Machine Learning using Python

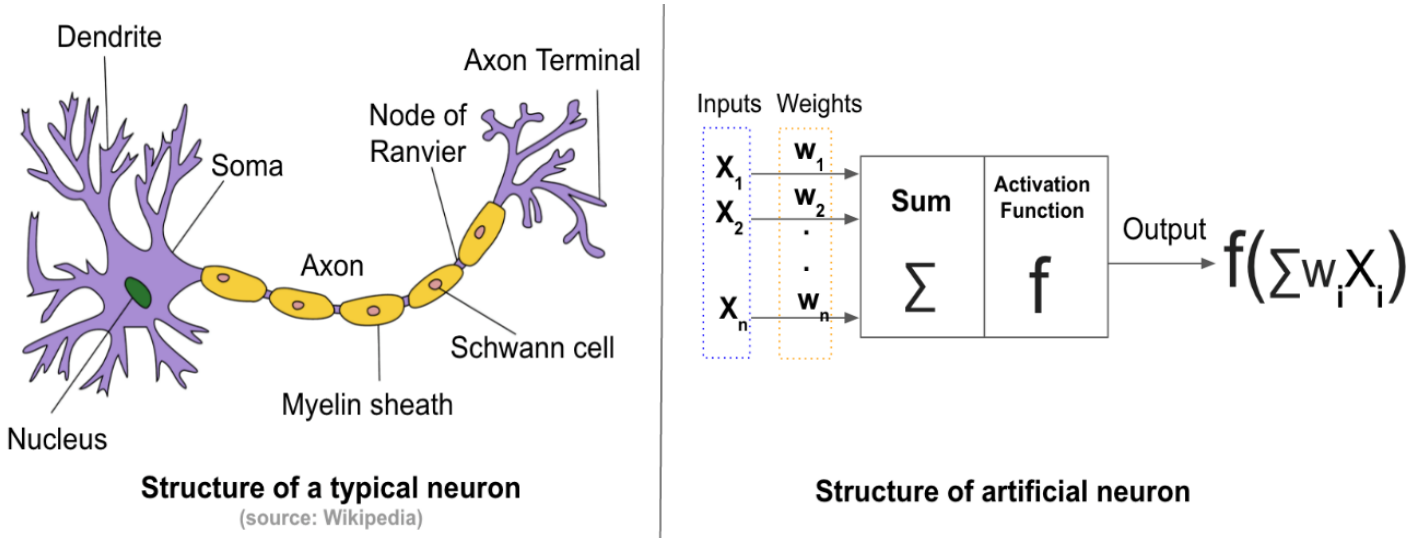
Neda Hantehzadeh

Week 2

Concepts related to DL

- Biological Inspiration
- When to use Neural Networks?
- Binary Classification
- Logistic Regression
- Logistic Regression Cost Function
- Loss functions(SVM, Softmax)
- Weight Regularization

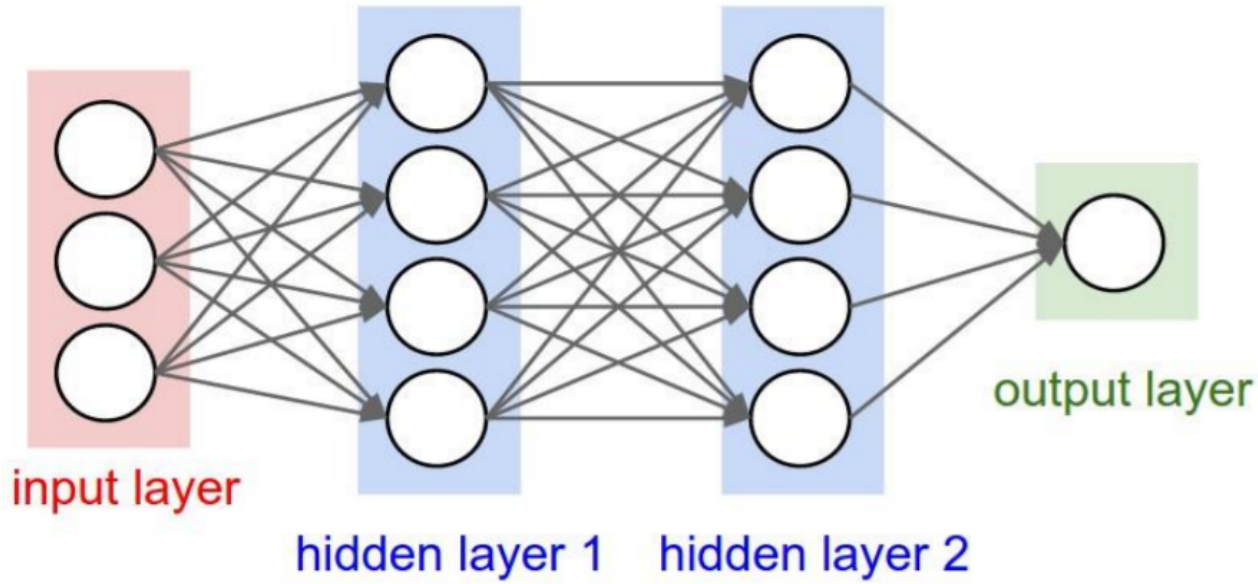
Defining Deep Learning -Biological inspirations



Source: http://adilmoujahid.com/posts/2016/06/introduction-deep-learning-python-caffe/

Neurons are trained to filter and detect specific features or patterns (e.g. edge, color, parts) by receiving weighted input, transforming it with the activation function and passing it through.

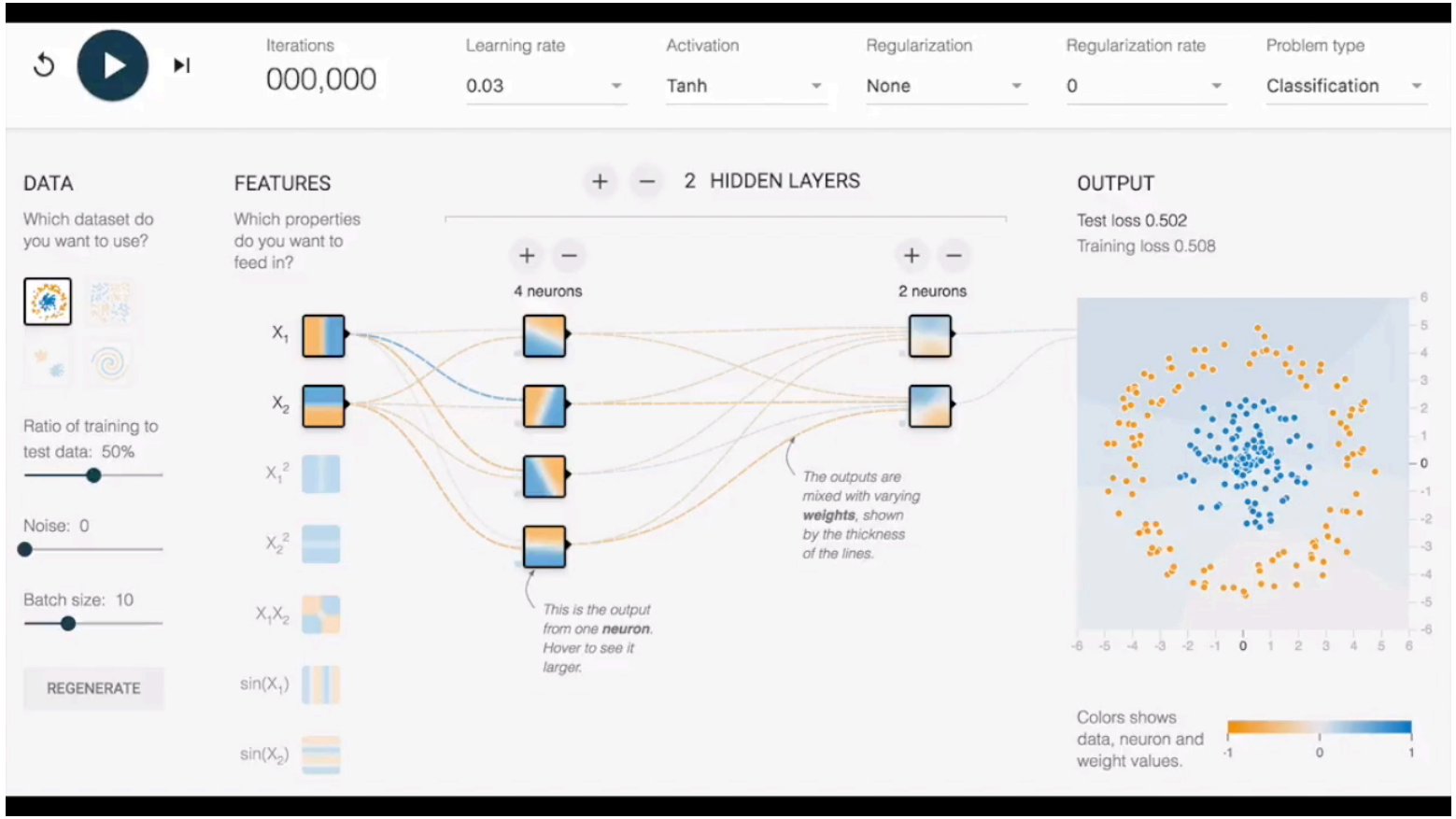
Defining Basic Neural Network



Each layer in a neural network represents a series of neurons and progressively extracts higher level features of input until final layer make a decision about the input category.

An Example of an Artificial Neural Network: Tensorflow

http://playground.tensorflow.org/#activation=tanh&batchSize=10&dataset=circle&regDataset=reg-plane&learningRate=0.03&regularizationRate=0&noise=0&networkShape=4,2&seed=0.00483&showTestData=false&discretize=false&percTrainData=50&x=true&y=true&xTimesY=false&xSquared=false&ySquared=false&cosX=false&sinX=false&cosY=false&sinY=false&collectStats=false&problem=classification&initZero=false&hideText=false



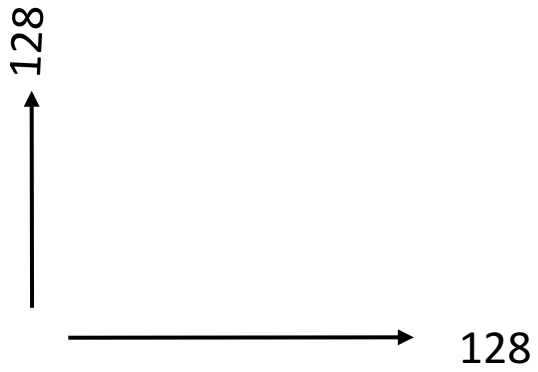
When to use Neural Networks?

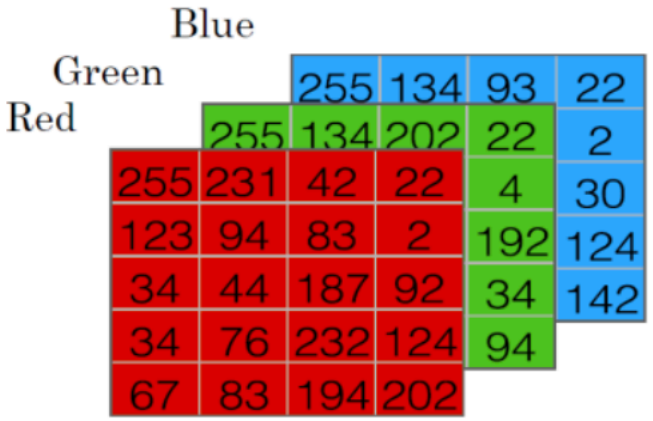
- Availability of big data (more than 1000s of example per category)
- Problem can’t be linearly solved
- Problem is complex and can’t be solved using traditional machine learning algorithms
- Availability of GPUs

Implementation of Neural Networks

- Reading the whole training set at input unlike traditional methods
- Feedforward and backward pass (back propagation) steps

Binary Classification and Logistic Regression





$$x = \begin{bmatrix} 255 \\ 231 \\ 42 \\ \vdots \\ 255 \\ 134 \\ 202 \\ \vdots \\ 255 \\ 134 \\ 93 \\ \vdots \end{bmatrix} \begin{matrix} \} \text{red} \\ \\ \\ \\ \} \text{green} \\ \\ \\ \\ \} \text{blue} \\ \\ \\ \end{matrix}$$

Dimension of feature vector:
128 by 128 by 3 = 49 152

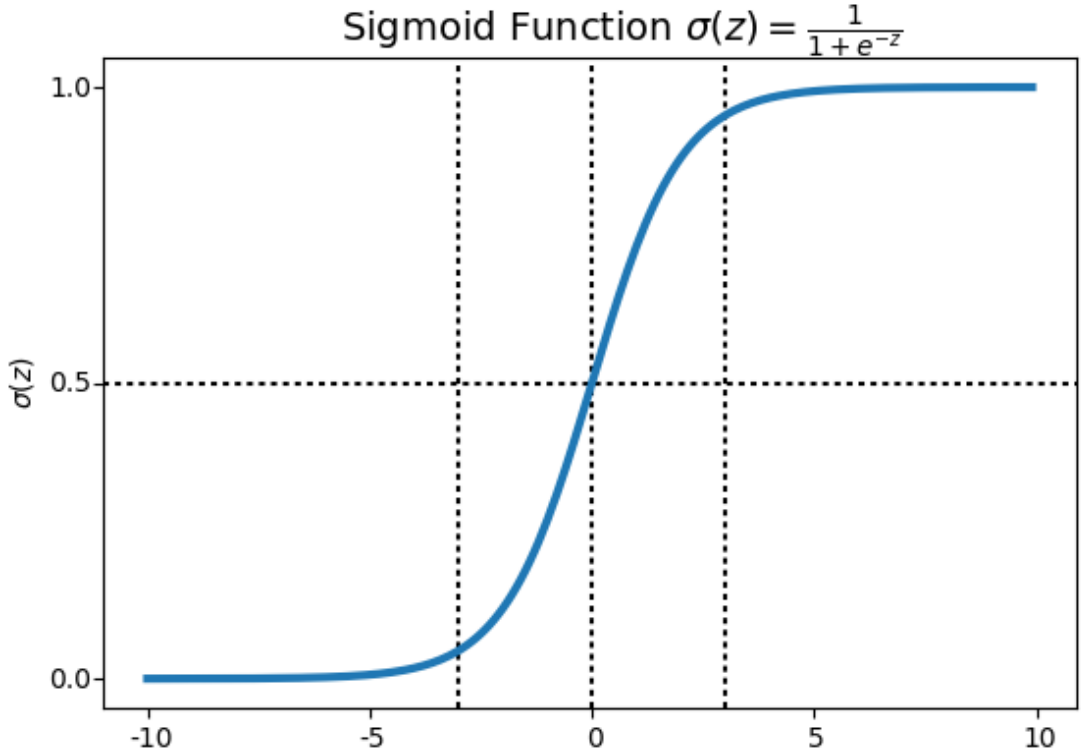
Binary Classification and Logistic Regression

$Given\ \ x\ ,\ \hat{y}=P(y=1|x)$,where. $\ 0 \leq \hat{y} \leq 1$

$$\hat{y} = \sigma(w^T x + b)$$

$$s = \sigma(w^T x + b) = \sigma(z) = \frac{1}{1+ e^{-z}}$$

- If z is a large positive number, then $\sigma(z) = 1$
- If z is small or large negative number, then $\sigma(z) = 0$



Binary Classification and Logistic Regression

$\hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

$x^{(i)}$ the i-th training example

$Given \{(x^{(1)}, y^{(1)}), \cdots, (x^{(m)}, y^{(m)})\},\ we\ want\ \hat{y}^{(i)} \approx y^{(i)}$

$$L(\hat{y}^{(i)}, y^{(i)}) = \frac{1}{2}(\hat{y}^{(i)} - y^{(i)})^2$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

- If $y^{(i)} = 1$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(\hat{y}^{(i)})$ where $\log(\hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 1
- If $y^{(i)} = 0$: $L(\hat{y}^{(i)}, y^{(i)}) = -\log(1 - \hat{y}^{(i)})$ where $\log(1 - \hat{y}^{(i)})$ and $\hat{y}^{(i)}$ should be close to 0

Overall loss: $$-\frac{1}{m}\sum_{i=1}^{m}[(y^{(i)} \log(\hat{y}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$$

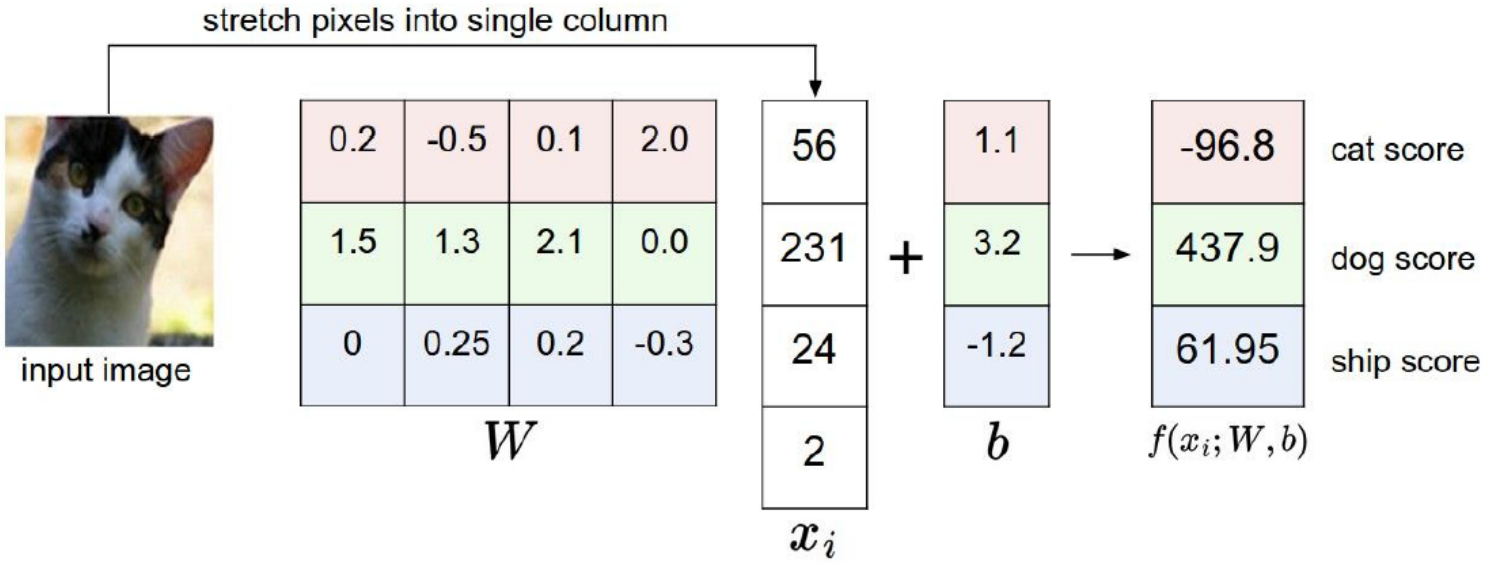
Loss functions

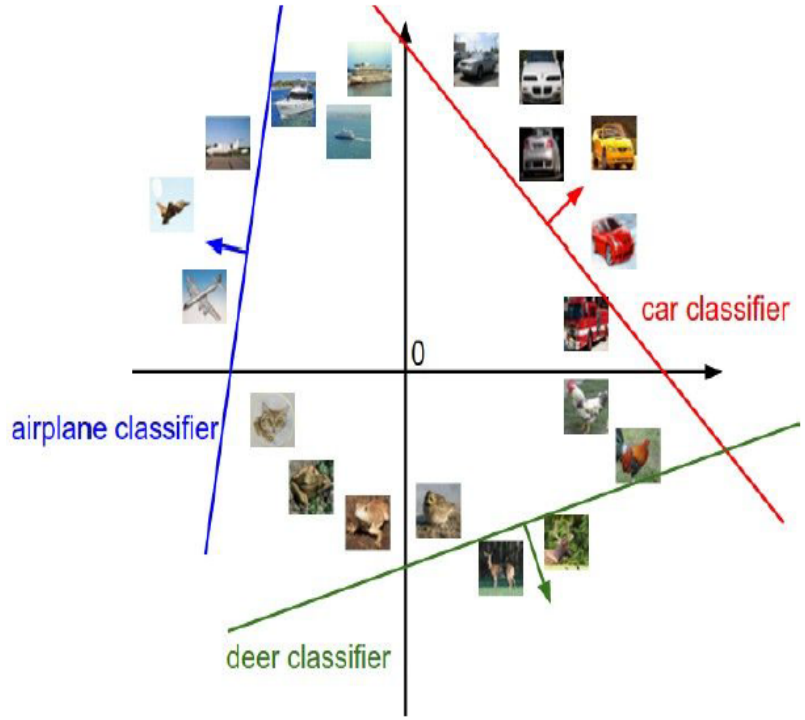
Linear **score function**: $f(x_i, W, b) = Wx_i + b$

Independent variable used to calculate the bias

Parameters, or "weights"

Input





Loss function: Quantifying what it means to have a "good" W

Optimization: Start with random W and find a W that minimizes the loss function

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

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cat	**3.2**	1.3	2.2
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frog	-1.7	2.0	**-3.1**
Losses:	2.9		

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the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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Losses:	2.9	0	

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the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
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where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 5.3) + max(0, 5.6)
= 5.3 + 5.6
= 10.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
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Multiclass SVM loss:

Given an example (x_i, y_i)
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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 10.9)/3
= 4.6

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: what if the sum was instead over all classes? (including j = y_i)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

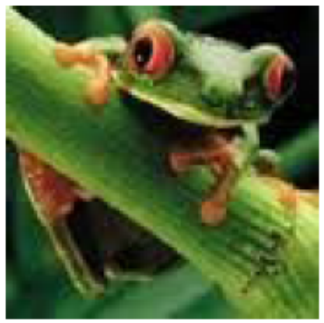
the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q2: what if we used a mean instead of a sum here?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:





cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
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the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: what if we used

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: what is the min/max possible loss?

There is a bug with the loss:

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

Suppose that we found a W such that L=0, Is this W unique?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Before:

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

With W twice as large:

= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

Weight Regularization **Secret sauce to higher accuracy**

$$L = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq y_i}\max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \lambda R(W)$$

Lambda: regularization strength

- L1 regularization: Penalized zero weights
- L2 regularization: Penalizes large weights
- Elastic net (L1+L2)
- Max norm regularization
- Dropout: Randomly ignore set of weights

Softmax (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

Want to maximize the log likelihood , or (for loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

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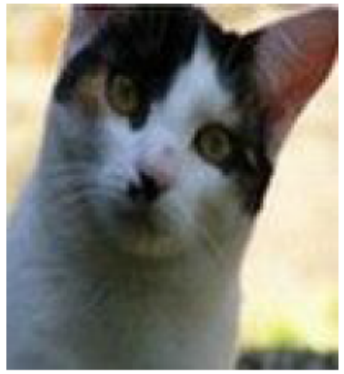
in summary: $L_i = -\log(\frac{e^{sy_i}}{\sum_j e^{s_j}})$

Softmax (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

Q: What is the min/max possible loss L_i?

Unnormalized exponent of scores

	Unnormalized scores	exp		normalize	probabilities
cat	3.2	→	24.5	→	0.13
car	5.1	→	164.0	→	0.87
frog	-1.7	→	0.18	→	0.00

L_i = -log(0.13)
= **0.89**

Softmax (cross-entropy loss) vs SVM (Hinge loss)

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right) \qquad L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:
[10, -2, 3]
[10, 9, 9]
[10, -100, -100]
and $y_i = 0$

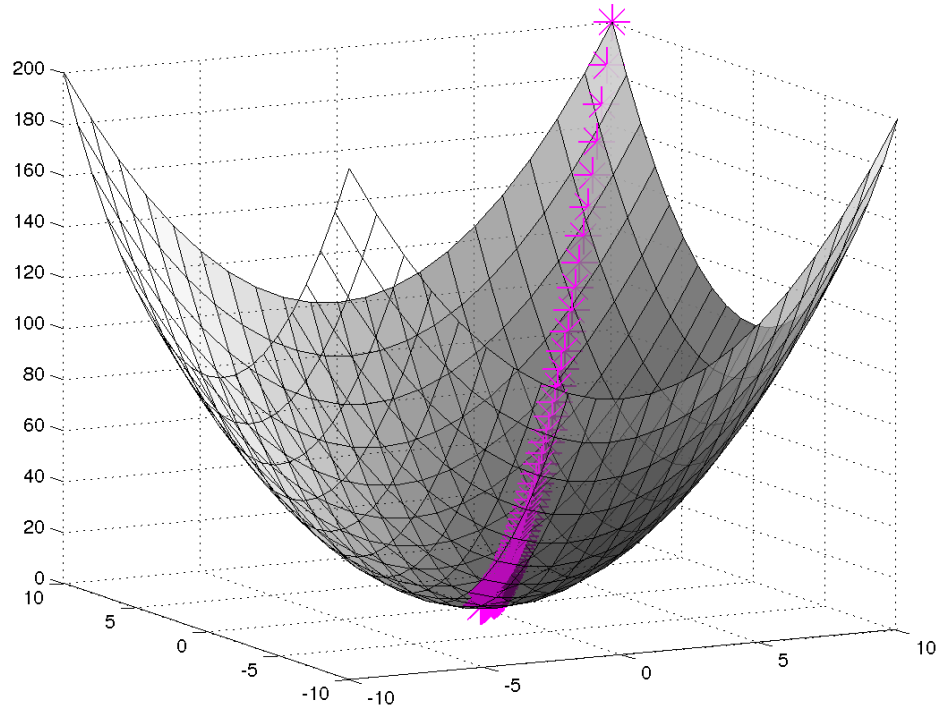
Q: Suppose I take a datapoint and I jiggle a bit (changing its score slightly). What happens to the loss in both cases?

Gradient Descent and Derivatives

$$\text{Recap: } \hat{y} = \sigma(w^T x + b), \ \sigma(z) = \frac{1}{1+e^{-z}}$$

$$C(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $C(w,b)$



Gradient Descent and Derivatives