UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech (All SoCSE Branches)**	**Semester**	**I**
Course	**Mathematics I**	**Course Code**	**MATH 1002**
Session	**July-Dec 2017**	**Topic**	**Matrices**

1. (i) Reduce the matrix $A=\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ to column echelon form and find its rank.

(ii) Find the rank of the matrix of the following matrix by reducing it to normal form.

$$A=\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

2. If $X_1 = [3,1,-4]$, $X_2 = [2,2,-3]$ and $X_3 = [0,-4,1]$, then show that

(i) The vectors X_1 and X_2 are linearly independent.

(ii) The vectors X_1, X_2 and X_3 are linearly dependent.

3. Find the values of k for which the system of equations

$(3k-8)\,x+3y+3z=0,\ 3x\ +(3k-8)\,y+3z=0, 3x+3y+\ (3k-8)\,z=0$

has a non-trivial solution.

4. Investigate for what values of λ and μ do the system of equations

$x\ +\ y\ +\ z=6, x+2y+3z=10,\ x+2y+\ \lambda\, z=\ \mu$ have

(i) No solution (ii) Unique solution (iii) Infinite solutions.

5. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Hence compute A^{-1}.

6. Find the Eigen values and corresponding Eigen vectors of the following matrices:

(i) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 0 & 2 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

7. Find a matrix P which transform the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form. Hence, find A^4.

8. Let $P = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ a & 2 & b \end{bmatrix}$ for some $a, b \in \mathbb{R}$. Suppose that 1 and 2 are eigenvalues of P and $P\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$. Find $P^4\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

9. Show that the following nonlinear system

$$\begin{aligned} \sin\alpha + 2\cos\beta + 3\tan\gamma &= 0 \\ 2\sin\alpha + 5\cos\beta + 3\tan\gamma &= 0 \\ -\sin\alpha - 5\cos\beta + 5\tan\gamma &= 0 \end{aligned}$$

has 18 solutions if $0 \le \alpha \le 2\pi$, $0 \le \beta \le 2\pi$, $0 \le \gamma \le 2\pi$.

10. The manufacturing of an automobile requires painting, drying and polishing. The Rome Motor Company produces three types of cars: the Centurion, the Tribune, and the Senator. Each Centurion requires 8 hours for painting, 2 hours for drying, and 1 hour for polishing. A Tribune needs 10 hours for painting, 3 hours of drying and 2 hours for polishing. It takes 16 hours of painting, 5 hours of drying and 3 hours of polishing to prepare a Senator. If the company uses 240 hours for painting, 69 hours for drying, and 41 hours for polishing in a given month, how many of each type of car are produced?