Graphic Era
HILL UNIVERSITY
Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act, 1956

End Term (Odd) Semester Examination December 2024

Roll No........................

Name of the Course and semester: Bachelor of Technology, I Semester
Name of the Paper: Engineering Mathematics-I
Paper Code: TMA-101
Time: 3 hours | Maximum Marks: 100

Note:
(i) All the questions are compulsory.
(ii) Answer any two sub questions from a, b and c in each main question.
(iii) Total marks for each question is 20 (twenty).
(iv) Each sub-question carries 10 marks.

Q1. (2x10=20 Marks) (CO1)

a. Using elementary transformations, find the inverse of the matrix

$$A=\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{bmatrix}.$$

b. Test the consistency of the following system of linear equations:

$$2x+3y+4z=11, x+5y+7z=15, 3x+11y+13z=25.$$

If consistent, find the solution.

c. Find the eigen values and corresponding eigen vectors for the matrix

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}.$$

Q2. (2x10=20 Marks) (CO2)

a. Verify Lagrange's mean value theorem for the following functions:

(i) $f(x)=2x^2-10x+29$ in $[2,7]$

(ii) $f(x)=\sqrt{x^2-4}$ in $[2,4]$

b. State and verify Rolle's theorem for the function $f(x)=(x-a)^m(x-b)^n$ in the interval $[a,b]$, where m and n are positive integers.

c. Evaluate the following limits:

(i) $\lim_{x\to 0}\frac{\log(\sin 2x)}{\log(\sin x)}$ (ii) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$

Q3. (2x10=20 Marks) (CO3)

a. If $u=\tan^{-1}(x^2+2y^2)$, show that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$ (ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cos 3u$

b. If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

c. Find the point upon the plane $ax+by+cz=p$ at which the function $f(x,y,z)=x^2+y^2+z^2$ has a minimum value and find this minimum value of f.

End Term (Odd) Semester Examination December 2024

Q4. (2x10=20 Marks) (CO4)

a. Prove that $\beta(l,m)=\dfrac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$.

b. Sketch the region bounded by the curves $y=x^2$ and $x+y=2$. Express the area of the given region as a double integral in two different ways and evaluate the area using any one way.

c. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$.

Q5. (2x10=20 Marks) (CO5)

a. Apply Green's Theorem to evaluate $\int_C (2x^2-y^2)dx+(x^2+y^2)dy$, where C is the boundary of the area enclosed by the $x-$ axis and the upper half of the circle $x^2+y^2=1$.

b. Evaluate $\oint_C \overline{F}.\overline{dr}$ by Stoke's Theorem, where $\overline{F}=y^2\hat{i}+x^2\hat{j}-(x+z)\hat{k}$ and C is the boundary of triangle with vertices at $(0,0,0),(1,0,0)$ and $(1,1,0)$.

c. Prove that $(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ is both solenoidal and irrotational.