UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2018**	**Topic**	**Probability & Statistics**

1. A student is given a true false examination with 8 questions. If he corrects at least 7 questions, he passes the examination. Find the probability that he will pass given that he guesses all questions.

2. If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit atleast once?

3. Assuming half the population of a town consumes chocolates and that 100 investigators, each take 10 individuals to see whether they are consumers, how many investigators you expect to report that three people or less were consumers?

4. Assume that the probability of an individual coalminer being killed in a mine accident during a year is 1/2400. Calculate the probability that in a mine employing 200 miners there will be atleast one fatal accident in a year.

5. Suppose that a book of 600 pages contains 40 printing mistakes. Assume that three errors are randomly distributed throughout the book and r , the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free from errors?

6. Suppose the number of telephone call on an average received from 9:00 to 9:05 follow a Poisson distribution with a mean 3. Find the probability that
   (a) The operator will receive no calls in that time interval tomorrow.
   (b) In the next three days, the operator will receive a total of 1 call in that time interval.

7. In a sample of 1000 cases, the mean of a certain test is 14 and S.D. is 2.5. Assuming the distribution to be normal, find
   (a) how many students score between 12 and 15?
   (b) how many score above 18?
   (c) how many score below 8?
   (d) how many score 16?

8. A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x} = 12\ hours, \sigma = 3\ hours$. Assuming the data to be normally distributed what percentage of battery cells are expected to have life (a) more than 15 hours (b) less than 6 hours (c) between 10 and 14 hours?

9. Probability mass function of a discrete random variable X is given by

$$p(x) = \begin{cases} k, \ x = 0 \\ 2k, x = 1 \\ 3k, x = 2 \\ 0, otherwise. \end{cases}$$

(a) Find k. (b) Evaluate $P(X \geq 2), P(0 < X \leq 2)$ (c) find cumulative distribution function of X.

10. Probability density function of a continuous random variable X is given by $f(x) = ke^{-x}, 0 \leq x < \infty$. (a) Find k. (b) Evaluate $P(0 < X < 1), P(0 \leq X < 1), P(X \geq 5)$ (c) Find distribution function of X.

11. Distribution function of a random variable X is $F(x) = \begin{cases} 1 - e^{-2x}, x \geq 0 \\ 0, \quad\quad x < 0. \end{cases}$

(a) Find density function of X (b) Evaluate $P(X > 2), P(-3 < X \leq 4)$.

12. Find Mean, Variance, Skewness and Kurtosis for the following data.

x	3	7	9	20

13. Calculate first four moments of the following data.

Values	$10 - 20$	$20 - 30$	$30 - 40$	$40 - 50$
Frequency	5	10	20	10

Hence, evaluate Skewness and Kurtosis.

14. Find mean of (a) Poisson distribution and (b) $N(0,1)$.

15. A continuous r.v. has probability density function $f(x) = ae^{-ax}, 0 < x < \infty, a > 0$. Calculate moment generating function, and obtain moment of order k about origin.

16. The number of admissions per day at an emergency room has a Poisson distribution and the mean is 5. Find the probability of at most 3 admissions par day and the probability of at least 8 admissions per day.

17. Suppose that 15% of the population is left-handed. Find the probability that in group of 50 individuals there will be (a) at most 10 left handers (b) at least 5 left handers (c) between 3 and 6 left handers inclusive (d) exactly 5 left handers.

18. Out of 2000 families with 4 children each, how many would you expect to have (a) at least 1 boy (b) 2 boys (c) 1 or 2 girls and (d) no girls ?

19. The probability that an engineering student will graduate is 0.4. Determine the probability that out of 5 students (a) none will graduate (b) 1 will graduate (c) at least 1 will graduate and (d) all will graduate.

20. The time spent watching TV per week by middle-school students has a normal distribution with mean 20.5 hours and standard deviation 5.5 hours. Find the percent who watch less than 25 hours per week and who watch over 30 hours per week. Sketch a curve representing these two groups.

21. Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random exactly 2 will be defective by using (a) the binomial distribution and (b) the Poisson approximation to the binomial distribution.

22. If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals (a) exactly 3 and (b) more than 2 individuals will suffer a bad reaction.

23. An insurance salesperson sells policies to 5 men, all of identical age and in good health. According to the actuarial tables, the probability that a man of this particular age will be alive 30 years hence is $\frac{2}{3}$. Find the probability that in 30 years (a) all 5 men, (b) at least 3 men, (c) only 2 men, and (d) at least 1 man will be alive.

24. The mean grade on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive A's. What is the minimum grade that a student must get in order to receive an A?

25. The incidence of an occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random, four or more will suffer from the disease?

26. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

27. Find the maximum n such that the probability of getting no head in tossing a coin n times is greater than 0.1.

28. The probability of a man hitting a target is $\frac{1}{3}$

(a) If he fires 5 times, what is the probability of his hitting the target at least twice?

(b) How many times must he fire so that the probability of his hitting the target at least once is more than 90% ?

29. Fit a binomial distribution to the following frequency distribution:

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

30. The mean of binomial distribution is 3 and the variance is $\frac{9}{4}$. Find

(a) The value of n (b) $P(x \geq 7)$ (c) $P(1 \leq x \leq 6)$

31. A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days (a) on which there is no demand (b) on which demand is refused.

32. A manufacturer of cotter pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet the guaranteed quality?

33. If $X \sim P(\lambda)$ such that $P(1) = P(2)$, find (a) mean of the distribution (b) $P(4)$ (c) $P(x \geq 1)$ and (d) $P(1 < x < 4)$.

34. If the variance of a Poisson variate is 3, then find the probability that (a) $x = 0$ (b) $0 < x \leq 3$ and (c) $1 \leq x < 4$.

35. Fit a Poisson distribution for the following data and calculate the expected frequencies.

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

36. The distribution of typing mistakes committed by a typist is given below. Assuming the distribution to be Poisson, find the expected frequencies

x	0	1	2	3	4	5
$f(x)$	42	33	14	6	4	1

37. If $X \sim P(\lambda)$ such that $3\,P(x = 4) = \frac{1}{2}P(x = 2) + P(x = 0)$, find (a) the mean of X and (b) $P(x \leq 2)$.

38. The marks obtained in Mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine

(a) How many students got marks above 90%

(b) What was the highest mark obtained by the lowest 10% of the students

(c) Within what limits did the middle of 90% of the students lie.

39. The marks obtained in Statistics in a certain examination found to be normally distributed. If 15% of the students secured ≥ 60 marks, 40% < 30 marks, find the mean and standard deviation.

40. X is normally distributed and the mean of X is 12 and S.D. is 4

(a) find out the probability of the following:

(i) $X \geq 20$, (ii) $X \leq 20$ and (iii) $0 \leq x \leq 12$

(b) Find x' when $P(X > x') = 0.24$

(c) Find $x_0{}'$ and $x_1{}'$ when $P(x_0' < X < x_1') = 0.5$ and $P(X > x_1') = 0.25$

41. In an examination it is laid down that a student passes if he secures 30% or more marks. He is placed in the 1st , 2nd or 3rd division according as he secures 60% or more, between 45% & 60% and between 30% and 45% marks respectively. He gets distinction in case he secures 80 % or more marks. It is noticed from the result that 10% of the students failed in examination, where as 5% of them obtained distinction. Calculate the % of students placed in the 2nd division. (Assume normal distribution of marks)

42. Find the first four moments of the set 2, 3, 7, 8, 10 about (a) origin, (b) an arbitrary number 4 (origin 4) and (c) mean.

43. Following data gives three different distributions for the variable X. The frequencies for the three distributions are given by f_1, f_2 and f_3. Find moment coefficient of skewness and kurtosis for the three distributions.

x	f_1	f_2	f_3
0	10	1	1
1	5	2	2
2	2	14	2
3	2	2	5
4	1	1	10

44. Let X be a random variable having m.g.f. is $M_X(t)$ then find the $M_U(t)$ where $U = \frac{X-a}{h}$ and hence find the m.g.f of a standard normal variate Z.

45. Let the random variable X assumes the values 1,2,3,… with probability function $P(X = x) = pq^{x-1}$ find moment generating function and hence obtain mean & variance.

46. Prove that mean is always greater than variance in binomial distribution.

47. Suppose $X \sim B(n_1, p_1)$ and $Y \sim b(n_2, p_2)$ and X & Y are independent then $X + Y$ need not be a binomial variate.

48. Suppose X is a random variable follows binomial distribution with mean 4 and variance $\frac{4}{3}$ find $P(X < 1)$.

49. Verify whether there exists $X \sim B(n, p)$ with mean 3 and variance 4 and justify your answer.

50. Prove that Poisson(λ) distribution is the limiting case of binomial $B(n, p)$ distribution with $\lambda = np$ as $n \to \infty$.