UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2018**	**Topic**	**Differential Equations**

CO1:

1. Find the value of k for which the given equations are exact.
   (a) $(xy^4 + y^2)dx + k(x^2y^3 + xy + y^5)dy = 0$
   (b) $(xy^2 + kx^2y)dx + (x + y)x^2dy = 0$
   (c) $(ye^{2xy} + x)dx + kxe^{2xy}dy = 0$
   (d) $(6xy^3 + cosy)dx + (2kx^2y^2 - xsiny)dy = 0$
   (e) $(y^3 + kxy^4 - 2x)dx + (3xy^2 + 20x^2y^3)dy = 0$

2. Find the complementary function for the following differential equations:
   (a) $y'' - (a + b)y' + aby = e^{ax} + e^{bx}$
   (b) $(D^2 - 3D + 2)y = e^{3x}$
   (c) $(D^3 - 3D^2 - 2D + 2)y = 0$
   (d) $(D^3 + 1)y = 0$
   (e) $(D^2 + D + 1)y = 0$
   (f) $(D^3 + 2D^2 + D)y = 0$
   (g) $(D^2 + 2)y = 0$
   (h) $y_4 + 2y_2 + y = 0$

3. Find the particular integral for the following differential equations:
   (a) $y'' - (a + b)y' + aby = e^{ax} + e^{bx}$
   (b) $(D^2 - 3D + 2)y = e^{3x}$ *given that* $y(0) = 0 = y(ln2)$
   (c) $(D^3 + 1)y = sin3x$
   (d) $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$
   (e) $(D^2 + 2)y = x^2e^{3x} + e^x cos2x$
   (f) $y_4 + 2y_2 + y = x^2cosx$

CO2:

4. Show that the given equations are not exact but becomes exact when multiplied by the given integrating factor $\mu(x, y)$.
   (a) $(2t^2 + 3xsin^2t)dx + 2x(t + x\, sint\, cost)dt = 0$ $\quad\mu(x, y) = x$
   (b) $(2x^2t^2 + 3x^3 \sin^2 t)dx + (2x^3t + 2x^4\, sint\, cost)dt = 0$ $\quad\mu(x, y) = \frac{1}{x}$
   (c) $(2xy^2 - 2y)dx + (3x^2y - 4x)dy = 0$ $\quad\mu(x, y) = y$

5. Explain the method of variation of parameters to solve the differential equation $f(D)y = R(x)$ where $f(D)$ is a linear second order differential operator.

6. Explain the following methods to solve the differential equation $y'' + P(x)y' + Q(x)y = R(x)$.
   (a) When a part of C.F is known
   (b) Reduction to normal form or removal of the first derivative
   (c) Change of independent variable

7. Show that the solution of $(D^3 + 1)y = sin3x - cos^2\left(\frac{x}{2}\right) + cos2x$ is
   $y = c_1e^{-x} + e^{\frac{x}{2}}\left[c_2 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_3 \sin\left(\frac{\sqrt{3}}{2}x\right)\right] + \frac{sin3x}{730} + \frac{cos2x}{65} + \frac{27\cos 3x}{730} - \frac{cosx}{4} + \frac{sinx}{4} - \frac{1}{2} - \frac{8sin2x}{65}$

8. Show that $y = e^{\frac{-3x^{\frac{2}{3}}}{4}}[c_1e^{-2x} + c_2e^{3x}]$ is the solution of $y'' + x^{\frac{-1}{3}}y' + \left[\frac{1}{4x^{\frac{2}{3}}} - \frac{1}{6x^{\frac{4}{3}}} - \frac{6}{x^2}\right]y = 0.$

9. Show that the solution of $y'' + \left(1 - \frac{1}{x}\right)y' + 4x^2e^{-2x}y = 4(x^2 + x^3)e^{-3x}$ is
   $y = c_1\ cos[-2e^{-x}(x+1)] + c_2\ sin[-2e^{-x}(x+1)] + e^{-x}(x+1)$

10. Show that the solution of $(1 - x^2)y'' + xy' - y = x(1 - x^2)^{\frac{3}{2}}$ is
    $y = -c_1[\sqrt{1-x^2} + xsin^{-1}x] + c_2\,x - \frac{1}{9}\,x(1 - x^2)^{\frac{3}{2}}$

CO3:

11. Solve the following differential equations:
    (a) $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left\{1 - \left(\frac{x}{y}\right)\right\}dy = 0$
    (b) $\left(y^2e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$
    (c) $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$
    (d) $ydx - xdy + (1 + x^2)dx + x^2siny\ dy = 0$
    (e) $(x^3 + xy^2 + k^2y)dx + (y^3 + yx^2 - k^2x)dy = 0$ where k is a constant
    (f) $(x^4e^x - 2mxy^2)dx + 2mx^2ydy = 0$ where m is a constant
    (g) $(x + y)siny\ dx + (x\ siny + cosy)dy = 0$

12. Solve the following differential equations:
    (a) $y_2 - 4y_1 + 4y = 8x^2e^{2x}sin2x$
    (b) $x^2D^2y - 3xDy + 5y = x^2\ sin(lnx)$
    (c) $x^6y'' + 3x^5y' + a^2y = \frac{1}{x^2}$
    (d) $x^3y''' + x^2y'' = 1 + x + x^2$
    (e) $(x + 2)y_2 - (2x + 5)y_1 + 2y = (x + 1)e^x$
    (f) $y_2 + 2xy_1 + (x^2 + 1)y = x^3 + 3x$
    (g) $y'' - 4xy' + (4x^2 - 1)y = -3e^{x^2}sin2x$
    (h) $x^2y_2 - 2(x^2 + x)y_1 + (x^2 + 2x + 2)y = 0$
    (i) $x^2y'' - (x^2 + 2x)y' + (x + 2)y = x^3e^x$
    (j) $x(xcosx - 2sinx)y_2 + (x^2 + 2)sinx\ y_1 - 2(xsinx + cosx)y = 0$
    (k) $xy'' - (2x + 1)y' + (x + 1)y = (x^2 + x - 1)e^{2x}$
    (l) $xy_1 - y = (x - 1)(y_2 - x + 1)$
    (m) $y'' + 4y = 4\sec^2 2x$