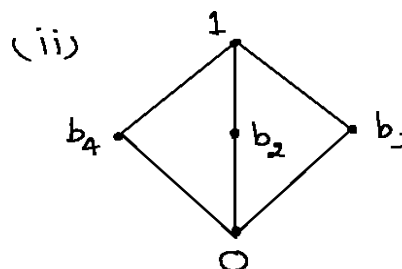
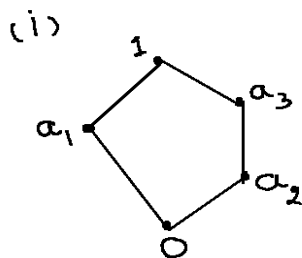


## UNIVERSITY OF PETROLEUM &amp; ENERGY STUDIES, DEHRADUN

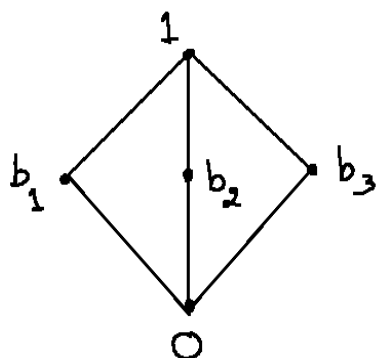
Program	B. Tech SCS	Semester	II
Course	Mathematics II	Course Code	MATH 1005
Session	Jan-May 2018	Topic	Posets and Lattices

1. Show that if  $S$  be the set of all numbers and  $R$  is a relation in  $S$  defined by “ $a$  is less than or equal to  $b$  ( $a \leq b$ )” is a partial order relation.
2. Define Hasse diagram.
  - (a) Let  $A = \{2, 3, 4, 6, 12, 36, 48\}$  be a non-empty set and  $R$  be a partial order relation of divisibility on  $A$ . i.e. if  $a, b \in A$  then  $a$  divides  $b$ . draw the Hasse diagram of the Relation.
  - (b) Let  $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$  be a non-empty set and  $R$  be a partial order relation of divisibility on  $A$ . i.e. if  $a, b \in A$  then  $a$  divides  $b$ . draw the Hasse diagram of the Relation.
3. Define partial order relation with example.
4. Show that if  $N$  be the set of all natural numbers. The relation  $R$  in  $N$  defined by  $aRb \Leftrightarrow a$  divides  $b$  is a Partial order relation.
5. Let  $A = \{2, 3, 4, 6, 8, 12, 24, 48\}$  and  $R$  be a partial order relation of divisibility on  $A$ . Let  $B = \{4, 6, 12\}$  be a subset of  $A$ . Find
  - (a) All upper bounds of  $B$
  - (b) All lower bounds of  $B$
  - (c) The least upper bound of  $B$
  - (d) The greatest lower bound of  $B$
6. Show that the set  $L$  of all factors of 12 under divisibility forms a lattice.
7. Show that the lattices given by the following diagrams are not distributive.



8. Define complemented and distributive lattice.

9. Prove that the lattice given by the following diagram is modular.



10. Determine whether the posets represented by following Hasse diagrams are lattices.

