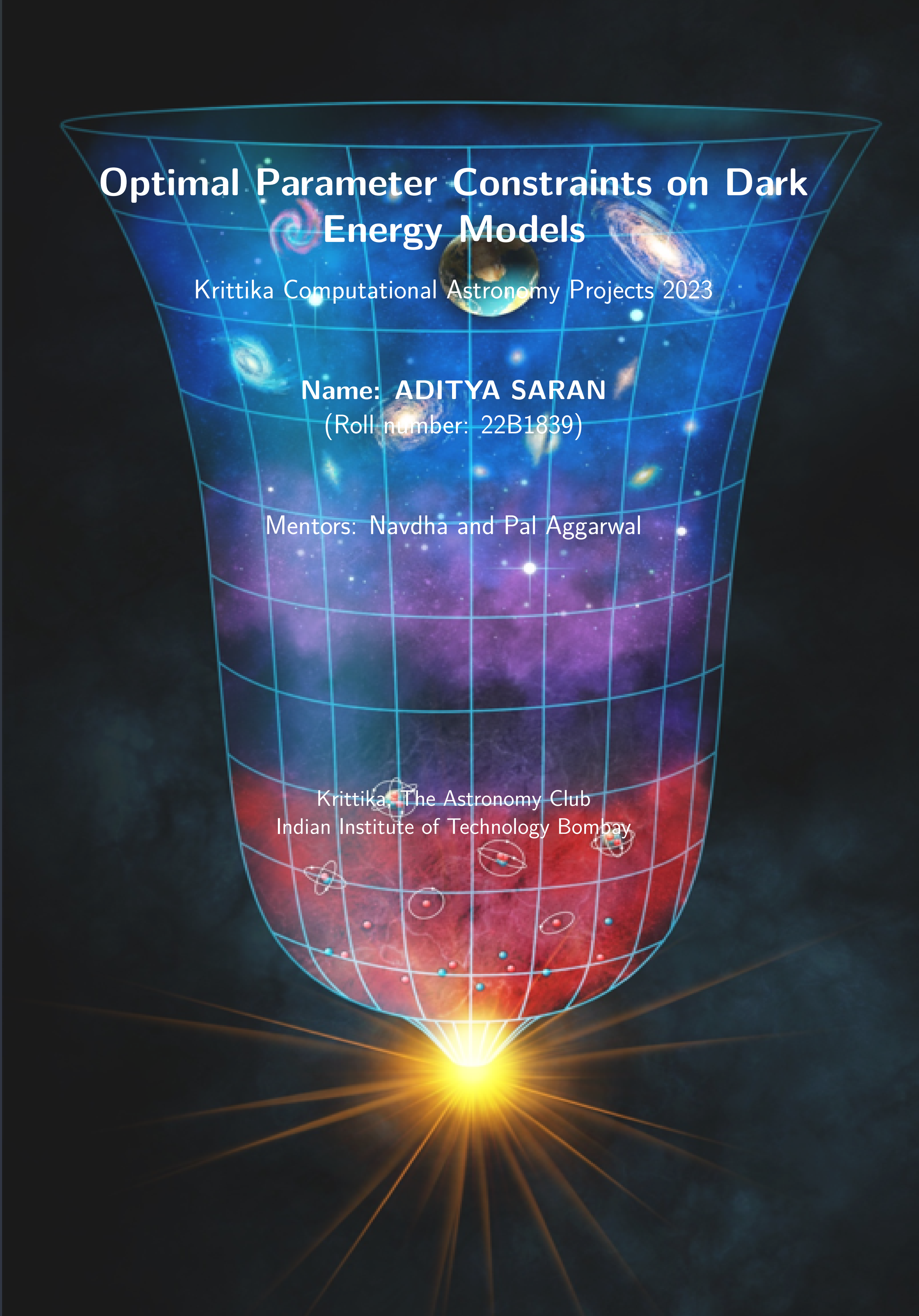

Optimal Parameter Constraints on Dark Energy Models

Krittika Computational Astronomy Projects 2023

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1 Introduction

1.1 Cosmology

Cosmology, an exciting realm within astronomy, delves into the intricate origins and progression of the universe, spanning from its primordial Big Bang to its current state and beyond. Throughout history, our comprehension of the universe has undergone remarkable transformations.

Early astronomical beliefs centered around Earth as the pivotal point of existence, with celestial bodies orbiting it. The groundbreaking theories of Polish scientist Nicolas Copernicus in the 16th century revolutionized this perception, proposing that Earth and the planets revolve around the sun, known as the heliocentric model, fundamentally altering our cosmic understanding.

Isaac Newton's 17th-century calculations of gravitational forces between planets further enriched this evolving narrative. Edwin Hubble's groundbreaking calculations of distant celestial entities, proving their existence beyond our Milky Way, unveiled the vastness of our universe. His use of General Relativity unveiled a universe not static but expanding, with galaxies hurtling away from us.

Pioneers like Stephen Hawking, leveraging concepts like General Relativity, contributed to this journey, unveiling the universe's finite nature, debunking the notion of infinity.

The study of Cosmology stands as a beacon of curiosity, inviting exploration into the profound questions that have captivated humankind for ages. By merging disciplines like astronomy and physics, Cosmology offers a unified approach to unravel the mysteries of our physical universe, continually broadening its horizons and propelling the quest for fundamental truths.

1.2 Metric

In the realm of mathematics and geometry, metrics serve as fundamental tools to measure and define distances, angles, and other properties within spaces. These mathematical structures provide a way to quantify notions of 'closeness' or 'distance' between elements in a set. Metrics play a pivotal role in various fields, from pure mathematics, where they help define spaces like Euclidean or non-Euclidean spaces, to applications in physics, computer science, and beyond. They enable precise calculations and formulations, offering a framework to explore the geometric properties of spaces and objects, making them a cornerstone in diverse areas of study and practical applications.

1.3 Friedmann Equation

The Friedmann equations are a set of fundamental equations in cosmology, devised by Alexander Friedmann based on Einstein's theory of general relativity. These equations form the cornerstone of our understanding of the dynamics and evolution of the universe on a large scale.

At their core, the Friedmann equations describe how the universe's expansion rate evolves over time by relating the rate of expansion (Hubble parameter), the energy density of the universe, and the curvature of space. They are essential in modeling the behavior of the cosmos and predicting its fate.

The equations emerge from Einstein's field equations of general relativity when applied to a homogeneous and isotropic universe, a reasonable assumption based on observations suggesting the universe's uniformity on large scales. These equations are:

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G\rho(t)}{3} + \frac{2U}{r_s^2}(\frac{1}{a(t)})^2 \tag{1}$$

The solutions derived from these equations indicate possible scenarios for the universe's fate: expansion, contraction, or a steady state. They have also played a crucial role in establishing the framework for the Big Bang theory, providing insights into the universe's early moments and its subsequent evolution

1.4 Redshift

2 Cosmological Models

In this project, to explain the constraints on dark energy dynamics we studied different cosmological models stated as follows:

2.1 ΛCDM model

The ΛCDM model stands as the prevailing cosmological framework describing the composition and behavior of the universe. It combines two key components: Λ, representing the cosmological constant associated with dark energy, and CDM (cold dark matter).

In this model, the universe is envisioned as predominantly composed of dark energy, denoted by Λ, which drives its accelerated expansion. This mysterious form of energy generates a repulsive force, counteracting gravity's attractive pull and contributing to the universe's observed expansion rate. The expansion rate can be written in terms of the density parameters as:

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \Omega_\Lambda)(1+z)^2 + \Omega_\Lambda} \tag{2}$$

Here, Ω_{m0} is the current value of the non-relativistic matter density parameter, Ω_Λ is the cosmological constant energy density parameter.

If we consider a flat universe, then the $\Omega_\Lambda$is taken to be 0. Additionally, the model incorporates CDM, a type of matter that interacts weakly with electromagnetic forces and behaves non-relativistically (or "coldly") at cosmic scales. Although invisible and elusive to direct detection, the presence of cold dark matter is inferred through its gravitational effects on galaxies and cosmic structures.

2.2 XCDM Model

The XCDM model expands upon the ΛCDM model by introducing a more versatile framework to explore various forms of dark energy beyond the cosmological constant (Λ). In this model, the "X" represents a dynamic component of dark energy characterized by an equation of state (EoS) parameter, usually denoted as w. In this model the expansion rate is written as:

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{k0}(1+z)^2 + \Omega_{X0}(1+z)^{3(1+w_x)}} \tag{3}$$

Unlike the fixed nature of Λ, the XCDM model allows for a variable equation of state for dark energy, enabling exploration into different behaviors of this energy component throughout cosmic history. The parameter "w" governs the ratio of pressure to energy density for dark energy and determines how it influences the universe's expansion rate.

By adjusting the value of the equation of state parameter "w," the XCDM model accommodates a broader range of scenarios for dark energy, such as quintessence (w $>$ -1), phantom energy (w $<$ -1), or even scenarios where dark energy's behavior changes over time. When w = -1, XCDM model becomes ΛCDM model. This flexibility enables cosmologists to test various theoretical predictions against observational data to better understand the nature and dynamics of dark energy in shaping the universe's evolution.

2.3 ϕCDM Model

ϕCDM models are cosmological frameworks extending beyond the standard ΛCDM model. Here, ϕ represents a scalar field, introducing dynamism to dark energy, unlike the constant nature of Λ in the ΛCDM model. This scalar field, similar to quintessence models, possesses a varying energy density that evolves with the expansion of the universe. Here,potential energy density is given by:

$$V(\phi) = \frac{1}{2}\kappa m_p^2 \phi^\alpha \tag{4}$$

Here $\alpha > 0, m_p^2 = G^{-1}$, where G is a gravitational constant and

$$\kappa = \frac{8}{3}\left(\frac{\alpha+4}{\alpha+2}\right)\left[\frac{2}{3}\alpha(\alpha+2)\right]^{\alpha/2} \tag{5}$$

In ϕCDM, the scalar field introduces an additional degree of freedom, influencing the universe's expansion rate through its dynamics. The behavior of the scalar field is often characterized by its potential function, dictating how it evolves over cosmic time.

These models explore scenarios where dark energy's behavior deviates from a cosmological constant, allowing for a wider range of possible dynamics and addressing some shortcomings of the ΛCDM model. The investigation into ϕCDM models aims to understand the nature of dark energy and its potential role in shaping the universe's evolution.

3 Data

I have used 31 H(z) data extracted from various research paper.

Table 1: Observational Data(H(z) and σ_H have units of $kms^{-1}Mpc^{-1}$)

Redshift (z)	Hubble Parameter (Hz)	Error(σ_H)
0.07	69.0	19.6
0.09	69.0	12.0
0.12	68.6	26.2
0.17	83.0	8.0
0.179	75.0	4.0
0.199	75.0	5.0
0.20	72.9	29.6
0.27	77.0	14.0
0.28	88.8	36.6
0.352	83.0	14.0
0.3802	83.0	13.5
0.4	95.0	17.0
0.4004	77.0	10.2
0.4247	87.1	11.2
0.4497	92.8	12.9
0.47	89.0	50.0
0.4783	80.9	9.0
0.48	97.0	62.0
0.593	104.0	13.0
0.68	92.0	8.0
0.781	105.0	12.0
0.875	125.0	17.0
0.88	90.0	40.0
0.90	117.0	23.0
1.037	154.0	20.0
1.3	168.0	17.0
1.363	160.0	33.6
1.43	177.0	18.0
1.53	140.0	14.0
1.75	202.0	40.0
1.965	186.5	50.4

It was known that the first six BAO data is correlated and also the last two data is correlated and the corelation matrix of those are given below:

$$\begin{bmatrix} 624.707 & 23.729 & 325.332 & 8.34963 & 157.386 & 3.57778 \\ 23.729 & 5.60873 & 11.6429 & 2.33996 & 6.39263 & 0.968056 \\ 325.332 & 11.6429 & 905.777 & 29.3392 & 515.271 & 14.1013 \\ 8.34963 & 2.33996 & 29.3392 & 5.42327 & 16.1422 & 2.85334 \\ 157.386 & 6.39263 & 515.271 & 16.1422 & 1375.12 & 40.4327 \\ 3.57778 & 0.968056 & 14.1013 & 2.85334 & 40.4327 & 6.25936 \end{bmatrix}$$

(6)

$$\begin{bmatrix} 0.0841 & -0.183396 \\ -0.183396 & 3.4596 \end{bmatrix} \tag{7}$$

Table 2: BAO Data

Redshift (z)	Measurement	Value	Reference
0.38	$D_M(r_{s,fid}/r_s)$	1512.39	Alam et al. (2017)
0.38	$H(z)(r_s/rs, fid)$	81.2087	Alam et al. (2017)
0.51	$D_M(r_{s,fid}/r_s)$	1975.22	Alam et al. (2017)
0.51	$H(z)(r_s/r_{s,fid})$	90.9029	Alam et al. (2017)
0.61	$D_M(r_{s,fid}/r_s)$	2306.68	Alam et al. (2017)
0.61	$H(z)(r_s/rs, fid)$	98.9647	Alam et al. (2017)
0.122	$D_V(r_{s,fid}/r_s)$	539 $\pm$ 17	Carter et al. (2018)
0.81	D_A/r_s	10.75 $\pm$ 0.43	DES Collaboration (2019)
1.52	$D_V(r_{s,fid)}/r_s$	3843 $\pm$ 147	Ata et al. (2018)
2.34	D_H/r_s	8.86	de Sainte Agathe et al. (2019)
2.34	D_M/r_s	37.41	de Sainte Agathe et al. (2019)

4 Method

Chi-Minimization:

In cosmological parameter estimation, one common approach is to find the best-fit values for parameters by minimizing the chi-squared (χ^2) statistic. The chi-squared is a measure of the goodness of fit between a theoretical model and observational data.

The theoretical model is used to predict the expected values of observables (e.g., Hubble parameter, BAO data) for a given set of cosmological parameters. The chi-squared statistic is then calculated by comparing the predicted values with the observed values, considering uncertainties in the observations.

$$\chi^2 = \Sigma((O_i - E_i)^2/\sigma_i^2)$$

where O_i is the observed value, $E_i$is the expected (model-predicted) value, and σ_i is the uncertainty associated with the observation.

Minimization:

The parameter values are adjusted iteratively to minimize the chi-squared statistic. This is often done using optimization algorithms (such as the Levenberg-Marquardt algorithm or the scipy.optimize.minimize function in Python) that systematically explore the parameter space to find the minimum χ^2 Best-Fit Parameters: The parameter values that result in the minimum χ^2 are considered the best-fit parameters for the given model.

Markov Chain Monte Carlo (MCMC) Methods:

MCMC methods are powerful statistical techniques used for exploring and sampling from complex probability distributions. In the context of cosmological parameter estimation, MCMC is employed to obtain probability distributions of model parameters. MCMC methods aim to explore the parameter space by generating a Markov chain of samples, where each sample corresponds to a set of parameter values. The distribution of these samples provides information about the likelihood of different parameter values.

In the context of the project, the emcee library in Python is used for MCMC analysis. This library efficiently implements the Ensemble Sampler, a variant of MCMC, to explore the parameter space.

By combining chi-minimization and MCMC methods, the project aims to determine optimal parameter constraints for different cosmological models based on observational data. These methods allow for a robust statistical analysis that considers uncertainties and explores the entire parameter space.

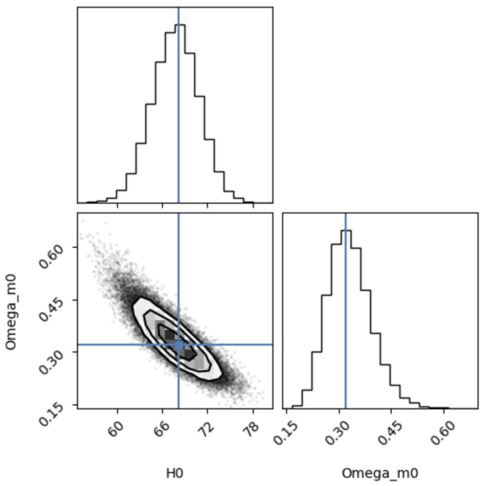
5 Results

Insert a table, and qualitatively explain your results [**?**]

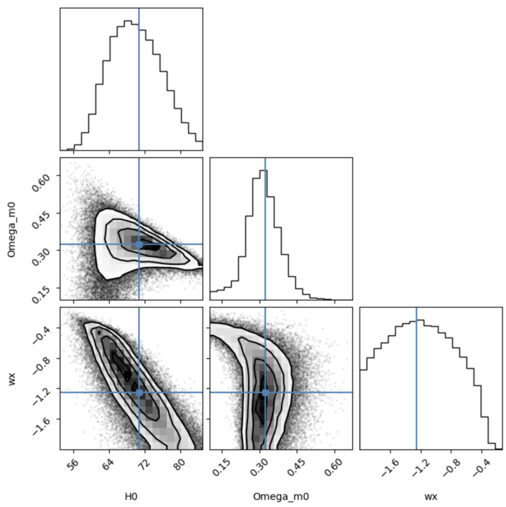
5.1 H(z) constraints

Table 3: Observational Data (H(z) and σ_H have units of $kms^{-1}Mpc^{-1}$)

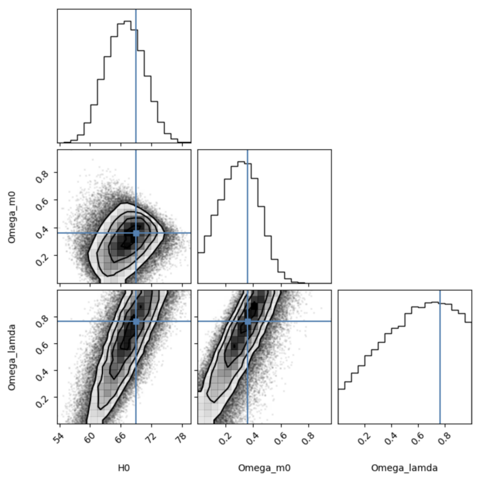
Models	**Hubble constant (H_0)**	Ω_{m0}	Ω_Λ	Ω_k	w_x
Flat ΛCDM Model	$67.786^{+3.052}_{-3.122}$	$0.326^{+0.066}_{-0.055}$	-	-	-
Non-Flat ΛCDM Model	$66.799^{+3.754}_{-3.829}$	$0.301^{+0.139}_{-0.151}$	$0.601^{+0.262}_{-0.322}$	-	-
Flat XCDM Model	$69.610^{+6.375}_{-5.658}$	$0.311^{+0.067}_{-0.060}$	-	-	$-1.210^{+0.501}_{-0.488}$
Non-Flat XCDM Model	$65.347^{+5.905}_{-4.457}$	$0.188^{+0.102}_{-0.107}$	-	$0.405^{+0.348}_{-0.276}$	$-1.231^{+0.642}_{-0.529}$



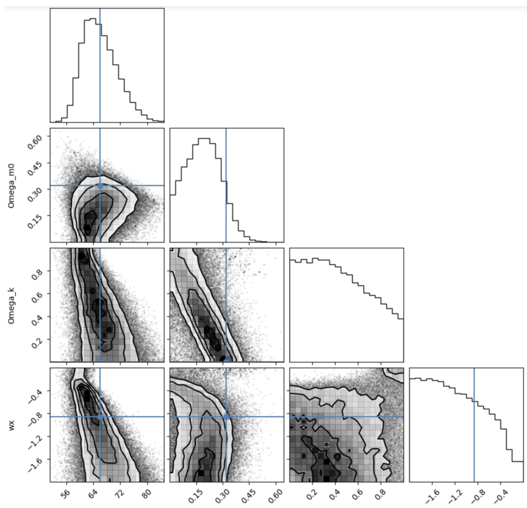
(a) Contour plot of flat LCDM model



(b) Contour plot of flat XCDM model



(c) Contour plot of non-flat LCDM model



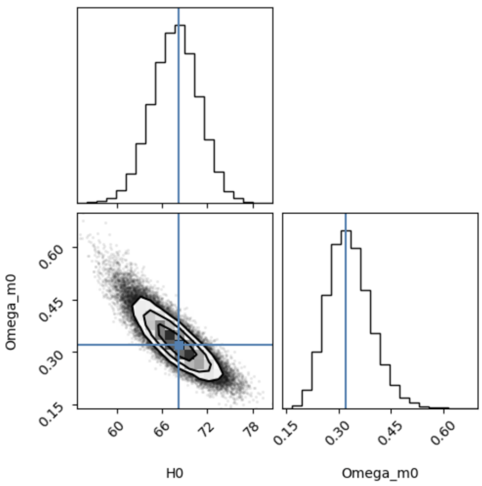
(d) Contour plot of non-flat XCDM model

Figure 1: Contour plots of different cosmological models

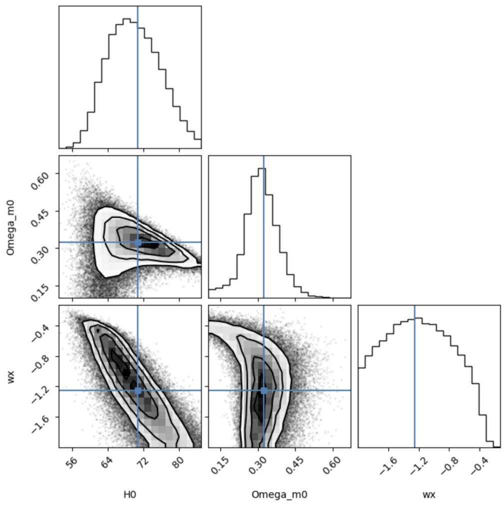
5.2 H(z) + BAO constraints

Table 4: Observational Data (H(z) and σ_H have units of $kms^{-1}Mpc^{-1}$)

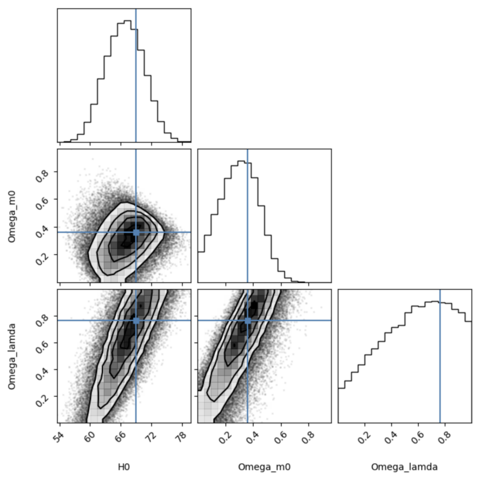
Models	**Hubble constant (H_0)**	Ω_{m0}	Ω_Λ	Ω_k	w_x
Flat ΛCDM Model	$68.035^{0.679}_{-0.677}$	$0.315^{0.014}_{-0.013}$	-	-	-
Non-Flat ΛCDM Model	$67.312^{1.483}_{-1.505}$	$0.316^{0.014}_{-0.013}$	$0.651^{0.064}_{-0.071}$	-	-
Flat XCDM Model	$66.485^{2.477}_{-2.045}$	$0.319^{0.015}_{-0.015}$	-	-	$-0.899^{0.125}_{-0.157}$
Non-Flat XCDM Model	$66.793^{2.752}_{-2.219}$	$0.314^{0.017}_{-0.017}$	-	$0.067^{0.067}_{-0.046}$	$-1.028^{0.180}_{-0.263}$



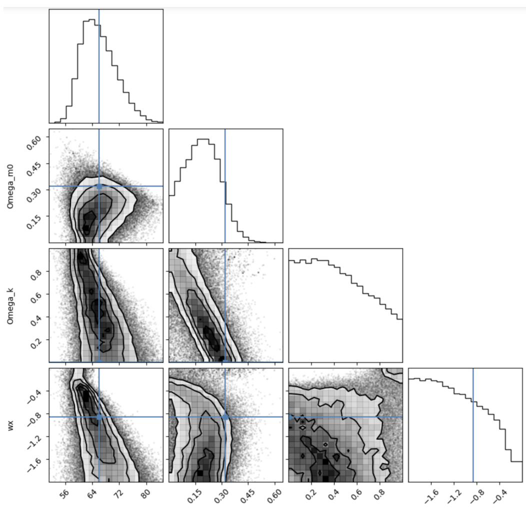
(a) Contour plot of flat LCDM model



(b) Contour plot of flat XCDM model



(c) Contour plot of non-flat LCDM model



(d) Contour plot of non-flat XCDM model

Figure 2: Contour plots of different cosmological models

6 Conclusion

Our mission: to understand the universe's accelerated expansion beyond the traditional ΛCDM and XCDM models. This project served as our launchpad, using observational data (Hubble parameter and Baryon Acoustic Oscillations) to constrain these models' parameters.

First, we built a foundation of knowledge. The Friedmann equations, distance measures, and MCMC analysis became our tools to navigate the cosmos. Armed with this theoretical framework, we delved into the practical phase.

Python's emcee library became our spaceship, guiding us through MCMC simulations to constrain parameters in various ΛCDM models (flat and non-flat) using H(z) data. We even explored chi-squared minimization with Scipy.optimize.minimize, honing our parameter optimization skills.

But the universe whispers its secrets from multiple sources. We couldn't stop at H(z) data. Incorporating BAO data brought even greater clarity, refining our understanding and parameter estimates. MCMC once again played its key role, helping us map out uncertainties and confidence intervals with precision.

This wasn't just a scientific quest; it was a learning journey. We grappled with the complexities of cosmological parameter estimation, realizing the power of diverse datasets for robust results. References became our fuel, immersing us in theory, data, and statistics.

The journey doesn't end here. New datasets like higher-Redshift gamma-ray bursts, H II starburst galaxies, and quasar data beckon. The ϕ-CDM model could even become our next target, depending on our discoveries.

This project wasn't just about the universe; it was about equipping ourselves. We honed our Python programming, data analysis, and statistical skills within the awe-inspiring realm of cosmology. More importantly, this hands-on experience instilled a deeper appreciation for the universe's intricate wonders, leaving us forever fascinated by its vastness and mysteries.

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