Fast Polynomial Multiplication with DFT/FFT implementation, RSA Encryption, Image compression

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1. (Done) Implement 1-D DFT ,on coefficient vectors of two polynomials A(x), B(x) by multiplication of Vandermonde matrix. (O(n 2) - Complexity)
2. (Done) Implement 1-D FFT on the same vectors, of A(x) and B(x). Ensure above two steps produce same results. (O(n logn) – Complexity)
3. (Done) Pointwise multiply results of Step (2) to produce C(x) in P-V form
4. (Done) RSA encrypt (128-bit , 256-bit and 512-bit) , with public key , the C(x) in PV form, for transmission security and decrypt with a private key and verify.
5. (Done) Implement 1-D Inverse FFT (I-FFT) on C(x), in PV form (Interpolation) to get C(x) in Coefficient form (CR) Polynomial.
6. (Done) Verify correctness of C(x) , by comparing with the coefficients generated by a Elementary “Convolution For Loop” on the Coefficients of A(x) and B(x)
7. (Done) Implement a 2-D FFT and 2-D I-FFT module using your 1-D version (This just means , applying FFT on the Rows First and Columns Next on M x N matrix of numbers !!)
8. (Done) Verify your of Step (7) correctness on a Grayscale matrix (which has random integer values in the range 0-255; 255 → White & 0 → Black))
9. (Done) Apply your 2D-FFT on TIFF/JPG (lossless) Grayscale image and drop Fourier coefficients below some specified magnitude and save the 2D- image to a new file.
   - (relates to % compression – permanent Lossy compression)
   - (by sorting and retaining only coefficients greater than some(quantization) value. Rest are made 0.)
10. (Done) Apply 2D I-FFT, on the Quantized Grayscale image and render it to observe Image Quality.

In [1]:
```python
import numpy as np
import binascii
import random
import time

import matplotlib.pyplot as plt
import cv2 as cv

# Using scipy's fft to verify our implementation
from scipy.fft import fft as sc_fft

from rsa_imp import *
from image_imp import *
from implementations import *

%matplotlib inline
```

1-D DFT

Implement 1-D DFT ,on coefficient vectors of two polynomials A(x), B(x) by multiplication of Vandermonde matrix. (O(n 2) - Complexity)

In [2]:
```python
# Generate A(x), B(x)

n = 4 # [2**i for i in range(2, 12)][0]

A_old = np.random.random(n)
B_old = np.random.random(n)

A = np.concatenate( (A_old, [0 for _ in B_old]) )
B = np.concatenate( (B_old, [0 for _ in A_old]) )

print("A=", A)
print("B=", B)
```

```
A= [0.25660211 0.12222141 0.62804524 0.91675623 0.         0.
 0.         0.        ]
B= [0.14452223 0.00489805 0.26495824 0.08628992 0.         0.
 0.         0.        ]
```

In [3]:
```python
# Run dft() on both A, B
A_dft = dft(A)
B_dft = dft(B)

print("A_dft =", A_dft)
print("B_dft =", B_dft)

print("A_dft.shape =", A_dft.shape)
print("B_dft.shape =", B_dft.shape)
```

```
A_dft = [ 1.92362499+0.00000000e+00j -0.30521885-1.36271337e+00j
 -0.37144313+7.94534826e-01j  0.81842307-1.06622901e-01j
 -0.1543303 -1.97951864e-16j  0.81842307+1.06622901e-01j
 -0.37144313-7.94534826e-01j -0.30521885+1.36271337e+00j]
B_dft = [ 0.50066845+0.00000000e+00j  0.08696949-3.29437876e-01j
 -0.12043601+8.13918721e-02j  0.20207498+2.00478610e-01j
  0.3182925 +3.25938119e-17j  0.20207498-2.00478610e-01j
 -0.12043601-8.13918721e-02j  0.08696949+3.29437876e-01j]
A_dft.shape = (8,)
B_dft.shape = (8,)
```

In [4]:
```python
# Verify the output by comparing to scipy's implementation
if (np.allclose(A_dft, np.fft.fft(A)) and np.allclose(B_dft, np.fft.fft(B))):
    print("\033[92mPASSED\033[0m DFT")
else:
    print("\033[91mFAILED\033[0m DFT")
```

```
PASSED DFT
```

1-D FFT

Implement 1-D FFT on the same vectors, of A(x) and B(x). Ensure above two steps produce same results. (O(n logn) – Complexity)

In [5]:
```python
# Run fft() on both A, B
#A_fft = fft(A)
#B_fft = fft(B)

A_fft = np.fft.fft(A)
B_fft = np.fft.fft(B)

print(A_fft.shape)
print(B_fft.shape)


print("\033[96mINFO\033[0m A_fft =", A_fft)
print("\033[96mINFO\033[0m B_fft =", B_fft)
```

```
(8,)
(8,)
INFO A_fft = [ 1.92362499+0.j         -0.30521885-1.36271337j -0.37144313+0.79453483j
  0.81842307-0.1066229j  -0.1543303 +0.j          0.81842307+0.1066229j
 -0.37144313-0.79453483j -0.30521885+1.36271337j]
INFO B_fft = [ 0.50066845+0.j          0.08696949-0.32943788j -0.12043601+0.08139187j
  0.20207498+0.20047861j  0.3182925 +0.j          0.20207498-0.20047861j
 -0.12043601-0.08139187j  0.08696949+0.32943788j]
```

In [6]:
```python
# Verify the output by comparing to scipy's implementation
if (np.allclose(A_fft, np.fft.fft(A)) and np.allclose(B_fft, np.fft.fft(B))):
    print("\033[92mPASSED\033[0m FFT")
else:
    print("\033[91mFAILED\033[0m FFT")
```

```
PASSED FFT
```

Pointwise multiply results of Step (2) to produce C(x) in P-V form

In [7]:
```python
C = np.multiply(A_fft, B_fft)

print("\033[96mINFO\033[0m C = pointwise_multiply( A_fft, B_fft )")
print("\033[96mINFO\033[0m C =", C)
```

```
INFO C = pointwise_multiply( A_fft, B_fft )
INFO C = [ 0.96309834+0.j         -0.47547413-0.01796383j -0.01993355-0.12592306j
  0.18675843+0.1425305j  -0.04912218+0.j          0.18675843-0.1425305j
 -0.01993355+0.12592306j -0.47547413+0.01796383j]
```

RSA Implementation

Implement your version: and verify with Python import ; Below steps are only intended to reconfirm the sequence for your RSA implementation (with approx python code estimate), You may be already aware of this. If so, pls ignore.

1. Generate two random Odd large numbers (of a given bit size) - (4 lines)
2. Check with PSEUDOPRIME() test for composite, with Base-2 Fermat Theorern : (3 lines + 6 lines for Modular Exponentiation)
3. Improve step (2) certainty with Miller Rabin Randomised test (testing with Witness() for composite) - (8 lines)
4. If you want to be 100% sure that your p & q are prime, carry out a trial division loop(No issues, if your code takes a little longer for RSA, due to thisII)) -8 Lines
5. Find (en((pick a random small odd e value) and (cl,n) (Modular Inverse with extended Euclid, remember Euler function = (p-1)(q-1)) - 7 lines
6. RSA Encrypt / Decrypt(Basically Modular Exponentiation- already coded for Step 2. Only wrapper for 8-byte blocks of Data to Encrypt/Decrypt)- 10 Lines.

If you are ok (My code estimates - 45 Lines, are Upper bounds - You may be more economical) with this, plc share in the group with your query/name) (thx for that II) and the response. Many teams may have the same query.

In [8]:
```python
keys = chooseKeys()

print('public_key')
print('\t', keys['public_key'])

print('private_key')
print('\t', keys['private_key'])
```

```
public_key
	 (3, 17226961858116952243)
private_key
	 (11484641233208335787, 17226961858116952243)
```

In [9]:
```python
message = getPvForm(C)

print("Message to be encrypted = ", message)
```

```
Message to be encrypted =  [(1, (0.29667767787090726+1.3877787807814457e-17j)), (2, (-55.51963685528969+6.398094362911825j)), (3, (-1008.590316517904+99.11102434016638j)), (4, (-7682.455738877192+671.1843785467879j)), (5, (-36883.5268882489+2940.1428337578477j)), (6, (-132606.0410036778+9821.618156048185j)), (7, (-390836.9277719591+27255.792682297288j)), (8, (-996356.9756631973+66077.19600208125j))]
```

In [10]:
```python
print('Encrypting...')
encrypted_message = encrypt(message, keys['public_key'])
print(encrypted_message)
```

```
Encrypting...
753571 64000 117649 85184 32768 64000 110592 97336 125000 185193 157464 157464 166375
166375 157464 166375 166375 175616 166375 110592 185193 110592 166375 125000 157464 79
507 117649 97336 132651 175616 166375 166375 166375 175616 166375 175616 110592 166375
175616 117649 140608 140608 148877 166375 1030301 91125 117649 166375 1191016 68921 68
921 85184 32768 64000 125000 85184 32768 64000 91125 148877 148877 97336 148877 117649
185193 157464 132651 157464 175616 148877 148877 125000 175616 185193 157464 185193 79
507 157464 97336 132651 185193 175616 110592 185193 140608 132651 157464 125000 185193
117649 117649 175616 125000 148877 1191016 68921 68921 85184 32768 64000 132651 85184
32768 64000 91125 117649 110592 110592 175616 97336 148877 185193 110592 132651 117649
157464 148877 117649 166375 185193 110592 140608 79507 185193 185193 97336 117649 1176
49 117649 110592 125000 140608 132651 140608 110592 117649 157464 157464 132651 175616
1191016 68921 68921 85184 32768 64000 140608 85184 32768 64000 91125 166375 157464 175
616 125000 97336 140608 148877 148877 166375 132651 175616 175616 166375 166375 117649
185193 125000 79507 157464 166375 117649 97336 117649 175616 140608 132651 166375 1756
16 148877 140608 157464 166375 175616 166375 185193 1191016 68921 68921 85184 32768 64
000 148877 85184 32768 64000 91125 132651 157464 175616 175616 132651 97336 148877 125
000 157464 175616 175616 175616 125000 140608 175616 185193 79507 125000 185193 140608
110592 97336 117649 140608 125000 175616 132651 132651 166375 148877 166375 175616 140
608 166375 166375 1191016 68921 68921 85184 32768 64000 157464 85184 32768 64000 91125
117649 132651 125000 157464 110592 157464 97336 110592 140608 117649 110592 110592 132
651 157464 166375 166375 175616 79507 185193 175616 125000 117649 97336 157464 117649
175616 117649 148877 157464 110592 140608 175616 117649 175616 148877 1191016 68921 68
921 85184 32768 64000 166375 85184 32768 64000 91125 132651 185193 110592 175616 13265
1 157464 97336 185193 125000 166375 166375 166375 117649 185193 148877 185193 117649 7
9507 125000 166375 125000 148877 148877 97336 166375 185193 125000 157464 175616 12500
0 125000 185193 166375 125000 175616 175616 1191016 68921 68921 85184 32768 64000 1756
16 85184 32768 64000 91125 185193 185193 157464 132651 148877 157464 97336 185193 1663
75 148877 157464 157464 132651 117649 185193 166375 132651 79507 157464 157464 110592
166375 166375 97336 117649 185193 157464 110592 110592 125000 110592 175616 117649 125
000 148877 1191016 68921 68921 804357
```

In [11]:
```python
print('Decryption...')
decrypted_message = decrypt(encrypted_message, keys['private_key'])
print(decrypted_message)
```

```
Decryption...
[(1, (0.29667767787090726+1.3877787807814457e-17j)), (2, (-55.51963685528969+6.398094362911825j)), (3, (-1008.590316517904+99.11102434016638j)), (4, (-7682.455738877192+671.1843785467879j)), (5, (-36883.5268882489+2940.1428337578477j)), (6, (-132606.0410036778+9821.618156048185j)), (7, (-390836.9277719591+27255.792682297288j)), (8, (-996356.9756631973+66077.19600208125j))]
```

In [12]:
```python
if (np.allclose(message, decrypted_message)):
    print("\033[92mPASSED\033[0m RSA Encryption+Decryption")
else:
    print("\033[91mFAILED\033[0m RSA Encryption+Decryption")
```

```
PASSED RSA Encryption+Decryption
```

Inverse FFT

Implement 1-D Inverse FFT (I-FFT) on C(x), in PV form (Interpolation) to get C(x) in Coefficient form (CR) Polynomial.

In [13]:
```python
A_ifft = ifft(A_fft)
B_ifft = ifft(B_fft)

print("A_ifft = ", A_ifft)
print()
print("B_ifft = ", B_ifft)
```

```
A_ifft =  [(0.2566021088752648+0j), (0.12222140832322603-6.080355409323674e-19j), (0.6280452351011461-6.972702805154118e-17j), (0.9167562338619437-1.6718097384370783e-16j), (2.7755575615628914e-17+0j), (-6.938893903907228e-18+4.796422178641823e-17j), (5.551115123125783e-17+6.972702805154118e-17j), 1.103747823125813e-16j]

B_ifft =  [(0.14452223287023103+0j), (0.004898049646460915+7.636285451249868e-18j), (0.26495824327518613-2.941627422483452e-17j), (0.08628992175514771-1.5455463217337568e-17j), 0j, (-1.474514954580286e-17+8.06996632024407e-18j), 2.941627422483452e-17j, 1.9239006302198574e-17j]
```

In [14]:
```python
# Verify that ifft( fft(A) ) == A
if np.allclose(A, A_ifft) and np.allclose(B, B_ifft):
    print("\033[92mPASSED\033[0m IFFT")
else:
    print("\033[91mFAILED\033[0m IFFT")
```

```
PASSED IFFT
```

In [15]:
```python
#print(fast_multi(A_old, B_old))
#print(np.polymul(A_old, B_old))
#print(ifft(C))

if np.allclose(fast_multi(A_old, B_old), ifft(C)):
    print("\033[92mPASSED\033[0m IFFT")
else:
    print("\033[91mFAILED\033[0m IFFT")
```

```
PASSED IFFT
```

2D FFT and 2-D I-FFT

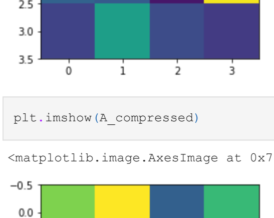
Implement a 2-D FFT and 2-D I-FFT module using your 1-D version (This just means , applying FFT on the Rows First and Columns Next on M x N matrix of numbers !!)

Verification

Verify previous step (2D FFT and iFFT) correctness on a Grayscale matrix (which has random integer values in the range 0-255; 255 → White & 0 → Black))

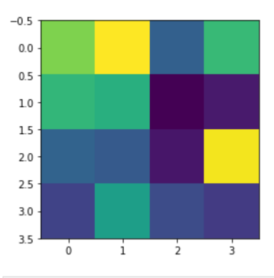
In [16]:
```python
A_img = np.reshape(np.random.random(4*4),(4,4))

y_A = fft2(A_img)
y_c = np.fft.fft2(A_img)
A_compressed = ifft2(y_A).real

plt.imshow(A_img)
```

Out[16]: <matplotlib.image.AxesImage at 0x7f013bec9c40>



In [17]:
```python
plt.imshow(A_compressed)
```

Out[17]: <matplotlib.image.AxesImage at 0x7f0133ddeca0>



In [18]:
```python
if np.allclose(A_img, A_compressed):
    print("\033[92mPASSED\033[0m IFFT")
else:
    print("\033[91mFAILED\033[0m IFFT")
```

```
PASSED IFFT
```

FFT on Image

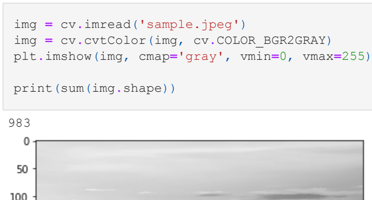
Apply your 2D-FFT on TIFF/JPG (lossless) Grayscale image and drop Fourier coefficients below some specified magnitude and save the 2D- image to a new file.

- (relates to % compression – permanent Lossy compression)
- (by sorting and retaining only coefficients greater than some(quantization) value. Rest are made 0.)

Apply 2D I-FFT, on the Quantized Grayscale image and render it to observe Image Quality.

In [19]:
```python
img = cv.imread('sample.jpeg')
img = cv.cvtColor(img, cv.COLOR_BGR2GRAY)
plt.imshow(img, cmap='gray', vmin=0, vmax=255)

print(sum(img.shape))
```

```
983
```



In [20]:
```python
b_img = Jpeg_img(img, 50)
plt.imshow(b_img.render(), cmap='gray', vmin=0, vmax=255)

print(sum(b_img.shape))
```

```
Starting conversion
Conversion Done
768
```

Time Analysis

In [21]:
```python
print(bcolors.HEADER + "Brute VS FFT VS DFT: " + bcolors.ENDC)

print(bcolors.OKCYAN + bcolors.UNDERLINE + "Brute force multiplication:" + bcolors.ENI
start_time = time.time()
brute = polynomial_multiplication(A, B)
print("--- %s seconds ---" % (time.time() - start_time))

print(bcolors.OKCYAN + bcolors.UNDERLINE + "FFT Numpy:" + bcolors.ENDC + bcolors.ENDC)
start_time = time.time()
# Numpy FFT
arr_a1=np.pad(A,(0,len(A)),'constant')
arr_b1=np.pad(B,(0,len(B)),'constant')
a_f=np.fft.fft(A)
b_f=np.fft.fft(B)

c_f = np.multiply(a_f, b_f)

C = np.fft.ifft(c_f)
C = [i.real for i in C]
print("--- %s seconds ---" % (time.time() - start_time))

print(bcolors.OKCYAN + bcolors.UNDERLINE + "FFT custom:" + bcolors.ENDC + bcolors.ENDC
start_time = time.time()
# Custom Multiply

a_p2 = np.pad(A, (0, len(A)), 'constant')
b_p2 = np.pad(B, (0, len(B)), 'constant')
y_c = np.multiply(fft(a_p2), fft(b_p2))

C = [i.real for i in ifft(y_c)]

print("--- %s seconds ---" % (time.time() - start_time))

print(bcolors.OKCYAN + bcolors.UNDERLINE + "DFT:" + bcolors.ENDC + bcolors.ENDC)
start_time = time.time()
dft_A = dft(A)
dft_B = dft(B)
dft_C = np.multiply(dft_A, dft_B)
dft_y = idft(dft_C)
print("--- %s seconds ---" % (time.time() - start_time))
```

```
Brute VS FFT VS DFT:
Brute force multiplication:
--- 0.00012087821960449219 seconds ---
FFT Numpy:
--- 0.0007305145263671875 seconds ---
FFT custom:
--- 0.0018849372863769531 seconds ---
DFT:
--- 0.00044155120849609375 seconds ---
```

In [ ]: