H **Roll No. ....**

TCS–343

B. TECH. (CSE) (THIRD SEMESTER) END SEMESTER EXAMINATION, Dec., 2023

MATHEMATICAL FOUNDATIONS FOR ARTIFICIAL INTELLIGENCE

Time : Three Hours

Maximum Marks : 100

Note : (i) All questions are compulsory.

(ii) Answer any *two* sub-questions among (a), (b) and (c) in each main question.

(iii) Total marks in each main question are **twenty**.

(iv) Each sub-question carries 10 marks.

1. (a) Using Crout's L–U Decompostion method solve the system equations : (CO1)

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8.$$

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(b) Show that the vectors : (CO1)

$$\alpha_1 = (1,0,-1), \alpha_2 = (1,2,1), \alpha_3 = (0,-3,2)$$

form a basis of R^3. Express each of the standard basis vectors a linear combination of α_1, α_2 and α_3.

(c) Show that the mapping : (CO1)

$$T : V^2(R) \to V^3(R)$$

defined as

$$T(a,b) = (a+b, a-b, b),$$

is a linear transformation (linear mapping) from $V^2(R)$ into $V^3(R)$. Find the range, rank and null space of T.

2. (a) Let V (C) be the vector space of all polynomials in t with coefficients in C. If : (CO2)

$$f(t), g(t) \in V,$$

defined as

$$(f(t), g(t)) = \int_0^1 f(t)\overline{g(t)}\,dt.$$

Prove that :

$$(f(t), g(t))$$

is an Inner Product Space on V (C).

(b) Let V be a vector space over F. Show that the sum of two inner products on V in inner product space on V. Is the difference of two inner products an inner product ? Show that a positive multiple of an inner product is an inner product. (CO2)

(c) Suppose that α and β are vectors in an inner product space V. If :

$$|(\alpha,\beta)| = \|\alpha\|\,\|\beta\|,$$

then α and β are linearly dependent. (CO2)

3. (a) Obtain the Singular Value Decomposition of the Matrix : (CO3)

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}$$

(b) Solve the following system of equations by Cholesky method : (CO3)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}.$$

(c) Let T be a linear operator on R^3 which is represented in the standard basis by the matrix : (CO3)

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable.

4. (a) Define scalar and vector point functions. Also give the geometrical meaning of gradient. (CO4)

(b) Verify : (CO4)

$$\vec{\Delta} \times \left[f(r)\vec{r} \right] = 0$$

(c) Calculate the second degree polynomial of : (CO4)

$$f(x, y) = e^{-\left(x^2+y^2\right)}$$

at the point (0, 0) and at the point (1, 2).

5. (a) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. Find the probability of the lost card being a heart. (CO5)

(b) Define Gaussian distribution and Gaussian Function. (CO5)

The average height of 5th graders in a given school district is 52 inches with a standard deviation of 2.4 inches. Assuming that the heights of the 5th graders in the district are normally distributed, find the probability that a 5th grade chosen at random is taller than 56 inches.

(c) $$f(x) = \begin{cases} 0 & x < 2, \\ (2x+3)/18 & 2 \le x \le 4, \\ 0 & x > 4 \end{cases}$$

Show that it is a probability density function. (CO5)

The diameter of an electric is assumed to be continuous random variate with probability density function

$$f(x) = 6x(1-x), 0 \le x \le 1$$

(i) Verify that above is a p.d.f.

(ii) Find the mean and variance.