UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech SCS**	**Semester**	**II**
Course	**Mathematics II**	**Course Code**	**MATH 1005**
Session	**Jan-May 2018**	**Topic**	**Numerical Methods**

1. Find the root of $\tan x + x = 0$, up to four decimal places, which lies between 2 and 2.1 using Bisection method.
2. Find a real root of the equation $x^2 - \log_e x - 12 = 0$ using Regula-Falsi method correct to three decimal places.
3. Find the root of the equation $\tan x + \tanh x = 0$, which lies in the interval (1.6, 3.0) correct to four significant digits using the method of false position.
4. Determine the real root of $\cos x - 3x + 1 = 0$, by iteration method correct to six decimal places.
5. Using Newton-Raphson method, find the real root of $f(x) = x\sin x + \cos x = 0$ which is near $x = \pi$ correct to three decimal places.
6. Solve the following system of equations by using Gauss-Jacobi and Gauss-Seidel iterative methods correct to 4 decimal places

(i) $\begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 19 \\ -20 \\ 27 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 4 & 1 & 2 \\ 3 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 10 & 4 & -2 \\ 1 & -10 & -1 \\ 5 & 2 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ -3 \end{bmatrix}$ **(iv)** $\begin{bmatrix} 5 & 1 & -2 \\ 3 & 4 & -1 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 10 \end{bmatrix}$

7. Following are the marks obtained by 492 candidates in a certain examination.

Marks	0 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
No. of Candidates	210	43	54	74	32	79

By using appropriate formula, find out the number of candidates who secured
(i). more than 40 but not more than 60 marks
(ii). less than 48 but not less than 45 marks

8. From the table, estimate the number of students who obtained scores between 40 and 45.

Scores:	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of students:	31	42	51	35	31.

9. Using Newton's divided difference formula, find a polynomial function satisfying the following data:

$$\begin{array}{llllll} x: & -4 & -1 & 0 & 2 & 5 \\ f(x): & 1245 & 33 & 5 & 9 & 1335 \end{array}.$$

Hence, find $f(1)$.

10. Find the first and second derivatives of $f(x)$ at $x = 1.2,\ 1.6$ and $x = 2.2$ from the following table:

x:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$f(x)$:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

11. The velocity distribution of a fluid near a flat surface is given below:

$Distance(\,x\ cm)$	0.1	0.3	0.5	0.7	0.9
$Velocity(v\ cm/s)$	0.72	1.81	2.73	3.47	3.98

Using suitable formulae obtain the velocity at $x = 0.2\ cm$ and $0.8\ cm$.

12. A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of the time *t* (seconds).

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and acceleration of the rod when *t* = 0.6 sec.

13. Evaluate the following integral by Simpson's(1/3) rule.

$(i)\ \int_0^1 x^x dx,\quad$ (ii) $\int_0^1 \frac{x^2}{1+x^2} dx,\quad$ (iii) $\int_0^1 \frac{\tan^{-1} x}{x^{3/2}} dx,$ for n=10.

14. Evaluate the following integrals by Trapezoidal and Simpson's rule and compare the result with the result obtained by direct integration

$(i)\ \int_0^{1.2} \ln(1+x^2)$, for n=6 $\qquad$ (ii) $\int_0^1 \cos x^2 dx$,for n=10.

15. Using Picard's method, solve $y' = -2xy$ with $x_0 = 0, y_0 = 1$ up to third approximation.

16. Find an approximate value of y when x=0.1, if $\frac{dy}{dx} = x^2 - y^2$ and y=1 at x=0, using Taylor's series method.

17. Use Euler's modified method to find y(0.25) given that $y' = xy$, y(0)=1.

18. Find y for x=0.1 and 0.2 for $\frac{dy}{dx} = \frac{y^2 + x}{y^2 + 2x}$ given that y(0)=1 by Runge-Kutta method of fourth order.

19. If Δ, ∇, δ, E and μ denote forward, backward, central, shift and average difference operators, respectively, in the analysis of data with equal spacing h, then prove the following relations:

I. $1+\delta^2\mu^2 \cong \left(1+\frac{\delta^2}{2}\right)^2$, $\qquad$ II. $E^{1/2} \cong \mu + \frac{\delta}{2}$ $\qquad$ III. $\Delta \cong \frac{\delta^2}{2} + \delta\sqrt{\left(1+\frac{\delta^2}{4}\right)}$,

IV. $\mu\delta \cong \frac{\Delta E^{-1} + \Delta}{2}$, $\qquad$ V. $\mu\delta \cong \frac{\Delta + \nabla}{2}$