Fuzzy Inference Systems for Emergency Medical Assessment

Introduction

"Life's not black and white. There is some grey aspect to it" - Pilou Asbaek. This statement perfectly defines fuzzy behavior in real-world data not everything can be calculated in 0 and 1. The study focuses on the development of 2 fuzzy systems and their comparison where these Fuzzy Systems[1] help family doctors assess the urgency of the patient's condition provided the illness. The Fuzzy Inference Systems (FIS models) take 3 antecedents i.e. Body Temperature (in degrees Celcius), Severity of Headache, and Age of patient. Based on these inputs we need to calculate the urgency of patients' condition and whether they need hospital assistance or not. These systems use linguistic terms and a ruleset to enhance the model's decision.

1. Fuzzification Models and Their Justification

In this study we have used two Fuzzy Inference Models - 1) Mamdani and 2) Sugeno(specifically 0th-order Sugeno). We have used both models for both cases.
Case 1 - All inputs are singleton numeric
Case 2 - All inputs are non-singleton interval-based

"Mamdani Fuzzy Inference" [2] was introduced as a way of producing a control system using some rules for the steam engine, boiler combination collected from the operants. The Mamdani Fuzzy Inference System takes fuzzy sets as inputs along with the rules and gives back a fuzzy set as output. We defuzzify the output set to get the crisp output.
Why Mamdani? It is suitable in cases where the relation between input and output is complex and cannot be explained mathematically. The rule base helps us interpret the data, which helps for better human understanding.

"Sugeno Fuzzy Inference" [3] is similar to *"Mamdani Inference"* considering the antecedents but the outputs in Sugeno are singleton fuzzy sets. In the zeroth order, the singleton is a constant, and in the first order, as the order of the polynomial increases, it becomes a linear function. Based on these singleton outputs we can say that Sugeno defuzzification is computationally efficient.
Why Sugeno? This modeling approach is suitable when we need to precisely calculate the output. It is easier to understand mathematically and is computationally efficient.

1.1. Linguistic Terms for Antecedents and Consequents

Here we discuss the linguistic terms used in both of the models for each antecedent and consequent.
Linguistic Terms - Antecedent:

a) **Body Temperature *(degrees Celsius)***[4] **-** Here we have used four linguistic terms
   - Frost-bite **[30-35.5]** - The condition when the human body is exposed to very low temperatures for a long time.
   - Normal **[34-37.5]** - The temperature where the body is perfectly healthy and functions smoothly.

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- Higher Than Normal **[36-38]** - This condition arrives when the body temperature is above normal but not as high as fever.
- Fever **[38-41] -** The situation when the body heats up and we feel uneasy..

a) Headache - Here we have used three linguistic terms:
- Less **[0-5]** - None to very minor headache
- Moderate **[3-7]** - Minor as slightly more than an average headache.
- Severe **[5⁺-10]** - When the headache is extreme and unbearable (headache caused in migraine).

b) Age - Age has three linguistic terms [5]:
- Young **[0-40]** - People who have existed in this world for a short amount of time
- Midlife **[30-80]** - Age where most people are on the verge of becoming old.
- Old **[70-130]** - In this age bracket, most people need mental and physical support

Linguistic Terms - Consequent

a) Urgency - Here we have used three linguistic terms to define the output:
- None **[0-50]** - The patient is perfectly fine.
- Urgent **[25-75]** - The patient needs family attention
- Priority **[50-100]** - The patient needs to be submitted to a hospital

1.2. Membership Functions and Parameters for Both Models

In this section, we will discuss the membership functions and their parameters that are being used for both model-1 and model-2 along with why are they used here.

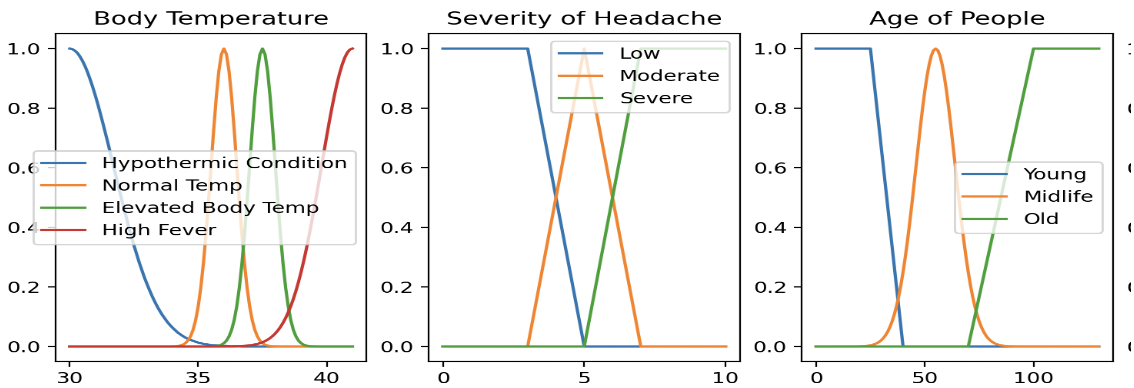

Fig 1 - Membership Functions - Antecedents

As we can see, for **Body Temperature** we have used the Gaussian membership function for all linguistic terms, as Temperature can most appropriately be viewed as Normal Distribution. Gaussian is used because data in normal distribution or population is distributed equally.

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

Fig 2 - Gaussian MF

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	Mean	Standard Deviation
Frost-bite	30	1.7
Normal	36	0.5
Higher Than Normal	37.5	0.5
Fever	41	1.2

Table 1 - Parameters - Body Temperature

For **Headache**, we took trapezoidal and triangular functions - where low and severe linguistic terms have open-left trapezoidal and open-right trapezoidal functions respectively and moderate being a triangular membership. We have taken so because certain values of headache must be strictly 1 and strictly 0. Triangular and Trapezoidal membership are stricter than a Gaussian

$$f(x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

Fig 3 - Triangular MF

$$f(x;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

Fig 4 - Trapezoidal MF

	a	b	c	d
Less	0	0	3	5
Moderate	3	5	7	-
High	5	7	10	10^+

Table 2 - Parameters - Headache

Similarly, for **Age**, we have taken trapezoidal and triangular where young and old ages are represented by open-left trapezoidal and open-right trapezoidal to represent the two extremes, and the midlife term is represented by Gaussian.

	Mean	Standard Deviation
Midlife	55	9

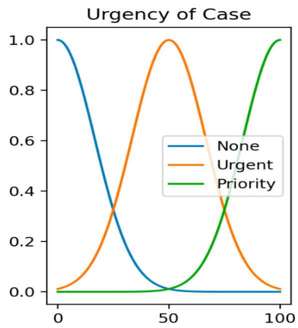
	a	b	c	d
Young	0	0	25	40
Old	70	100	130	130^+

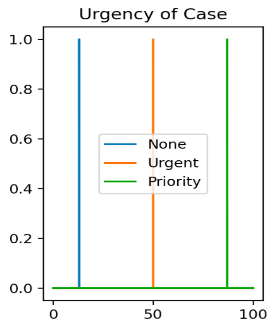
Tables 3 & 4 - Parameters - Age

For **Urgency**, it’s different for both models in the Mamdani Model, we have considered Gaussian membership functions and for the Sugeno Model, we have considered Singleton fuzzy sets for all linguistic terms.

Sugeno	z	Mamdani	Mean	Standard Deviation
None (z1)	12.986	None	0	16.67
Urgent (z2)	50	Urgent	50	16.67
Priority (z3)	87.014	Priority	100	16.67

Table 5 - Parameters - Urgency





Mamdani and Sugeno Consequent

1.3. Fuzzification Approaches and Reasoning

Here in this paper, we have taken two approaches to assign membership values or fuzzify the crisp values:

1) **Intuition -** This method uses human intelligence to develop membership functions using their own understanding of the terms. This involves the knowledge of context and concepts that we have acquired through experience and studies. The functions might change in accordance with the person developing them.

2) **Inference -** This approach deduces the membership function based on pre-existing knowledge or by collecting data from experts in order to create functions. These functions completely depend on the data. The more precise the data the better the membership functions will be.

1.4. Defuzzification Approaches

One of the models, taken in this study requires the output fuzzy sets to be defuzzified for the prediction of crisp output. This is the last step of the **Mamdani Approach**. We can defuzzify the values of output sets by a variety of methods -

1) **Centroid Defuzzification -** We calculate the centroid for the aggregated output membership functions in order to find the defuzzified output.
2) **First, Mean, and Last of Maxima -** First value where maxima occurred, mean of all the maximum output membership values, and last value where maxima occurred respectively.

For **Sugeno**, we just take the weighted average of all values to get crisp output as there is no concept of defuzzification in Sugeno

2. Final Models with Examples

2.1. Operators

Implication:
Min Operator: The min operator or t-norm is used in fuzzy inference while evaluating fuzzy rules. It is applied to each rule in order to define the degree of activation of each rule or firing strength. This firing strength is then taken into account while calculating the defuzzified output from the aggregated output membership.

Aggregation:
Max Operator: The max operator is used in the aggregation step of fuzzy inference, it is applied to find the aggregated degrees of memberships. This is important for determining the contribution of each rule in the final output.

2.2 Ruleset

Here we consider 24 if-then rules in order to build the inference system in which the AND operator is used to implicate the inputs.
For Example - If Body Temp is Normal AND Headache is Less AND Age is Young then the Severity is None.
Similarly, all rules use the AND operator.

Body Temp	Headache	Age	Severity of Case
-	Moderate	Old	Urgent
-	Severe	Old	Priority
Frost-bite	-	-	Priority
Fever	-	Old	Priority
Normal	Less	Old	None
Higher Than Normal	-	Old	Urgent
Normal	Less	Young	None
Normal	Moderate	Young	None
Normal	Severe	Young	Urgent
Higher Than Normal	Less	Young	None
Higher Than Normal	Moderate	Young	None
Higher Than Normal	Severe	Young	Urgent
High	Less	Young	Urgent
High	Moderate	Young	Urgent

High	Severe	Young	Priority
Normal	Less	Midlife	None
Normal	Moderate	Midlife	None
Normal	Severe	Midlife	Urgent
Higher Than Normal	Less	Midlife	None
Higher Than Normal	Moderate	Midlife	None
Higher Than Normal	Severe	Midlife	Urgent
High	Less	Midlife	Urgent
High	Moderate	Midlife	Urgent
High	Severe	Midlife	Priority

Table 6 - Ruleset

2.3 Illustrative Examples

In this section, we will discuss the cases, and how inputs are taken with the help of a few examples - here we are taking two cases into account.

Case 1 - Inputs are numeric

In case 1, as the inputs are singleton it will cut all the antecedent terms at exactly one place, then we take min (t-norm) among all antecedents to find the firing strength of the rule, these firing strengths shows the contribution of each rule and we then aggregate all the rules combined and defuzzify to find the crisp output.

Examples -

First Example - If Temperature is 32.5*C, Headache is 4 and Age is 56 then as per defuzzified output Severity is Extreme (Priority), similar for all examples.

Body Temperature	Headache	Age	Centroid Defuzzification of Mamdani	Sugeno Weighted Average
32.5	4	30	82.844	87.013
36	6	45	40.250	31.61341
40	8	68	82.990	87.013

Table 7 - Examples - Case 1

Case 2 - Inputs are interval-based.

When encountering interval-based inputs in scenario 2, the approach involves forming membership functions for each range using the Non-Singleton Approach to manage input uncertainties. This process entails intersecting input FSs with antecedents MF [6] and calculating the centroid of the intersected area to obtain centroids for each antecedent linguistic term. These centroids facilitate the fuzzification process, similar to that described in case 1.

Body Temperature	Headache	Age	Centroid Defuzzification of Mamdani	Sugeno Weighted Average
30-35	2-7	15-45	55.174	39.051
34-38	4-8	40-50	40.274	31.907
39-41	6-10	50-86	66.795	71.756

Table 8 - Examples - Case 2

Even though we are taking the mean of Case 2 ranges in Case 1, the outputs differ because in Case 2 the intersection of input range MF and antecedents MF might differ from the actual centroids or mean.

3. Comparison, Strengths, and Weaknesses of Both Models

As we have discussed above, in this study, we are using Mamdani and Sugeno as our inference systems for both Case 1 and Case 2 -

3.1 Comparative Analysis of Both Cases

In **Case 1,** inputs are singleton values and because these inputs remain same for each rule, it is easy to find firing strength and evaluate the rules as values do not differ for each linguistic terms because the intersection of singleton MF with antecedent MF is always a single output.
Example - for the body temperature input, the same, will be considered in all rules where the body temperature antecedent is being used.

Strengths - Precise, Computationally Efficient, Easy Implementation, etc.
Weaknesses - Ineffective when data is uncertain, Less efficient in real-world data, etc

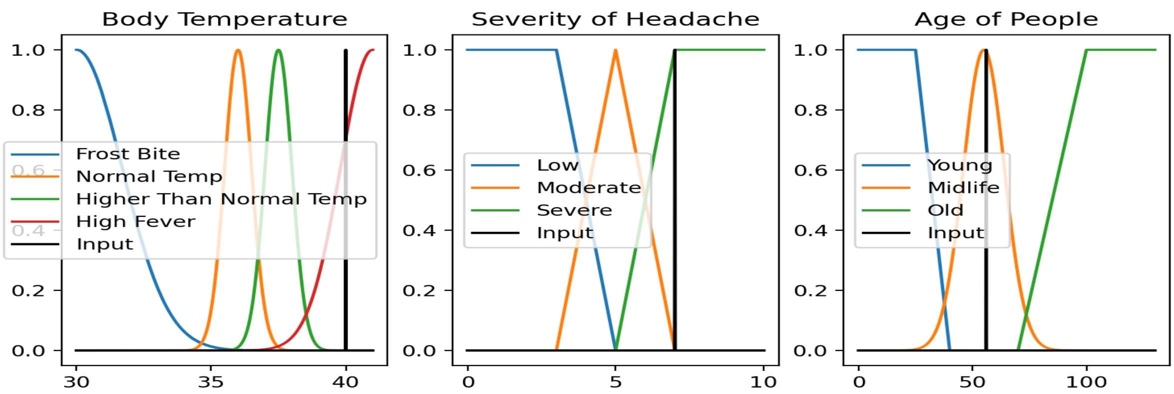


Fig 3 - All Antecedents with singleton Inputs

Whereas, in **Case 2,** we consider Non-Singleton MF as input is an interval that defines the uncertainty of input, to get a particular input value for further analysis of rules we take the intersection of input MF and each of the linguistic term's MF in the antecedent. After doing so we calculate the centroid of that intersected area which is used as an input to be implicated over the rules of that particular linguistic term.

Example - The centroid calculated for the Body Temperature Input Range and the frostbite MF will be implicated in the rules where body temperature is specified as Frost-Bite

Strengths - Interpretable by Humans, Human Knowledge Considered, High-Applicability, etc
Weaknesses - Computationally Expensive, Difficulty in Fuzzification, Dependent on Input Range, etc

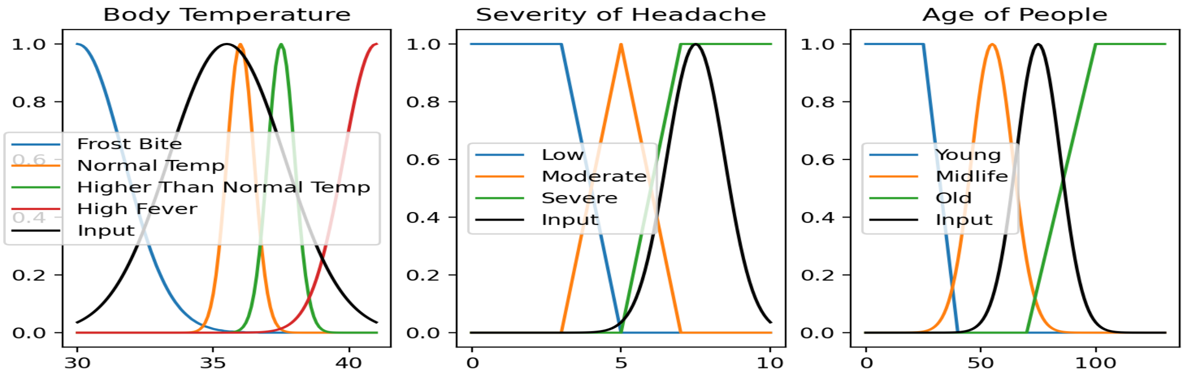


Fig 4 - All Antecedents with Input Ranges

3.2 Personal Reflection

My takeaway from this study is, understanding fuzzy logic and how can they be implemented in applications where detecting precise outcome is difficult or where people are uncertain about making decisions. I also learned how we can tackle problems where inputs are themselves uncertain, and how one system can be better than another in predicting the outcomes. I strongly feel that in Case 2 the inputs are more interpretable but the Case 1 will be more precise.

We faced problems while deducing an approach to tackle interval-based inputs. But with the help of [6] we were able to complete Case 2

4. Self-Reflection and Team Contribution

According to me, Fuzzy Systems are of utmost importance in tackling uncertain data, especially in medical cases where data is very uncertain and we need fast and precise outputs in order to save lives. While discussing this topic we decided and deduced the rules together by combining some data from online and our own knowledge. We derived the rules and built two systems for each Case.

In the study, I worked on developing the fuzzification approach of Case 1 and Case 2 for both models, my teammate Anasuya helped me with defuzzifying the output aggregate for both models and plotting the graph. Overall, I think we did a good job as a team.

Conclusion

In conclusion, I would like to express that Fuzzy Logic is the best approach when dealing with uncertain data. The models before fuzzy sets are used to be binary {0, 1}, and hence every problem could have had only 2 solutions whereas while using fuzzy logic we can get many different solutions and approaches to a single problem.

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