Q1: Let $f_i(x,y) = a_i x^2 + 2h_i xy + b_i y^2 + 2g_i x + 2f_i y + c_i = 0$; $h_i^2 = a_i b_i$ $(i = 1, 2, 3)$ be 3 parabolas with a common directrix.

Let $f_1(0,1) = 0 = f_2(0,1) = f_1(4,7) = 0 = f_2(4,7)$
$f_2(4,0) = 0 = f_3(3,2) = f_2(3,2) = f_3(4,0)$

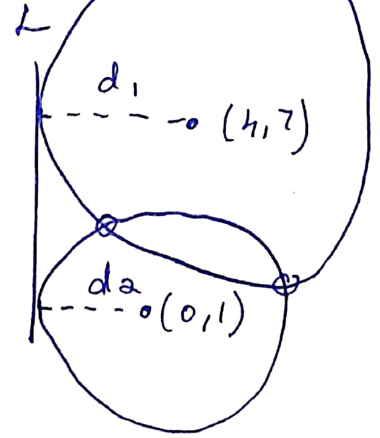
If (α, β) are the coordinates of the circumcentre of the triangle formed by the foci of these parabolas, then $[\alpha + \beta] = ?$

Credits: Sourced from an incorrect question posed on Narayona's AOPS forum and assigned to me. I realized that given the constraints, the circumcenter was one of the few identifying points of the triangle that could be uniquely determined logically and quickly.

Level: Mathathon / Math Bazinga 1st round, maybe second round.

Answer: 6

Idea: Assume an arbitrary directrix L and consider the foci of f_1, f_2



The foci can only be on the points of intersection of the two circles. It is easy to prove that the perpendicular bisector of the foci is the line joining

(4, 7) and (0, 1) (since they are the 2 points common to both circles).

Similarly, the perpendicular bisector of the line joining the foci of b_2 and b_3 passes through (3, 2) and (4, 0).

This gives 2 equations which can be solved.