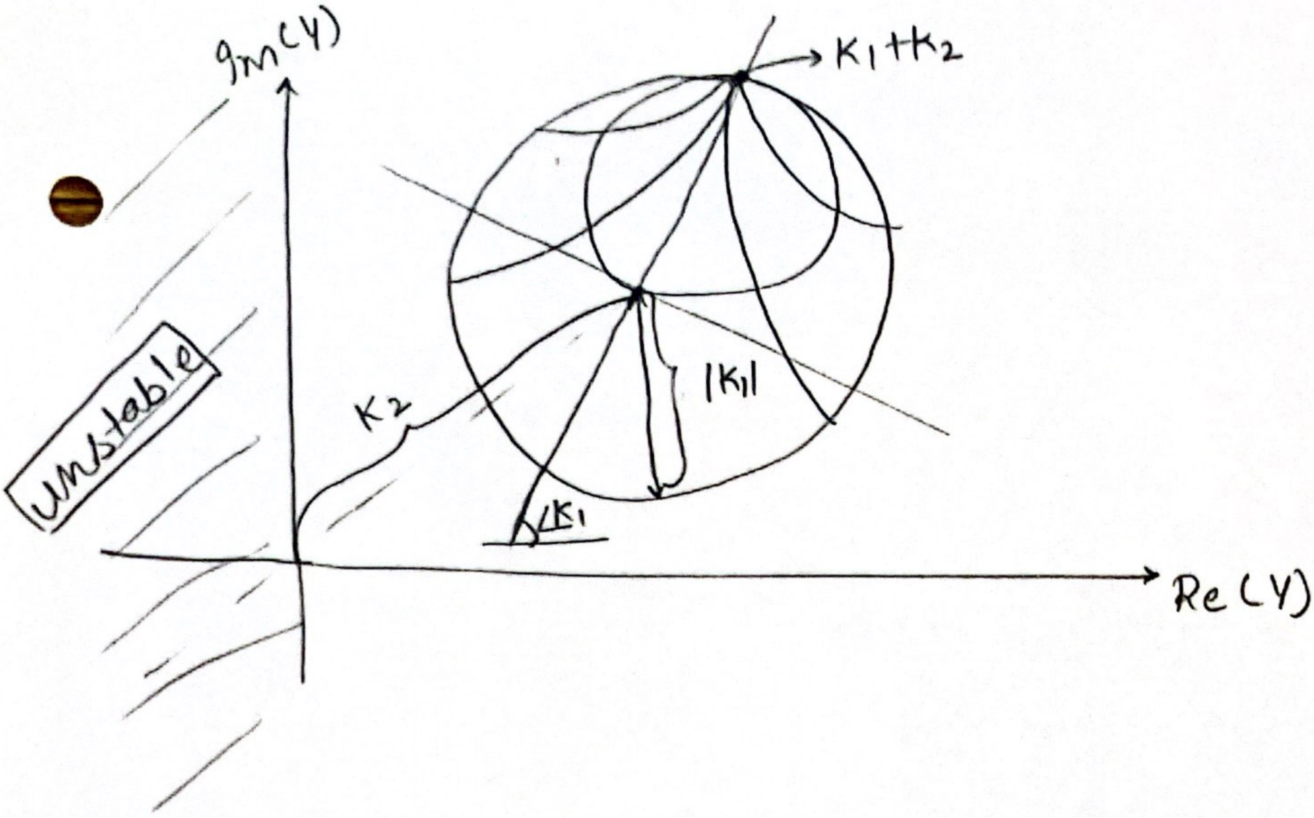
y_N plane.



λ plane.



- y to λ is a pure bilinear transform as shown above ↑
- λ to Y_{IN} is a scaling & shifting as shown by 3.61.
- The unit circle is scaled by $|k_1|$, rotated by $\angle k_1$ & shifted or translated by k_2.

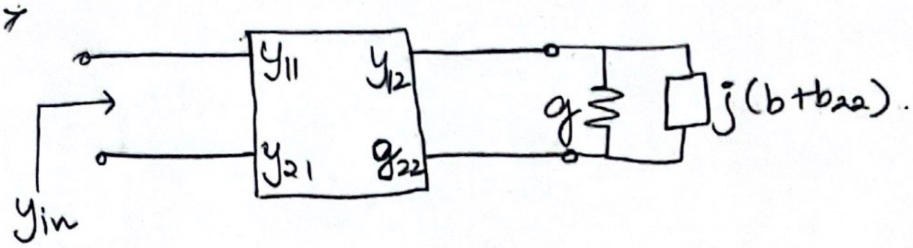


Im(Y)
K1+K2
Unstable
K2
|K1|
∠K1
Re(Y)

Example: Alignability of amplifier

> Changing output reactance to change input impedance.

>



> $g+jb$ is load.
> $g_{22}+jb_{22}$ is part of circuit
> Here we have lumped $b+b_{22}$.

> We would like to see the effect of $j(b+b_{22})$ on y_{in}.

> $$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & g_{22}+g+j(b_{22}+b) \end{bmatrix}$$

$$\Rightarrow \Delta^{0} = \Delta\big|_{b+b_{22}=0} = y_{11}(g_{22}+g) - y_{12}y_{21}$$

$$\Delta_{22} = y_{11}$$

$$\Delta_{11}^{0} = g_{22}+g.$$

$\Delta_{22,11}$ = unity (∵ matrix is empty)

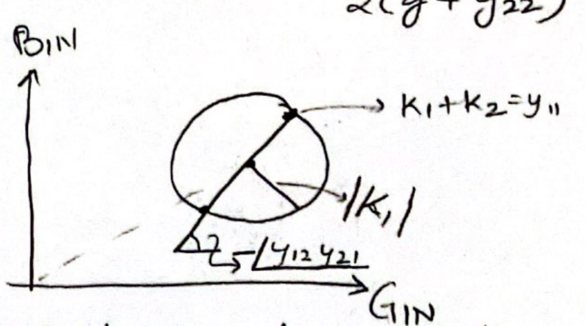
$$\Rightarrow Y_{IN} = Y_{11} = \frac{y_{11}(g_{22}+g) - y_{12}y_{21} + y_{11}j(b+b_{22})}{(g_{22}+g)+j(b+b_{22})}$$

Here, $m = g_{22}+g$

> Transforming from λ plane to Y_{IN} plane ⇒ $k_1+k_2 = y_{11}$

$$k_1 = \frac{-y_{12}y_{21}}{2(g+g_{22})}$$

> $b+b_{22} \rightarrow \pm\infty \Rightarrow Y_{IN} = y_{11} = k_1+k_2$
> which should be intuitive since this implies no load reactance. ⇒ <u>independant of g!</u>



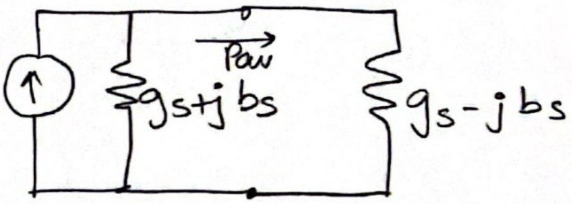
> $\angle k_1 = -\angle y_{12}y_{21}$ is also indepedant of g.
> Stability depends on $|k_1| \to g$, on $-\angle y_{12}y_{21}$ and on real part of y_{11}
$\Rightarrow g_{11}$ ... ⇒ unstable when there is no load.

Power Gain

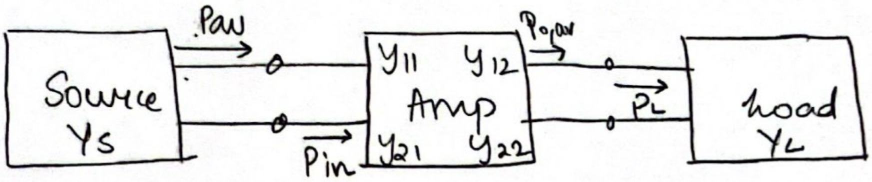
Available power

"Power delivered to a matched load".

$$P_{av} = \frac{|I_s|^2}{4g_s}$$



> This is also the maximum power delivered.
> If we need higher power delivered we need power gain.
> A matched load is used to calculate P_{av}.



(I) Transducer power gain G → (most meaningful)

> $$G = \frac{P_L}{P_{av}} = \frac{\text{Power delivered to load}}{\text{Max. power the source can delivered}}$$

> $G \leq 1 \Rightarrow$ passive.

> $G = 1$ for matched load or ideal transformer matching.

> $$G = \frac{P_L}{P_{av}} = 4 g_s g_L \left| \frac{V_L}{I_s} \right|^2$$

$$= \frac{4|y_{21}|^2 g_s g_L}{|(y_{11}+y_s)(y_{22}+y_L) - y_{12}y_{21}|^2}$$

$$\because \frac{V_L}{I_s} = \frac{y_{21}}{(y_{11}+y_s)(y_{22}+y_L) - y_{12}y_{21}}$$

(II) Power gain (G_p):

$$G_p = \frac{P_L}{P_{in}} = \frac{\text{power delivered to load}}{\text{power supplied to amp i/p by the source}}$$

> $P_{in} \leq P_{av}$ always. $P_{in} = P_{av}$ if Amp i/p is matched to source

$\therefore$ $G_P \geq G$ $\Rightarrow$ Can be deceptive.

> G_P also is ≤ 1 for passive networks.

III Available power gain. (G_A)

> If Amp. output is not matched, the power at the output $\neq P_L$

$$\therefore \quad G_A = \frac{P_{o,av}}{P_{av}} = \frac{\text{Power delivered to load matched at O/P of Amp}}{\text{power delivered by source matched at I/p of Amp}}$$

> It is also deceptive.

> $G_A = G \Rightarrow$ output is conjugate matched.

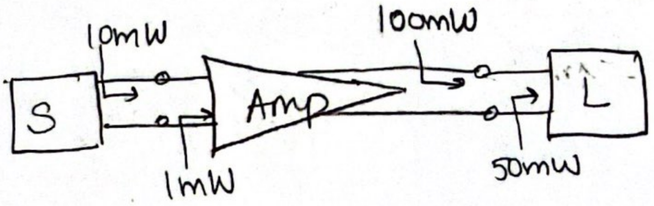
$\Rightarrow$ $G_A \geq G$.

IV Conjugate matched power gain (G_C)

> $G_C = G_A = \dfrac{P_{o,av}}{P_{av}}$ = power gain when both I/p & O/p are matched

> Conjugate matching may lead to instability at very high freq.

Eg:



$\Rightarrow$ $G = 5$; $G_P = 50$; $G_A = 10$

Ch:4

Signal Power

If $v(t) = \hat{V}\cos\omega t$
$i(t) = \hat{I}\cos(\omega t + \phi)$ } Delivered to a one port.

① **Instantaneous power:** $p(t) = v(t)\,i(t)$

$$= \frac{\hat{V}\hat{I}}{2}\cos\phi + \frac{\hat{V}\hat{I}}{2}\cos(2\omega t + \phi)$$

constant & max when $\phi = 0$

- Varies with time.
- Even when load is purely resistive it varies since $v(t)$ & $i(t)$ vary. Varies at 2x freq.

② **Average power (P_{av})**

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt = \frac{\hat{V}\hat{I}}{2}\cos\phi$$

avg. power is constant.

Decomposing $i(t)$ into inphase & quadrature components:

$$p(t) = \frac{\hat{V}\hat{I}}{2}\cos\phi + \underbrace{\frac{\hat{V}\hat{I}}{2}\cos 2\omega t\cos\phi}_{\text{(Inphase I)}} - \underbrace{\frac{\hat{V}\hat{I}}{2}\sin 2\omega t\sin\phi}_{\text{(out of phase I)}}$$

$$= \underbrace{\frac{\hat{V}}{2}(1+\cos 2\omega t)\,\hat{I}\cos\phi}_{} - \underbrace{\frac{\hat{V}}{2}\sin 2\omega t \cdot \hat{I}\sin\phi}_{}$$

V & I are in phase & this power is dissipated in the circuit

Active signal power

avg value is $\frac{1}{2}\hat{V}\hat{I}\cos\phi$

V & I are out of-phase & power is stored in reactive components.

Reactive signal power.

avg value is zero

Amplitude is $\frac{1}{2}\hat{V}\hat{I}\sin\phi$

In phasors.

$$V = \hat{V} e^{j\theta}$$
$$I = \hat{I} e^{j(\theta+\phi)}$$

$$\Rightarrow \quad V^* I = \hat{V} e^{-j\theta} \hat{I} e^{j(\theta+\phi)}$$

$$\boxed{V^* I = \hat{V}\hat{I}\cos\phi + j\hat{V}\hat{I}\sin\phi}$$

complex signal power (P) — Real signal power (P_R) — Imaginary signal power (P_I)

> Notice complex conjugate product gives what we got earlier.

But $P_R = 2 P_{active}$ & $P_I = 2 \times$ amplitude of $P_{reactive}$.

To avoid carrying this $\frac{1}{2}$ everywhere we assume V & I in phasors are rms values. $V_{rms} = \frac{V}{\sqrt{2}}$.

$$P = V^* I = P_R + j P_I$$

Avg. power dissipated — peak value of reactive power oscillation.

> We care about real power. If real power delivered "to" a device is negative $\Rightarrow$ device is active since $P_{out} > P_{in}$ & $P_{in} - P_{out}$ is $-ve$.

> Active device $\Rightarrow$ Real signal power supplied "to" the element is "capable" of being negative.

> Given a 3 terminal (2 port) device: The total signal power (real) entering it is:

$$\frac{P_R}{|V_1||V_2|} = a g_{11} + a^{-1} g_{22} + \mathrm{Re}\left[(y_{21} + y_{12}^*) e^{j\alpha}\right]$$

$$\Rightarrow \frac{P_R}{|V_1||V_2|} = (a g_{11} + a^{-1} g_{22}) + |y_{21} + y_{12}^*| \cos[\angle y_{21}+y_{12}^* + \alpha]$$

> If $g_{11} < 0$ and/or $g_{22} < 0$, P_R would be negative. This type of activity is rarely used.

> If g_{11} & g_{22} are positive, $\frac{P_R}{|V_1||V_2|}$ minimum must be negative.

$$\Rightarrow \left(\frac{P_R}{|V_1||V_2|}\right)_{min} = +\sqrt{g_{11} g_{22}} \cdot 2 - |y_{21} + y_{12}^*|$$

$$\Rightarrow \boxed{|y_{21} + y_{12}^*|^2 > 4 g_{11} g_{22}}$$

Subtracting $g_{12} g_{21}$ on both sides $\Rightarrow$

$$|y_{21} - y_{12}|^2 > 4(g_{11} g_{22} - g_{12} g_{21})$$

$$\Rightarrow \boxed{U = \frac{|y_{21} - y_{12}|^2}{4(g_{11} g_{22} - g_{12} g_{21})}}$$ Mason's Invariant.

> $U > 1 \Rightarrow$ active!

> $U < 0 \Rightarrow$ also active but this type is not used.

> Why use U? It has interesting properties!

> U is invariant under lossless, reciprocal embedding.

> U is invariant w.r.t CE, CC or CB (CD, CS or CG).

> If a device is unilateral ($\Rightarrow y_{12} = 0$) and is matched at i/p & o/p the power gain is equal to the value of U. A device can be made unilateral with lossless reciprocal embedding.

Hermitian matrix

$$\underbrace{A^t}_{\text{Transpose}} = \underbrace{A^*}_{\text{Conjugate}} \Rightarrow \text{Hermitian!} \rightarrow A_H.$$

Skew hermitian matrix

$$A^t = -A^* \longrightarrow A_{SH}$$

> $j \cdot A_{SH} \longrightarrow$ gives a Hermitian matrix.

> Any matrix can be written as a sum of A_H & A_{SH}.

$$\Rightarrow A = A_H + A_{SH}.$$

$$A_H = \frac{1}{2}(A + A^{*t})$$

$$A_{SH} = \frac{1}{2}(A - A^{*t})$$

Matrix description of activity.

$$P = V_1^* I_1 + V_2^* I_2 = [V_1^* \quad V_2^*] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = V^{*t} I = V^{*t} Y V.$$

$$\Rightarrow P = \underbrace{V^{*t} Y_H V}_{\substack{\text{Hermitian} \\ \text{Quadratic} \\ \text{form}}} + \underbrace{V^{*t} Y_{SH} V}_{\substack{\text{purely} \\ \text{imaginary}}}$$

Hermitian Quadratic form → purely real ⇒ real power

purely imaginary } imaginary power.

$\therefore \; P_R = V^{*t} Y_H V$

$jP_I = V^{*t} Y_{SH} V$

$V^{*t} Y_H V \geq 0 \Rightarrow$ positive semidefinite

$V^{*t} Y_H V > 0 \Rightarrow$ positive definite.

> Since device activity is independant of V, we only test to see if $Y_H \geq 0$ or not

** $\Rightarrow$ Necessary and sufficient condition that Y_H is a positive matrix is that all it's principal minors are nonnegative.

Eg:

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$ Find activity condition for 2 port.

Sol:

$$Y_H = \begin{bmatrix} g_{11} & \frac{1}{2}(y_{12}+y_{21}^*) \\ \frac{1}{2}(y_{21}+y_{12}^*) & g_{22} \end{bmatrix}$$

Principal minor of order 0 is $g_{11}g_{22} - \frac{1}{4}(y_{12}+y_{21}^*)(y_{21}+y_{12}^*)$

" " " 1st order are g_{11} & g_{22}.

$\Rightarrow$ Passivity conditions:

$$g_{11}g_{22} - \frac{1}{4}\underbrace{|y_{12}+y_{21}^*|^2 \geq 0}_{①}$$

& either $\underbrace{g_{11} \geq 0}_{②}$ or $\underbrace{g_{22} \geq 0}_{③}$.

(because ① & ② $\Rightarrow$ ③)

These are identical to the ones proved earlier.

> matrix description of activity is more general to n-port.

Isoclinic power element

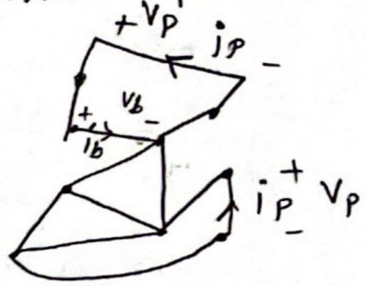
- $\frac{P_R}{P_I}$ is fixed $\Rightarrow$ independant of terminal voltages.
- $Y = e^{j\theta} Y_{herm}$ → general form of isoclinic power element

---

Ch:5 Actual power theorem

- Given a certain graph with m branches and n ports,

$$\boxed{\sum_{b=1}^{m} i_b v_b = \sum_{p=1}^{n} i_p v_p}$$

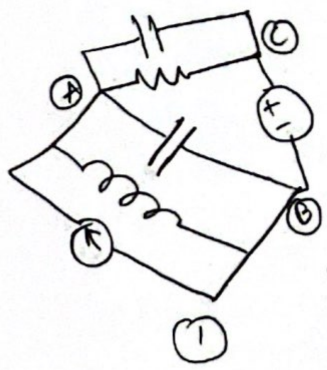


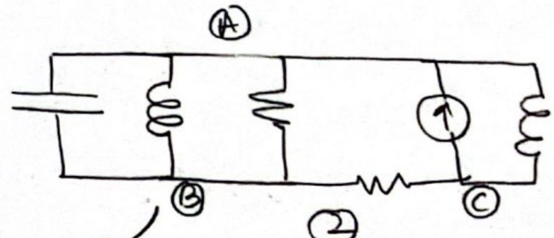
- Notice directions of i in port & brach are opposite.
- This suggests that total actual instantaneous power in a network is zero.
- Here i & v are actual i & v and they are found by solving "both" KVL & KCL.

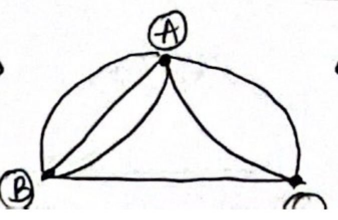
- However KVL & KCL can individually give an infinite set of voltages & currents respectively. These voltages which satisfy only KVL & currents which satisfy only KCL are called virtual voltages & current.

- Two different circuits can have identical topologies. Actual V&I values of ① are virtual values for ② & vice versa.

Eg:







- <u>Tellegen's theorem</u> (aka virtual power theorem).

- $\sum_b i_b' V_b'' = \sum_p i_p' V_p''$ - where i' are virtual currents from state 1 & V'' are virtual voltages from state 2.

- A much more general form of actual power theorem and depends only on topology.

- Valid for nonlinear, time variant, passive/active etc. as long as Kirchoff's laws are satisfied.

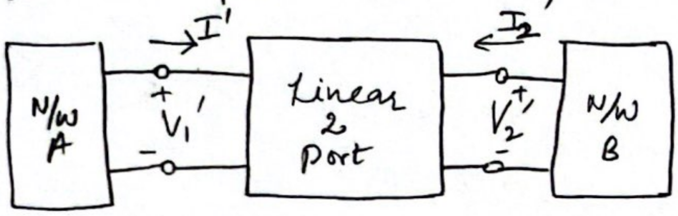
<u>For linear networks</u>

$$\sum_b I_b' V_b'' = \sum_p I_p' V_p'' \rightarrow \text{(aka strong tellegen's theorem)}$$

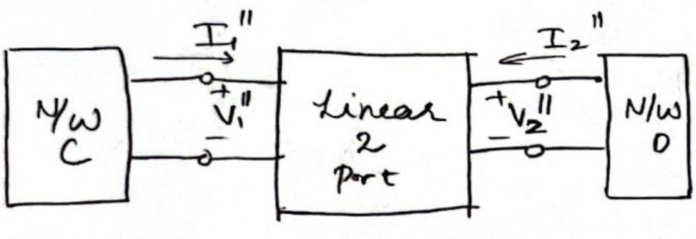
<u>Reciprocity</u>

Given a linear 2 port connected to 2 networks.

<u>State ①</u>



<u>State ②</u>



If $\sum_b (I_b' V_b'' - I_b'' V_b') = 0$, (5.15)

Then, $\boxed{I_1' V_1'' + I_2' V_2'' = I_1'' V_1' + I_2'' V_2'}$ is satisfied.

→ Where I_b & V_b are branch currents & voltages inside the 2port.

→ Eq. 5.15 is the most general statement of 2port reciprocity.

> If $V_2' = 0$ & $V_1'' = 0$ ⇒ $\boxed{y_{21} = y_{12}}$

> Similarly, $\boxed{h_{21} = -h_{12}}$

} Special tests of reciprocity.

Condition for reciprocity

$\sum_b (I_b' V_b'' - I_b'' V_b') = 0$. This is true if each branch also satisfies this condition. ⇒ $\sum_{b=1}^{N} (I_b' V_b'' - I_b'' V_b') = 0$.

> Any resistor, capacitor, inductor or transformer satisfies $I_b' V_b'' - I_b'' V_b'$. ⇒ they are reciprocal.

** ⇒ Any 2 port with reciprocal elements is also reciprocal.

* → An n-port is reciprocal if it satisfies the condition:

$$\sum_p (I_p' V_p'' - I_p'' V_p') = 0$$

↳ derived from the weak tellegen's theorem.

→ A reciprocal element has a symmetric admittance or impedance matrix.

→ A reciprocal network also has a symmetric admittance or impedance matrix

Note: Negative resistances are also reciprocal.

⇒ Passivity ≠ Reciprocity

Weak Tellegen's theorem

$$\sum_b (I_b' V_b'' - I_b'' V_b') = \sum_p (I_p' V_p'' - I_p'' V_p').$$

Gyrator $\rightarrow$ (A passive nonreciprocal element)

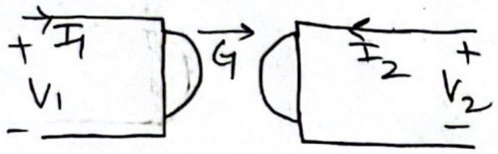
> Antireciprocal element.

$$Y = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \qquad G \rightarrow \text{gyration conduction.}$$

> R, L, C, mutual inductors & gyrators can model any nonreciprocal network.

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \underbrace{\begin{bmatrix} g_{11} & \frac{1}{2}(g_{12}+g_{21}) \\ \frac{1}{2}(g_{12}+g_{21}) & g_{22} \end{bmatrix}}_{\text{reciprocal}} + \underbrace{\begin{bmatrix} 0 & \frac{1}{2}(g_{12}-g_{21}) \\ -\frac{1}{2}(g_{12}-g_{21}) & 0 \end{bmatrix}}_{\text{antireciprocal}}$$

> Circuit symbol.



> Recall input admittance of a 2port is $y_{IN} = y_{11} - \dfrac{y_{12}y_{21}}{y_{22}+y_L}$

if a gyrator is used as the 2port $\Rightarrow$ $\boxed{y_{IN} = \dfrac{G^2}{Y_L}}$

> which is an impedance inverter.

$\Rightarrow$ Cap $\longleftrightarrow$ inductor for ICs.

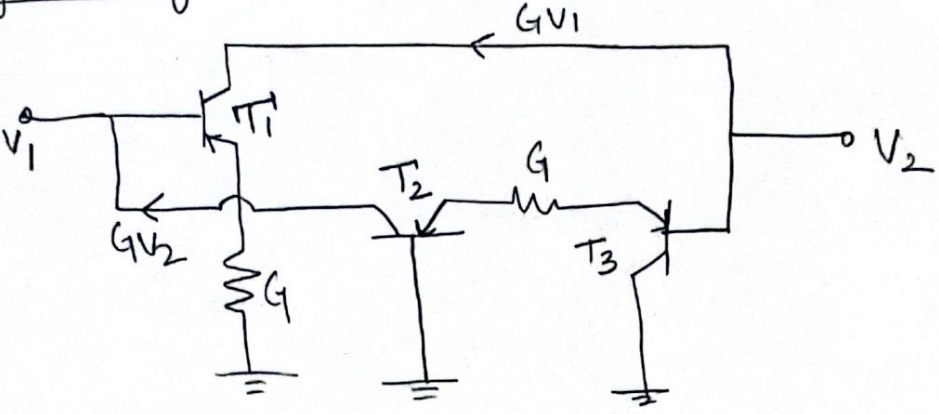
> H matrix of gyrator is $\begin{bmatrix} 0 & 1/G_1 \\ G_1 & 0 \end{bmatrix}$.

$\Rightarrow$ Cascaded gyrators can act as ideal transformers.

$$\begin{bmatrix} 0 & 1/G_1 \\ G_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1/G_2 \\ G_2 & 0 \end{bmatrix} = \begin{bmatrix} \frac{G_2}{G_1} & 0 \\ 0 & \frac{G_2}{G_1} \end{bmatrix}$$

$\rightarrow$ Turns ratio $n = \dfrac{G_1}{G_2}$

Gyrator from transistors.



Antireciprocity

> An N branch element is antireciprocal if :

$$\sum_{b=1}^{N} (I_b' V_b'' + I_b'' V_b') = 0$$ → Same as reciprocal except for sign change.

> An N port network is antireciprocal if:

$$\sum_{p=1}^{M} (I_p' V_p'' + I_p'' V_p') = 0$$

> A network of antireciprocal elements is antireciprocal.

Complex gyrator: $Y = \begin{bmatrix} 0 & y_0 \\ -y_0 & 0 \end{bmatrix}$ where y_0 is complex.

> Ideal transformer, short & open are both reciprocal & antireciprocal.

> Skew symmetric admittance matrix => antireciprocal.

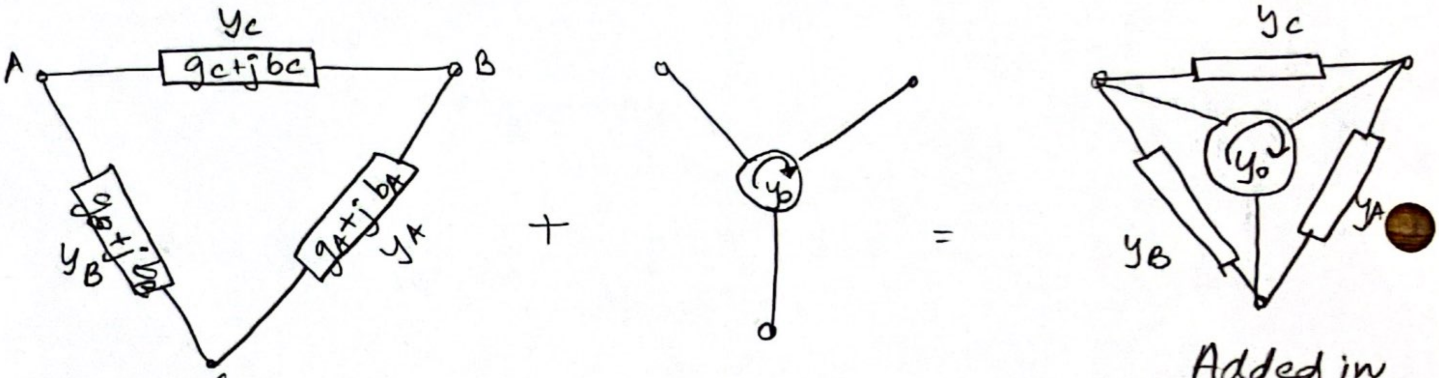
> splitting indefinite admittance matrix into reciprocal and antireciprocal components makes oscillator model easier.

> Splitting port admittance matrix makes amplifier model easier.

3 terminal symmetrical model.

$$\begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix} = \underbrace{\begin{bmatrix} y_B+y_C & -y_C & -y_B \\ -y_C & y_C+y_A & -y_A \\ -y_B & -y_A & y_A+y_B \end{bmatrix}}_{\text{reciprocal part}} + \underbrace{\begin{bmatrix} 0 & y_0 & -y_0 \\ -y_0 & 0 & y_0 \\ y_0 & -y_0 & 0 \end{bmatrix}}_{\text{3terminal complex gyrator part.}}$$

where,
$$y_A = g_A + j\,b_A = -\frac{1}{2}(y_{bc}+y_{cb}) = \frac{1}{2}(y_{bb}+y_{cc}-y_{aa})$$
$$y_B = g_B + j\,b_B = -\frac{1}{2}(y_{ac}+y_{ca}) = \frac{1}{2}(y_{cc}+y_{aa}-y_{bb})$$
$$y_C = g_C + j\,b_C = -\frac{1}{2}(y_{ab}+y_{ba}) = \frac{1}{2}(y_{aa}+y_{bb}-y_{cc})$$
$$y_0 = \frac{1}{2}(y_{ab}-y_{ba}) = \frac{1}{2}(y_{bc}-y_{cb}) = \frac{1}{2}(y_{ca}-y_{ac}).$$



reciprocal part

3 terminal antireciprocal part

Added in parallel.

> Same can be done with z parameters.

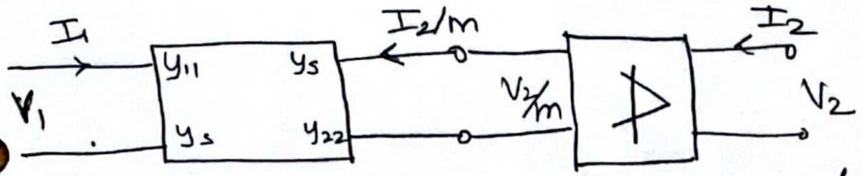
Two port model using antireciprocal transitions.

> Above we saw how 3terminal elements can be modelled.
> Now let's look at 2port. It uses a cascaded model.

$$[I] = [Y][V]$$

Let us define. $y_s \triangleq (y_{12}\,y_{21})^{1/2}$
$$m \triangleq (y_{21}/y_{12})^{1/2}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2/m \end{bmatrix} = \begin{bmatrix} y_{11} & y_s \\ y_s & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2/m \end{bmatrix}$$



Antireciprocal transition.

> The antireciprocal transition introduces a voltage gain of 'm' and current gain of $-m$. $\Rightarrow$ power gain of $|m|^2$.

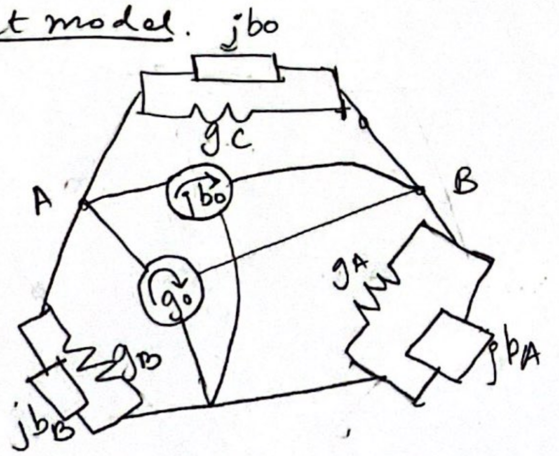
Non reciprocity and Activity.

> Disassemble the given matrix into $\to$ Symmetric $\to$ Hermitian, SkewHermitian; $\to$ Skew symm. $\to$ Hermitian, SkewHermitian.

$$\begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix} = \underbrace{\begin{bmatrix} g_B+g_C & -g_C & -g_B \\ -g_C & +g_C+g_A & -g_A \\ -g_B & -g_A & g_A+g_B \end{bmatrix}}_{\text{nonconservative, reciprocal} \} \text{ Resistor}} + j \underbrace{\begin{bmatrix} b_B+b_C & -b_C & -b_B \\ -b_C & b_C+b_A & -b_A \\ -b_B & -b_A & b_A+b_B \end{bmatrix}}_{\text{conservative, reciprocal} \} \text{ L, C, transform}}$$

$$+ \underbrace{\begin{bmatrix} 0 & g_0 & -g_0 \\ -g_0 & 0 & g_0 \\ g_0 & -g_0 & 0 \end{bmatrix}}_{\text{conservative. nonreciprocal (real gyrator)}} + \underbrace{\begin{bmatrix} 0 & jb_0 & -jb_0 \\ -jb_0 & 0 & jb_0 \\ jb_0 & -jb_0 & 0 \end{bmatrix}}_{\text{nonconservative nonreciprocal. (imaginary gyrator).}}$$

Circuit model.



Conditions for activity

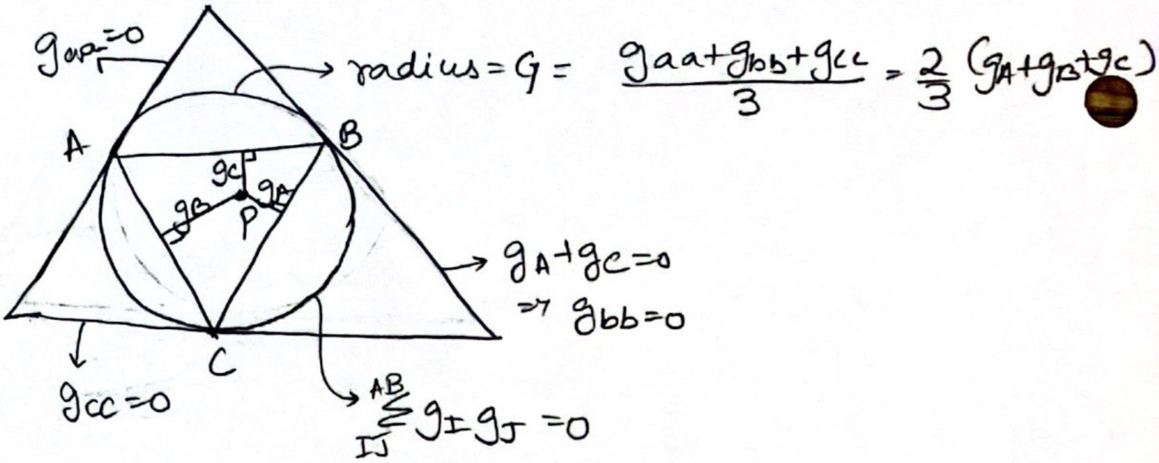
$g_B + g_C < 0$ — (1)

$g_C + g_A < 0$ — (2)

$g_A + g_C < 0$ — (3)

$\sum_{I,J}^{A,B} g_I g_J - b_0^2 < 0$ — (4)

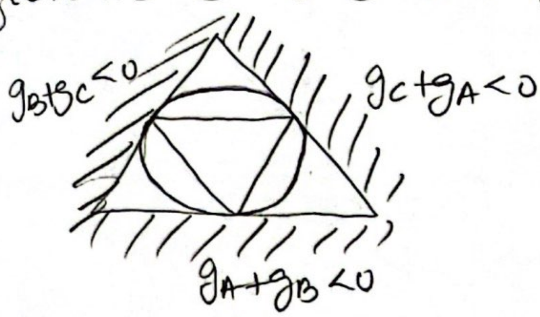
can be represented by the Mason's Conductance diagram.



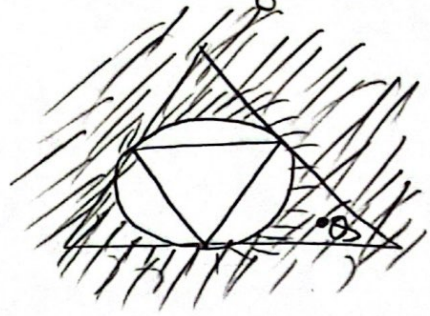
> 'P' represents the 3 terminal device.

> g_A, g_B, g_C are arm conductances in the model. They could be negative.

> If $b_0 = 0$; activity conditions. ①-③ correspond to shaded region in ⓐ & ④ corresponds to shaded region in Ⓑ.



Ⓐ



Ⓑ

> A device at pt. Q is reciprocal & has no negative self conductance but it is active due to negative arm conductance.

> If $b_0 \neq 0$, the circle of passivity has a radius $= \sqrt{G^2 - \frac{4}{3}b_0^2}$.

Chapter 6

Stability.

Return difference.

> RD $F(x)$ of a network w.r.t an element x is the ratio of the network determinant (Δ) and the det when $x=0$. Δ°

$$\Rightarrow F(x) = \frac{\Delta}{\Delta^\circ}$$

> Given a network with 2 accessible ports G & H, where H is driven by a VCCS.

The port admittance matrix is

$$\begin{array}{c|c} & \cdots\ G \quad H\ \cdots \\ \hline \begin{array}{c} G \\ H \end{array} & \begin{bmatrix} & & \\ & Y_{gg} \quad Y_{gh} & \\ & Y_{hg}+x \quad Y_{hh} & \\ & & \end{bmatrix} \end{array}$$

R.D of this network w.r.t x:

$$F(x) = 1 + x\frac{\Delta_{hg}}{\Delta^\circ}$$

where Δ_{hg} is the cofactor of $(Y_{hg}+x)$ & Δ° is the det of the matrix when $x=0$

Here $-x\frac{\Delta_{hg}}{\Delta^\circ}$ is a voltage ratio & called the "loop transmission"

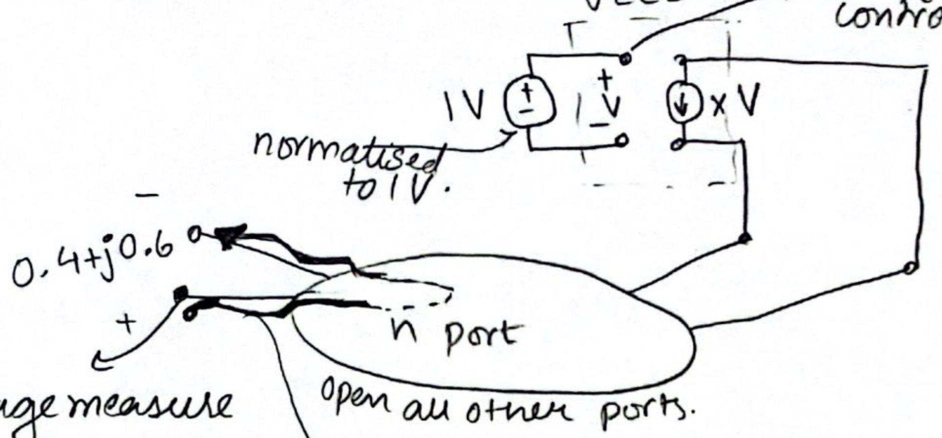
"Return difference is the difference, normalized to one volt, b/w the voltage applied to the controlling terminals of the controlled source and the voltage measured, left of the point at which the network is broken for the purpose of defining the measurement".

Return ratio → "Fraction of the applied voltage returned to the controlling terminals".

$$T(x) = x \frac{\Delta_{ng}}{\Delta}$$

$$\Rightarrow F(x) = 1 + T(x)$$

Eg:



voltage applied to the controlling terminals of the controlled source.

Voltage measure left of the pt. where n/w is broken.

open all other ports.

$$\Rightarrow F(x) = 1 - (0.4 + j0.6)$$

$$\boxed{F(x) = 0.6 - j0.6}$$

One loop transmission.

$$\Rightarrow x\frac{\Delta_{ng}}{\Delta} = 0.4 + j0.6. \Rightarrow \boxed{T(x) = -0.4 - j0.6.}$$

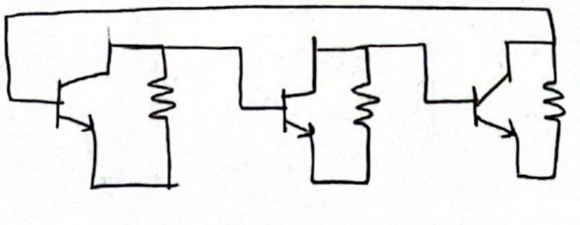
Nyquist Criterion.

Prerequisite: Single loop feedback ⇒ nulling any one active element must stabilize the network.

Consequences of single loop amplifier feedback

① > $\Delta = abc + d$ → parameters of the circuit appear in simple form in the determinant.

Eg:

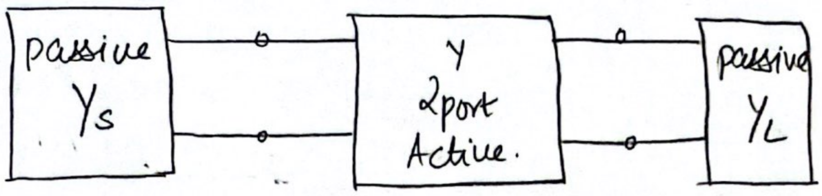


$$\Rightarrow \Delta = Y_1 Y_2 Y_3 - g_{m1} g_{m2} g_{m3}$$

② > Return difference is same for each active element (Since Δ° is same).

> Criterion: If the locus of the return ratio encloses (−1, 0) on the Re(T), Im(T) plane, the network is unstable.

2 Port Stability – Llewellyn's Conditions.



> An active two port terminated in passive one ports is considered absolutely (or inherently) stable if it satisfies :

$$\boxed{\begin{array}{c} g_{11} \geq 0 \\ g_{22} \geq 0 \\ 2g_{11}g_{22} - M \geq L \end{array}}$$

where, $M + jN = y_{12} y_{21}$ & $L^2 = M^2 + N^2$

> Absolutely stable at f ⇒ the network is stable for all possible Y_S & Y_L. Which should be clear since the conditions only depend on the 2 port parameters.

> Potentially unstable ⇒ conditions are not satisfied & some values of Y_S & Y_L could lead to instability.

> A network can be absolutely stable at one frequency and potentially unstable elsewhere.

> Switching the terminals (of a transistor) of the 2 port may cause instability ⇒ conditions must be rechecked.

> In general $g_{11} \geq 0$ & $g_{22} \geq 0$ so we define $\eta = \frac{2g_{11}g_{22} - M}{L}$

> If $\eta \geq 1 \Rightarrow$ inherently stable

$\eta < 1 \Rightarrow$ potentially unstable

> η is a real number.

> η is invariant to immittance transformation. [w.r.t $\vec{Z}$, h & k]

$$\Rightarrow \eta = \frac{2g_{11}g_{22} - M_y}{L_y} = \frac{2r_{11}r_{22} - M_z}{L_z}$$ where $M_Z = Re(Z_{12}Z_{21})$, $L_Z = |Z_{12}Z_{21}|$

> η is not invariant with terminal transformation.

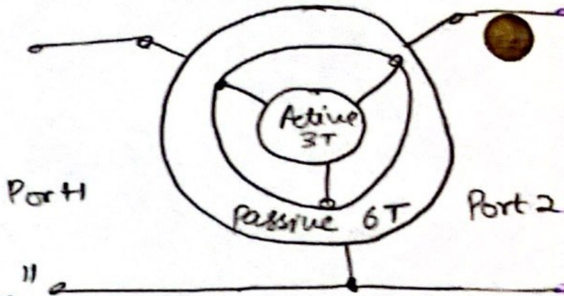
Stabilization.

> We care about 3rd condition $2g_{11}g_{22} - M \geq L$.

> Add a 6 terminal passive reciprocal embedding to make it stable.

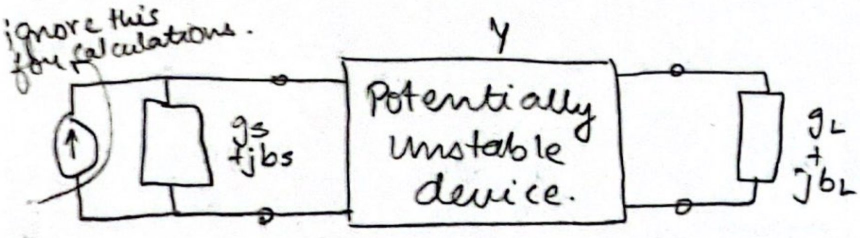
> Special case is "unilateralization".

$\Rightarrow$ Make $y_{12} = 0 \Rightarrow M = L = 0 \Rightarrow$ Stable!



→ However more widely used is a technique called "mismatch stabilization".

→ Mismatch Stabilization.



> The values of Y_s & Y_L are so chosen that the network is stable over a narrow band. The values of Y must remain relatively constant over this band!

> For stability,

$$\boxed{\frac{2\,(g_{11}+g_s)\,(g_{22}+g_L)}{M+L} > 1}$$

$L = Re(y_{12}y_{21})$
$M = |y_{12}y_{21}|$

> We define $\boxed{K \triangleq \frac{2(g_{11}+g_s)(g_{22}+g_L)}{M+L}}$ as the Stern's stability factor.

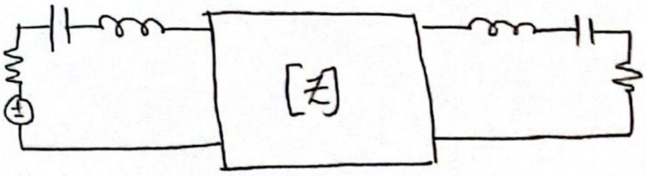
> Recall that $\frac{2g_{11}g_{22}-M}{L} < 1 \Rightarrow$ potential instability. This can be rewritten as $\frac{2g_{11}g_{22}}{M+L} < 1$ & we define K_i as the inherent stability factor. $\boxed{K_i = \frac{2g_{11}g_{22}}{M+L}}$

> Both $K_i < 1$ & $\eta < 1$ are equivalent expressions of inherent instability but $K_i \neq \eta$ and is not immittance invariant.

> Note that potential instability is independant of Y_s & Y_L but here we are dealing with instability in general.

> A potentially unstable 2 port device is made stable over a small bandwidth using Y_s & Y_L to satisfy $K > 1$.

<u>In case of series tuning:</u>

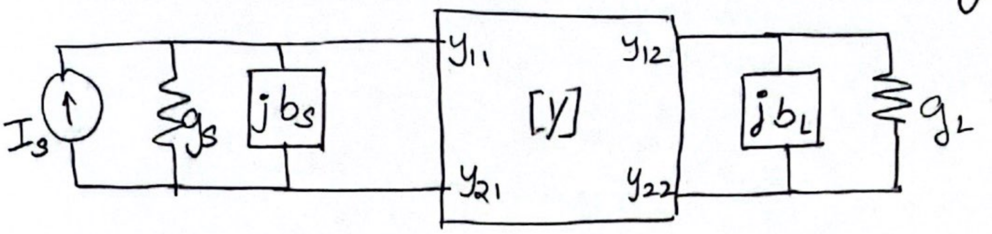


$$\underbrace{\frac{2(r_{11}+r_s)(r_{22}+r_L)}{L_Z+M_Z}}_{\boxed{K_Z}} > 1$$

> Activity is a necessary condition for instability but not a sufficient condition.

Power gain I

Conjugate matching: → Conjugate matching is only possible at the frequencies where the 2port is "inherently" stable. This can be changed if a suitable embedding is used. In which case it can always be matched.



Conjugate match in terms of Y

$$\boxed{\begin{aligned} g_s &= R g_{11}, & b_s &= -b_{11} + \frac{N}{2g_{22}} \\ g_L &= R g_{22}, & b_L &= -b_{22} + \frac{N}{2g_{11}} \end{aligned}}$$

where $M + jN = y_{12}y_{21}$; $R = \sqrt{1 - \frac{M}{g_{11}g_{22}} - \frac{N^2}{4g_{11}^2 g_{22}^2}}$

unilateral $\Rightarrow y_{12} = 0 \Rightarrow M = N = 0$; $R = 1 \Rightarrow$ $\left.\begin{aligned} g_s &= g_{11} ; & b_s &= -b_{11} \\ g_L &= g_{22} ; & b_L &= -b_{22} \end{aligned}\right\}$ obvious.

> It only makes sense when R is real since g_s, g_L can only be real. Therefore a conjugate match is not always possible.

Power Gain.

> Recall, $$G = \frac{4 g_s g_L |y_{21}|^2}{|(y_{11} + y_s)(y_{22} + y_L) - y_{12}y_{21}|^2}$$

when conjugate matched,

$$G_c = \frac{|y_{21}|^2}{2 g_{11} g_{22}(1+R) - M}.$$

max-available power gain. + also the transducer power gain.

What happens if R is not real?

> $R = \frac{L}{2g_{11}g_{22}}\sqrt{\eta^2-1}$ where, $\eta = \frac{2g_{11}g_{22}-M}{L}$.

> η must be >1 for stability $\Rightarrow$ conjugate matching is only possible for a stable 2 port (inherently stable).

Upper bound on G_c

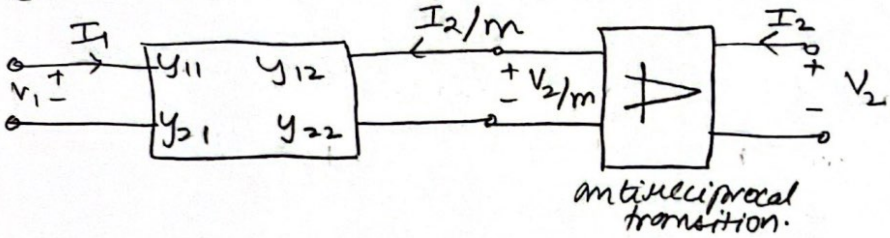
Rewriting, $G_c = \left|\frac{y_{21}}{y_{12}}\right| \frac{1}{\eta+\sqrt{(\eta^2-1)}}$, let $A = \frac{y_{21}}{y_{12}}$

$$\Rightarrow G_c = \frac{|A|}{\eta+\sqrt{(\eta^2-1)}} \quad -(7.14)$$

> G_c is not invariant to terminals unlike U. However. G_c of all configurations is $=1$ at f_{max} (like U).

> $G_c < 1$ at $f > f_{max}$.

Using the cascade model



> We know $m^2 = \frac{y_{21}}{y_{12}} = A$. where $|m^2|$ is the power gain of ⊳.

> Power gain of $[Y]$ is $\frac{1}{\eta+\sqrt{\eta^2-1}}$

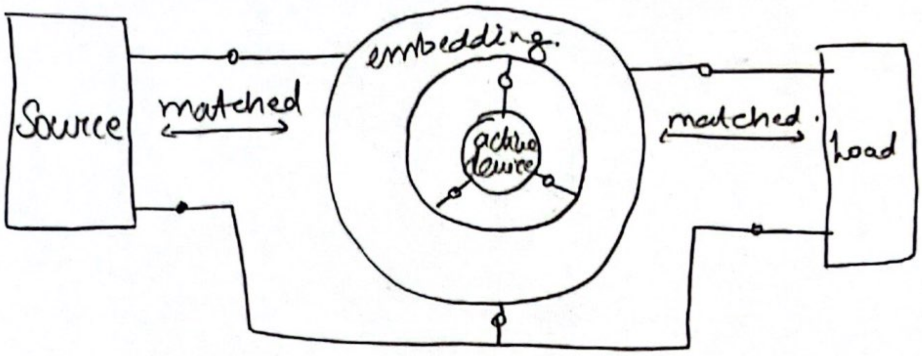
> The ⊳ is impedance transparent $\Rightarrow$ can be ignored during stability analysis.

> In equation 7.14 η & G_c are invariant to immittance & hence $|A|$ must be so too.

$$A = \frac{y_{21}}{y_{12}} = -\frac{h_{21}}{h_{12}} = -\frac{k_{21}}{k_{12}} = \frac{z_{21}}{z_{12}}$$

> Similarly $G = \dfrac{4|y_{21}|^2 g_s g_L}{|(y_{11}+y_s)(y_{22}+y_L) - y_{12}y_{21}|^2}$ can also be expressed interms of z, h & K parameters with care.

Power gain of Embedded device



> Assume source and load are passive.
> The embedding ensures conjugate matched => inherently stable.

$$U = \frac{|y_{21} - y_{12}|^2}{4[g_{11}g_{22} - g_{12}g_{21}]}$$

Rewriting & substituting $G_c = \dfrac{|A|}{\eta + \sqrt{\eta^2 - 1}}$ — (7.11)

We get,

$$\boxed{\frac{U}{G_c} = \left|\frac{A-1}{A-G_c}\right|^2}$$

> U, G_c are real numbers.
> A is complex.

$$\boxed{\sqrt{\frac{G_c}{U}} = \left|\frac{A-G_c}{A-1}\right|} \text{ — (7.19).}$$

Assuming $|A| \gg 1$ which is usually the case.

$$\underbrace{\sqrt{\frac{G_c}{U}}}_{\text{normalised gain. (real)}} = \left|1 - \overbrace{\frac{U}{A}}^{\text{complex value}} \cdot \frac{G_c}{U}\right| \text{ — (7.22)}$$

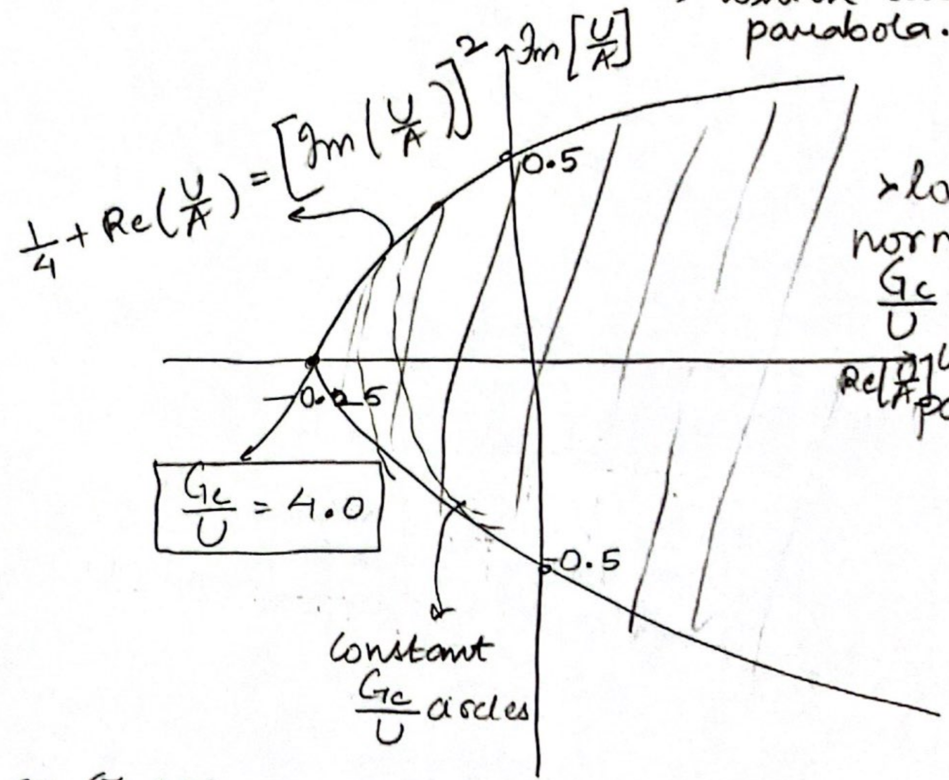
$\hookrightarrow$ basis for gain plane.

> We plot loci of constant $\frac{G_c}{U}$ on the $\frac{U}{A}$ plane.
To do that we must find the bounds of G_c ⇒ when conjugate matching is possible. ⇒ area where device is inherently stable.

> Inherent stability ⇒ $\eta \geq 1$ ⇒ $G_c \leq |A|$ {from (7.11)} & Substituting in (7.22) gives:

$$\boxed{\left[Im\left(\frac{U}{A}\right)\right]^2 < Re\left(\frac{U}{A}\right) + \frac{1}{4}}$$

↳ which describes a parabola.



> loci of constant normalized gain $\frac{G_c}{U}$ are only valid within this parabola.

Rewriting eq. (7.22)

$$\frac{G_c}{U} = \left| 1 - \frac{G_c}{U} Re\left(\frac{U}{A}\right) - j \frac{G_c}{U} Im\left(\frac{U}{A}\right)\right|^2$$

$$\boxed{\frac{U}{G_c} = \left[Re\left(\frac{U}{A}\right) - \frac{U}{G_c}\right]^2 + \left[Im\left(\frac{U}{A}\right)\right]^2}$$

↳ circles of radius $\sqrt{\frac{U}{G_c}}$ and centre at $(U/G_c, 0)$

> Constant gain circles are incomplete since they don't exist outside the parabola.

> Embedding can change the position on the gain plane.

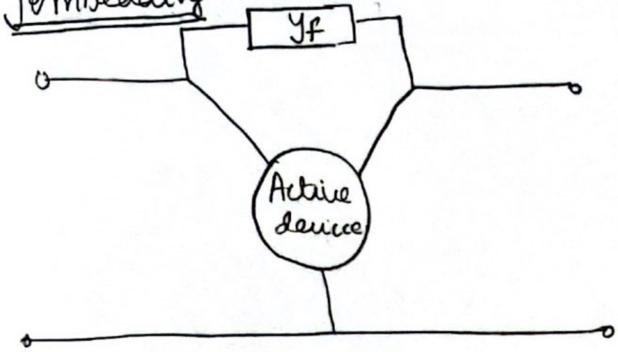
> $$\boxed{G_{C_{max}} = (2U-1) + 2\sqrt{U(U-1)}} \quad \text{(7.27)}$$

When $U \gg 1 \Rightarrow G_{C_{max}} = 4U$.

At frequencies close to f_{max} $U \simeq 1$ & we use Eq 7-27

Movement in gain plane

Y embedding.

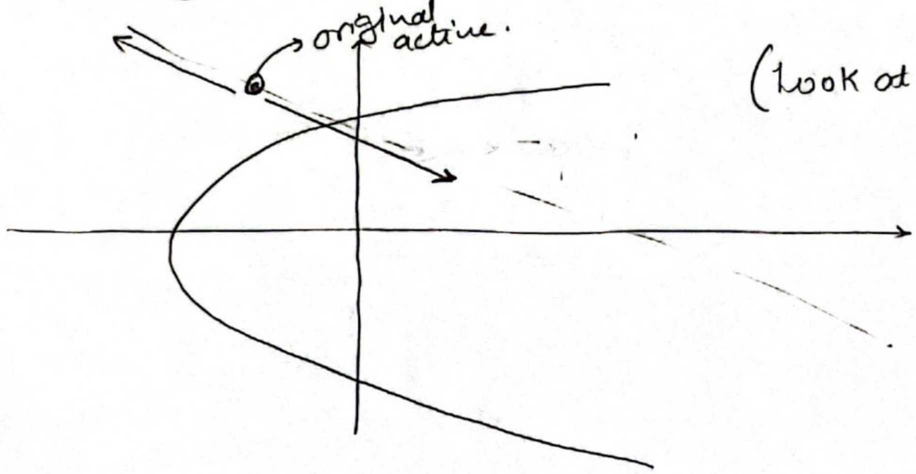


Assumptions: $|Y_f| \ll |Y_{21}|$
& Y_f is lossless.

$$\Rightarrow \frac{U'}{A'} = \frac{U}{A} - \frac{U|Y_f|}{|Y_{21}|} \angle \pm \pi/2 - \angle Y_{21}$$

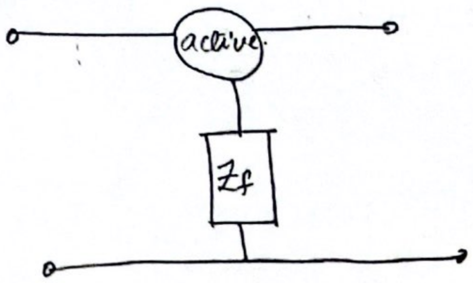
$-$ve $\Rightarrow$ capacitor.
$+$ $\Rightarrow$ inductor

> Draw a straight line through the point (original active 2port) at an angle $\mp \frac{\pi}{2} - \angle Y_{21}$ w.r.t $Re(U/A)$. Then Y embedding moves along this line.



(Look at attached example.)

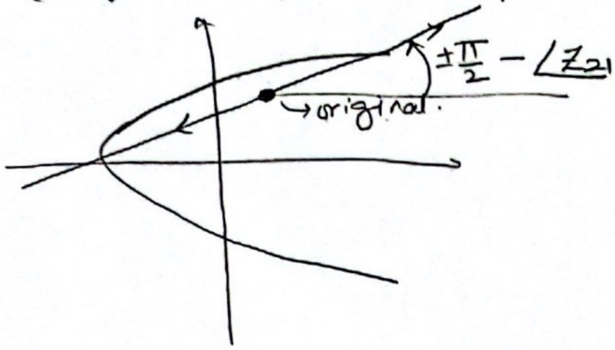
Z embedding.



$$\Rightarrow \begin{bmatrix} Z_{11}+Z_f & Z_{12}+Z_f \\ Z_{21}+Z_f & Z_{22}+Z_f \end{bmatrix}$$

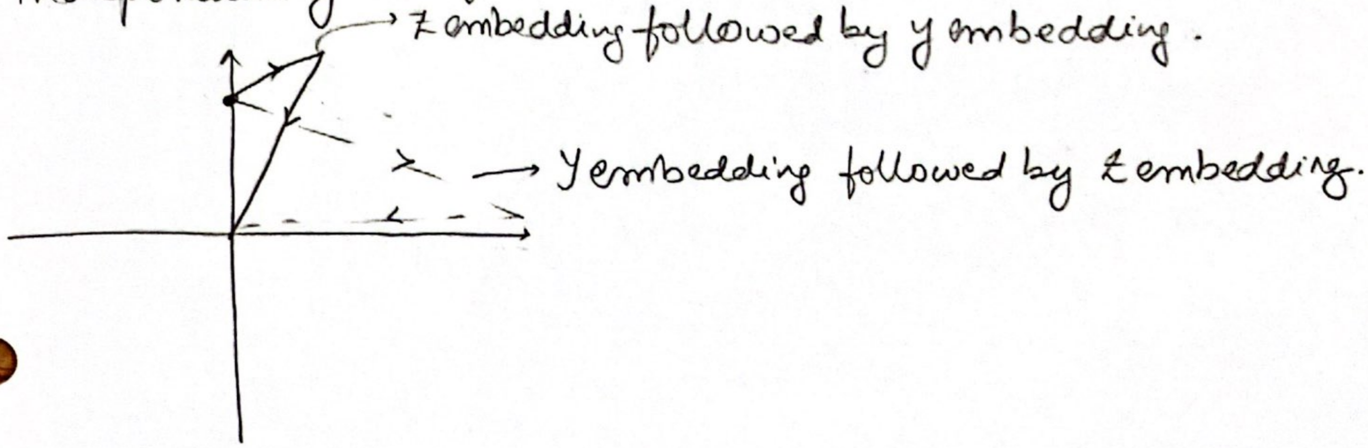
$$\left(\frac{U}{A}\right)' = \frac{U}{A} + U\left|\frac{Z_f}{Z_{21}}\right| \underline{/\pm\frac{\pi}{2} - \angle Z_{21}}$$

$+ \Rightarrow$ inductive.
$- \Rightarrow$ capacitive.



> Note that unilateralization $\Rightarrow$ $y_{12}=0 \Rightarrow \frac{1}{A}=0 \Rightarrow$ the origin! Therefore at the origin the normalized gain $\frac{G_c}{U} = 1$ $\left\{\text{since } \sqrt{\frac{G_c}{U}} = \left|\frac{1-\frac{G_c}{U}\cdot\frac{U}{A}}{1-\frac{1}{U}\cdot\frac{U}{A}}\right|\right\}$.

> Since Y embedding changes Z_{21}, it also changes the slope of the Z embedding. Therefore the two are coupled & cannot be independantly designed to reach the origin.



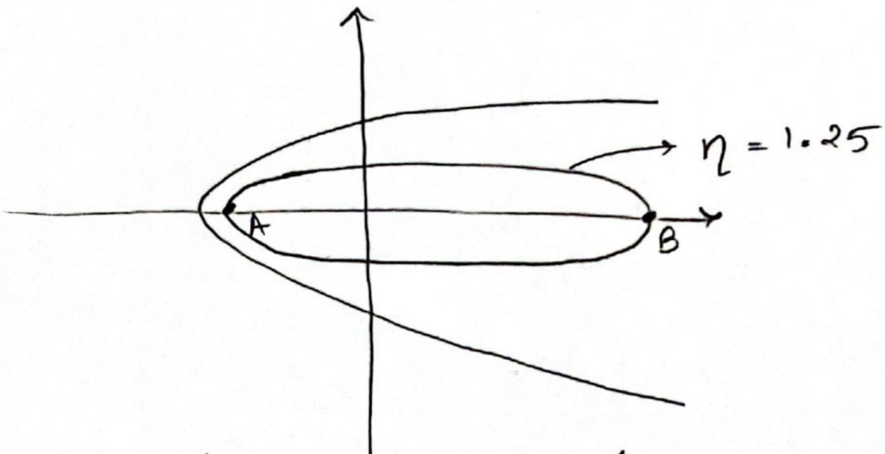
Stability of conjugate matched amplifiers.

$$\eta = \frac{2g_{11}g_{22} - M}{L}$$ is a measure of stability.

We know, $$G_c = \frac{|A|}{\eta + \sqrt{\eta^2 - 1}}$$

$$\Rightarrow \frac{G_c}{U} = \frac{|A|}{U} \cdot \frac{1}{\eta + \sqrt{\eta^2 - 1}}$$

$\Rightarrow$ Constant stability loci form ellipses in the gain plane.



> But η does not serve as a good measure of stability. It is only an indicator of stability.

> Also note that information is lost in the gain plane $\Rightarrow$ Location in gain plane is insufficient to derive Y params.

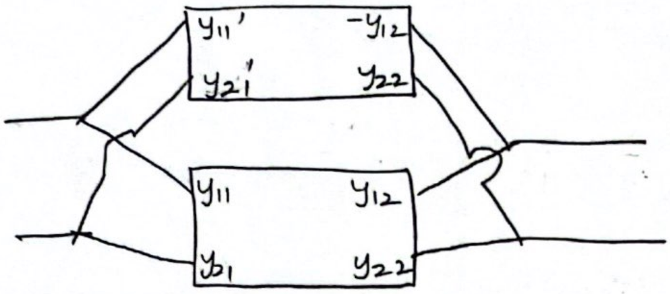
$\Rightarrow$ Two identical points on the gain plane may respond differently to identical embedding since their U, $|y_{21}|$ & $\angle y_{21}$ may differ.

> Did not understand section 7.7.

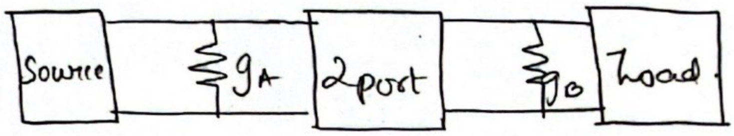
Lossy Embedding.

① > For unilateralization of a 2 port a lossy parallel connection is a trivial solution.

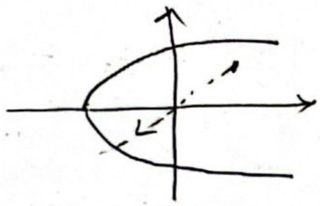
(See example)



② Resistive port padding.



> Since g_A & g_{00} do not change A & only change U, we see that the locus is a straight line through the origin.



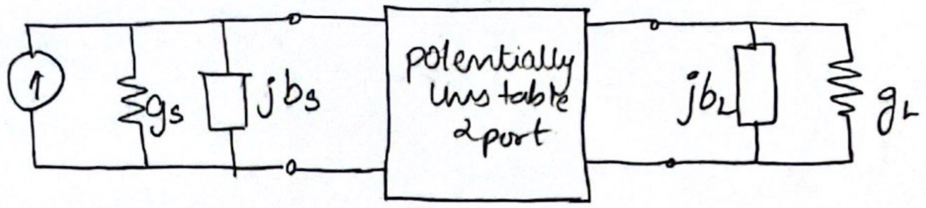
(See example).

> Note that numerator of U is invariant to port padding.

> This form of embedding can improve stability and also is used for cascaded stages.

Ch: 8 : Power Gain II

Mismatch stabilization.



> Choose y_s & y_L to satisfy $K = \dfrac{2(g_{11}+g_s)(g_{22}+g_L)}{L+M} > 1$ to ensure stability. But it will not be conjugate matched.

> mismatch ⇒ use transducer gain.

$$G = \frac{4|y_{21}|^2 (g_1 - g_{11})(g_2 - g_{22})}{(b_1 b_2 - g_1 g_2 + M)^2 + (b_1 g_2 + b_2 g_1 - N)^2}$$

where,

$M + jN = y_{12} y_{21}$; $g_1 = g_s + g_{11}$; $g_2 = g_L + g_{22}$; $b_1 = b_s + b_{11}$; $b_2 = b_L + b_{22}$

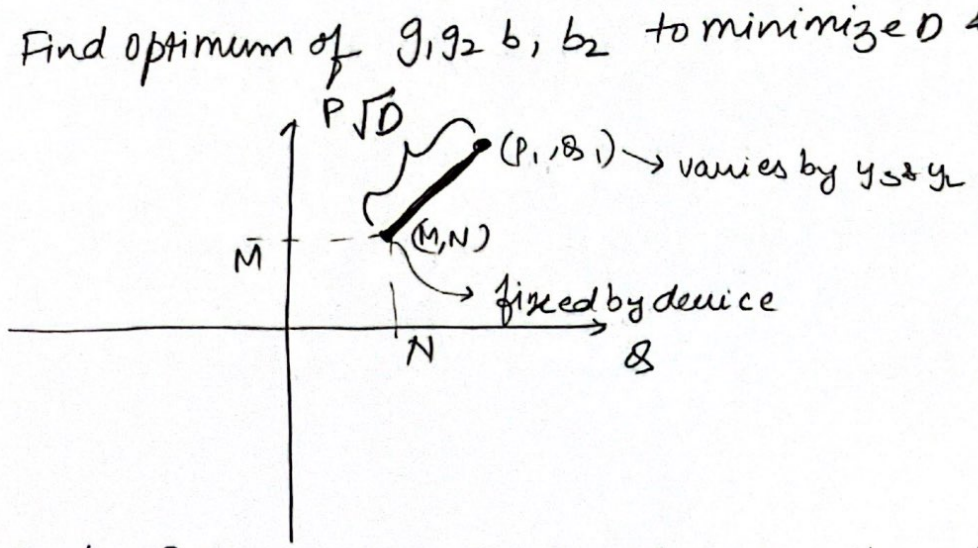
Let denominator of $G = \boxed{D = (P-M)^2 + (Q-N)^2}$

where,

$P = -b_1 b_2 + g_1 g_2$ & $Q = b_1 g_2 + b_2 g_1$

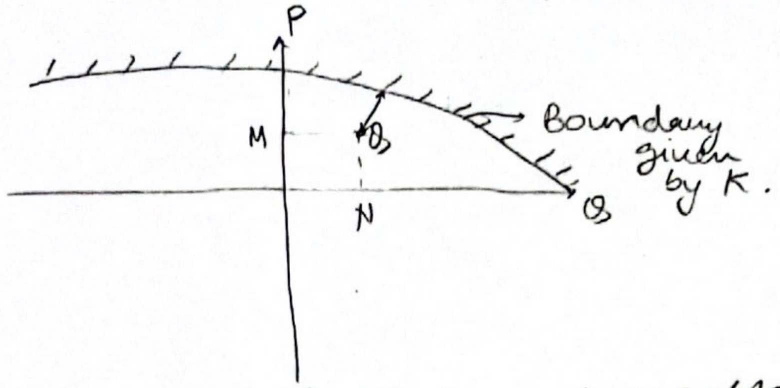
> M, N are device specific ⇒ constant. P, Q vary with source & load.

> Find optimum of $g_1 g_2$ b_1, b_2 to minimize D & maximize G.



> Find P, Q, to minimize D. However, the values of P, Q are constrain
by $Q^2 - 4g_1^2 g_2^2 + 4g_1 g_2 P \geq 0$ to keep it stable.

> The parabola defining P_1, Q_1 to keep the amplifier stable, gives us an optimum of P_1, Q_1 on it that lies closest to M,N.
> Parabola position is defined solely by $K \Rightarrow$ the device itself.



> An analytical solution is possible.

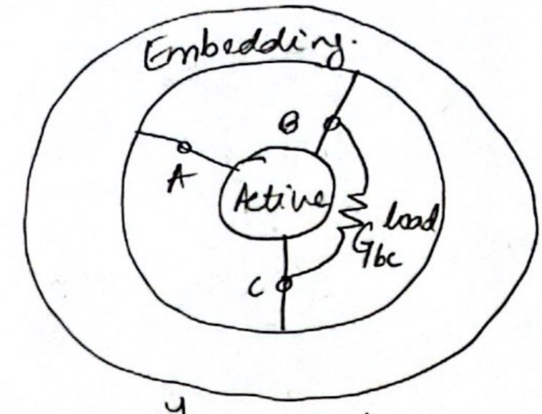
>
$$g_{S\,opt} = g_{11}\left[\sqrt{K/K_i} - 1\right] \quad ; \quad g_{L\,opt} = g_{22}\left[\sqrt{K/K_i} - 1\right]$$

where $K_i = \dfrac{2g_{11}g_{22}}{L+M}$ & K is the desired stability factor.

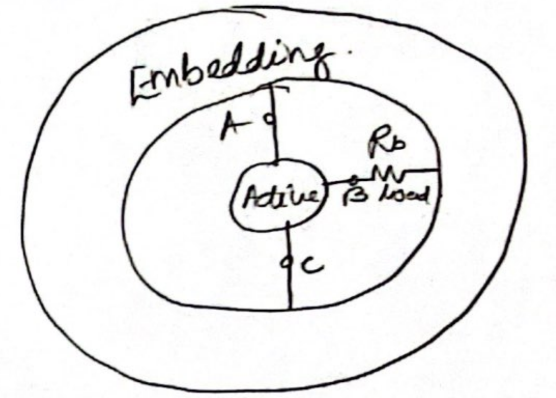
Oscillators.

Maximum loading: Load resistance is increased until the oscillations die out. For practical cases a fixed amplitude is selected and the load conductance is increased until the desired amplitude is reached.

Oscillator models.



Y oscillator.



Z oscillator.

- → We use terminal description. Assuming g_A, g_B, g_C are >0.
  Oscillation condition: $g_A g_B + g_B g_C + g_C g_A - b_0^2 < 0$.
- > G_{bc} is parallel to g_A (don't see why). ⇒ $(g_A + G_{bc}) g_B + g_B g_C + g_C (g_A + G_{bc}) - b_0^2 = 0$

is the maximal loading condition.

- > let $G_{BC} = \max(G_{bc})$

$$\Rightarrow \boxed{G_{BC}\, g_{aa} = G_{CA}\, g_{bb} = G_{AB}\, g_{cc} = \mathcal{G}^2}$$ → invariant to permutations!

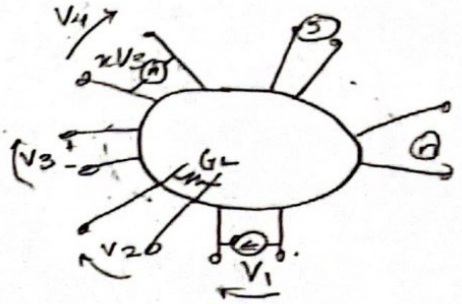
- > G_{BC}, G_{CA}, G_{AB} are the maximum load conductances that can be tolerated.

Network Sensitivity.

> Let $w(x)$ be a transfer function within the network w.r.t x.

Sensitivity

$$S(x) \triangleq \left.\frac{\frac{\delta w}{w}}{\frac{\delta x}{x}}\right|_{\delta x \to 0} = \frac{x}{w} \cdot \frac{dw}{dx} = \frac{d(\ln(w))}{dx} \cdot x$$



$$S(x) = -x \cdot \frac{\Delta_{13}\,\Delta_{42}}{\Delta_{12} \cdot \Delta}$$

$$S(x) = \frac{1}{F(x)} \left[1 - \frac{w(0)}{w(x)} \right]$$

return diff.