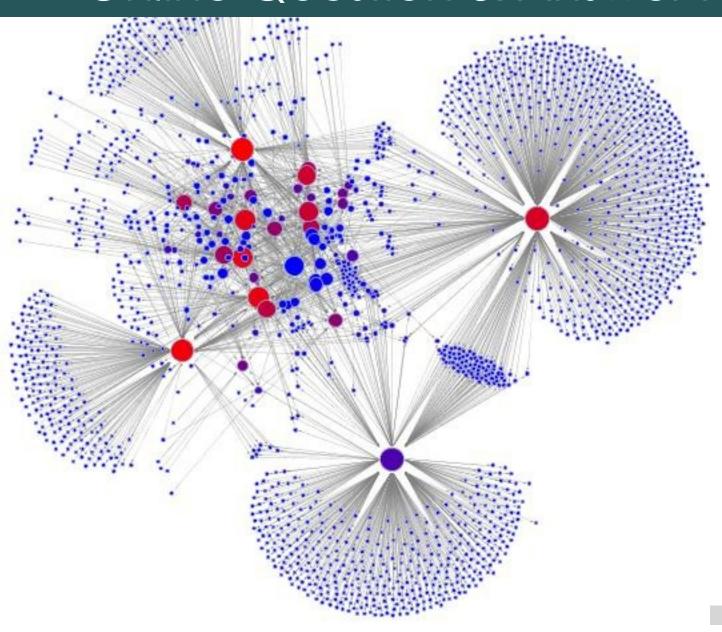


# SNA 2C: Growth & Preferential Attachment Models

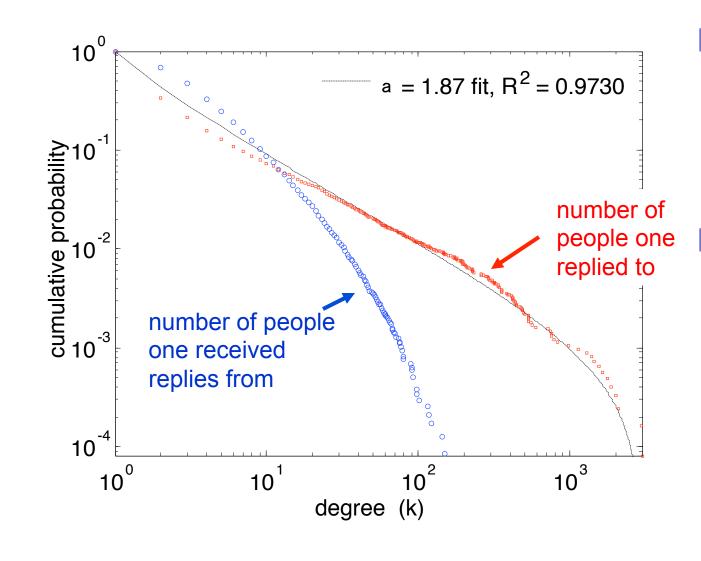
Lada Adamic



### Online Question & Answer Forums



### **Uneven participation**

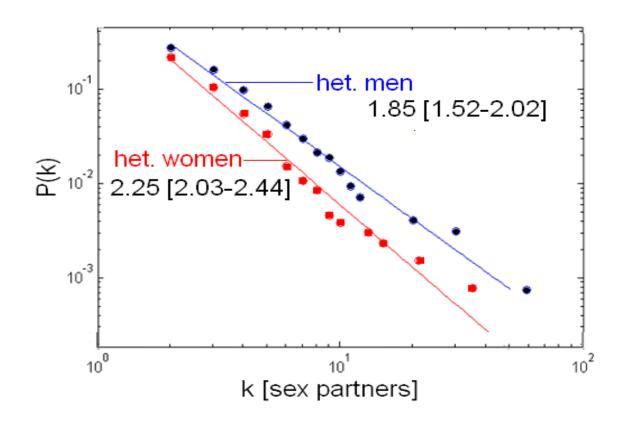


- answer people' may reply to thousands of others
- 'question people' are also uneven in the number of repliers to their posts, but to a lesser extent

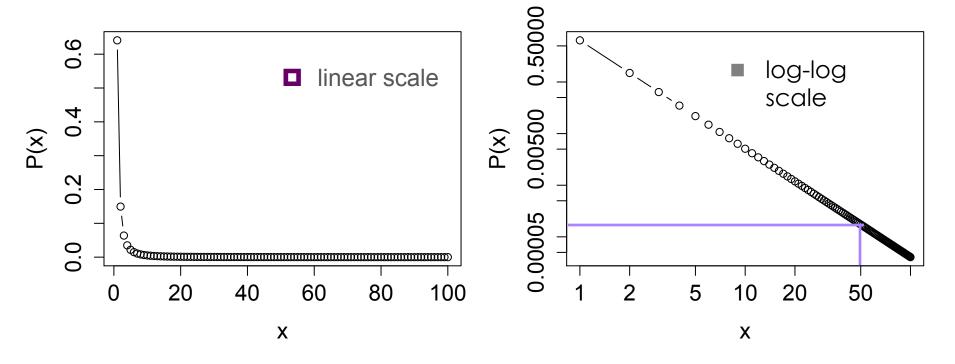
### Real-world degree distributions

■ Sexual networks

■ Great variation in contact numbers

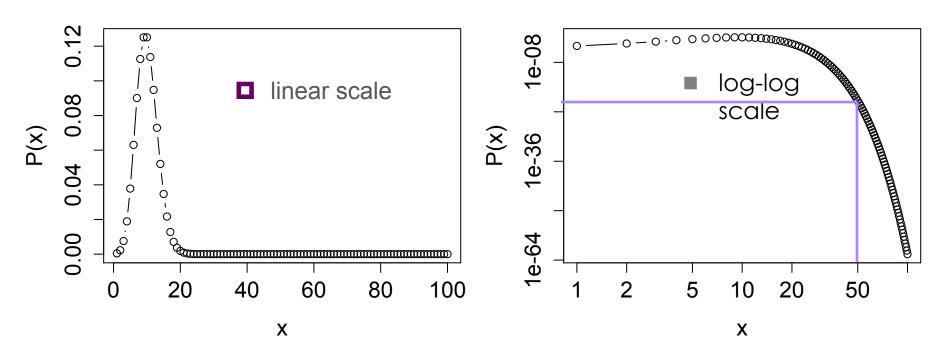


### Power-law distribution



- high skew (asymmetry)
- straight line on a log-log plot

### Poisson distribution



- little skew (asymmetry)
- curved on a log-log plot

#### Power law distribution

■ Straight line on a log-log plot

$$ln(p(k)) = c - \alpha \ln(k)$$

Exponentiate both sides to get that p(k), the probability of observing an node of degree 'k' is given by

$$p(k) = Ck^{-\alpha}$$

normalization constant (probabilities over all *k* must sum to 1)

power law exponent a

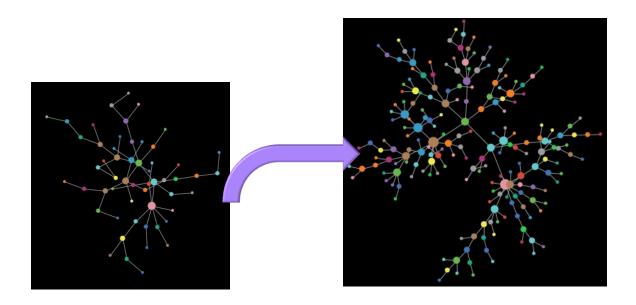
### Quiz Q:

- lacktriangle As the exponent lpha increases, the downward slope of the line on a log-log plot
  - stays the same
  - becomes milder
  - becomes steeper

$$ln(p(k)) = c - \alpha \ln(k)$$

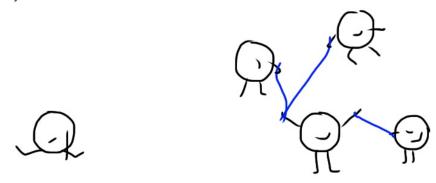
### 2 ingredients in generating power-law networks

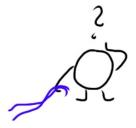
□ nodes appear over time (growth)



### 2 ingredients in generating power-law networks

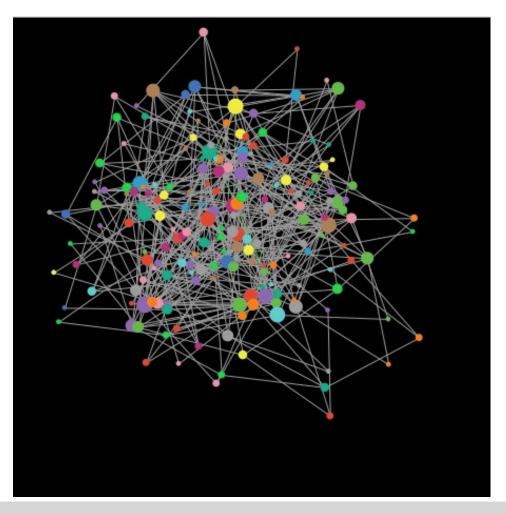
nodes prefer to attach to nodes with many connections (preferential attachment, cumulative advantage)





### Ingredient # 1: growth over time

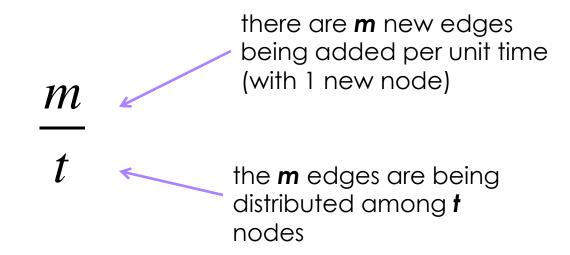
■ nodes appear one by one, each selecting *m* other nodes at random to connect to



m = 2

### random network growth

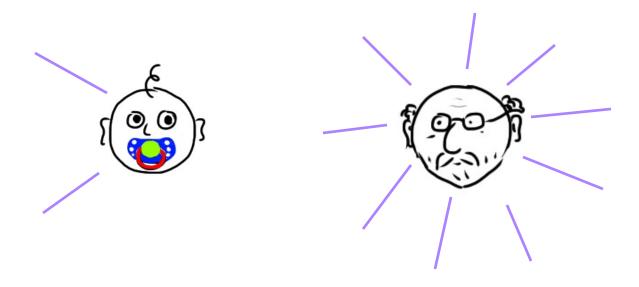
- one node is born at each time tick
- **a**t time *t* there are *t* nodes
- change in degree  $k_i$  of node i (born at time i, with 0 < i < t)



### age and degree

on average 
$$k_i(t) > k_j(t)$$
 
$$i < j$$

i.e. older nodes on average have more edges



### Quiz Q:

- How could one make the growth model more realistic for social networks?
  - old nodes die
  - some nodes are more sociable
  - friendships vane over time
  - □ all of the above

### 2<sup>nd</sup> ingredient: preferential attachment

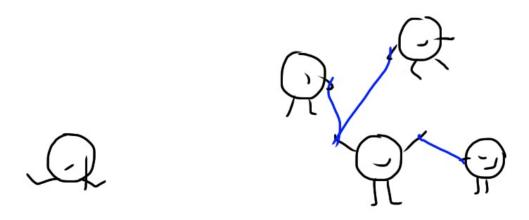
- Preferential attachment:
  - new nodes prefer to attach to well-connected nodes over less-well connected nodes

- Process also known as
  - cumulative advantage
  - □ rich-get-richer
  - Matthew effect

### Price's preferential attachment model for citation networks

- [Price 65]
  - each new paper is generated with m citations (mean)
  - new papers cite previous papers with probability proportional to their indegree (citations)
  - what about papers without any citations?
    - each paper is considered to have a "default" citation
    - probability of citing a paper with degree k, proportional to k+1
- $\blacksquare$  Power law with exponent  $\alpha = 2+1/m$

### Preferential attachment





### Cumulative advantage: how?

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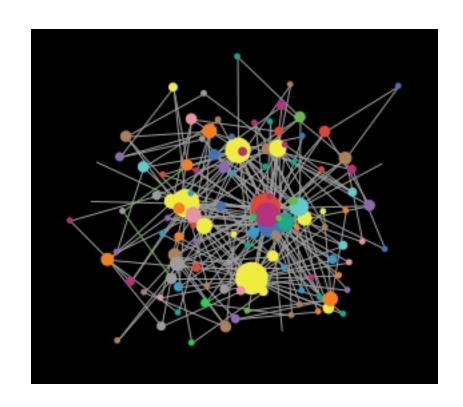
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- copying mechanism
- visibility

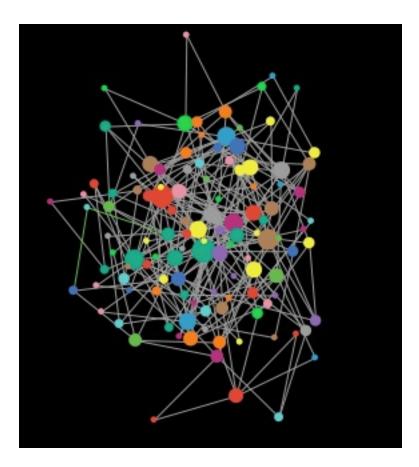
#### Barabasi-Albert model

- First used to describe skewed degree distribution of the World Wide Web
- Each node connects to other nodes with probability proportional to their degree
  - the process starts with some initial subgraph
  - each new node comes in with m edges
  - probability of connecting to node i
- $\blacksquare$  Results in power-law with exponent  $\alpha = 3$

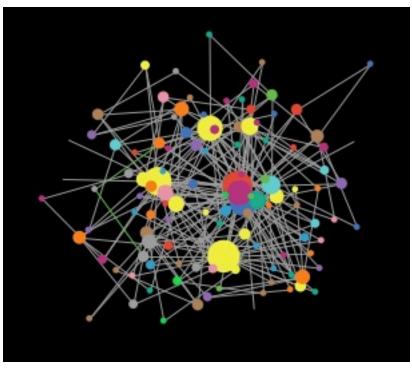
### after a while...



## contrasting with random (non-preferential) growth



m = 2



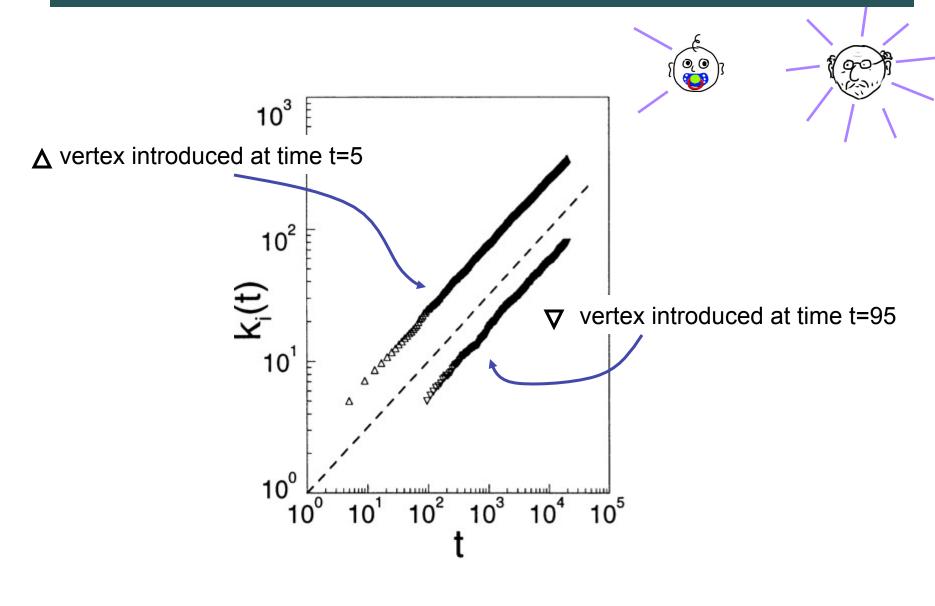
random

preferential

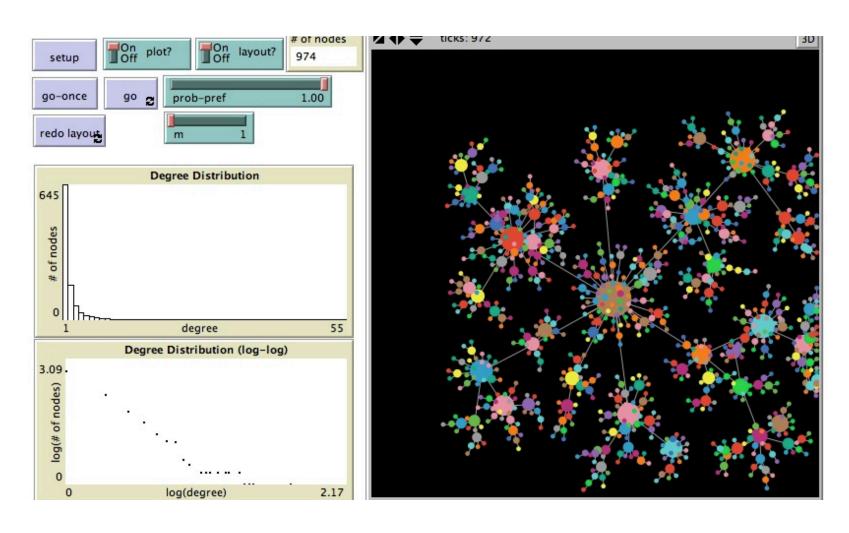
### Properties of the BA graph

- The distribution is scale free with exponent α = 3 $P(k) = 2 \text{ m}^2/\text{k}^3$
- The graph is connected
  - Every new vertex is born with a link or several links (depending on whether m = 1 or m > 1)
  - It then connects to an 'older' vertex, which itself connected to another vertex when it was introduced
  - And we started from a connected core
- The older are richer
  - Nodes accumulate links as time goes on, which gives older nodes an advantage since newer nodes are going to attach preferentially – and older nodes have a higher degree to tempt them with than some new kid on the block

#### Young vs. old in BA model



### try it yourself



http://ladamic.com/netlearn/NetLogo501/RAndPrefAttachment.html

### Quiz Q:

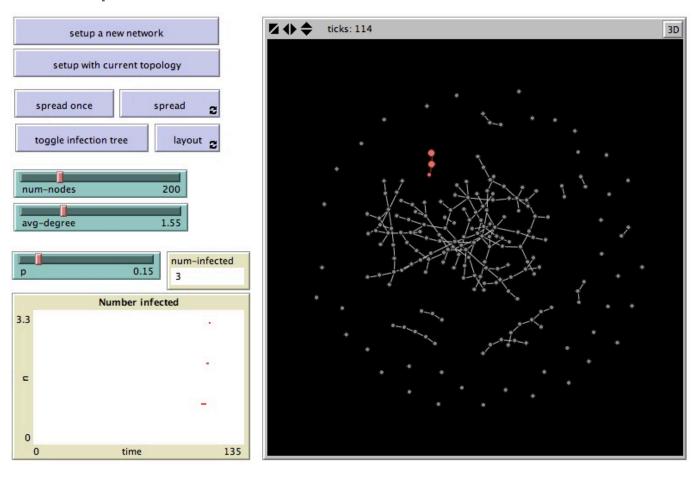
- Relative to the random growth model, the degree distribution in the preferential attachment model
  - resembles a power-law distribution less
  - resembles a power-law distribution more

### Summary: growth models

- Most networks aren't 'born', they are made.
- Nodes being added over time means that older nodes can have more time to accumulate edges
- Preference for attaching to 'popular' nodes further skews the degree distribution toward a power-law

### Assignment: implications for diffusion

■ How does the size of the giant component influence diffusion?



### Assignment: implications for diffusion

How do growth and preferential attachment influence diffusion?

