Roll No. …………………..

# BCA–A-104(G1)

## Bachelor of Computer Applications

## (First Semester)

## EXAMINATION, 2022-23

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

*Time : $2\frac{1}{2}$ Hours*

*Maximum Marks : 60*

**Note :** All questions have to attempted.

**Section—A**

1. Multiple Choice questions : 1 each

(a) A function is said to be ............. iff $x \neq y$ implies that $f(x) \neq f(y)$ for all $x$ and $y$ in the domain of *f*.

(CO1, BL-3)

(i) injective

(ii) surjective

(iii) many-one

(iv) None of the above

(b) If $f(x) = \sin x$ and $g(x) = x^2$, then $f \circ g$ is :

(CO1, BL-5)

(i) $\sin 2x$

(ii) $2\sin x$

(iii) $\sin x^2$

(iv) $(\sin x)^2$

(c) The function $f : \mathrm{R} \to \mathrm{R}$ given by $f(x) = x^2$ is :

(CO1, BL-4)

(i) injective

(ii) surjective

(iii) bijective

(iv) None of the above

(d) What is the order of the recurrence relation $5a_n - 3a_{n-1} + 4a_{n-2} = n^2 + 7$ ? (CO2, BL-5)

(i) 1

(ii) 2

(iii) 3

(iv) 4

(e) .................. is the additive identity of integers.

(CO3, BL-1)

(i) 0

(ii) 1

(iii) – 1

(iv) 2

(f) A compound preposition that is neither a tautology nor a contradiction is called :

(CO4, BL-1)

(i) equivalence

(ii) condition

(iii) inference

(iv) contingency

(g) Which of the following prepositions is a tautology ? (CO4, BL-3)

(i) $(p \vee q) \to p$

(ii) $p \vee (q \to p)$

(iii) $p \vee (p \to q)$

(iv) $p \to (q \to p)$

(h) Every finite subset of a lattice has : (CO5, BL-2)

(i) a LUB and GLB

(ii) many LUB's and a GLB

(iii) many LUB's and many GLB's

(iv) None of the above

(i) If every two elements of a poset are comparable then the poset is called : (CO5, BL-4)

(i) subordered poset

(ii) totally ordered poset

(iii) sublattice

(iv) semigroup

(j) ................ and .............. are the two binary operations defined for lattices. (CO5, BL-1)

(i) join, meet

(ii) addition, subtraction

(iii) union, intersection

(iv) multiplication, modulo division

(k) According to the principle of mathematical induction, if $P(k+1) = 3(k+1)^2 + 2$ is true then ................ must be true. (CO2, BL-4)

(i) $3(k+1)^3 + 2$

(ii) $4(k+1)^2 + 2$

(iii) $3(k+2)^2 + 2$

(iv) $3k^2 + 2$

(l) Let R be the set of real numbers. From the following determine which of the binary operation * does not satisfy commutativity :

(CO3, BL-3)

(i) $a * b = \frac{ab}{3}$

(ii) $a * b = 3ab$

(iii) $a * b = 2a + 3b$

(iv) $a * b = \min(a, b)$

2. Attempt any *four* of the following : 3 each

(a) Check the injectivity and surjectivity of the function $f : \mathrm{R} \to \mathrm{R}$ given by $f(x) = 3x^2 - 1$.

(CO1, BL-3)

(b) Write a short note on Peano's axioms.

(CO2, BL-1)

(c) Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7. Find the multiplication table of G. (CO3, BL-5)

(d) With the help of truth table, prove that $p \vee q = \sim(\sim p \wedge \sim q)$. (CO4, BL-5)

(e) Define Lattice using suitable examples.

(CO5, BL-2)

## Section—B

3. Attempt any *two* of the following : 6 each

(a) Let A be a set of non-zero integers and let $\approx$ be the relation on A × A defined by $(a, b) \approx (c, d)$ whenever $ad = bc$. Prove that $\approx$ is an equivalence relation. (CO1, BL-6)

(b) Explain the following using suitable examples :

(CO1, BL-2)

(i) Reflexive relation

(ii) Symmetric relation

(iii) Transitive relation

(c) Solve the recurrence relation : (CO2, BL-5)

$$a_{n+2} - 5a_{n+1} + 6a_n = n.$$

4. Attempt any *two* of the following : 6 each

(a) Find the generating functions of the following : (CO2, BL-5)

(i) 2, – 2, 2, – 2, .........

(ii) 1, 0, 1, 0, .........

(iii) 0, 1, – 2, 4, – 8, ........

(b) Let $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutations of degree 3. Show that $f$ o $g$ is not commutative. (CO3, BL-3)

(c) Consider the lattice $D_{24}$ = {1, 2, 3, 4, 6, 8, 12, 24} the divisors of 24 ordered by divisibility. Draw the Hasse Diagram of $D_{24}$. (CO5, BL-4)

5. Attempt any *two* of the following : 6 each

(a) Prove the a group is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. (CO3, BL-3)

(b) Verify that the preposition $p \vee \sim (p \wedge q)$ is a tautology. (CO4, BL-3)

(c) Let $f$, $g$ and $h$ be functions from R to R so that :

$f(x) = \cos x$, $g(x) = e^x$ and $h(x) = x^2$.

Determine : (CO1, BL-5)

(i) $f \circ f$

(ii) $f \circ g$

(iii) $g \circ f$

(iv) $g \circ h$

(v) $h \circ g$

(vi) $h \circ f$