UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech (All SoCSE Branches)**	**Semester**	**I**
Course	**Mathematics I**	**Course Code**	**MATH 1002**
Session	**July-Dec 2017**	**Topic**	**Differential Calculus and Multiple Integrals**

1. Find n^{th} derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$.

2. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.

3. If $y = sin\ (m\sin^{-1}x)$, find $y_n(0)$.

4. If $u=x\left(1-r^2\right)^{-\frac{1}{2}}$, $v=y\left(1-r^2\right)^{-\frac{1}{2}}$ and $w=z\left(1-r^2\right)^{-\frac{1}{2}}$,

where $r^2 = x^2 + y^2 + z^2$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \left(1-r^2\right)^{-\frac{5}{2}}$.

5. If $\phi(x, y, z) = 0$, show that $\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -1$.

6. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u *and* v functionally related? If so, find the relationship.

7. Change the order of integration and hence evaluate $I = \int_0^a \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax-y^2}}\,dxdy$.

8. A tapering log has a square cross section whose side varies uniformly and is equal to a at the top and $b(b > 3a/2)$ at the bottom. Show that the volume of the greatest conical frustum that can be obtained from the log is $\frac{\pi b^3 l}{27(b-a)}$, where l is the length of the log.

9. Find the total area included between the curve $y^2x = 4a^2(2a-x)$ and its asymptote.

10. Find the volume bounded by the elliptic paraboloids $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$.