Image Gradients

Introduction

Gradient operator is mostly used type of edge detection process. The gradient of an image holds two pieces of information. The gradient magnitude and gradient direction informations are important quantaties to find edges in the image. The magnitude of an edge gives maximum rate of increase of (x, y) per unit distance in the direction of changing, while the direction of an edge at (x, y) is perpendicular to the direction of the gradient vector at that point.[1]

Mathematical Concept of Gradient

Mathematically, an image I, function $I(x, y)$, the gradient magnitude of the image is $g(x, y)$ and gradient direction $\theta(x, y)$. To form magnitude and direction, partial derivative of the image in both x direction and y direction. Partial derivate I with respect to x direction represents rate of change of image intensity is changed, by smilar reasoning, image intensity changing in y direction can be computed.

$$Gx = \frac{\partial I(x,y)}{\partial x} = \lim_{\substack{\Delta x \\ \rightarrow}} \frac{I(x+n,y) - I(x-n,y)}{\Delta x}$$

$$Gy = \frac{\partial I(x,y)}{\partial y} = \lim_{\substack{\Delta y \\ \rightarrow}} \frac{I(x,y+n) - I(x,y-n)}{\Delta y}$$

, n is usually unity.

By combining partial derivative of the image in the x and y direction, gradient vector of the image is written like that :

$$\nabla I = (Gx, Gy)$$

Using gradient vector, gradient magnitude and direction are calculated below equations:

$$g(x,y) = (\Delta x^2 + \Delta y^2)^{1/2}$$

and,

$$\theta(x,y) = atan(\frac{\Delta y}{\Delta x})$$

Properties and Interpretation of the Gradient for Edge Detection

A colored scale image is converted into the grayscale to intensity information and compute the directional change of the image in the x and y direction. Figure1 and Figure 2 show the colored and grayscale images, respectively.

Figure 1: Colored image. Source: Author.

Figure 2: Grayscale image. Source: Author.

Directional derivatives of the grayscale image are computed by using Sobel gradient operator. The results are shown in the Figure 3 and Figure 4, respectively. They show how much the gray levels in image change in the positive x and y directions, this change in the intensity is encoded in the gray level of the image of the horizontal and vertical components.

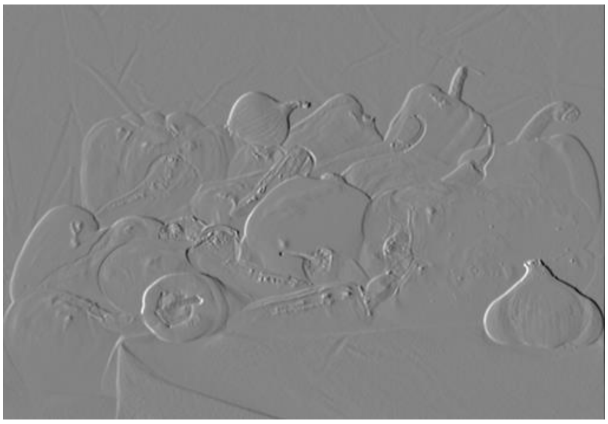

Figure 3: x direction gradient. Source: Author.

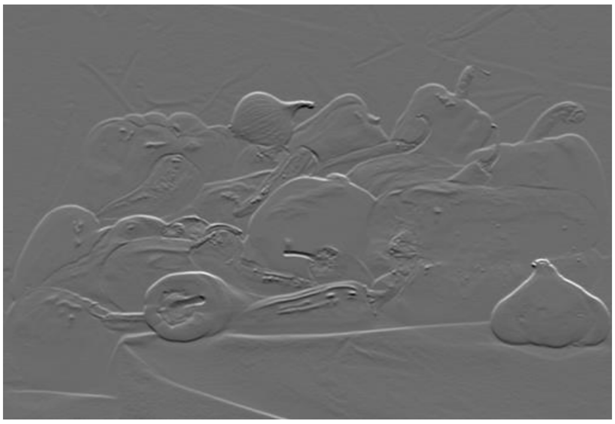

Figure 4: y direction gradient. Source: Author.

After directional changes of the image are computed, gradient magnitude and gradient direction can be found by using above mathamatical equations. (can be written number of the equation here). Again, Figure 5 and 6 represent magnitude and direction values as an image in the respect to minumum and maximum values. In the magnitude image edge pixels has more value than background and direction image edge pixels has less value than background (In the intensity image, the minumum pixel values are representing by black, on the contrary maximum values are representingby white as the same idea in the binary images).

Figure 5: Gradient magnitude. Source: Author

Figure 6: Gradient direction. Source: Author

To more explicity, magnitude and gradient analysis will be done in the specific range of image that includes edge and background. In addition to these images, pixel values will be added to explain the ralationship between edges and gradient. Figure 7 and 8 shows the selected subcorners. Figure 9 and Figure 10 represents the pixel values of the subcorners magnitude and direction, respectively.

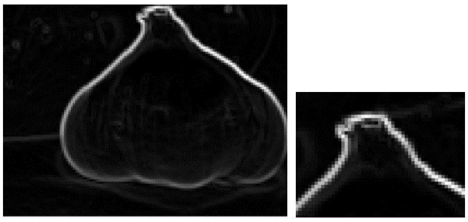

Figure 7: Magnitude image subcorners

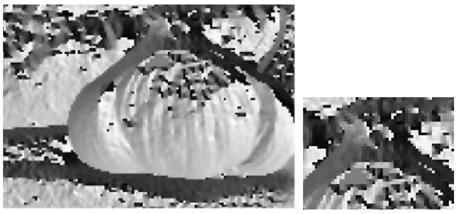

Figure 8: Direction image subcorners

	443	444	445	446	447	448	449	450	451	452	453
236	0.0124	0.0235	0.0620	0.1860	0.3529	0.7734	1.5087	2.0442	2.4299	2.8423	2.9597
237	0.0111	0.0447	0.0499	0.5354	1.2454	2.0940	2.6781	2.7278	2.4823	1.5518	0.6445
238	0.0314	0.2628	0.4638	0.5700	1.3526	1.9049	1.7933	1.1336	0.7058	0.7848	0.4510
239	0.0620	0.9308	1.7623	1.8142	1.1979	2.1452	1.2812	0.2753	0.1948	1.0511	1.4296
240	0.0808	1.4707	2.2887	2.1168	2.3171	1.7548	1.0083	0.2335	0.1577	0.1143	0.3959
241	0.0649	1.2973	2.5647	1.5651	0.3186	0.5259	0.4991	0.3666	0.2071	0.1569	0.1534
242	0.0675	0.5187	1.8737	2.8856	1.9090	0.4169	0.2611	0.4540	0.2544	0.1277	0.0832
243	0.0555	0.0299	0.3553	1.8683	2.4029	0.9157	0.3882	0.5222	0.2513	0.1502	0.0471
244	0.0277	0	0.0299	1.2293	2.2598	0.9922	0.5017	0.6163	0.1690	0.1403	0.0723
245	0.0323	0.0200	0.0277	1.2429	2.1904	0.9371	0.5669	0.6323	0.0941	0.1090	0.1004
246	0.0447	0.0477	0.1036	1.4590	2.2063	0.8932	0.4985	0.4749	0.1055	0.0889	0.1231

Figure 9: Pixel values of the magnitude subcorners. Source: Author

	443	444	445	446	447	448	449	450	451	452	453
236	-161.5651	-90	-55.3048	-55.3048	-75.1916	-63.8247	-66.5668	-81.8388	-80.4313	-76.9224	-86.1253
237	45.0000	-37.8750	-45.0000	-31.8274	-56.5601	-63.5309	-63.0973	-77.4623	-83.5599	-82.1566	-97.3412
238	-5.0690e-14	-37.7250	-71.2587	-34.3461	-9.6808	-25.6155	-42.3411	-61.4845	-104.8084	-167.0054	151.4293
239	-34.6952	-31.2121	-59.9546	-92.9737	-45.2653	-4.9289	-7.3860	-10.6698	-130.1009	-127.7250	-104.4556
240	-29.0546	-10.2912	-26.7846	-86.9203	-93.1045	-42.1014	-5.3558	40.9144	34.8753	-95.9061	-123.6901
241	-25.0169	19.4208	27.8973	33.9688	-76.4768	-72.6460	8.1301	34.5401	24.6236	36.8699	147.5288
242	-54.4623	38.8627	49.0735	46.9825	46.9979	73.6105	32.7352	19.6871	24.5901	42.5104	135.0000
243	-45.0000	-66.8014	47.6838	27.9098	25.7287	42.9174	45.0000	14.3493	22.9638	40.7636	90.0000
244	-45	0	-156.8014	3.1089	6.0767	17.7188	21.5519	2.5529	3.9909	26.5651	49.3987
245	-104.0362	-101.3099	-98.1301	-6.3402	-6.4761	-12.8149	-14.4208	-7.1250	-1.3517e-13	30.2564	38.6598
246	-105.2551	-99.4623	-60.5241	-9.4370	-8.7926	-20.0261	-24.1455	-2.3662	41.9872	48.5763	37.2348

Figure 10: Pixel values of the direction subcorners. Source: Author

The edges can be predicted by pixel value, higher values in the magnitude image and less change over 3x3 adjasent neighbourhood in the gradient direction. For instance, at the (242,446) pixel, in the magnitude image has 2.8856 that is closer to the maximum value as shown Figure 11,

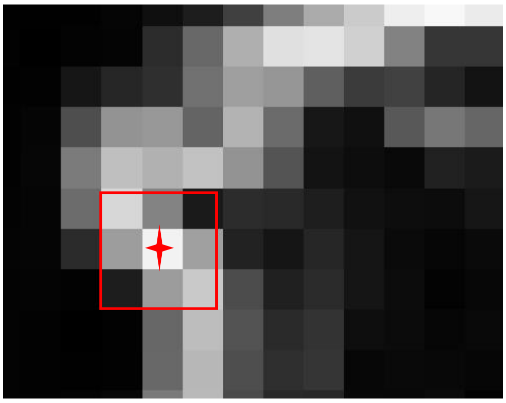

Figure 11: 3x3 adjasent neighbourhood

and on the direction image side, differences between the center pixel and other pixels are less in the direction of edge that is D1 for that pixel value like that:

$$\begin{bmatrix} 27.8973 & 33.9688 & -76.4768 \\ 49.0735 & \mathbf{46.9825} & 46.9979 \\ 47.6838 & 27.9098 & 25.7287 \end{bmatrix} \longrightarrow \text{D1}$$

Proposed Algorithm for Edge Detection

Using gradient of image and fuzzy logic system to highlight all edges in an image. Gradient magnitude and gradient direction informations are used as inputs of the Mamdani Fuzzy Inference System. All inputs are mapped to the range of 0 to 100. The output of this system is to make decision about result pixel's value that can be in the range 0 to 255 and three fuzzy sets black, gray and white. The gaussian membership functions are used both for the inputs and the output.

In Figure 12, first input of the FIS is represented. This input is obtained from gradient magnitude and all fuzzy sets are designed by our intuation.

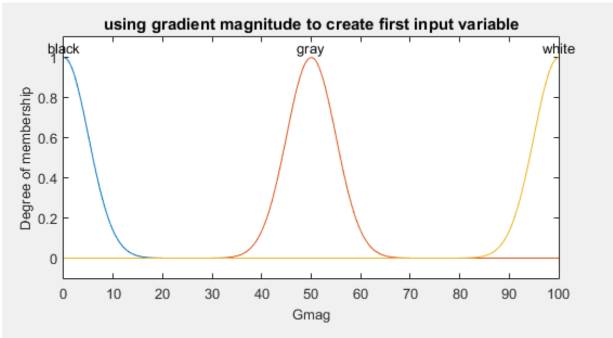

Figure 12: First input

Other inputs of the system are acquired by using 3x3 sliding window on the gradient direction. In the Figure 13, over the 3x3 sample matrix for the central pixel has 8 neighbourhoods and 4 possible edge directions as shown....

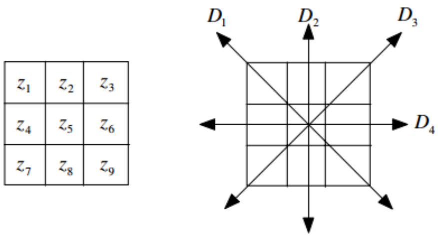

Figure 13: Coefficents and possible edge directions for 3x3 sample matrix. Source:...

$$z_1 = I(x-1, y-1) \quad z_2 = I(x-1, y) \quad z_3 = I(x-1, y+1)$$

$$z_4 = I(x, y-1) \; z_5 = I(x, y) \; z_6 = I(x, y+1)$$

$$z_7 = I(x-1, y+1) \; z_8 = I(x, y) \; z_9 = I(x+1, y+1)$$

Output of the sytem and fuzzy sets are shown in the Figure 14. Centroid method is used as a defuzzification method to obtain final pixel value.

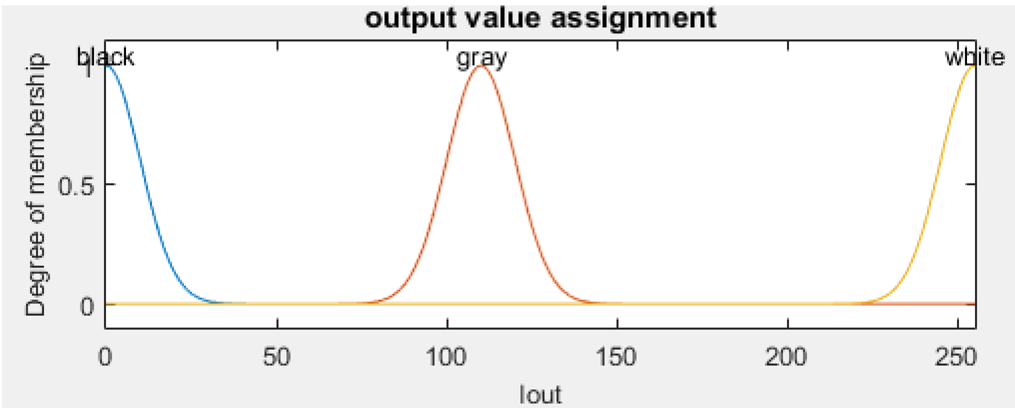

Figure 14: Outpu fuzzy sets

References:

[1] Gonzalez, R.C., and Woods, R.E., Digital Image Processing, Addison- Wesley, Reading, M.A., 1992.