H **Roll No. ....**

TBS–103

B. SC. (CS)

(FIRST SEMESTER) END SEMESTER EXAMINATION, Jan., 2023

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any ***two*** sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Prove that if R is an equivalence relation on set A, then R^{-1} is also an equivalence relation. (CO1)

(b) (i) What is the matrix representation of the relation : (CO1)

R = {(1, *a*), (3, *c*) (5, *d*) (1, *b*)} which is defined from X = {1, 2, 3, 4, 5} to Y = {*a*, *b*, *c*, *d*, *e*}.

(ii) Draw the directed graph representing relation : (CO1)

R = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)} on set A = {1, 2, 3, 4}.

(c) Consider A = {0, 1, 2, 3} and R = {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)}. Show that R is a partially ordered relation on A. (CO1)

2. (a) Show that the function

$$f : R \to R,$$

defined by $f(x) = 3x^3 + 5$ for all $x \in R$ is a bijection. (CO2)

(b) Consider $f : R \to R, g : R \to R$, defined as $f(x) = x + 1, g(x) = \sin x$, then find $g o f$, $f o f$, $f o g$, $g o g$. (CO2)

(c) (i) Explain the relationship between exponential and logarithmic functions. (CO2)

(ii) Find the inverse of function $f : R \to R$ which is defined as

$$f(x) = \frac{3x + 2}{4x - 1}.$$

3. (a) Find the number of arrangements of the letters of the word PERMUTATIONS. In how many of these arrangements, (CO3)

(i) do the words start with P and end with S,

(ii) do all the vowels occur together.

(b) Expand each of the expression using binomial theorem : (CO3)

(i) $(2x - 3)^6$

(ii) $(102)^5$

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(c) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has : (CO3)

(i) at least one boy and one girl

(ii) no girls

(iii) at least 3 girls ?

4. (a) Solve the recurrence relation using generating functions : (CO4)

$$a_{n+2} - 5a_{n+1} + 6a_n = 2$$

where initial conditions are $a_0 = 1, a_1 = 2$.

(b) Solve the recurrence relation $a_n = 4(a_{n-1} - a_{n-2})$ with initial conditions $a_0 = 1, a_1 = 1$ using characteristic roots. (CO4)

(c) Solve the non-homogenous recurrence relation using the method of undetermined coefficient : (CO4)

$$a_n - 6a_{n-1} \_\, 8a_{n-2} = 3$$

5. (a) Check the validity of the argument : (CO5)

$$\begin{array}{l} p \vee q \\ p \Rightarrow \sim q \\ p \Rightarrow r \\ \hline \therefore r \end{array}$$

(b) (i) Obtain the Conjunctive Normal Form (CNF) : (CO5)

$$[q \vee (p \wedge r)] \wedge \sim [(p \vee r) \wedge q]$$

(ii) Obtain Disjunctive Normal Form (DNF).

$$p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$$

(c) Construct the truth table for the following : (CO5)

$$p \Rightarrow [(q \vee r) \wedge \sim (p \Leftrightarrow \sim r)]$$

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