UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2018**	**Topic**	**Numerical Methods**

1. Prove the following relation:

$$D \equiv \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \cdots\right],$$

where h is the interval of differencing, D denotes the differentiation operator and Δ is the forward difference operator.

2. In a class of 100, the students are placed into following categories according to the marks they have obtained in a test out of 60.

Marks Obtained	**0 – 9**	**10 – 19**	**20 – 29**	**30 – 39**	**40 – 49**	**50 – 59**
No. of Students	**3**	**12**	**15**	**35**	**25**	**10**

Find the number of students who have secured 85% and above marks using Newton's backward difference interpolation formula.

3. (a). For a function $f(x)$ the divided difference table is given by:

x	$f(x)$	First divided difference	Second divided difference
$x_0 = 0.0$	$f(x_0) = ?$	$[x_0, x_1] = ?$	$[x_0, x_1, x_2] = \frac{50}{7}$
$x_1 = 0.4$	$f(x_1) = ?$	$[x_1, x_2] = 10$	
$x_2 = 0.7$	$f(x_2) = 6$		

Determine the missing terms.

(b). A curve $y = f(x)$ passes through the points $(0, 18), (1, 10), (3, -18)$ and $(6, 90)$. Find the slope of the curve at $x = 2$ by using Newton's divided difference interpolation formula.

4. Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0), (0.5, k), (1, 3)$ and $(2, 2)$. Find k if the coefficient of x^3 in $P_3(x)$ is 6, by using Newton's divided difference interpolation formula.

5. (a) Use Simpson's rule to find the value of the definite integral $\int_{-1}^{1} e^{-|x|}\ dx$ by dividing the range of integration $(-1,1)$ into four equal parts. Compare this result with the exact value of the integral and hence compute the approximate value of e.

(b) The value of the integral $\int_{1}^{9} x^2 dx$ by Trapezoidal rule is $2\left[\frac{1}{2}(1+9^2)+\alpha^2+\beta^2+7^2\right]$ for $n=4$. Find the value of α and β.

6. A curve $y=f(x)$ is drawn to pass through the points given by the following table:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Find the area bounded by the curve $y=f(x)$, the x-axis and the lines $x=1, x=4$ using Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule.

7. (a). Solve the following system of equations using Gauss-Jacobi iteration method

$$5x-y+z=10$$
$$2x+4y=12$$
$$x+y+5z=-1$$

Start the iterations with the initial solution $(2,3,0)$. Perform five iterations.

(b). Solve the following system of equations

$$3x+2y=4.5$$
$$2x+3y-z=5$$
$$-y+2z=-0.5$$

by using Gauss-Seidel method correct to two places of decimal, starting with the initial approximations as $x_0=0.4, y_0=1.6, z_0=0.4$.

8. Estimate $y(1)$ if $2yy'=x^2$ and $y(0)=2$ using Runge-Kutta method of fourth order by taking $h=0.5$. Also compare the result with the exact value.

9. Use Picard's method to obtain the value of y correct to three decimal places when $x=0.25$ given that

$$\frac{dy}{dx}=\frac{x^2}{y^2+1};\ y(0)=0.$$

10. (a). Derive the expression for Newton-Raphson formula

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)};\ \ n=0,1,2,....$$

to find a root of $f(x)=0$.

(b). The graph of the curve $y=f(x)$ where $f(x)=x^2-12$ crosses the *X*-axis twice. Use Newton- Raphson method to compute the abscissa of the point correct to 3 decimal places where it crosses the positive *X*-axis.