UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2018**	**Topic**	**Numerical Methods**

1. Prove the following relations where the operators have their usual meanings:

   (i) $\mu \equiv \cosh(u/2)$ where $u = hD$ **(ii)** $\Delta \equiv \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}}$

   (iii) $\nabla^2 \equiv h^2D^2 - h^3D^3 + \frac{7}{12}h^4D^4 - \ldots\ldots$ **(iv)** $1+\delta^2\mu^2 \equiv \left(1+\frac{\delta^2}{2}\right)^2$

2. Prove the following relations where the operators have their usual meanings

   (a) $\Delta \log f(x) = \log\left(1+\frac{\Delta f(x)}{f(x)}\right)$ (b) $e^x = \left(\frac{\Delta^2}{E}\right)e^x.\frac{E\,e^x}{\Delta^2 e^x}$

3. With usual notations, prove that, $\Delta^n\left(\frac{1}{x}\right) = (-1)^n \frac{n!\,h^n}{x(x+h)\ldots\ldots(x+nh)}$.

4. Prove: $(E-k)f(x) = k^{x+1}\Delta\left(\frac{f(x)}{k^x}\right)$

   where the symbols have their usual meanings and k is a constant.

5. Estimate the production of cotton in the year 1935 from the data given below:

Year, x	1931	1932	1933	1934	1935	1936	1937
Production, f(x) in millions	17.1	13	14	9.6	-------	12.4	18.2

6. The following table gives the pressure of a steam at a given temperature. Using Newton's formula, compute the pressure for a temperature of 142^0C

Temperature, ^{0}C	140	150	160	170	180
Pressure, kgf/cm^2	3.685	4.854	6.302	8.076	10.225

7. The following table give the marks secured by 492 students in Mathematics;

Marks	No. of Students
0 – 40	210
40 – 45	43
45 – 50	54
50 – 55	74
55 – 60	32
60 – 65	79

Find out; (i) Number of students who secured more than 48 but less than 50 marks.

(ii) Number of students who secured less than 48 but not less than 45.

8. Prove that the nth order divided difference of a polynomial of degree n is constant.

9. The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface :

Height (x)	100	150	200	250	300	350	400
Distance (y)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the value of y when x = 375 ft.

10. The following are the mean temperatures (°F) on three days, 30 days apart round the periods of summer and winter. Estimate the approximate dates and values of maximum temperature in summer and minimum temperature in winter.

Day	Summer		Winter	
	Date	Temperature	Date	Temperature
0	15 June	58.8	16 December	40.7
30	15 July	63.4	15 January	38.1
60	14 August	62.5	14 February	39.3

11. The function $y = f(x)$ is given at the points (7, 3), (8, 1), (9, 1) and (10, 9). Find the value of y for $x = 9.5$ using Newton Divided Difference interpolation formula.

12. Use Newton's divided difference interpolation to find the interpolating polynomial for the function y= f(x) given by

X	-1	1	4	6
f(x)	5	2	26	132

13. The following table gives some relations between steam pressure and temperature. Find the pressure at temperature 372.1^0 using interpolation formula.

T(in degree Celsius)	361	367	378	387	399
P	154.9	167.0	191.0	212.5	244.2

14. Apply Newton formula to find the value of $f(0.2)$ from the following table which gives the value of $f(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-x^2/2}\, dx$ at intervals of $x = 0.5$ from $x = 0$ to $x = 2$.

x	0	0.5	1.0	1.5	2.0
$f(x)$	0	0.191	0.341	0.433	0.477

15. Using appropriate formule for interpolation, estimate the number of persons earning wages between $ 60 and $ 70 from the following data:

Wages ($):	Below 40	40—60	60—80	80—100	100—120
Number of people: *(in thousands)*	250	120	100	70	50

16. Given the record of height from the earth's surface for a rocket at intervals of time in a laboratory of ISRO. Assuming that the height of atmosphere is 90 km, find the time when the rocket leaves the atmosphere, by using backward formula.

Height (km)	65	75	85	95
Time (sec)	30	34	36	37

17. Estimate the production for 1961 and 1971 from the following data:

Year:	1960	1962	1964	1966	1968	1970	1972
Production:	200	220	260	300	350	400	430

18. Apply Newton Divided difference interpolation formula to find $f(5)$ given that

$$f(1)=2,\ f(2)=4,\ f(3)=8,\ f(4)=16,\ f(7)=128\ .$$

19. Use Newton-Gregory forward difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data:

x:	1	2	3	4
f(x):	26	18	4	1

20. If *p*, *q*, *r*, *s* are the successive entries corresponding to equidistant arguments in a table, show that when the third differences are taken into account, the entry corresponding to the argument half way between the arguments at *q* and *r* is $\left[A+\left(B/24\right)\right]$, where *A* is the arithmetic mean of *q* and *r* and *B* is arithmetic mean of $3q-2p-s$ and $3r-2s-p$.

21. Given the following table, find $f(x)$ as a polynomial in power of $(x-5)$.

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

22. Estimate the production at the end of the year 1968 from the following data:

Year	1961	1962	1963	1964	1965	1966	1967
Production	210	215	222	230	245	260	270

23. A reservoir discharging water through sluices at a depth h below the water surface area A for various values of H is given below:

H (ft)	10	11	12	13	14
A (sq. ft)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by

$$\frac{dH}{dt} = \frac{-48H}{A},$$

estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices.

24. A solid of revolution is formed by rotating about the x axis, the lines $x = 0$ & $x = 1$ and a curve through the points with the following co-ordinates:

x	0	0.25	0.5	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

25. Find by Bisection's method, the positive root of the equation $x^3 + x^2 + x - 100 = 0$ correct to three decimal places.

26. Let $f(x)$ be a function satisfying
$f''(x) + f(x) = 0$ with the conditions $f(0) = 0$ and $f'(0) = 1$.
If $g(x) = f(x) - 5x + 2$ be another function depending on $f(x)$ then use Fixed point iteration method to determine the smallest positive real x for which $f(x) \approx 5x - 2$ is satisfied.

27. Let $f(x)$ be a non-zero function such that n^{th} derivative of it is equal to the function itself. Find the smallest positive root of the equation $xf(x) = 1$ by using method of False position.

28. The graph of $y = 2\sin x$ and $y = \log x + c$ touch each other in the neighborhood of point $x = 8$. Find c and the coordinates of point of contact.

29. Consider the van der Waal's equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \ .$$

Compute the specific volume V of carbon dioxide at a temperature of $T = 300^0 K$, given

$P = 1atom$, $R = 0.08254 J(Kg\ ^0K)$, $a = 3.592$, $b = 0.04267$ (Use suitable method).

30. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of time t seconds. Find the velocity of the slider and its acceleration when $t = 0.3$ seconds.

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x	3.013	3.162	3.287	3.364	3.395	3.381	3.324

31. Apply Regula False position method to solve the equation $x^3 - 8x^2 + 17x - 10 = 0$.

32. Find the smallest positive root of the equation $x^3 + 9x^2 - 18 = 0$ correct to three decimal places using Newton Rapshon method

33. Find $f'(2)$ and $f''(2)$ from the following table:

x:	0	1	3	6
$f(x)$:	18	10	-18	40

34. In celestial mechanics, *mean anomaly* is a parameter relating position and time for a body moving in a Kepler orbit and *eccentric anomaly* is a parameter that defines the position of a body that is moving along an elliptic Kepler orbit. The Kepler equation of motion relating the *mean anomaly* (M) and the *eccentric anomaly* (E) of an elliptic orbit with eccentricity *e* is given by $M = E - e\sin E$. Given $e = 0.0167$ (Earth's eccentricity) and $M = 1$ (in radians), compute E using (a) Bisection method (b) Fixed-point iteration method and (c) Newton's method (d) False position method.

35. Find a real root of the equation $3x + \sin x - e^x = 0$ by the method of False position correct to three decimal places.

36. Find the root of the equation $xe^x = \cos x$ by Bisection method correct to four decimal places.

37. Find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$ by using the fixed point iteration method correct to three decimal places.

38. A river is $80\ meters$ wide. The depth $d\,(in\ meters)$ of the river at a distance $'x'(meter)$ from the bank is given by the following table:

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find the approximate area of cross section of the river using Simpson's rule.

39. Find $f'(1.1)$ and $f''(1.1)$ from the following table:

x:	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$:	0.0	0.1280	0.5540	1.2960	2.4320	4.000.

40. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

(i) Simpson's 1/3 rule taking h = 1/4

(ii) Simpson's 3/8 rule taking h = 1/6

Hence compute an approximate value of π in each case.

41. A train is moving at the speed of 30 m/sec. suddenly brakes are applied. The speed of the train per second after t seconds is given by

Time (t):	0	5	10	15	20	25	30	35	40	45
Speed (v):	30	24	19	16	13	11	10	8	7	5

Apply Simpson's three-eighth rule to determine the distance moved by the train in 45 seconds

42. Compute

$$I_p = \int_0^1 \frac{x^p}{x^3+10}\,dx \text{ for } p = 0, 1.$$

Use Trapezoidal rule with number of points 3, 5 and 9.

43. A tank is discharging water through an orifice at a of depth x meter placed below the surface of the water whose area is $A\ m^2$. The following are the values of x for the corresponding values of A:

A	1.257	1.39	1.52	1.65	1.809	1.962	2.123	2.295	2.462	2.650	2.827
x	1.50	1.65	1.80	1.95	2.10	2.25	2.40	2.55	2.70	2.85	3.00

Using the formula

$$(0.018)T = \int_{1.5}^{3.0} \frac{A}{\sqrt{x}}\,dx\ ,$$

calculate time T, in seconds for the level of the water to drop from 3.0 m to 1.5 m above the orifice.

44. Find a real root of the equation $x^2 - \log_e x - 12 = 0$ using Regula-Falsi method correct to three decimal places.

45. Using Newton-Raphson method, find the real root of $f(x) = x\sin x + \cos x = 0$ which is near $x = \pi$ correct to three decimal places.

46. Solve the equation $AX = B$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1.5 \\ 3 \end{bmatrix}$ by Gauss Seidal method.

47. Solve the equations $10x+2y+z=9,\ 2x+20y-2z=-44,\ -2x+3y+10z=22$ by Gauss Jacobi method.

48. Solve the equation $AX=B$ where $A=\begin{bmatrix}4 & 1 & 1\\ 1 & 4 & -2\\ 3 & 2 & -4\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 4\\ 6\end{bmatrix}$ by Gauss Seidal method.

49. Solve the equations $2x+y-z=4,\ x-y+2z=-2,\ -x+2y-z=2$ by Gauss Jacobi method.

50. Using Runge-Kutta method of fourth order, calculate $y(0.1)$, and $y(0.2)$ of the differential equation $\frac{dy}{dx}=x+y^2,\ y(0)=1.$

51. Using Runge-Kutta method of fourth order, calculate $y(0.1)$, and $y(0.2)$ of the differential equation $\frac{dy}{dx}-\frac{2xy}{1+x^2}=1,\ y(0)=1.$

52. Using Picard's method, solve $y'=-2xy$ with $x_0=0, y_0=1$ up to third approximation.

53. Find an approximate value of y when x=0.1, if $\frac{dy}{dx}=x^2-y^2$ and y=1 at x=0, using Taylor's series method.

54. Use Euler's modified method to find y(0.25) given that $y'=xy$, y(0)=1.

55. Find y for x=0.1 and 0.2 for $\frac{dy}{dx}=\frac{y^2+x}{y^2+2x}$ given that y(0)=1 by Runge-Kutta method of fourth order.

56. Use Euler's method to find an approximate value of y(0.4) for the equation $y'=x+y$, y(0)=1 with h=0.1.

57. Using Runge-Kutta method of order four, find y for x=0.1, 0.2, 0.3 given that $\frac{dy}{dx}=2xy+3y^2$, y(0)=1.

58. Apply Euler's method to solve the differential equation $y'=3x-y^2$ at x=0.8, given that y(0)=2, with the step size of 0.2.

59. Solve the following system of equations by using Gauss-Jacobi and Gauss-Seidel iterative methods correct to 4 decimal places

(i) $\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 19 \\ -20 \\ 27 \end{bmatrix}$

(ii) $\begin{bmatrix} 4 & 1 & 2 \\ 3 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 10 & 4 & -2 \\ 1 & -10 & -1 \\ 5 & 2 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ -3 \end{bmatrix}$

(iv) $\begin{bmatrix} 5 & 1 & -2 \\ 3 & 4 & -1 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 10 \end{bmatrix}$

60. Determine the 7^{th} root of 30 by Bisection in the interval (1.62, 1.63) correct up to four decimal places.
61. Determine the 5^{th} root of 10 by Newton Raphson method correct up to six decimal places.
62. Determine the 4^{th} root of 20 by Regula False position method in the interval (2.11, 2.12).