## Common tests are linear models

See worked examples and more details at the accompanying notebook: <a href="https://lindeloev.github.io/tests-as-linear">https://lindeloev.github.io/tests-as-linear</a>

	Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
Simple regression: Im(y ~ 1 + x)	y is independent of x P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	Im(y ~ 1) Im(signed_rank(y) ~ 1)	√ for N >14	One number (intercept, i.e., the mean) predicts <b>y</b> (Same, but it predicts the <i>signed rank</i> of <b>y</b> .)	<u>;;</u>
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y <sub>1</sub> , y <sub>2</sub> , paired=TRUE) wilcox.test(y <sub>1</sub> , y <sub>2</sub> , paired=TRUE)	$Im(y_2 - y_1 \sim 1)$ $Im(signed_rank(y_2 - y_1) \sim 1)$	√ f <u>or N &gt;14</u>	One intercept predicts the pairwise $y_2$ - $y_1$ differences (Same, but it predicts the <i>signed rank</i> of $y_2$ - $y_1$ .)	<b>*</b>
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	$Im(y \sim 1 + x)$ $Im(rank(y) \sim 1 + rank(x))$	√ for N >10	One intercept plus <b>x</b> multiplied by a number (slope) predicts <b>y</b> .  - (Same, but with <i>ranked</i> <b>x</b> and <b>y</b> )	بعببب
	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y <sub>1</sub> , y <sub>2</sub> , var.equal=TRUE) t.test(y <sub>1</sub> , y <sub>2</sub> , var.equal=FALSE) wilcox.test(y <sub>1</sub> , y <sub>2</sub> )	$Im(y \sim 1 + G_2)^A$ $gls(y \sim 1 + G_2, weights=^B)^A$ $Im(signed_rank(y) \sim 1 + G_2)^A$	√ √ for N >11	An intercept for <b>group 1</b> (plus a difference if <b>group 2</b> ) predicts <b>y</b> .  - (Same, but with one variance <i>per group</i> instead of one common.)  - (Same, but it predicts the <i>signed rank</i> of <b>y</b> .)	+
X <sub>2</sub> +)	P: One-way ANOVA N: Kruskall-Wallis	aov(y ~ group) kruskal.test(y ~ group)	$Im(y \sim 1 + G_2 + G_3 + + G_N)^A$ $Im(rank(y) \sim 1 + G_2 + G_3 + + G_N)^A$	√ for N >11	An intercept for <b>group 1</b> (plus a difference if group ≠ 1) predicts <b>y</b> .  - (Same, but it predicts the <i>signed rank</i> of <b>y</b> .)	i <sub>4</sub> ++
Multiple regression: Im(y ~ 1 + x₁ +	P: One-way ANCOVA	aov(y ~ group + x)	Im(y ~ 1 + $G_2$ + $G_3$ ++ $G_N$ + x) <sup>A</sup>	✓	- (Same, but plus a slope on x.)  Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.	
	P: Two-way ANOVA	aov(y ~ group * sex)	$Im(y \sim 1 + G_2 + G_3 + + G_N + S_2 + S_3 + + S_K + G_2*S_2+G_3*S_3++G_N*S_K)$	1	Interaction term: changing <b>sex</b> changes the $\mathbf{y} \sim \mathbf{group}$ parameters.  Note: $\mathbf{G}_{2 \text{ to } N}$ is an <u>indicator (0 or 1)</u> for each non-intercept levels of the <b>group</b> variable. Similarly for $\mathbf{S}_{2 \text{ to } K}$ for sex. The first line (with $\mathbf{G}_i$ ) is main effect of group, the second (with $\mathbf{S}_i$ ) for sex and the third is the <b>group</b> × <b>sex</b> interaction. For two levels (e.g. male/female), line 2 would just be " $\mathbf{S}_2$ " and line 3 would be $\mathbf{S}_2$ multiplied with each $\mathbf{G}_i$ .	[Coming]
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	Equivalent log-linear model $glm(y \sim 1 + G_2 + G_3 + + G_N + G_2 + G_3 + + S_K + G_2*S_2+G_3*S_3++G_N*S_K, family=)^A$	~	Interaction term: (Same as Two-way ANOVA.)  Note: Run glm using the following arguments: $glm(model, family=poisson())$ As linear-model, the Chi-square test is $log(y_i) = log(N) + log(\alpha_i) + log(\beta_i) + log(\alpha_i\beta_i)$ where $\alpha_i$ and $\beta_i$ are proportions. See more info in the accompanying notebook.	Same as Two-way ANOVA
M	N: Goodness of fit	chisq.test(y)	glm(y ~ 1 + $G_2$ + $G_3$ ++ $G_N$ , family=) <sup>A</sup>	✓	(Same as One-way ANOVA and see Chi-Square note.)	1W-ANOVA

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation  $y \sim 1 + x$  is R shorthand for  $y = 1 \cdot b + a \cdot x$  which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they all are! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is  $signed_x = function(x) \cdot sign(x) \cdot rank(abs(x))$ . The variables  $G_i$  and  $G_i$  are "dummy coded" indicator variables (either 0 or 1) exploiting the fact that when  $\Delta x = 1$  between categories the difference equals the slope. Subscripts (e.g.,  $G_2$  or  $y_1$ ) indicate different columns in data. Im requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at <a href="https://lindeloev.github.io/tests-as-linear">https://lindeloev.github.io/tests-as-linear</a>.



<sup>&</sup>lt;sup>A</sup> See the note to the two-way ANOVA for explanation of the notation.

<sup>&</sup>lt;sup>B</sup> Same model, but with one variance per group:  $gls(value \sim 1 + G_2, weights = varIdent(form = ~1 | group), method="ML")$ .