

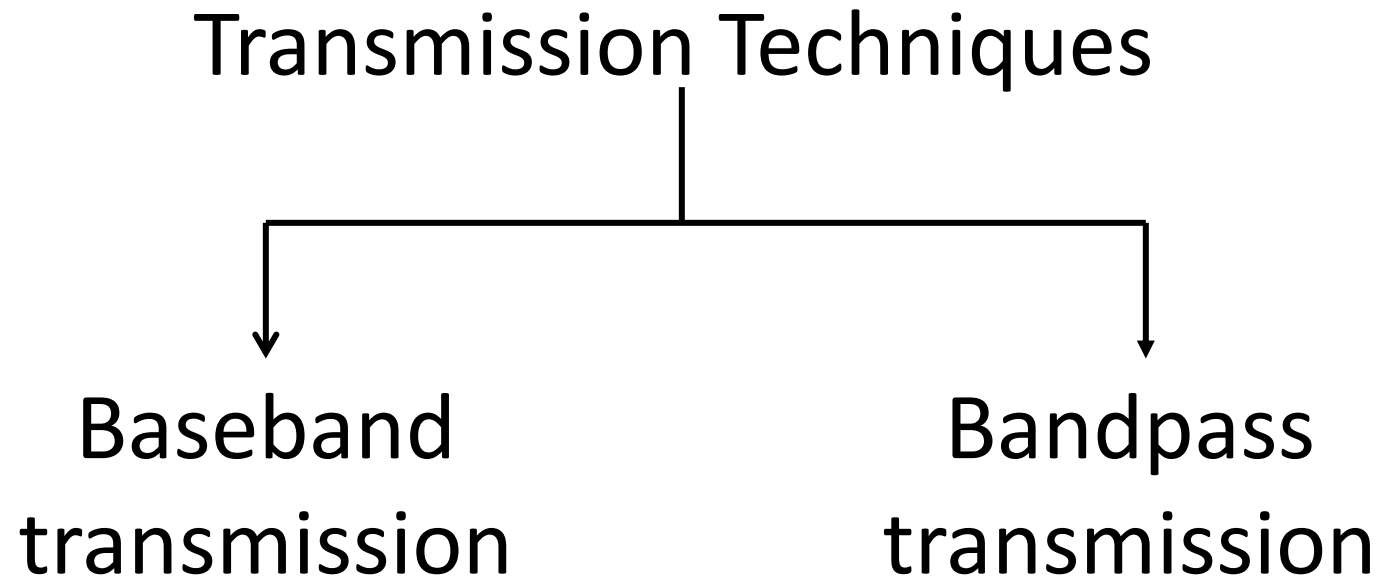
Module 3

Modulation & Demodulation

(AM & FM)

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Transmission Techniques



Baseband Transmission

The electrical equivalent of original information is known as the **baseband signal**.

The communication system in which the baseband signals are transmitted directly is known as **baseband transmission**.

Baseband transmission is effective only for wire communication.

Example, Telephone network, data communication in computer networks through coaxial cable.

But it is inefficient for wireless or radio communication.

Limitations of Baseband Transmission

- 1) Baseband signal having small frequency range from 20 Hz to 20 KHz only (so no large channel accommodation, mixing of signals).
- 2) Due to small frequency range, baseband signal cannot travel long distance in free space or air.
- 3) After a travel of short distance signal gets suppressed. So not used for radio communication. i.e. wireless communication.

To make the baseband signal efficient for radio communication modulation technique is used.

Modulation Technique

To overcome the drawbacks of baseband transmission and to transmit baseband signals by radio, modulation techniques must be used.

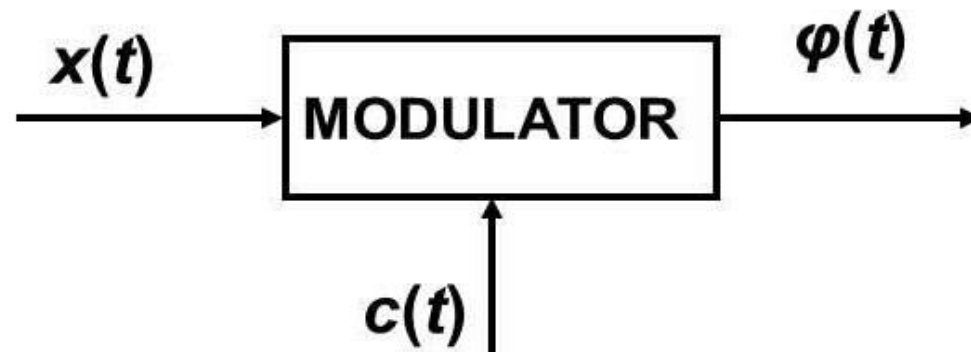
Baseband signal (Information signal) is a low-frequency signal and cannot travel longer distance. Just like we cannot walk at longer distance.

Modulation is the process of superimposing low-frequency information signal on a high-frequency carrier signal.

Definition of modulation

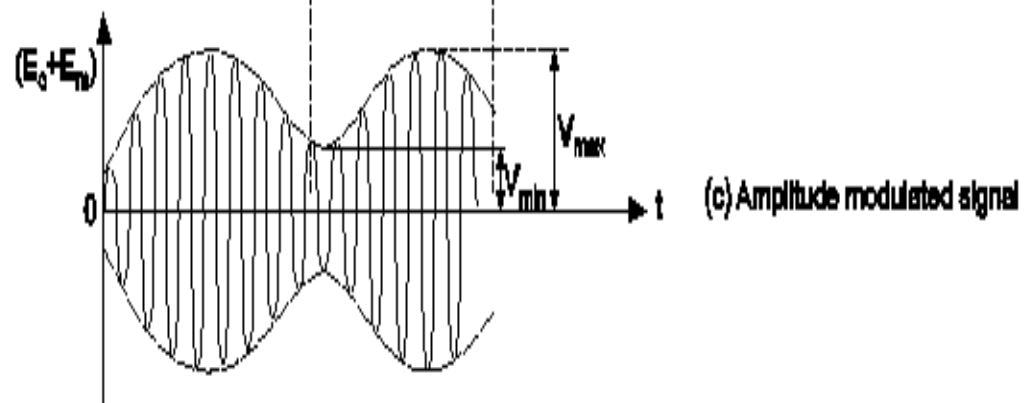
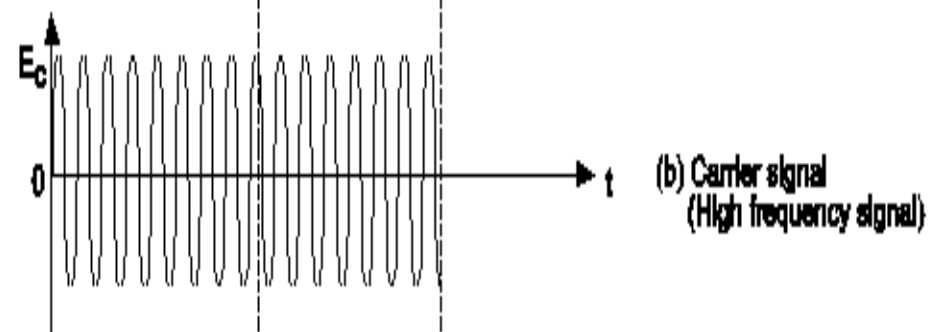
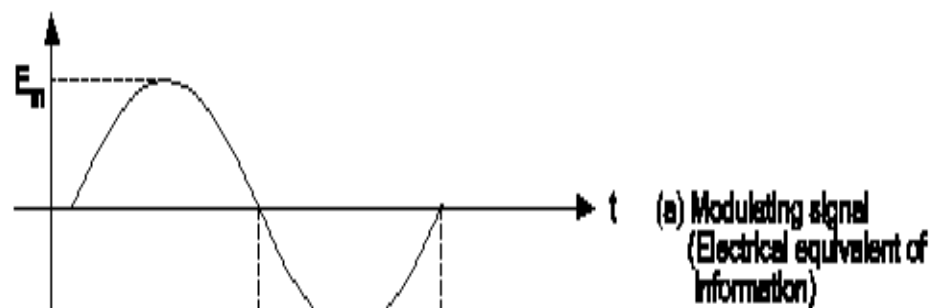
Modulation involves three signals:

- information carrying signal = **modulating signal** $x(t)$
 - **carrier** $c(t)$
 - **modulated signal** $\varphi(t)$
- **Modulation is mapping of the information signal into one of the parameters of the carrier.**
 - **The output signal is called the modulated signal.**

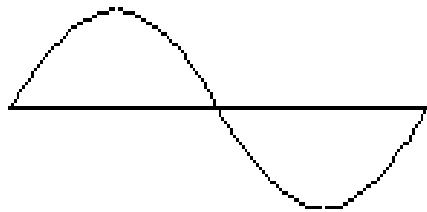


Modulation Technique





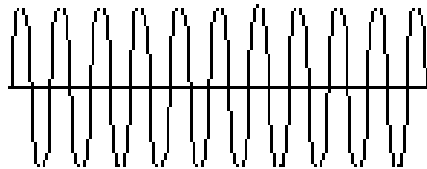
Modulation Technique



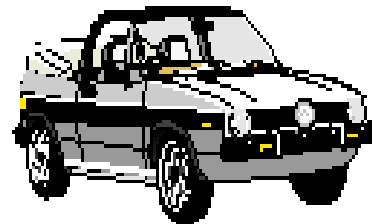
**Modulating
signal
(Information)**



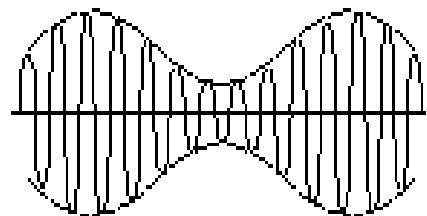
**Modulating signal
(Information)**



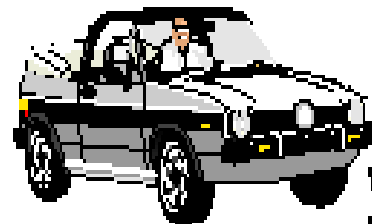
**Carrier
(High-Frequency signal)**



Carrier



**Modulated
signal**



**Modulated
signal**

**When man sits in car
becomes modulated signal**

Need of Modulation

Baseband signal transmission cannot be used for radio communication. To transmit the baseband signal for radio communication, modulation must be used.

Modulation is necessary because of following advantages:

1. Reduction in height of antenna.
2. Avoids mixing of signals.
3. Increase the range of communication.
4. Multiplexing is possible.
5. Improves quality of reception

Reduction in height of antenna

- For efficient transmission & reception, the antenna height should be at least one-fourth the signal wavelength ($\lambda/4$).
- To transmit a signal of $f=10\text{kHz}$.

$$\text{Minimum antenna height} = \lambda/4 = c/(4f) = 7500 \text{ m}$$

This is practically impossible !!

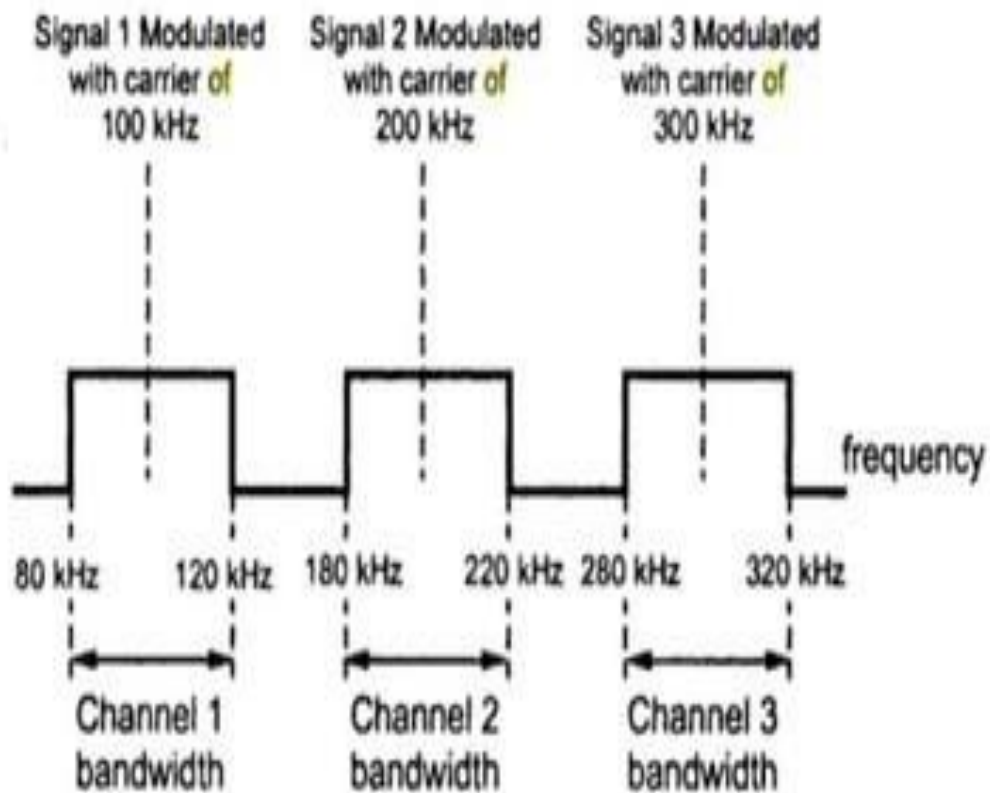
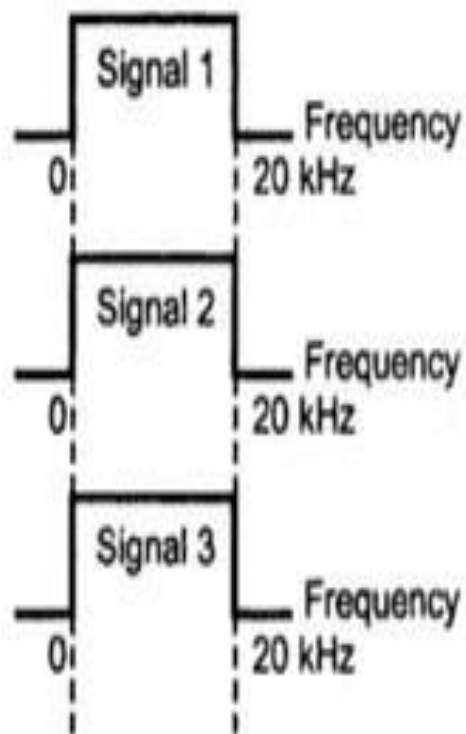
Consider a modulated signal at $f = 1 \text{ MHz}$

$$\text{Minimum antenna height} = \lambda/4 = c/(4f) = 75 \text{ m}$$

This can be installed practically.

Avoids mixing of signals

- All sound is concentrated within the range from 20 Hz to 20 KHz.
- If **no modulation** is used, all the signals would get mixed up. The receiver would not be able to separate them from each other.
- If each baseband signal is used to **modulate** a different carrier, then they will occupy different slots in the frequency domain.



Allows for multiplexing

- Multiplexing – Two or more signals can be transmitted over the same channel simultaneously.
- Multiplexing allows the same channel to be used by many signals.
- Therefore many TV channels can use the same frequency range without getting mixed up with each other.

Increases range of communication

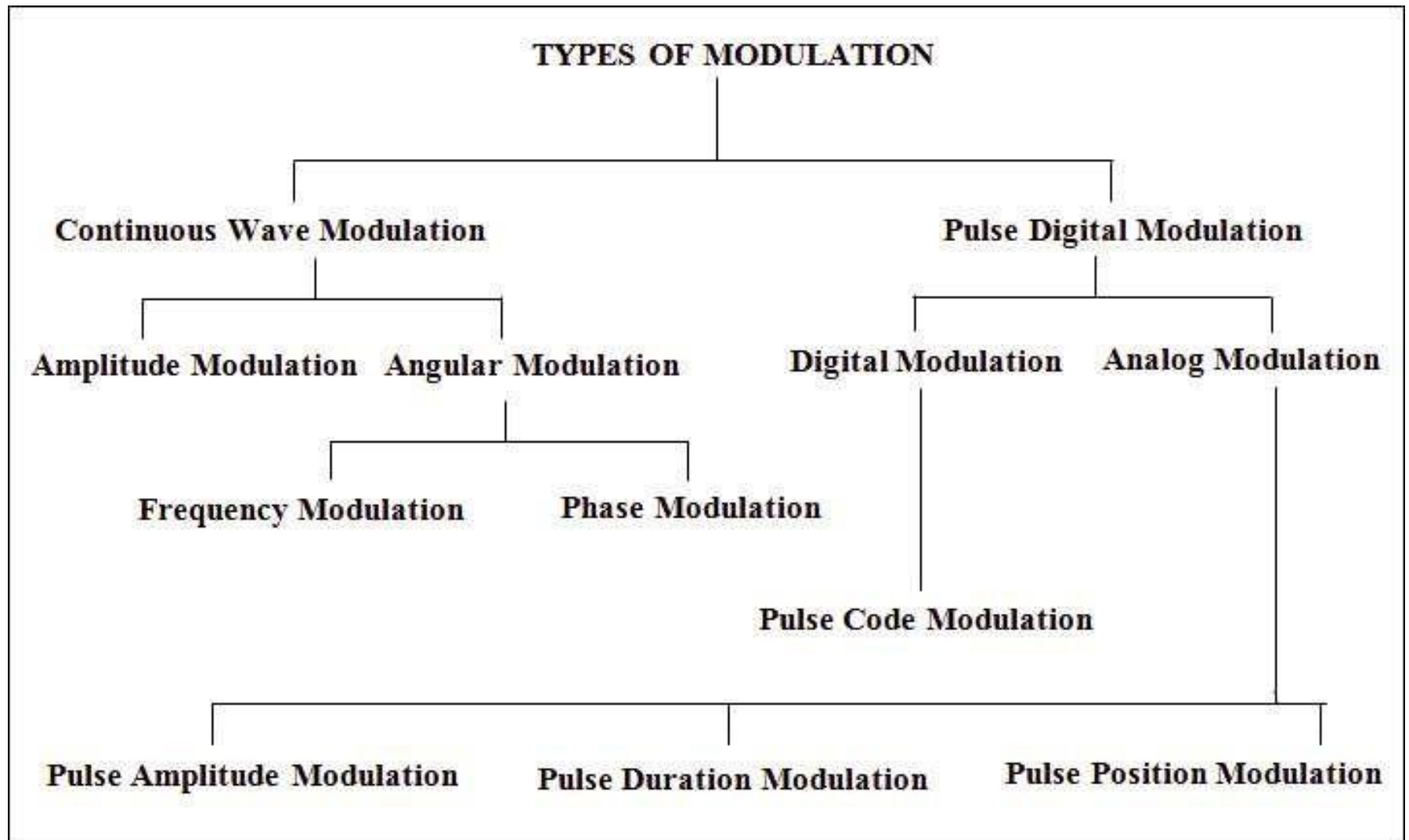
Low frequency signal - low energy -
travels small distances

High frequency signal - high energy -
travels longer distances.

Improves quality of reception

Digital modulation techniques reduce the effect of noise to a large extent.

Classification of Modulation



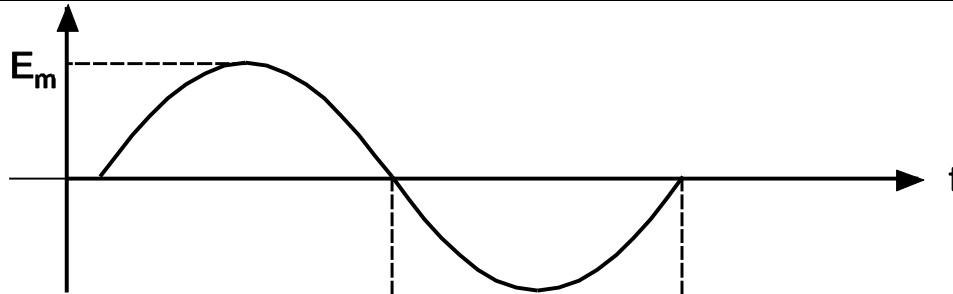
Amplitude Modulation

Definition:

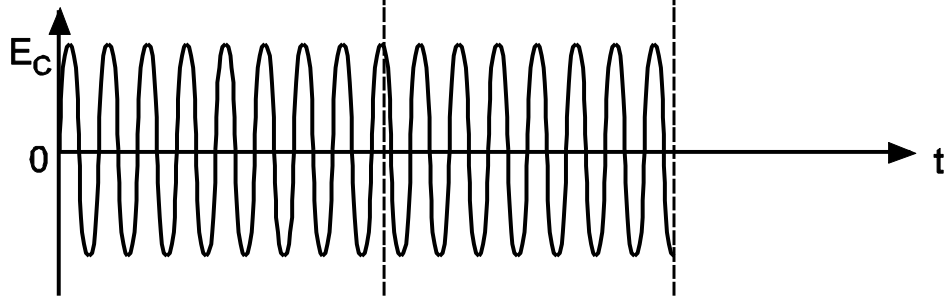
Amplitude modulation, is a technique of modulation in which the instantaneous amplitude of carrier signal varies in accordance with amplitude of modulating signal.

While frequency and phase of carrier remains constant. Nature of Amplitude Modulated waveform shown in Fig. below.

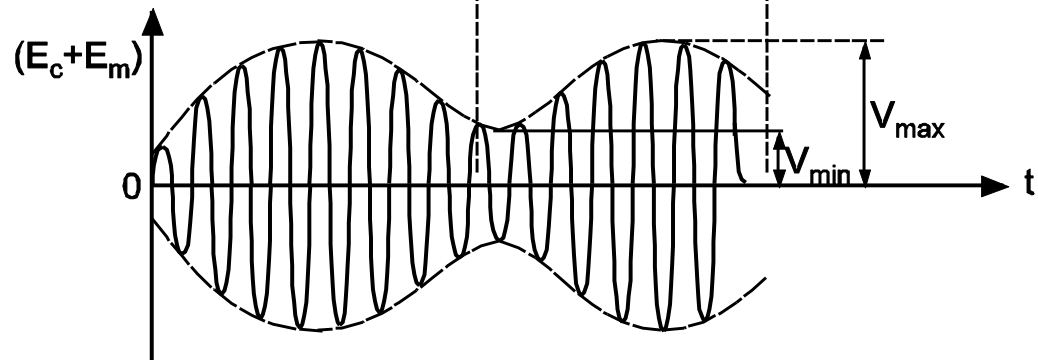
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(a) Modulating signal
(Electrical equivalent of
information)



(b) Carrier signal
(High frequency signal)



(c) Amplitude modulated signal

Time domain representation of AM wave

$$\text{Modulating signal } e_m = V_m \sin w_m t \quad \text{-----}(1)$$

$$\text{Carrier signal } e_c = V_c \sin w_c t \quad \text{-----}(2)$$

$$\text{Amplitude modulated signal} = e_{AM} = A \sin w_c t \quad \text{-----}(3)$$

$$\begin{aligned} \text{Where } A &= V_c + e_m \\ &= V_c + (V_m \sin w_m t) \quad \text{-----}(4) \end{aligned}$$

By putting equation (4) into (3)

$$\begin{aligned} e_{AM} &= A \sin w_c t \\ e_{AM} &= [V_c + (V_m \sin w_m t)] \sin w_c t \\ &= V_c \sin w_c t [1 + (V_m / V_c \sin w_m t)] \end{aligned}$$

$$\begin{aligned} e_{AM} &= V_c \sin w_c t [1 + m \sin w_m t] \\ \text{where } m &= V_m / V_c \quad [\text{modulation index}] \end{aligned}$$

This is called time domain representation of AM wave

Frequency Spectrum

Representation of AM wave in frequency domain is also known as **frequency spectrum of AM wave**.

Definition:

Frequency spectrum is a graph of amplitude versus frequency.

The frequency spectrum of **AM wave** tells us about **number of sidebands present in AM wave with corresponding amplitudes**.

Continued.....

- Consider equation of AM wave (equation 2.12).

$$e_{AM} = (E_C + E_m \sin \omega_m t) \sin \omega_c t \quad \dots (2.12)$$

$$e_{AM} = E_C \left(1 + \frac{E_m}{E_C} \sin \omega_m t \right) \sin \omega_c t$$

But,

$$m = \frac{E_m}{E_C}$$

$$\therefore e_{AM} = E_C (1 + m \sin \omega_m t) \sin \omega_c t \quad \dots (2.13)$$

Simplifying we get,

$$e_{AM} = E_C \sin \omega_c t + m E_C \sin \omega_m t \sin \omega_c t \quad \dots (2.14)$$

There is a trigonometric identity that says that the product of two sin waves is

$$\sin A \sin B = \frac{\cos (A - B)}{2} - \frac{\cos (A + B)}{2}$$

By substituting this identity into equation becomes

$$e_{AM} = \underbrace{E_C \sin \omega_c t}_{\text{Carrier}} + \underbrace{\frac{m E_C}{2} \cos (\omega_c - \omega_m) t}_{\text{LSB}} - \underbrace{\frac{m E_C}{2} \cos (\omega_c + \omega_m) t}_{\text{USB}} \quad \dots (2.15)$$

Carrier LSB USB

Features of Frequency spectrum

From equation (2.15) of AM wave, it consists of three terms:

- (i) The first term is sine term called unmodulated **carrier signal**.
 - (ii) The second term is cos term at frequency $(f_c - f_m)$ called Lower Side Band (LSB) with amplitude $\frac{mE_c}{2}$. ($\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$).
 - (iii) The third term is cos term at frequency $(f_c + f_m)$ called **upper sideband (USB)** with amplitude $\frac{mE_c}{2}$.
- **This shows that AM wave, having two sidebands which contains actual information and one carrier.**
 - From equation 2.15, plot of frequency spectrum shown in Fig. 2.13 (a).

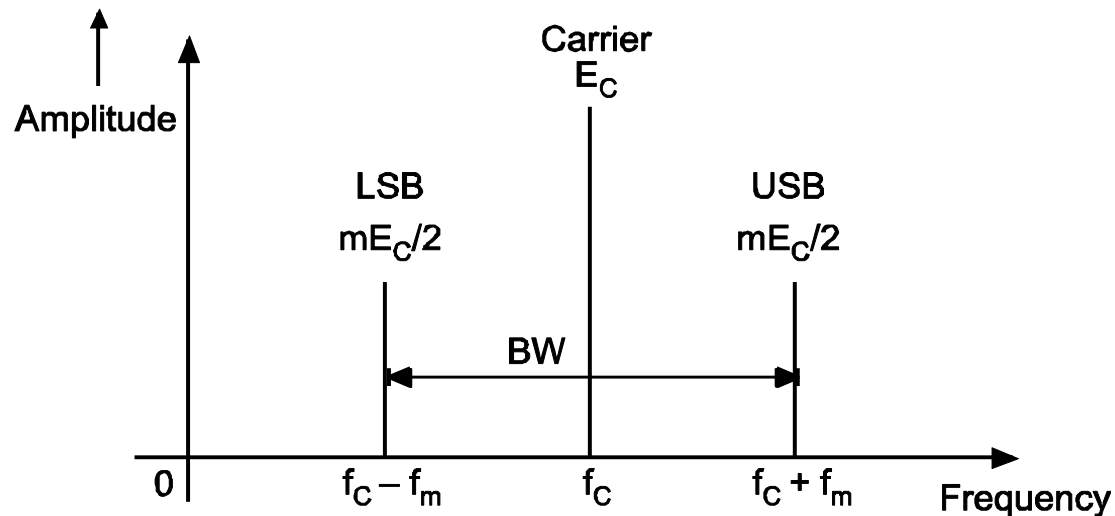


Fig. 2.13 (a): Frequency Spectrum of AM Wave

Ex If carrier freq. is 712 KHz, what is the freq. range occupied by the SB. if a carrier freq. is modulated by audio freq. upto 10 KHz.

$$\rightarrow f_m = 10 \text{ KHz}, f_c = 712 \text{ KHz}$$

$$\text{LSB} = f_c - f_m = 702 \text{ KHz}$$

$$\text{USB} = f_c + f_m = 722 \text{ KHz.}$$

Modulation Index

Definition:

In AM, the modulation index (m) is defined as the ratio of amplitudes of modulating signal to the carrier signal.

$$\text{M.I.} = \frac{\text{Modulating Signal Amplitude}}{\text{Carrier Signal Amplitude}}$$

$$\boxed{m = \frac{E_m}{E_c}} \quad \dots (2.4)$$

If modulation index is expressed in percentage, it is called '**percentage modulation**'.

i.e. $\boxed{\%m = \frac{E_m}{E_c} \times 100} \quad \dots (2.5)$

Modulation Index

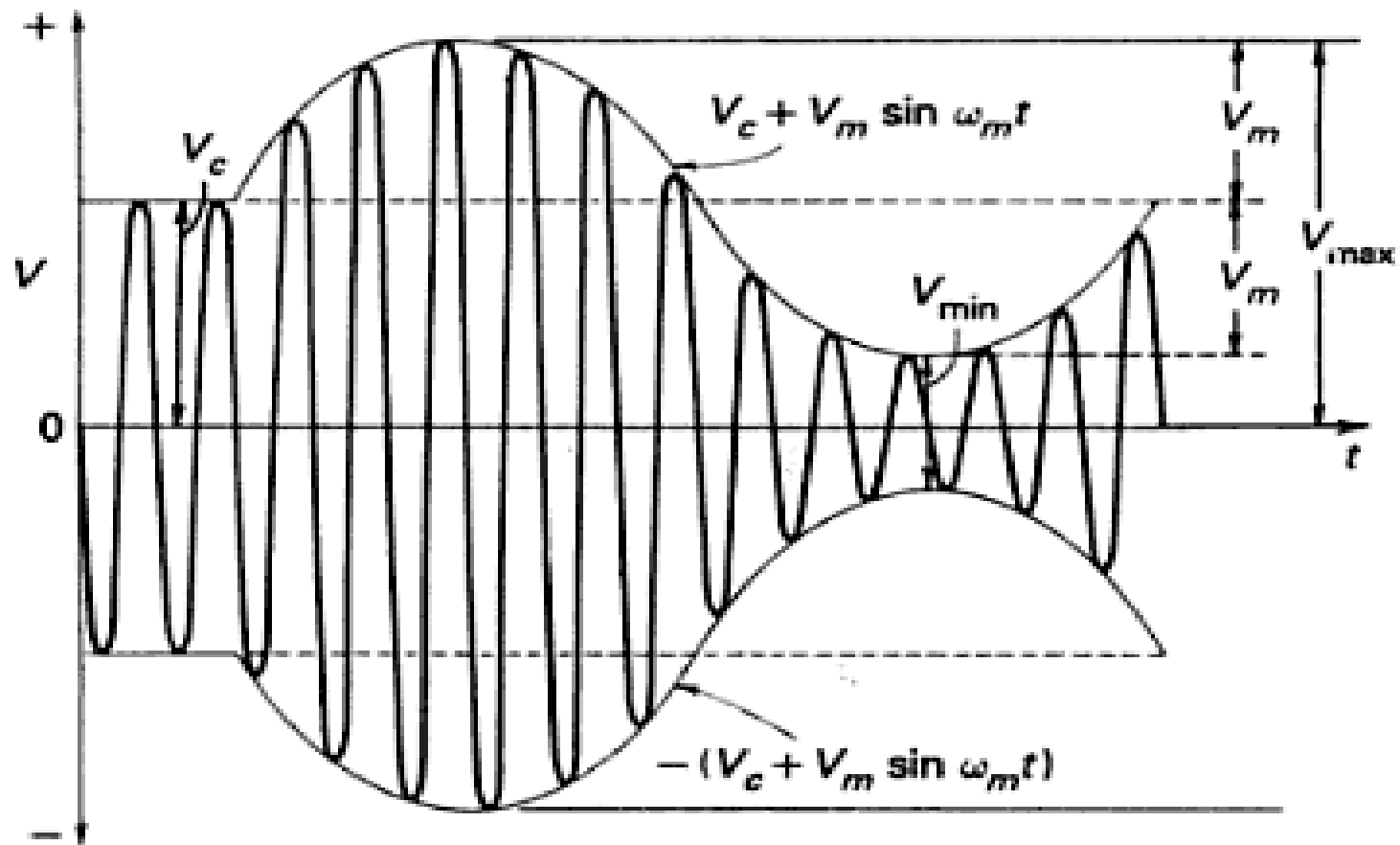


Fig. 3.1.1©

Modulation Index

- From the Fig. 3.1.1(c) we can write equation for maximum and minimum amplitude of AM signal.

Thus, $E_{\max} = E_c + E_m$

and $E_{\min} = E_c - E_m$

Therefore,

$$E_m = \frac{E_{\max} - E_{\min}}{2}$$

Modulation Index

$$\text{and } E_c = \frac{E_{\max} + E_{\min}}{2}$$

We know that, from definition of modulation index,

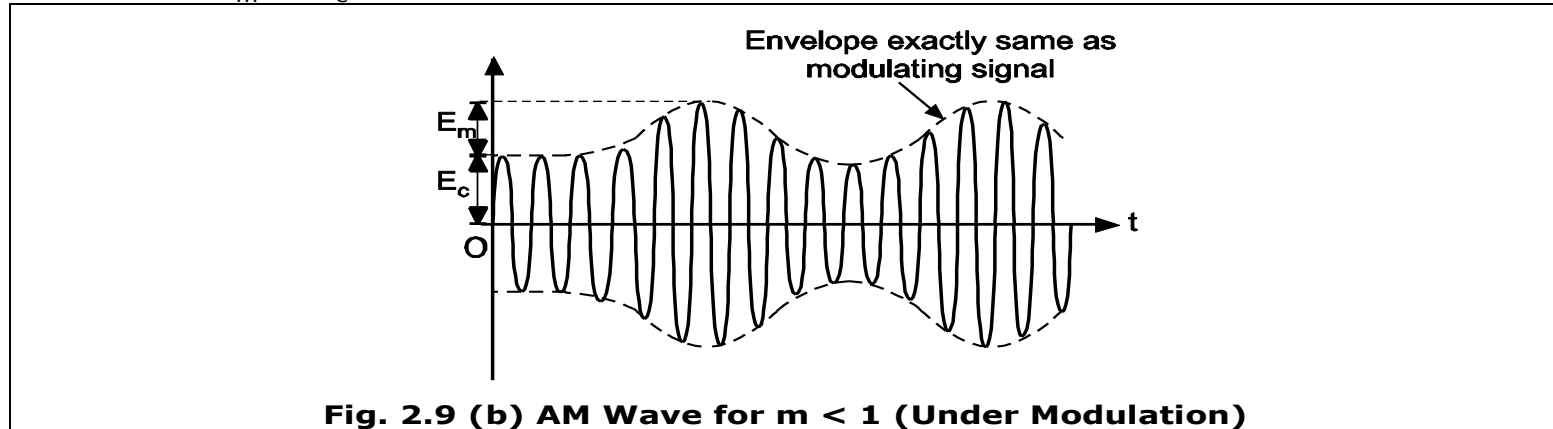
$$m = \frac{E_m}{E_c}$$

$$\therefore m = \frac{\frac{E_{\max} - E_{\min}}{2}}{\frac{E_{\max} + E_{\min}}{2}}$$

$$\therefore m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

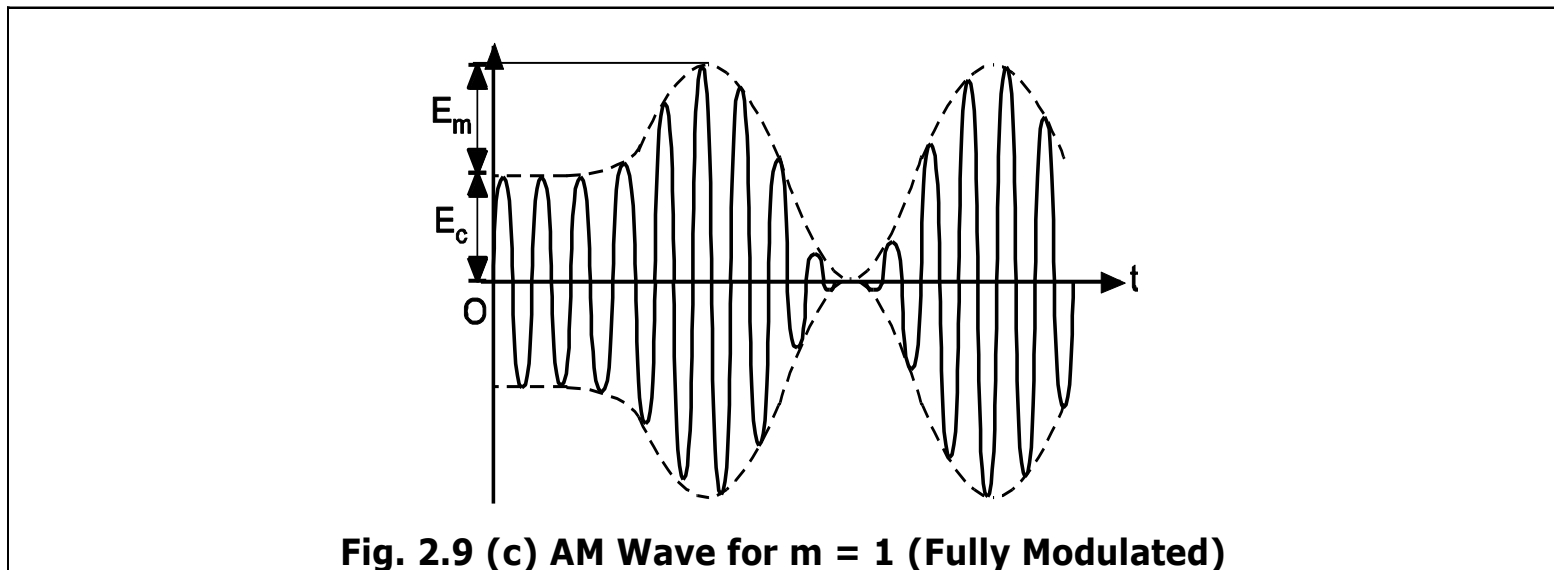
Effect of Modulation Index on Modulated Signal

1. For $m < 1$,
i.e. $E_m < E_c$



2. For $m = 1$

i.e. $E_m = E_c$. i.e. $m = 100\%$.



Continued....

3. For $m > 1$

i.e. $E_m > E_c$

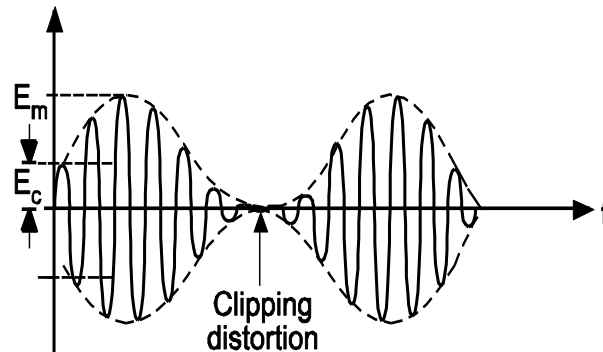


Fig. 2.9 (d) AM Wave at $m > 1$ (Over Modulation)

For $m = 0$

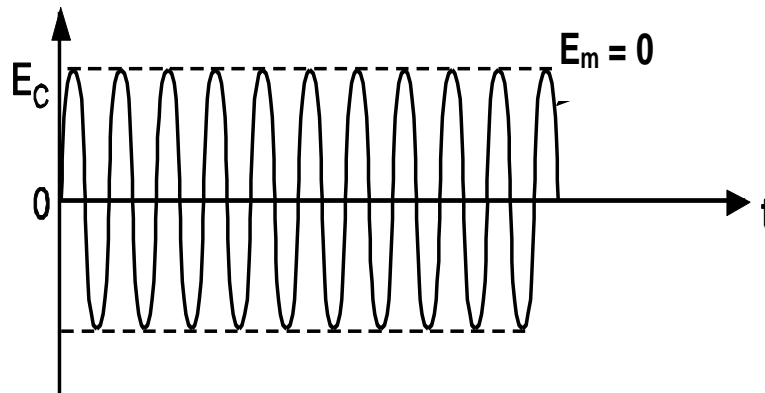
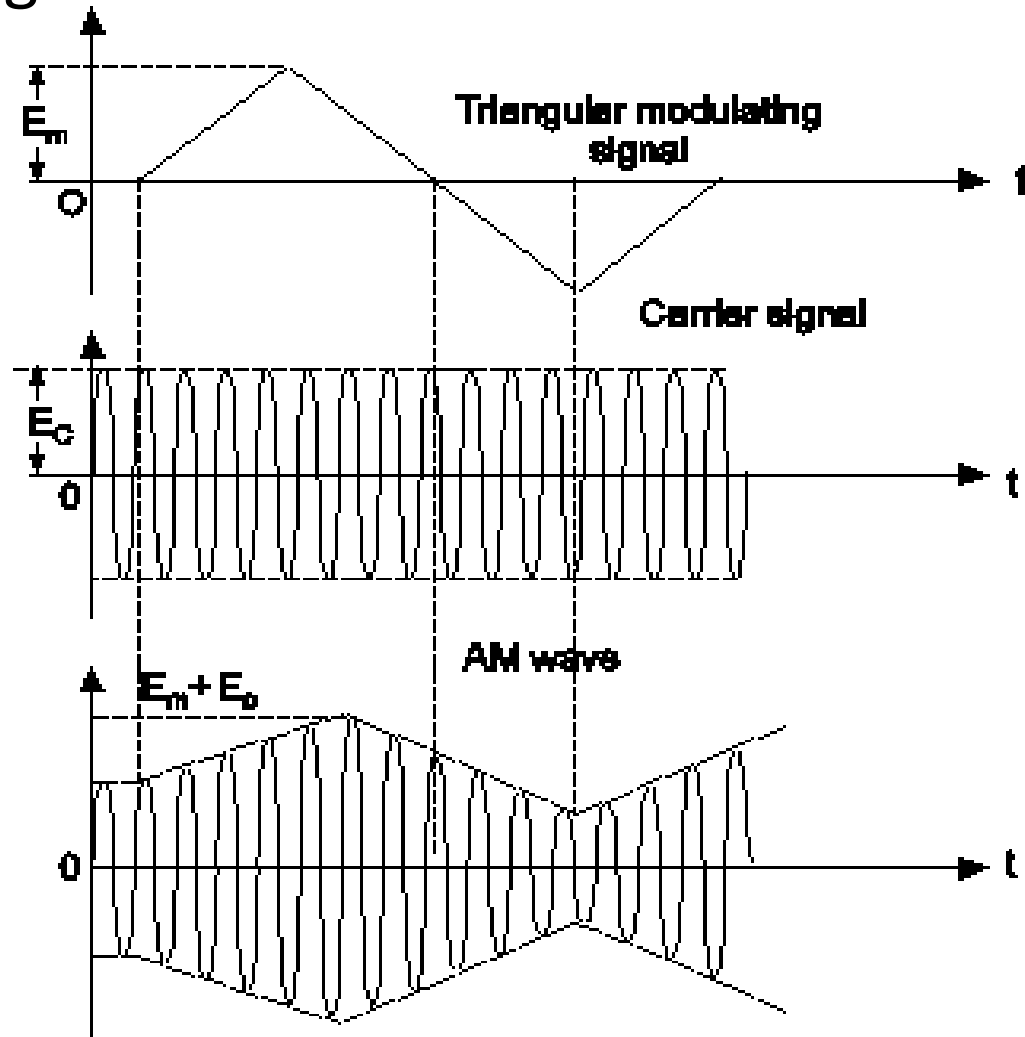


Fig. 2.9 (e) AM Wave at $m = 0$ (No Modulation Takes Place)

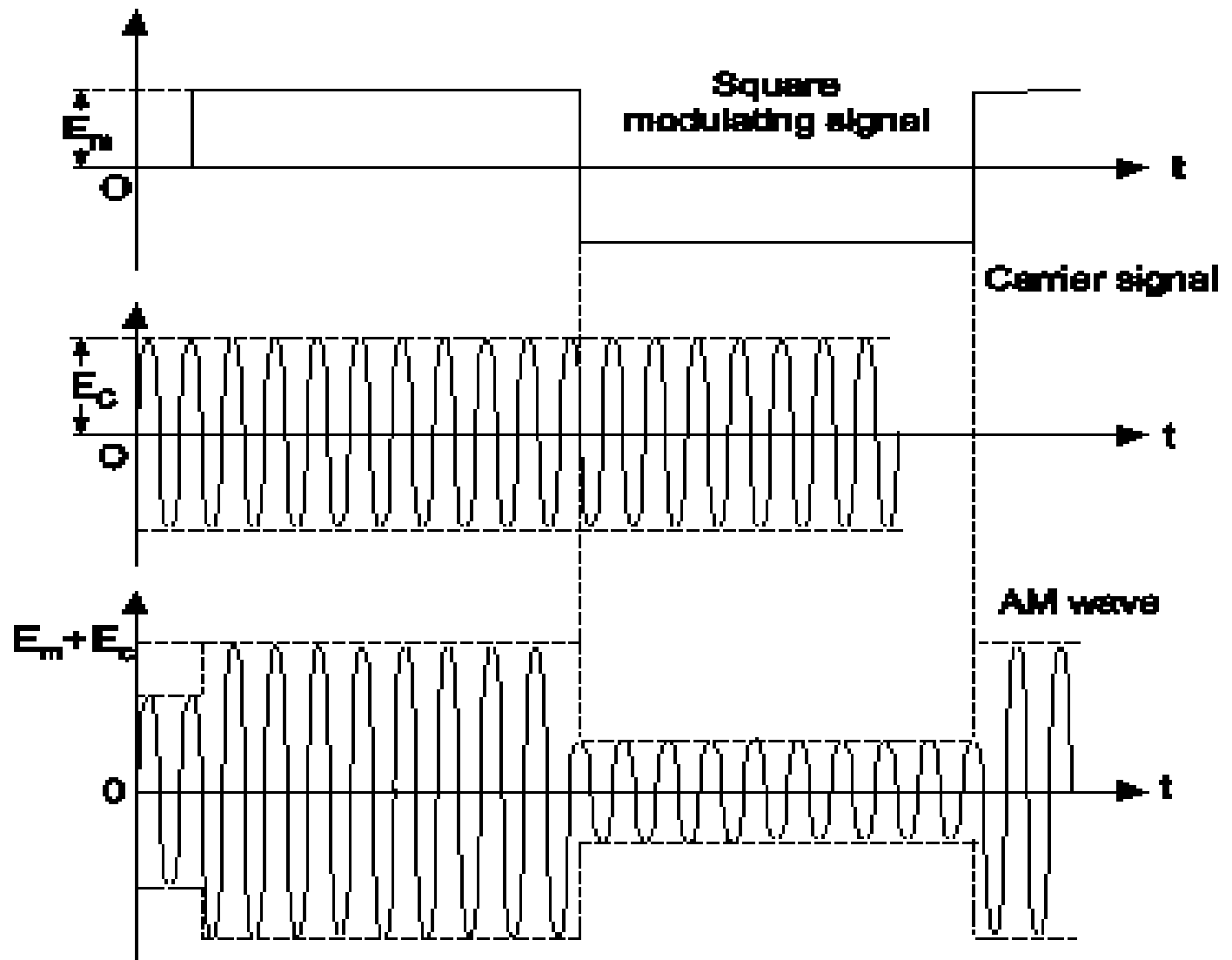
Example

Draw the AM wave for triangular and square wave modulating signal.

Solution:

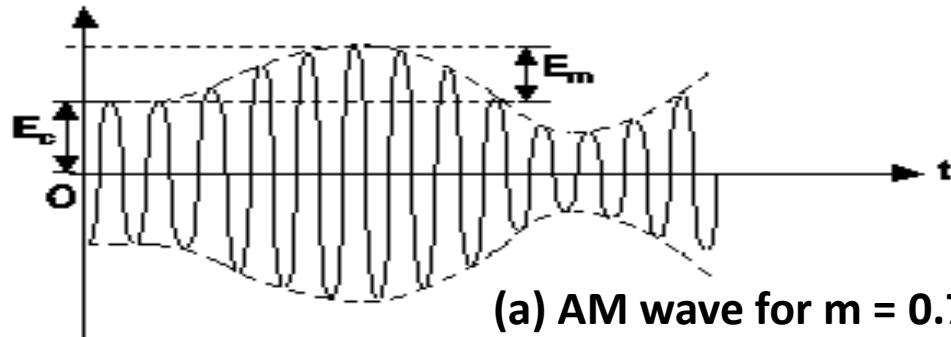


For square wave input.



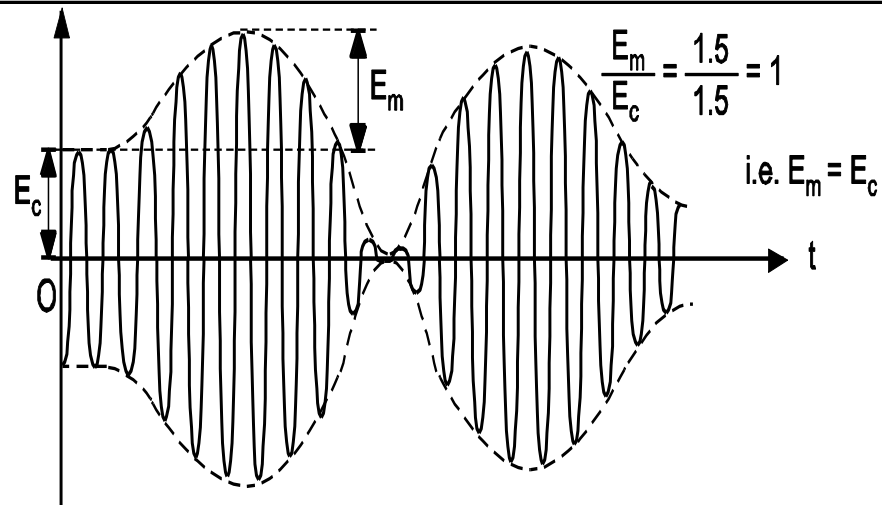
Example 2 Draw the AM waveform for the modulation index $m = 0.75$, $m = 1$ and $m = 1.25$.

Take a graph paper and adjust the value of E_m and E_c in such a way that $\frac{E_m}{E_c} = 0.75$, $\frac{E_m}{E_c} = 1$, $\frac{E_m}{E_c} = 1.25$.

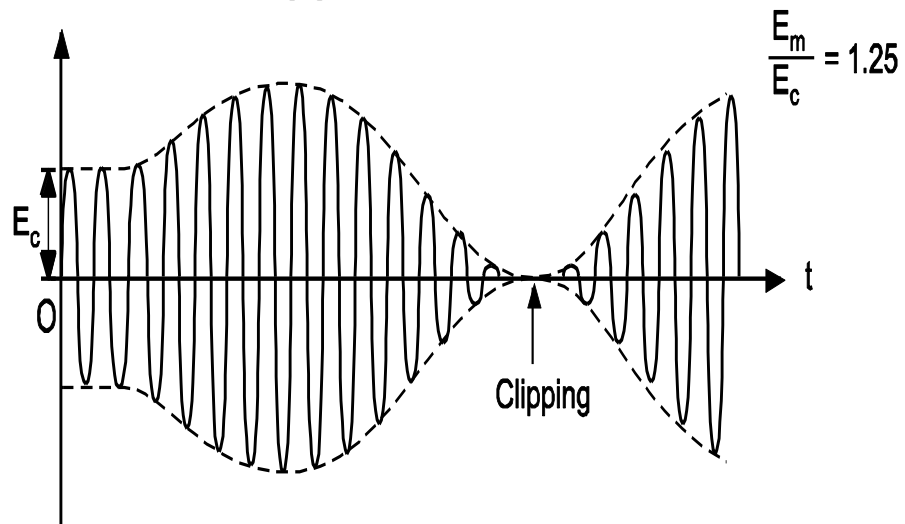


(a) AM wave for $m = 0.75$

Continued...



(b) AM Wave for $m = 1$



(c) AM Wave for $m = 1.25$

Bandwidth Requirement

The bandwidth of AM signal is defined as the frequency range from upper sideband to lower sideband frequency in frequency spectrum.

$$\begin{aligned} \text{BW} &= f_{\text{USB}} - f_{\text{LSB}} \\ &= (f_c + f_m) - (f_c - f_m) \quad \dots \text{(from Fig. 2.13)} \\ &= f_c + f_m - f_c + f_m \\ &= \mathbf{2 f_m} \end{aligned}$$

$\therefore \rightarrow$ BW required for AM signal.

Hence, bandwidth of AM signal is twice the modulating signal frequency.

Ex A modulating sig $10 \sin(2\pi \times 10^3 t)$ is used to modulate a carrier sig $20 \sin(2\pi \times 10^4 t)$. Find modulation index, % mod, freqs of the SB components & their amplitudes. What is the BW of the modulated sig? Draw the spectrum of AM wave.

$$V_m = 10 \sin(2\pi 10^3 t)$$

$$= E_m \sin(2\pi f_m t)$$

$$E_m = 10, V \quad f = 10^3 = 1 \text{ KHz}$$

$$V_c = 20 \sin(2\pi 10^4 t)$$

$$= E_c \sin(2\pi f_c t)$$

$$E_c = 20, V \quad f = 10 \text{ KHz}$$

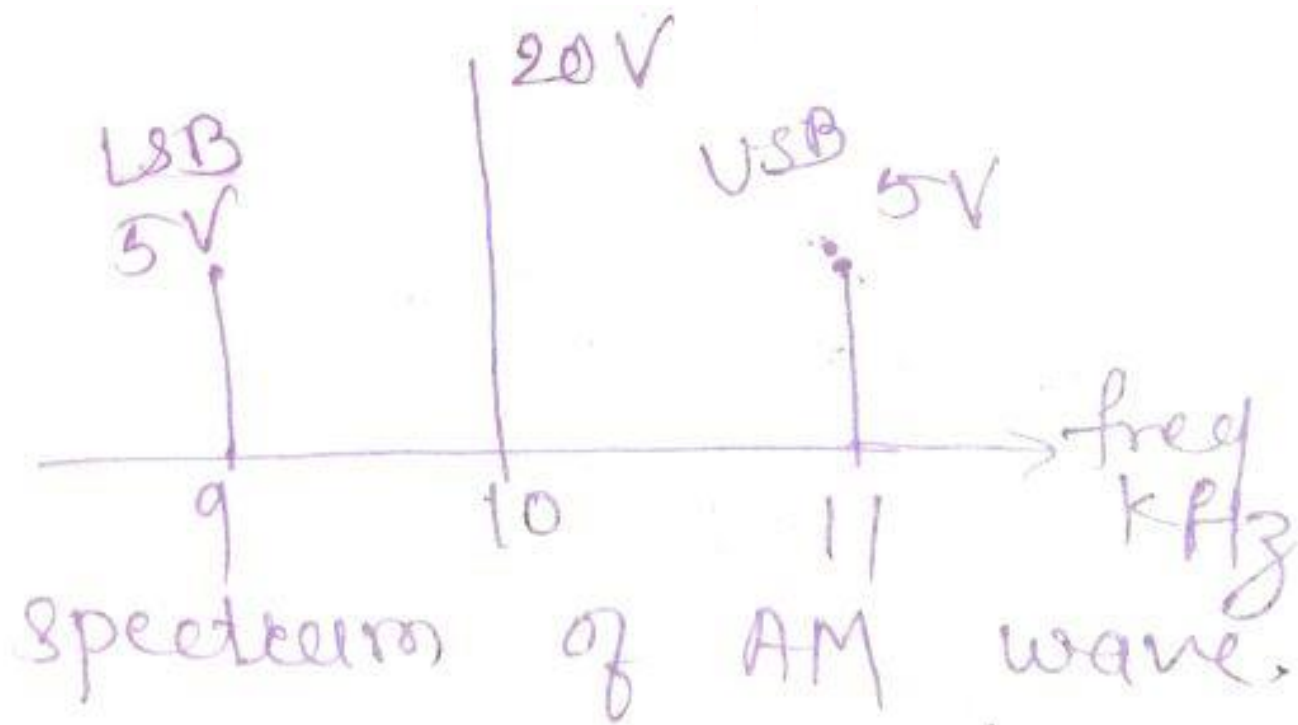
$$m = \frac{E_m}{E_c} = \frac{10}{20} = 0.5 \times 100 = 50\%$$

$$\text{freq. of SB} = f_c - f_m = 10 \text{ K} - 1 \text{ K} = 9 \text{ K}$$

$$f_c + f_m = 10 \text{ K} + 1 \text{ K} = 11 \text{ K}$$

$$\text{Amplitude of SB} = \frac{m E_c}{2} = \frac{0.5 \times 20}{2} = 5 \text{ Volt}$$

$$\text{BW} = 2 f_m = 2 \times 1 \text{ KHz} = 2 \text{ KHz}$$



3) Carrier amplitude of after A.M varies betⁿ 4 & 1 V. Calculate depth of mod?

$$m = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}}$$

$$= \frac{4 - 1}{4 + 1} = 0.6$$

$$= 60\%$$

4) Carrier wave is represented by the eqⁿ. $e_c(t) = 10 \sin \omega t$. draw w/f of an AM wave for $m = 0.5$.

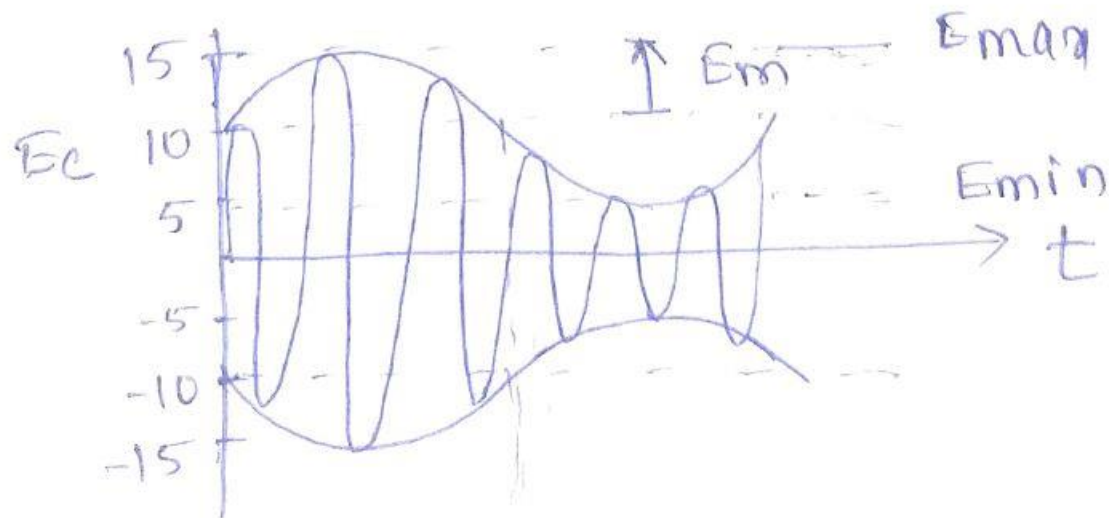
$$E_c = 10 \sin \omega t \quad \therefore E_c = 10 \text{ V}$$

$$m = \frac{E_m}{E_c}$$

$$0.5 = \frac{E_m}{10} \quad ; \quad E_m = 5 \text{ V}$$

$$E_{\max} = E_c + E_m = 10 + 5 = 15 \text{ V}$$

$$E_{\min} = E_c - E_m = 10 - 5 = 5 \text{ V}$$



An AM signal has the equation

$$v(t) = [15 + 4\sin(44 \times 10^3 t)] \sin(46.5 \times 10^6 t) \text{ volts.}$$

- (a) Find the carrier frequency
- (b) Find the frequency of modulating signal
- (c) Find modulation index
- (d) Find peak voltage of unmodulated carrier
- (e) Sketch the signal in time domain

Given : $v(t) = [15 + 4\sin (44 \times 10^3 t)] \sin (46.5 \times 10^6 t)$

To find : 1. f_c 2. f_m 3. m 4. E_c 5. AM wave

► **Step I :** Standard AM equation is

$$e_{AM} = E_c [1 + m \sin \omega_m t] \sin \omega_c t$$

Comparing it with given equation we get,

$$\omega_c = 2\pi f_c = 46.5 \times 10^6$$

$$\therefore f_c = \frac{46.5 \times 10^6}{2\pi}$$

$$\therefore f_c = 7.4 \times 10^6 \text{ Hz}$$

Step II $\omega_m = 2\pi f_m = 44 \times 10^3$

$$\therefore f_m = \frac{44 \times 10^3}{2\pi}$$

$$\therefore f_m = \mathbf{7.002 \text{ kHz}}$$

Step III : Given equation can be written as,

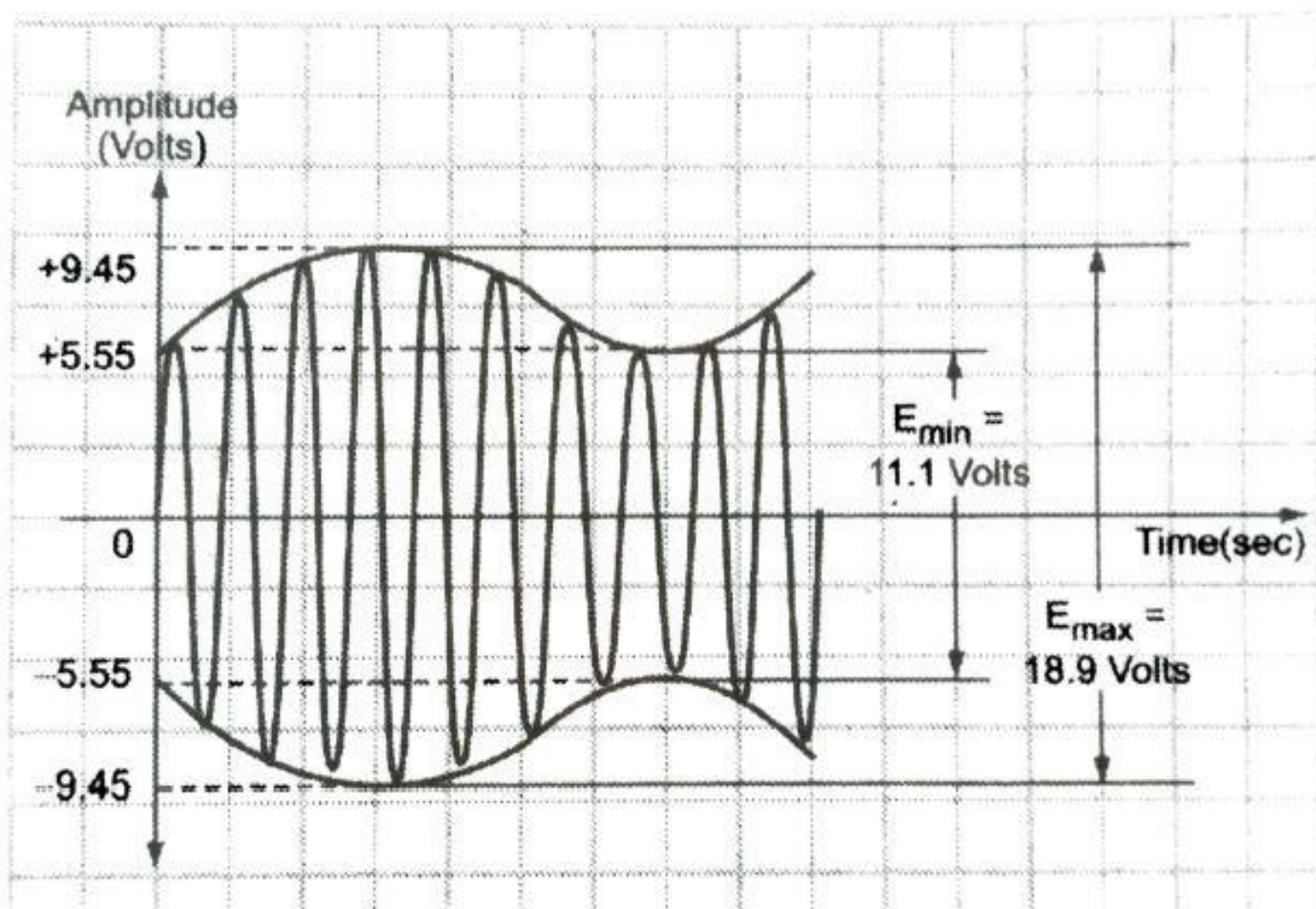
$$v(t) = 15 \left[1 + \frac{4}{15} \sin (44 \times 10^3 t) \right] \sin (46.5 \times 10^6 t)$$

$$\therefore E_c = \mathbf{15 \text{ volts}}$$

► **Step IV :** $m = \frac{4}{15} = 0.26$

$$\therefore \%m = \mathbf{26\% \text{ or } m = 0.26}$$

► **Step V**



$$m = \frac{E_m}{E_c}$$

$$\therefore E_m = m \times E_c = 0.26 \times 15 = 3.9 \text{ volts}$$

$$\therefore E_{\max} = E_c + E_m = 15 + 3.9 = 18.9 \text{ volts}$$

$$\therefore E_{\min} = E_c - E_m = 15 - 3.9 = 11.1 \text{ Volts}$$

Power Relations in AM Wave

(i) The Total Power in AM (P_t):

$$P_t = (\text{Carrier Power}) + (\text{Power in USB}) + (\text{Power in LSB})$$

$$P_t = P_c + P_{\text{USB}} + P_{\text{LSB}} \quad \dots(2.16)$$

$$\therefore P_t = \frac{E_{\text{carr}}^2}{R} + \frac{E_{\text{USB}}^2}{R} + \frac{E_{\text{LSB}}^2}{R}$$

where, $E_{\text{carr}}, E_{\text{USB}}, E_{\text{LSB}}$ = r.m.s. values of the carrier and side band amplitudes

R = Characteristic resistance of antenna in which total power is dissipated.

(ii) Carrier Power (P_c):

The carrier power is given by,

$$P_c = \frac{E_{\text{carr}}^2}{R}$$
$$= \frac{(E_c/\sqrt{2})^2}{R}$$

$$\boxed{P_c = \frac{E_c^2}{2R}} \quad \dots (2.17)$$

where, E_c = Peak carrier amplitude

Continued...

(iii) Power in sidebands:

The power in USB and LSB is same as,

$$P_{\text{USB}} = P_{\text{LSB}} = \frac{E_{\text{SB}}^2}{R}$$

From equation (2.15),

$$\text{Peak amplitude of sideband} = \frac{mE_c}{2}$$

$$\begin{aligned}\therefore P_{\text{USB}} = P_{\text{LSB}} &= \frac{(mE_c/2\sqrt{2})^2}{R} \\ &= \frac{m^2 E_c^2}{8R}\end{aligned}$$

$$\therefore P_{\text{USB}} = P_{\text{LSB}} = \frac{m^2}{4} \times \frac{E_c^2}{2R}$$

From equation (2.17),

$$\frac{E_c^2}{2R} = P_c$$

\therefore

$$P_{\text{USB}} = P_{\text{LSB}} = \frac{m^2}{4} P_c$$

A 400 watts carrier is modulated to a depth of 75%. Find the total power in AM wave. Assume sinusoidal modulating signal.

☑ **Soln. :**

Given : 1. $P_c = 400$ watts 2. $m = 75\% = 0.75$

To find : $P_t = ?$

$$P_t = P_c \left[1 + \frac{m^2}{2} \right]$$

$$\therefore P_t = 400 \left[1 + \frac{(0.75)^2}{2} \right]$$

$$\therefore P_t = 512.5 \text{ watts}$$

\therefore Total power in AM = 512.5 watts

A transmitter transmits 20 kW of power without modulation and 22 kW after amplitude modulation. What is the modulation index ?

✓ **Soln. :**

Given : 1. $P_c = 20 \text{ kW}$ 2. $P_t = 22 \text{ kW}$

To find : $m = ?$

$$P_t = P_c \left[1 + \frac{m^2}{2} \right]$$

$$\therefore \frac{P_t}{P_c} = 1 + \frac{m^2}{2}$$

$$\therefore m = \sqrt{2 \left[\frac{P_t}{P_c} - 1 \right]}$$

$$\therefore m = \sqrt{2 \left[\frac{22 \text{ kW}}{20 \text{ kW}} - 1 \right]}$$

$$\therefore m = \mathbf{0.447}$$

$$\therefore \text{Percentage } m = \mathbf{44.7\%}$$

Continued....

(iv) Total Power in AM:

From equation (2.16),

The total power in AM wave is,

$$\begin{aligned} P_t &= P_c + P_{USB} + P_{LSB} \\ &= P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c \end{aligned}$$

∴

$$\boxed{P_t = \left(1 + \frac{m^2}{2}\right) P_c} \quad \dots(2.19)$$

From this equation, we can say that as value of 'm' increases, total power also increases.

For $m = 1$, total power will be maximum. (i.e. for unity M.I.)

∴

$$\boxed{P_t = 1.5 P_c}$$

Current calculations

Let I_c be the carrier current (unmodulated) and I_t be the total current.

We know that,

$$\text{Power} = I^2 R$$

$$\text{Hence, } P_t = I_t^2 R$$

$$\text{and } P_c = I_c^2 R$$

Substituting in Equation

$$P_t = P_c \left[1 + \frac{m^2}{2} \right]$$

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2} \text{ we get,}$$

$$\therefore \frac{I_t^2 R}{I_c^2 R} = 1 + \frac{m^2}{2}$$

$$\therefore \left(\frac{I_t}{I_c} \right)^2 = 1 + \frac{m^2}{2}$$

Taking square roots of both sides,

$$\therefore I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

A broadcast radio transmitter radiates 15 kilowatts (15 kW) when the modulation percentage is 60. How much of this is carrier power ?

Given :

1. $P_t = 15 \text{ kW}$ 2. $m = 60\% = 0.6$

To find : $P_c = ?$

✓ Soln. :

$$P_t = P_c \left[1 + \frac{m^2}{2} \right]$$

$$15 \times 10^3 = P_c \left[1 + \frac{(0.6)^2}{2} \right]$$

$$\therefore P_c = \frac{15 \times 10^3}{\left[1 + \frac{(0.6)^2}{2} \right]} = \frac{15 \times 10^3}{1.18}$$

$$\therefore P_c = 12.711 \text{ kW}$$

An AM signal with a carrier of 1 kW has 200 watts in each sidebands. What is the percentage of modulation ?

Given :

1. $P_c = 1 \text{ kW}$ 2. $P_{\text{LSB}} = P_{\text{USB}} = 200 \text{ watts}$

To find : $m = ?$

► **Step I :** $P_t = P_c + P_{\text{LSB}} + P_{\text{USB}}$
 $= 1 \text{ kW} + 200 + 200$

$$\therefore P_t = 1.4 \text{ kW}$$

► **Step II :** $P_t = P_c \left[1 + \frac{m^2}{2} \right]$

$$\therefore m = \sqrt{2 \left[\frac{P_t}{P_c} - 1 \right]}$$

$$\therefore m = \sqrt{2 \left[\frac{1.4 \text{ kW}}{1 \text{ kW}} - 1 \right]}$$

$$\therefore m = 0.8944$$

$$\therefore \text{Percentage } m = 89.44\%$$

The antenna current of an AM transmitter is 8 amperes when only the carrier is sent; but it increases to 8.93 A when the carrier is modulated by single sine wave. Find the percentage modulation. Find the antenna current when the percent of modulation changes to 0.8.

To find

1. $m_1 = ?$ 2. $I_{t2} = ?$

$$\begin{aligned}\blacktriangleright \text{Step I: } m_1 &= \sqrt{2 \left[\left(\frac{I_{t1}}{I_c} \right)^2 - 1 \right]} \\ &= \sqrt{2 \left[\left(\frac{8.93}{8} \right)^2 - 1 \right]}\end{aligned}$$

$$m_1 = 0.701$$

$$\therefore \text{Percentage } m = 70.1 \%$$

$$\blacktriangleright \text{Step II: } I_{t2} = I_c \sqrt{1 + \frac{m^2}{2}} = 8 \sqrt{1 + \frac{(0.8)^2}{2}}$$

$$\therefore I_{t2} = 9.19 \text{ A}$$

Modulation by several sine waves:-

Modulation of a carrier by several sine waves simultaneously is the rule rather than the exception.

Let V_1, V_2, V_3 etc be the simultaneous modulation voltages. Total modulating voltage V_t will be equal to square root of the sum of the squares of the individual voltages.

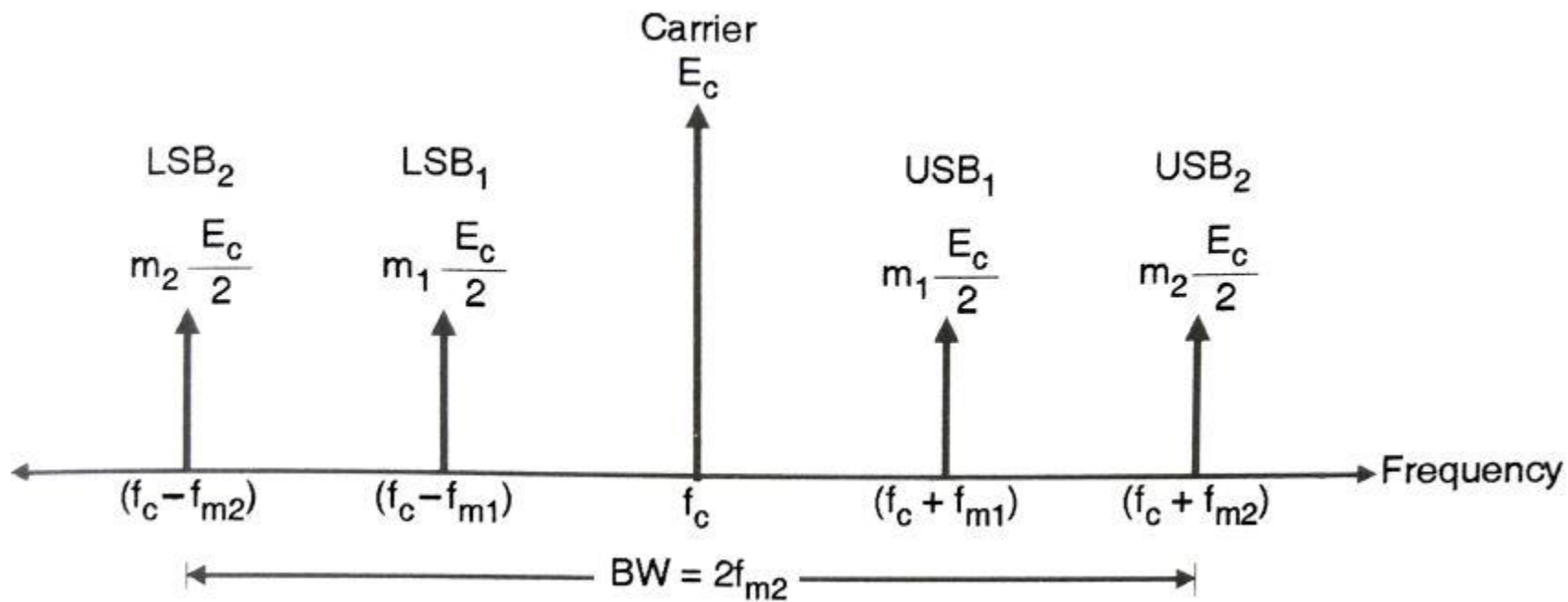
$$V_t = \sqrt{V_1^2 + V_2^2 + V_3^2 + \dots}$$

dividing both sides by v_c , we get

$$\frac{v_t}{v_c} = \frac{\sqrt{v_1^2 + v_2^2 + v_3^2 + \dots}}{v_c}$$

$$= \sqrt{\frac{v_1^2}{v_c^2} + \frac{v_2^2}{v_c^2} + \frac{v_3^2}{v_c^2} + \dots}$$

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \dots}$$



Ex. 4.7.3 : The antenna current of an AM transmitter is 10 Amp when it is modulated to a depth of 30 % by an audio signal. It increases to 11 Amp when another signal modulates the carrier. What is the modulation index due to second wave ?

Soln. :

1. It is given that $I_{t1} = 10$ Amp, $m_1 = 0.3$, $I_{t2} = 11$ Amp

$$\left[\frac{I_{t1}}{I_c} \right]^2 = 1 + \frac{m_1^2}{2}$$

$$\therefore I_c = \frac{I_{t1}}{\left[1 + \frac{m_1^2}{2} \right]^{1/2}} = \frac{10}{\left[1 + \frac{(0.3)^2}{2} \right]^{1/2}} = 9.78 \text{ Amp}$$

After modulating with the second signal,

$$I_{t2}^2 = I_c^2 \left[1 + \frac{m_t^2}{2} \right]$$

$$m_t = \sqrt{2 \left[\left(\frac{I_{t2}}{I_c} \right)^2 - 1 \right]} = \sqrt{2 \left[\left(\frac{11}{9.78} \right)^2 - 1 \right]}$$

$$\therefore m_t = 0.73$$

$$\text{But } m_t = [m_1^2 + m_2^2]^{1/2}$$

$$\therefore m_2 = [m_t^2 - m_1^2]^{1/2} = [(0.73)^2 - (0.3)^2]^{1/2} = 0.66 \text{ or } 66\%$$

Types of Amplitude Modulation

Depending on the frequency components present in spectrum, AM is categorized as __

Description	Abbreviation
Double Sideband Full Carrier ie. AM	DSBFC
Double Sideband Suppressed Carrier	DSBSC
Single Sideband Suppressed Carrier	SSBSC

Advantages of DSBFC (AM)

1. AM signals are reflected back to earth from ionosphere layer. Due to this, AM signals can reach far places which are thousands of miles from source. Hence, AM radio has wider coverage.
2. Design of AM transmitter and receiver is relatively simple and inexpensive.
3. Low cost and reliable.

Disadvantages of DSBFC (AM)

1. Noise immunity is poor.
2. Wastage of power is high.
3. Require high bandwidth.

Applications of DSBFC (AM)

1. Radio broadcasting.
2. Picture transmission in TV system.

% Power saving in DSB-SC

$$\begin{aligned} P_t &= (1+m^2/2) P_c \\ &= 1.5 P_c \text{ for } m = 1 \end{aligned}$$

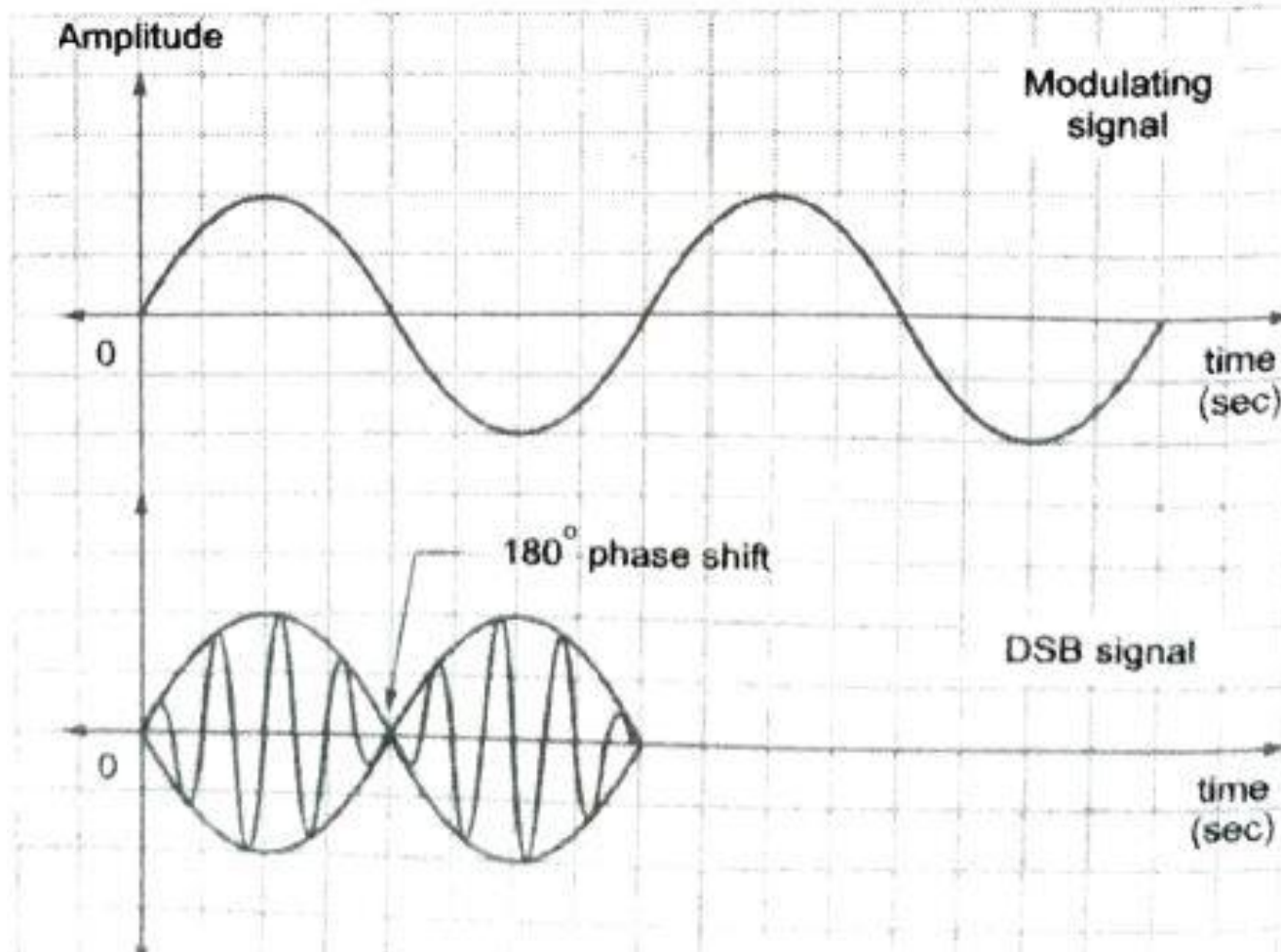
$$P_{USB} = P_{LSB} = 0.25 P_c$$

$$P_{SB} = (P_{USB} + P_{LSB}) = 0.5 P_c$$

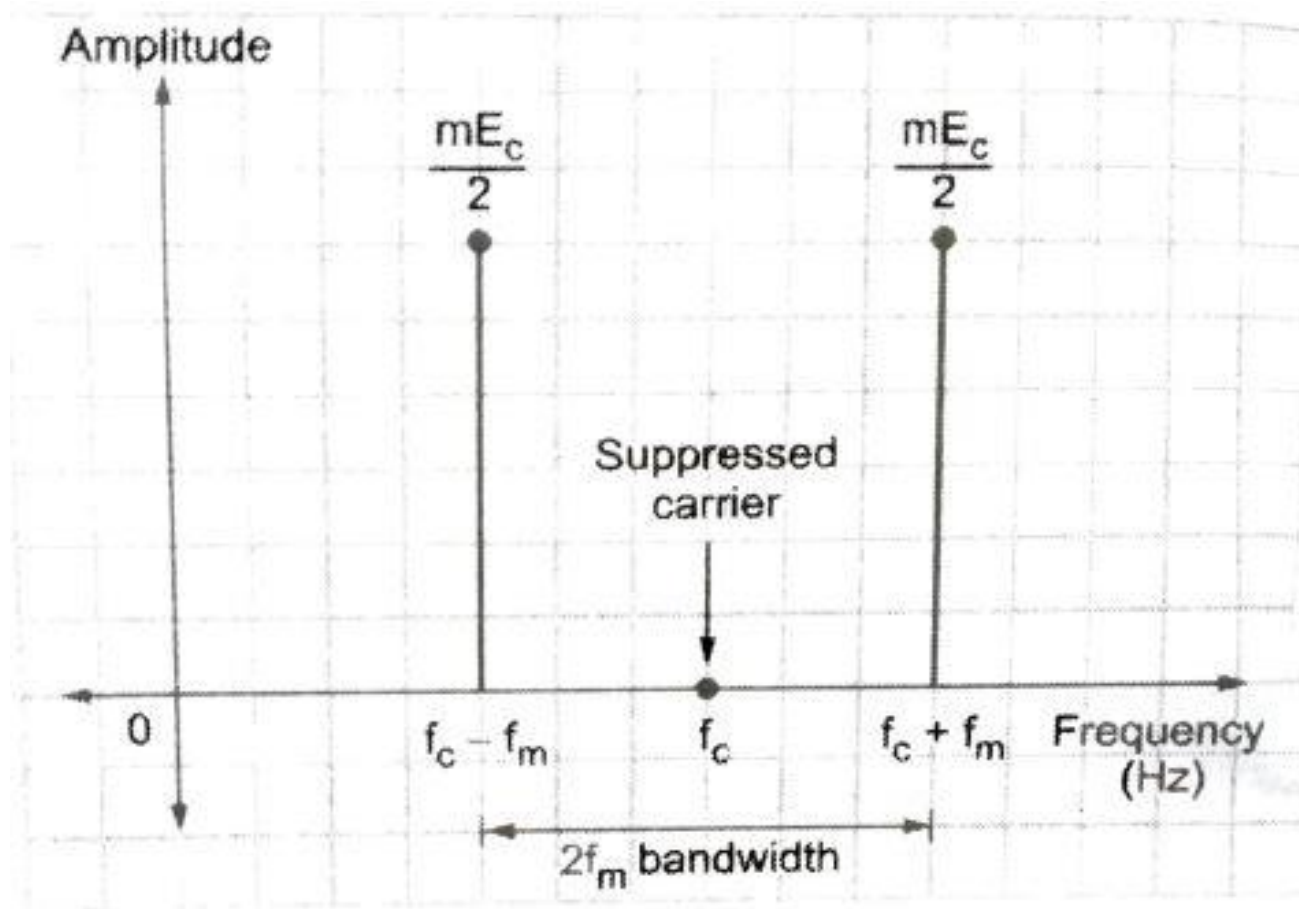
$$\text{Percentage Power saving} = (P_t - P_{SB}) / P_t * 100$$

$$= (1.5 - 0.5) P_c / 1.5 P_c * 100$$

$$= 66.66 \%$$



Time domain representation of DSBSC



Frequency domain representation of DSBSC

Advantages of DSBSC

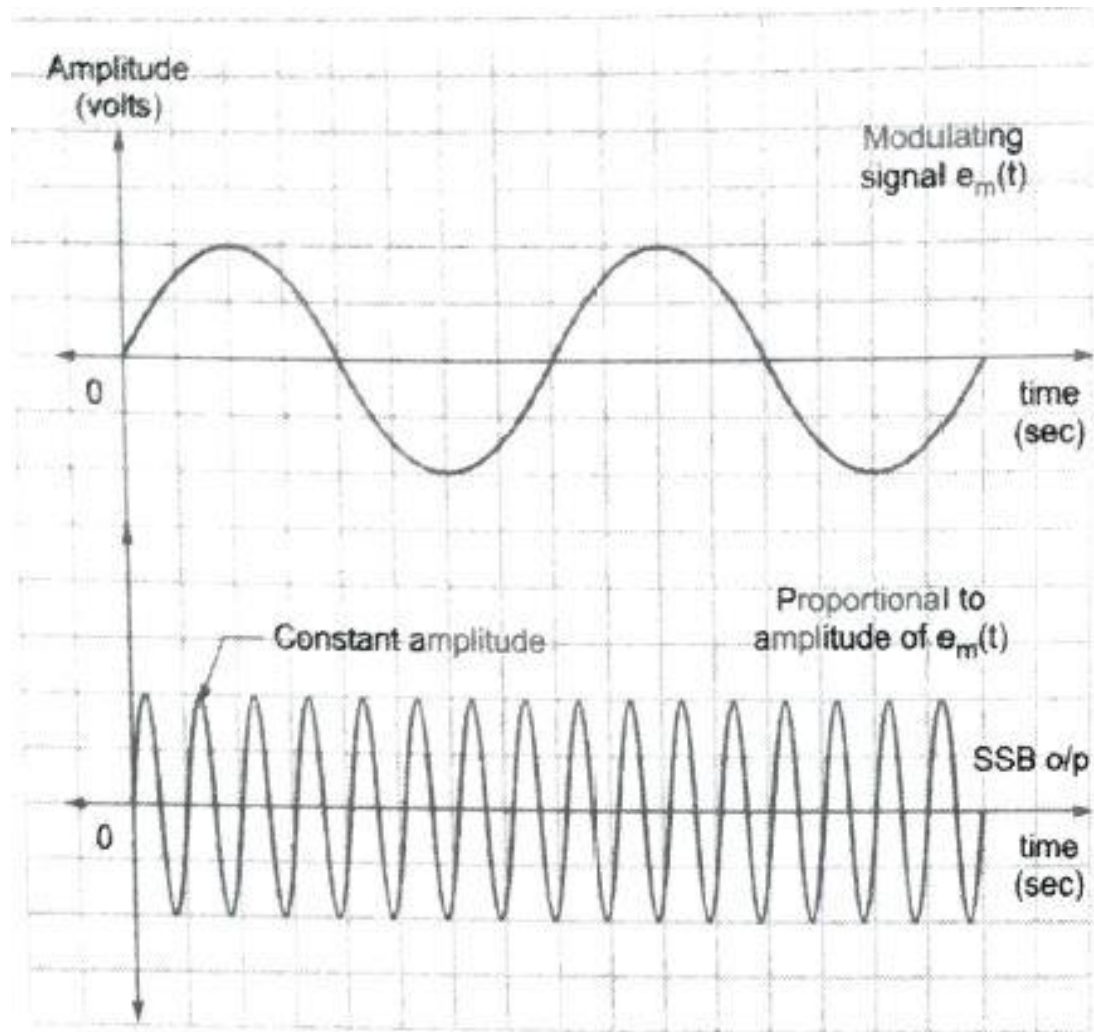
1. As the carrier is suppressed, transmission power is saved as compared with power transmission through AM.
2. Generation and detection is relatively simpler and inexpensive.

Disadvantages of DSBSC

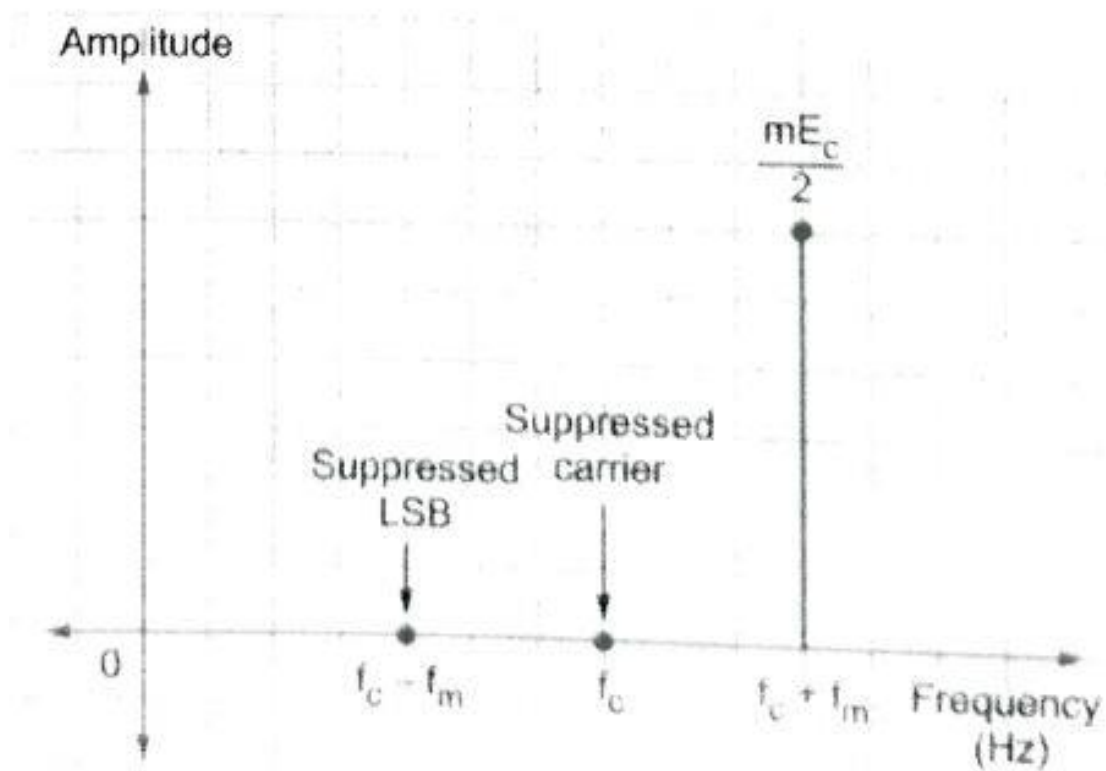
1. Two sidebands carry same information, so power wastage as compared to SSBSC
2. Less noise immunity.
3. It requires same BW as that of AM i.e. $2F_m$

Applications of DSB-SC

1. For transmitting stereo information, in FM sound broadcast at VHF.
2. CB radio
3. TV broadcasting
4. Air traffic control radios
5. Garage door opener keyless remotes
6. Two way radio communications.
7. Transmission of colour information in TV signals.



Time domain representation of SSBSC



Frequency domain representation of SSBSC

Advantages of SSBSC

1. Power is saved to great extent, as carrier and one of the sideband is suppressed.
2. SSB signal has more noise immunity as compared to DSB.

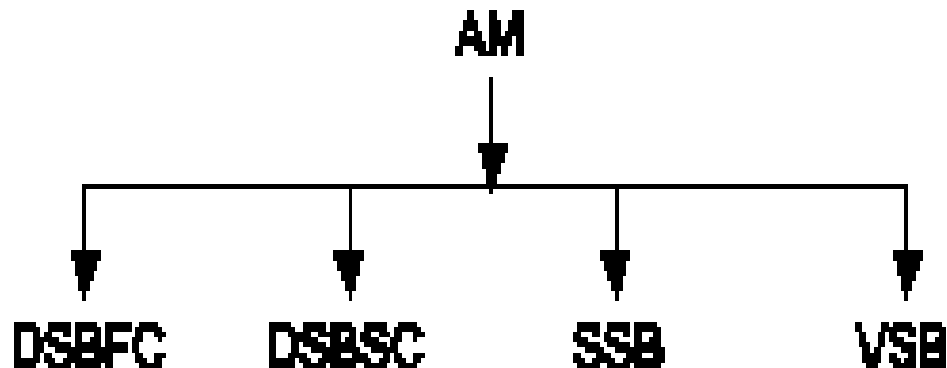
Disadvantages of SSBSC

1. SSB receivers are costly as compared to DSB receivers.

Applications of SSBSC

1. Applicable where power saving and low bandwidth is needed.
2. Wireless communication such as amateur radio.
3. Radar communications.
4. Telephone system.
5. Mobile Communication.
6. Point to Point Communication

Sideband Concept (DSB and SSB)

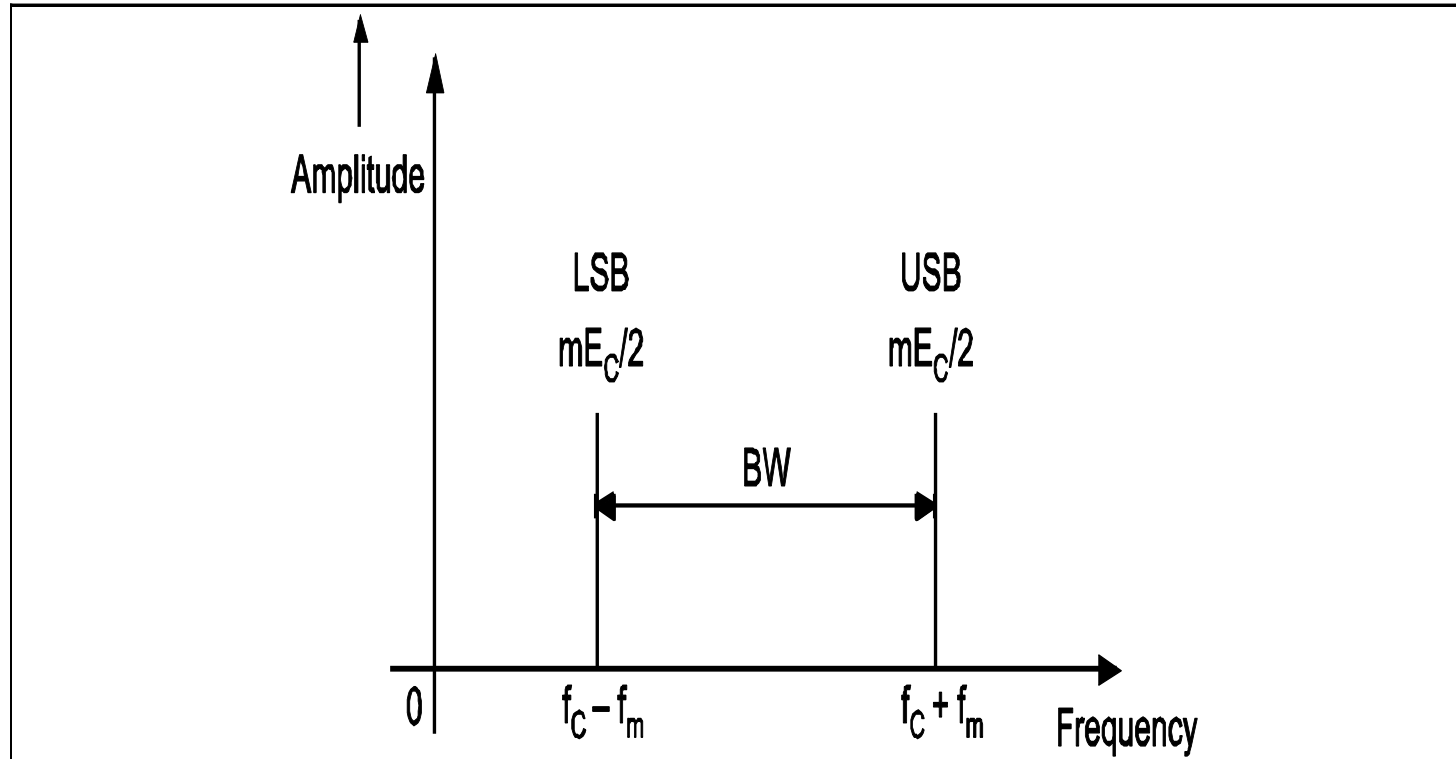


DSBFC:

Means double sideband full carrier as shown in Fig. 2.13 (a). Its $BW = 2f_m$.

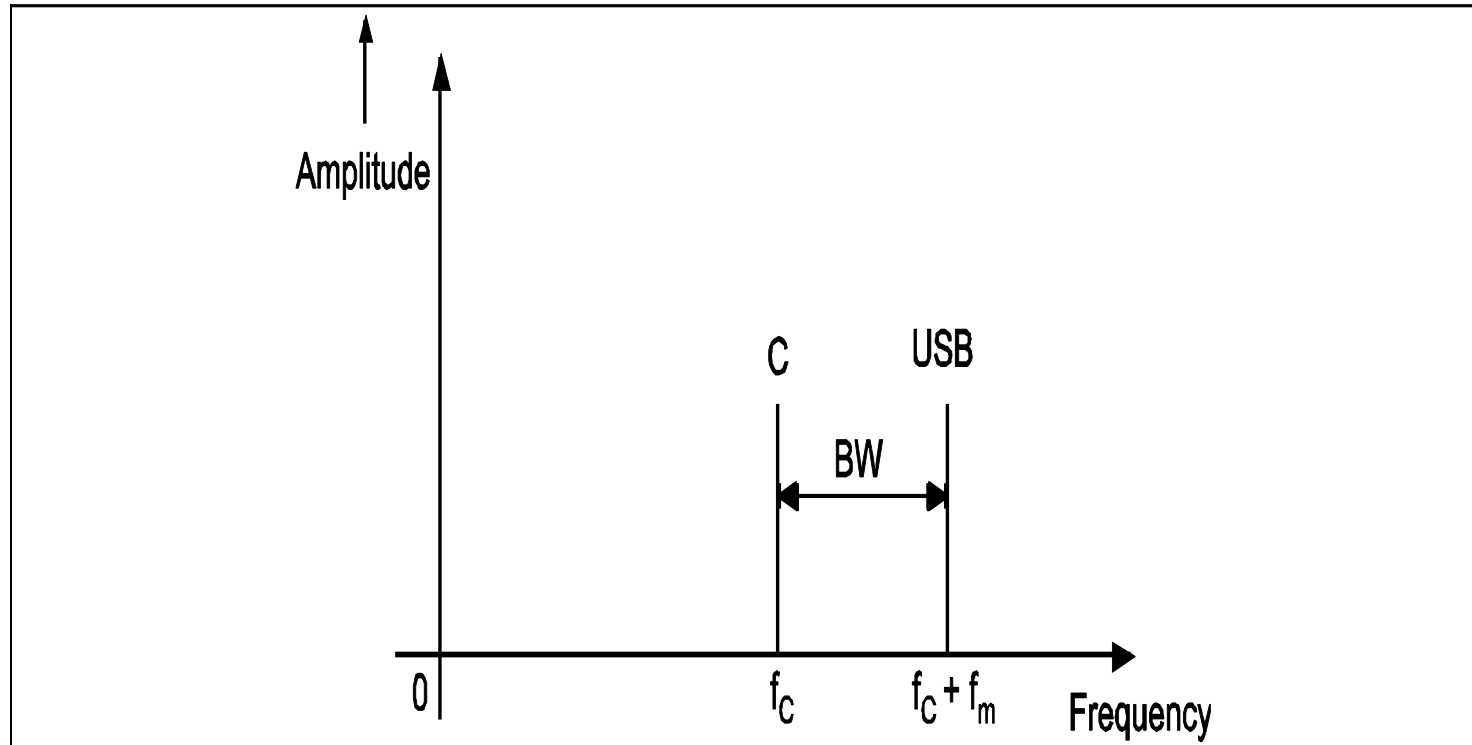
Continued...

DSBSC (or DSB):



Continued...

SSB:

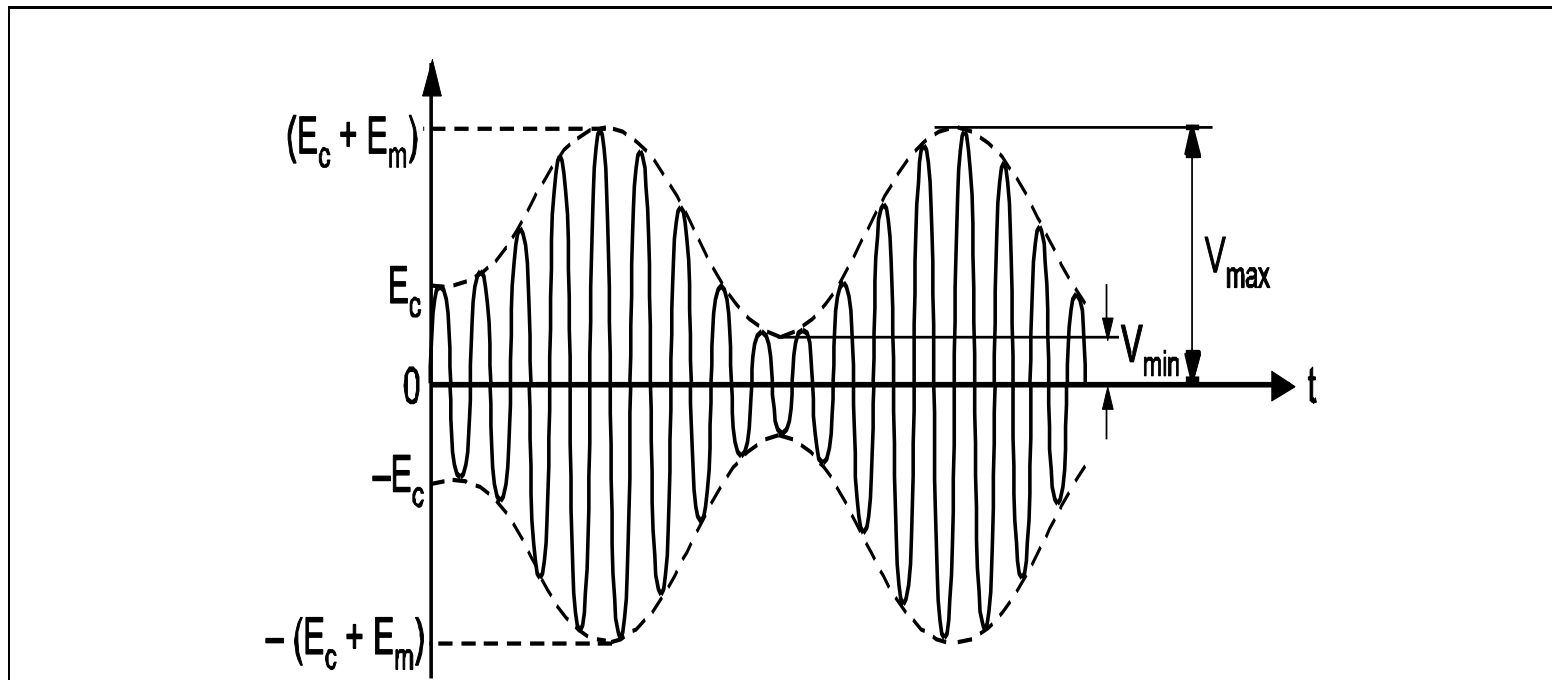


Representation of AM Wave

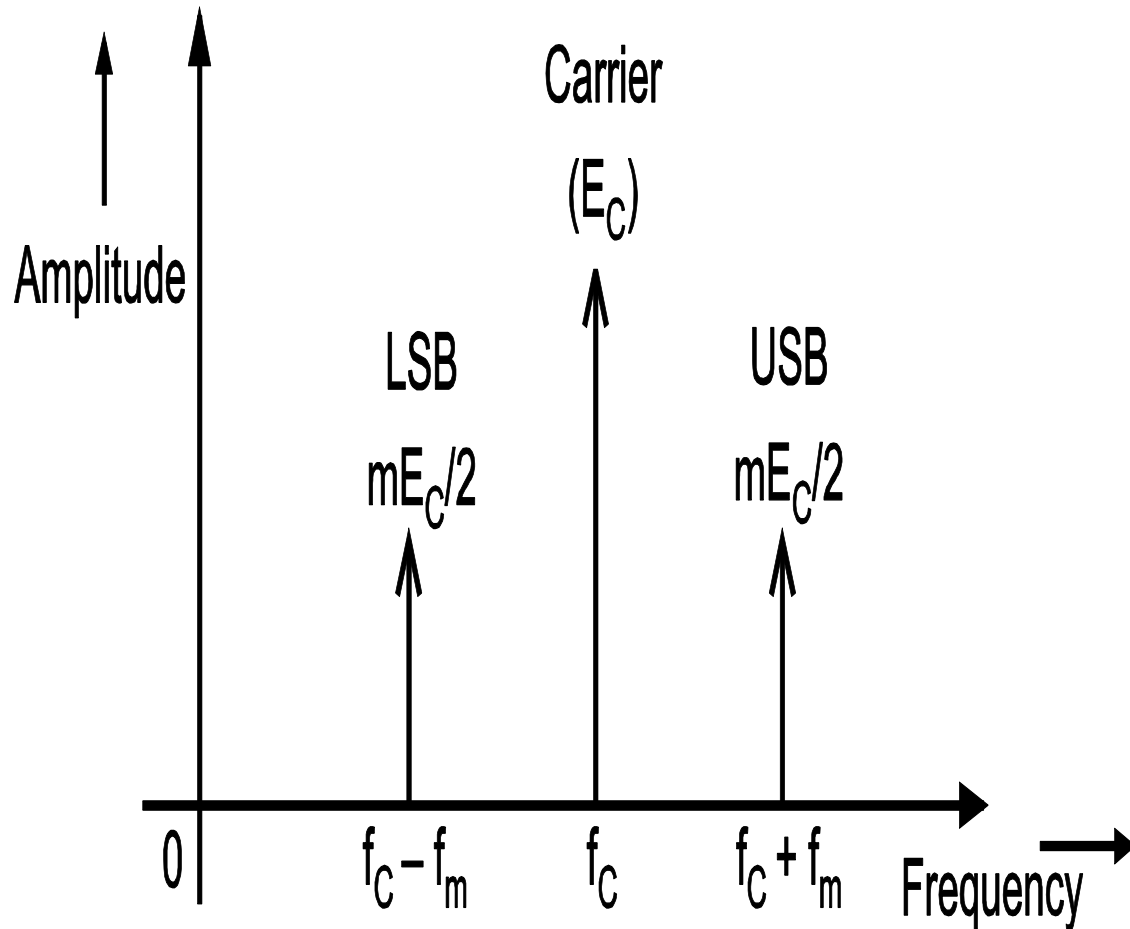
AM wave is represented in two ways:

- (i) In Time Domain
- (ii) In Frequency Domain

AM in Time Domain



AM in Frequency Domain



Example 1:

A modulating signal $20 \sin (2\pi \times 10^3 t)$ is used to modulate a carrier signal $40 \sin (2\pi \times 10^4 t)$. Find:

- (a) Modulation index
- (b) Percentage modulation
- (c) Sideband frequencies and their amplitude
- (d) Bandwidth of AM wave
- (e) Draw the frequency spectrum.

Solution:

Given: Modulating signal,

$$e_m = 20 \sin (2\pi \times 10^3 t) \dots (1)$$

$$e_m = E_m \sin (2\pi f_m t) \dots (2)$$

\therefore Compare equation (1) and (2), we get

$$E_m = 20 \text{ V}$$

$$f_m = 10^3 \text{ Hz} = 1 \text{ kHz}$$

Similarly, carrier signal

$$e_c = 40 \sin (2\pi \times 10^4 t) \dots (3)$$

But, $e_c = E_c \sin (2\pi f_c t) \dots (4)$

Compare equation (3) and (4), we get,

$$E_c = 40 \text{ v}$$

$$f_c = 10^4 \text{ Hz} = 100 \text{ kHz}$$

(a) Modulation Index:

$$m = 0.5$$

(b) Percentage modulation:

$$\% \text{ modulation} = m \times 100$$

$$= 0.5 \times 100$$

$$= 50\%$$

(c) Sideband frequencies and their amplitude:

$$\begin{aligned}\text{LSB} &= F_{\text{LSB}} &= f_c - f_m \\ &= 100 \text{ kHz} - 1 \text{ kHz} \\ &= 99 \text{ kHz}\end{aligned}$$

$$\begin{aligned}\text{USB} &= F_{\text{USB}} &= f_c + f_m \\ &= 100 \text{ k} + 1 \text{ kHz} \\ &= 101 \text{ kHz}\end{aligned}$$

$$\text{LSB amplitude} = \text{USB}$$

amplitude

$$\begin{aligned}&= 0.5 \times \\ &= 10 \text{ V}\end{aligned}$$

(d) Bandwidth of AM

$$\begin{aligned}\text{BW} &= 2 \times f_m \\ &= 2 \times 1 \text{ kHz}\end{aligned}$$

AM Transmitter

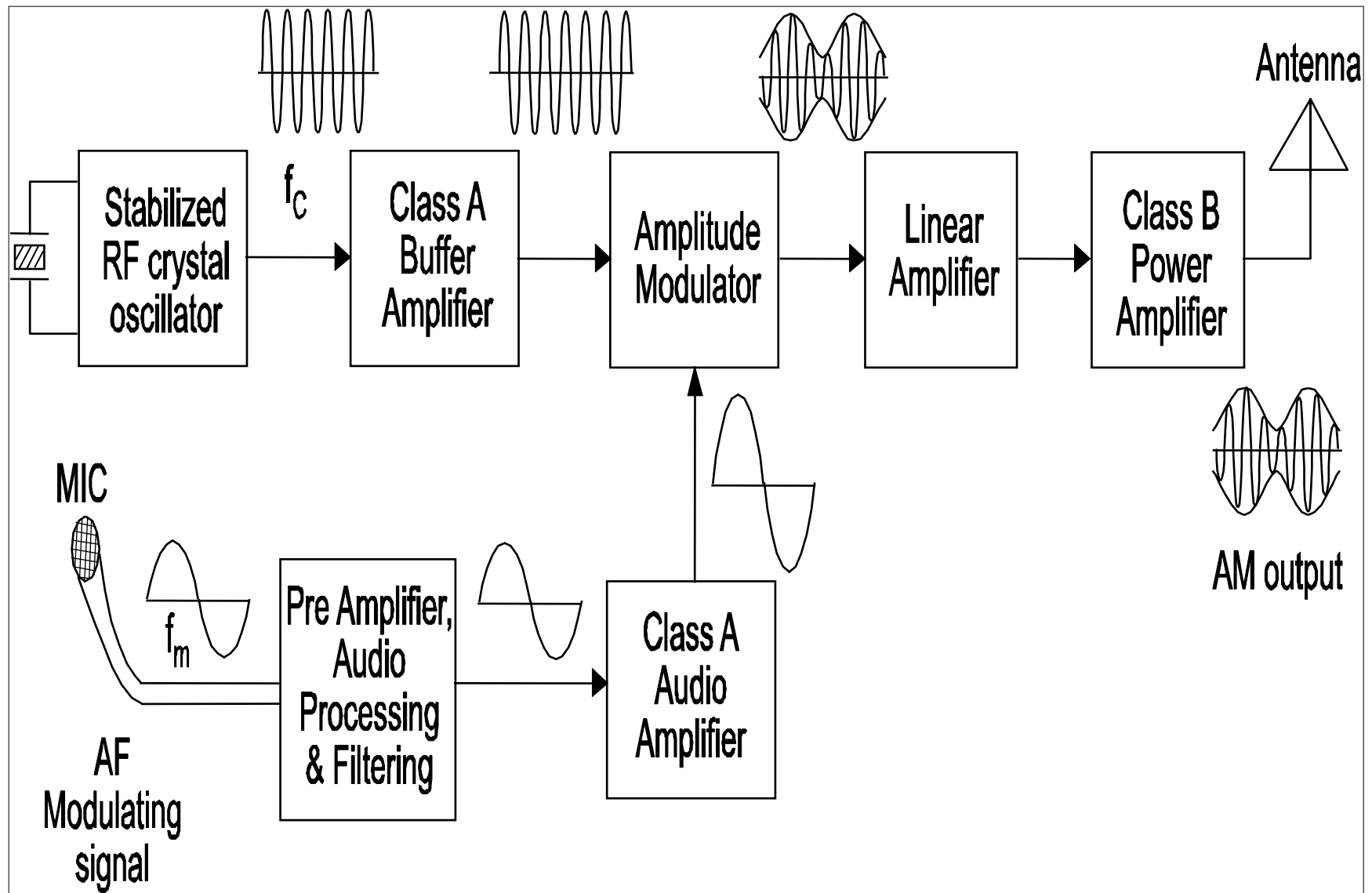
The functions of transmitter are:

1. To convert original information into electrical signal.
2. To amplify the weak signal.
3. To modulate the signal.
4. To increase the power level of modulated signal.
5. To transmit the signal through transmitting antenna.

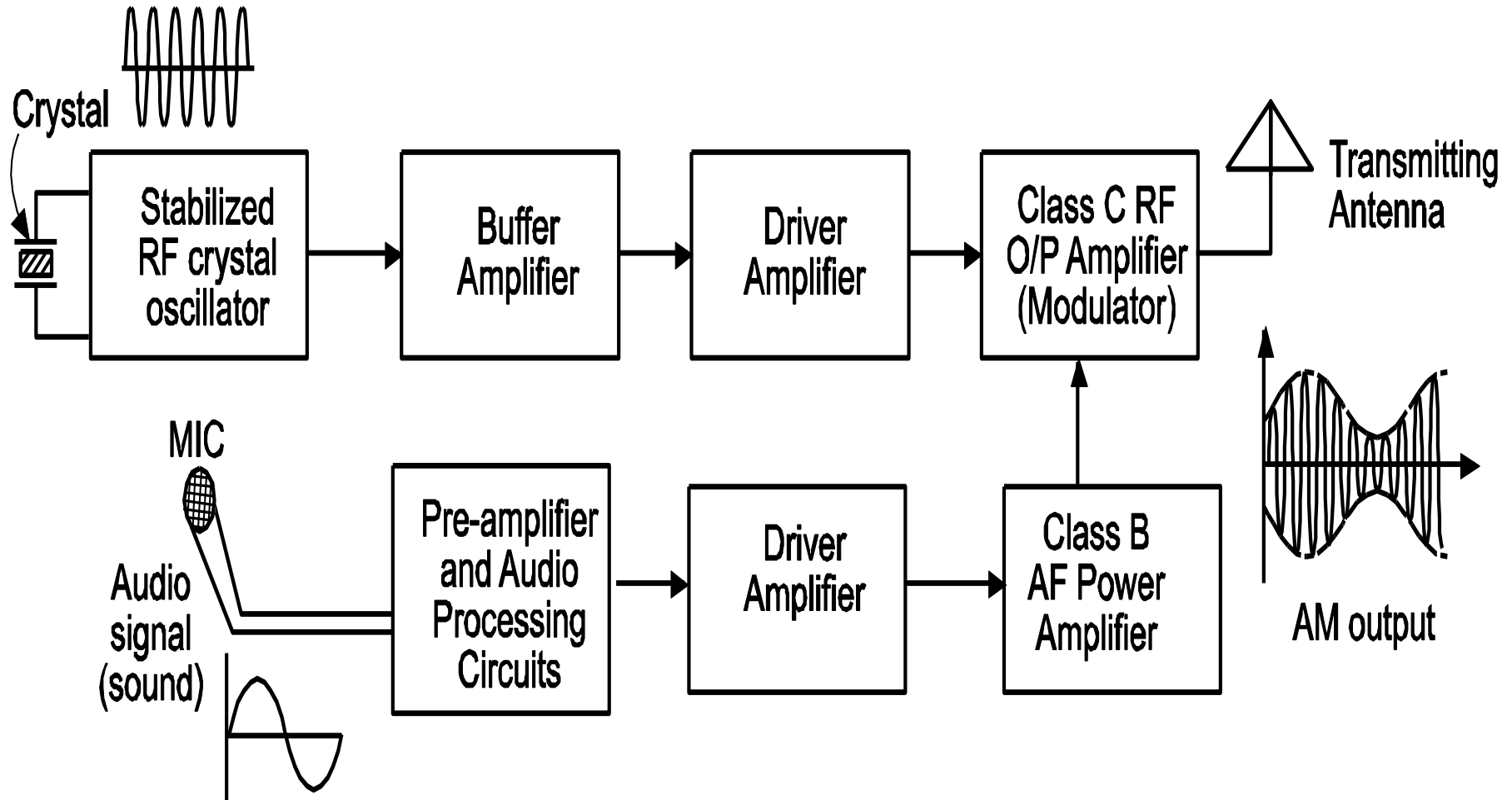
The AM transmitters are of two types:

1. Low level modulated transmitter.
2. High level modulated transmitter.

Low Level Modulated AM Transmitter



High Level Modulated AM Transmitter



Comparison between Low Level and High Level Modulation

Low Level Modulation	High Level Modulation
Modulation is carried out at low power level.	Modulation is carried out at high power level.
Needs lesser amplifier stages.	Needs more amplifier stages.
After modulation linear amplifiers can only be used. This gives lower power efficiency.	Nonlinear amplifiers can also be used which leads to higher power efficiency.
Class A, B, AB amplifiers are used	Class C amplifier is used
Used for high power broadcast transmitter	TV transmission

Generation of AM using Diode

There are two methods for generation of AM:

1. Square law modulator
2. Switching modulator

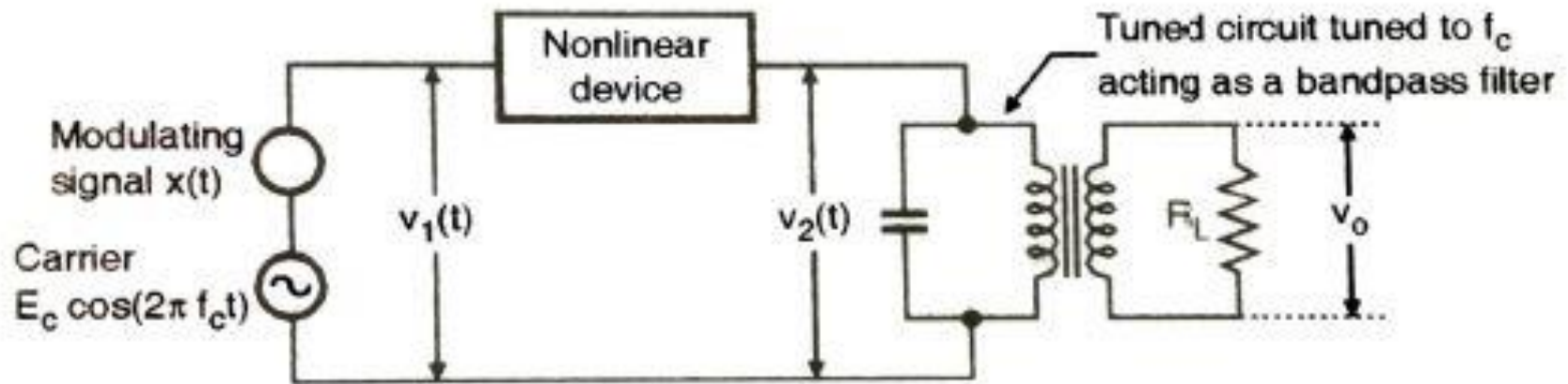
Both use a non-linear element such as diode for their implementation.

A nonlinear device is the device, with a nonlinear relation between its current and voltage.

Square law modulator

It consists of

1. A nonlinear device
2. A bandpass filter
3. A carrier source and modulating signal



A square law modulator

Square law modulator continue...

The modulating signal and carrier are connected in series with each other and their sum $v_1(t)$ is applied at the input of the nonlinear device, such as diode, transistor etc.

$$\therefore v_1(t) = x(t) + E_c \cos(2\pi f_c t)$$

The input output relation for nonlinear device is as follows :

$$v_2(t) = a v_1(t) + b v_1^2(t)$$

where a and b are constants.

Substituting the expression for $v_1(t)$ we get,

Square law modulator continue...

$$v_2(t) = a [x(t) + E_c \cos(2\pi f_c t)] + b [x(t) + E_c \cos(2\pi f_c t)]^2$$

$$\begin{aligned} \therefore v_2(t) &= ax(t) + aE_c \cos(2\pi f_c t) + b [x^2(t) + 2x(t)E_c \cos(2\pi f_c t) + E_c^2 \cos^2(2\pi f_c t)] \\ &= \underbrace{ax(t)}_{(1)} + \underbrace{aE_c \cos(2\pi f_c t)}_{(2)} + \underbrace{bx^2(t)}_{(3)} + \underbrace{2bx(t)E_c \cos(2\pi f_c t)}_{(4)} + \underbrace{bE_c^2 \cos^2(2\pi f_c t)}_{(5)} \end{aligned}$$

The five terms in the expression for $v_2(t)$ are as follows :

Term 1 : $ax(t) \rightarrow$ Modulating signal

Term 2 : $a E_c \cos(2 \pi f_c t) \rightarrow$ Carrier signal

Term 3 : $b x^2(t) \rightarrow$ Squared modulating signal

Term 4 : $2 b x(t) E_c \cos(2 \pi f_c t) \rightarrow$ AM wave with only sidebands

Term 5 : $b E_c^2 \cos^2(2 \pi f_c t) \rightarrow$ Squared carrier

Out of these five terms, terms 2 and 4 are useful whereas the remaining terms are not useful. Let us club terms 2,4 and 1,3,5 as follows to get.

$$\therefore v_2(t) = \underbrace{ax(t) + bx^2(t) + bE_c^2 \cos^2(2\pi f_c t)}_{\text{Unuseful terms}} + \underbrace{aE_c \cos(2\pi f_c t) + 2bx(t)E_c \cos(2\pi f_c t)}_{\text{useful terms}}$$

Square law modulator continue...

The LC tuned circuit acts as a band pass filter. This filter eliminates the unuseful terms from the equation.

Hence the output voltage $V_o(t)$ contains only the useful terms.

$$\begin{aligned}\therefore V_o(t) &= aE_c \cos(2\pi f_c t) + 2bx(t)E_c \cos(2\pi f_c t) \\ &= [aE_c + 2bx(t)E_c] \cos(2\pi f_c t)\end{aligned}$$

$$\therefore V_o(t) = aE_c \left[1 + \frac{2b}{a}x(t) \right] \cos(2\pi f_c t)$$

Compare this equation with the expression of standard AM wave. $e_{AM} = E_c(1 + m x(t))\cos(2\pi f_c t)$.

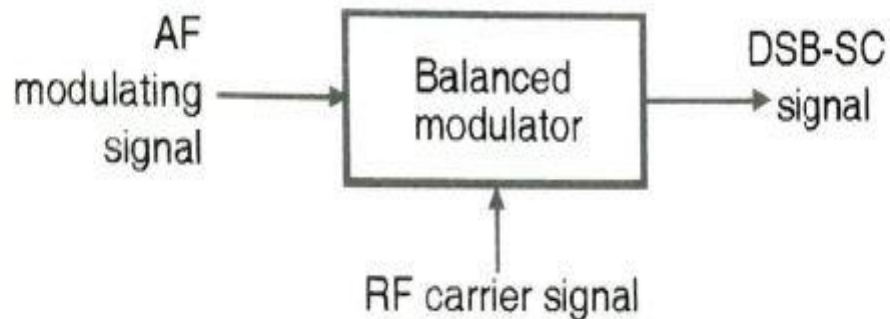
Thus $V_o(t)$ represents an AM wave. Thus the square law modulator produces AM wave.

Generation of DSBSC using Balanced Modulator

The balanced modulators are used to suppress the unwanted carrier in an AM wave.

The carrier and modulating signals are applied to the inputs of the balanced modulator and we get the DSB signal with suppressed carrier at the output of the balanced modulator.

Thus the output consists of the upper and lower sidebands only.



Principle of operation :

- The principle of operation of a balanced modulator states that if two signals at different frequencies are passed through a “nonlinear resistance” then at the output we get an AM signal with suppressed carrier.
- The device having a nonlinear resistance can be a diode or a JFET or even a bipolar transistor.

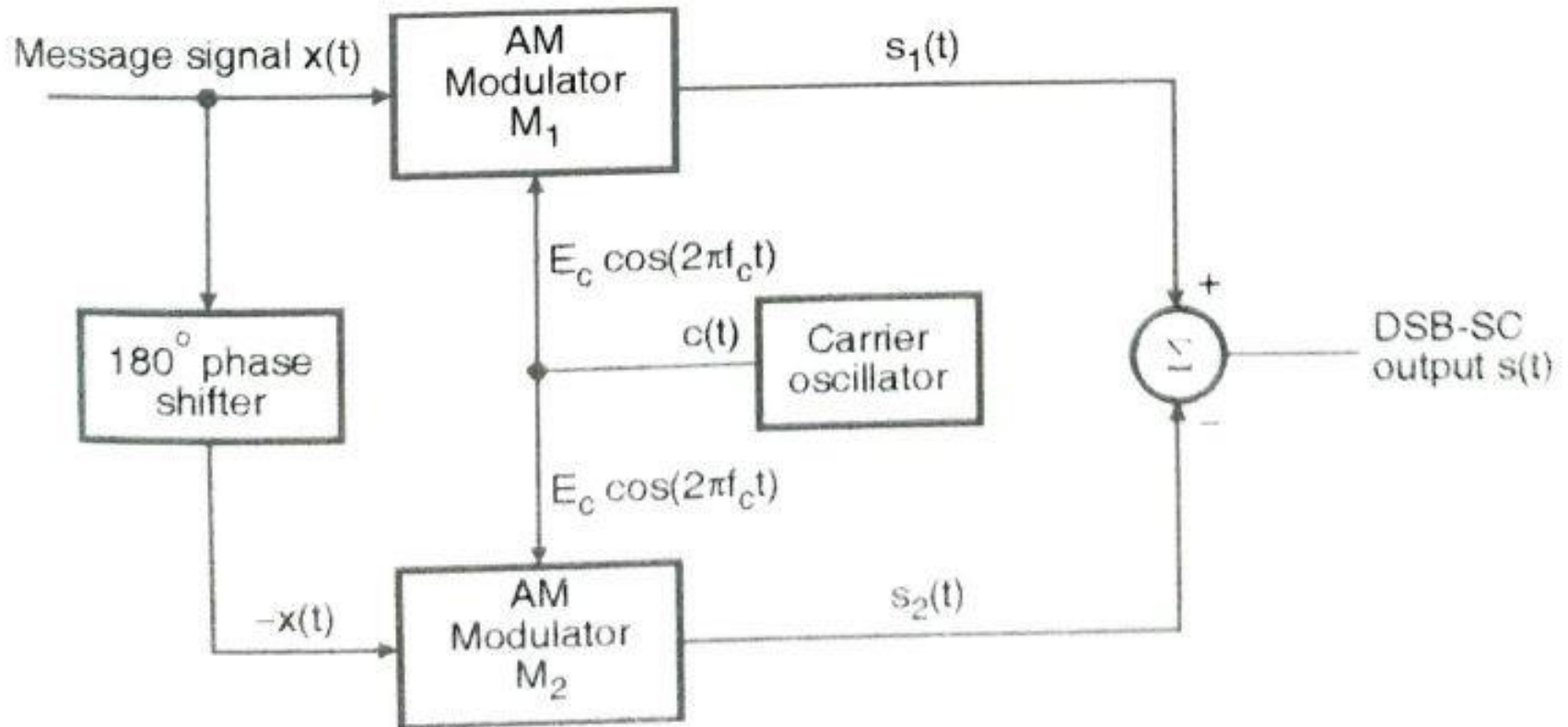
Types of balanced modulator :

The suppression of carrier can be done using the following two balanced modulators :

1. Using the diode ring modulator or lattice modulator.
2. Using the FET balanced modulator.

Block diagram for generation of DSBSC

BM using AM modulators



The carrier signal $c(t)$ is connected to both the AM modulators M_1 and M_2 . The message signal $x(t)$ is applied as it is to M_1 and its inverted version $-x(t)$ is applied to M_2 .

At the outputs of modulators M_1 and M_2 we get the standard AM signals $s_1(t)$ and $s_2(t)$ as follows :

Output of M_1 : $s_1(t) = E_c [1 + mx(t)] \cos(2\pi f_c t)$

Output of M_2 : $s_2(t) = E_c [1 - mx(t)] \cos(2\pi f_c t)$

These are then applied to a subtractor and the subtractor produces the desired DSB-SC signal as follows :

Subtractor output = $s_1(t) - s_2(t)$

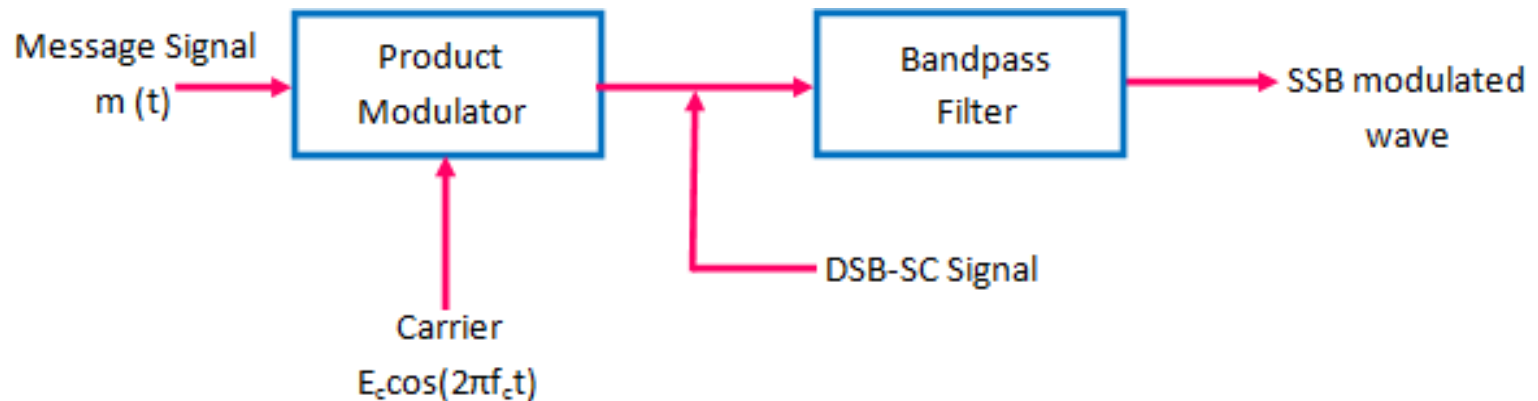
$$\begin{aligned}
 & E_c [1 + mx(t)] \cos(2\pi f_c t) - E_c [1 - mx(t)] \cos(2\pi f_c t) \\
 &= E_c \cos(2\pi f_c t) [1 + mx(t) - 1 + mx(t)] \\
 &= 2m E_c x(t) \cos(2\pi f_c t)
 \end{aligned}$$

The R.H.S. of this expression consists of product of $x(t)$ and $c(t) = E_c \cos(2\pi f_c t)$. Hence it represents a DSB-FC signal.

$$\begin{aligned}
 \therefore \text{Subtractor output} &= \text{DSB-SC signal} \\
 &= 2m E_c x(t) \cos(2\pi f_c t)
 \end{aligned}$$

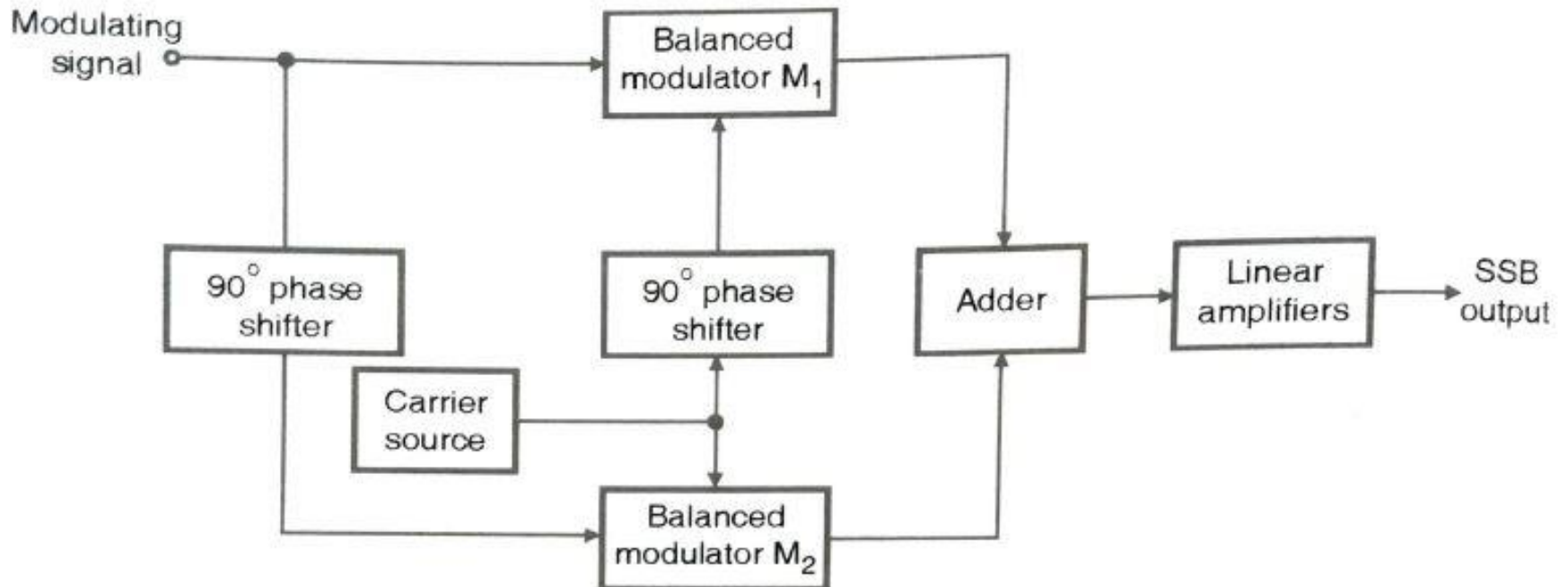
SSBSC Generation -

- 1) Filter method
- 2) Phase shift method



Filter method

Block diagram for generation of SSB using phase shift



Inputs to M_1 $\left\{ \begin{array}{l} \cos \omega_m t \\ \cos (\omega_c t + 90^\circ) \end{array} \right.$

....Modulating signal as it is
....90° phase shifted carrier.

And the inputs to balanced modulator M_2 are

Inputs to M_2 $\left\{ \begin{array}{l} \cos (\omega_m t + 90^\circ) \\ \cos \omega_c t \end{array} \right.$

....90° shifted modulating signal
....carrier as it is

So the output of $M_1 = \cos (\omega_c t) \cdot \cos \omega_m t$

$$= \frac{1}{2} \cos [\omega_c t + \omega_m t + 90^\circ] + \frac{1}{2} \cos [\omega_c t - \omega_m t + 90^\circ]$$

USB with 90° advance

LSB with 90° delay

And output of $M_2 = \cos \omega_c t \cdot \cos (\omega_m t + 90^\circ)$

$$= \underbrace{\frac{1}{2} \cos [\omega_c t + \omega_m t + 90^\circ]}_{\text{USB with } 90^\circ \text{ advance}} + \underbrace{\frac{1}{2} \cos [\omega_c t - \omega_m t - 90^\circ]}_{\text{LSB with } 90^\circ \text{ delay}}$$

USB with 90° advance

LSB with 90° delay

Output of the adder = output of M_1 + output of M_2

$$= \cos (\omega_c t + \omega_m t + 90^\circ)$$

- This output is obtained by adding Equations
- The LSBs in the outputs of M_1 and M_2 are 180° out of phase with respect to each other. Hence they are cancelled out when added.
- So the adder output contains only the upper sideband.

AM RECEIVERS

Electronic equipment which -

- picks up the desired signal,
- rejects the unwanted signal
- demodulates the modulated signal to get back the original information.



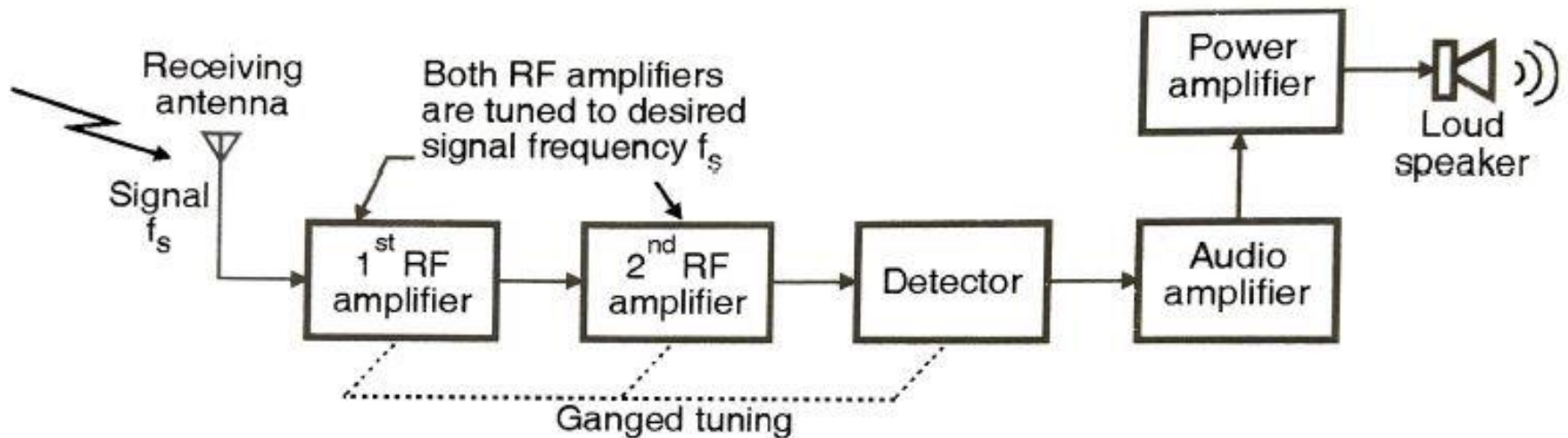
Functions of a Radio Receiver

- Intercept the incoming modulated signal
- Select desired signal and reject unwanted signals
- Amplify selected R.F signal
- Demodulate signal to get back original modulating signal
- Amplify original signal
- Apply amplified signal to loudspeaker

AM Receiver Types

1. Tuned Radio Frequency (TRF) Receiver
2. Superheterodyne Receiver

Tuned Radio Frequency (TRF) Receiver



Disadvantages of TRF

- It is very difficult to design at high frequency.
 - Difficult to design tunable RF stages.
 - Difficult to obtain high gain RF amplifiers
 - It has poor audio quality.
- This is mainly due to
 - Instability
 - Variation in BW
 - Poor Selectivity

Instability

- Overall gain of RF amplifier stages is very high. So a very small feedback signal from its output to input with correct phase can initiate oscillation in RF amplifier stage.
- Feedback takes place through stray capacitance in the circuit. Reactance of stray capacitance decreases at higher frequencies which result in the increased feedback.
- Thus the possibility of oscillatory behavior and hence instability will increase with increased frequency.
- Once oscillations begin, RF amplifier will stop to work as amplifier and operate as oscillator.

Variation in Bandwidth

- When the receiver is tuned, it is tuned to the carrier frequency (f_c), and the tuned circuit is expected to select the carrier and the sidebands of the desired signal.
- That means it must have adequate bandwidth (BW). For a tuned circuit,

$$BW = \frac{f_r}{Q}$$

- Where f_r is the resonant frequency which is f_c and Q is the quality factor.
- Let us assume that the required BW = 10 kHz. This should remain constant at all the carrier frequencies.
- Now at $f_r = f_c = 535$ kHz i.e. at the lowest frequency in the MW band the Q of the tuned circuit is,

$$Q = \frac{535}{10} = 53.5$$

- And at $f_r = f_c = 1640$ kHz, i.e. at the highest frequency in the MW band, for the same bandwidth of 10 kHz the required value of Q will be,

$$Q = \frac{1640}{10} = 164$$

- This value of Q is practically unobtainable due to various losses taking place at high frequency.
- At the most we can obtain a Q of 120 at this frequency. Now the corresponding bandwidth will be,

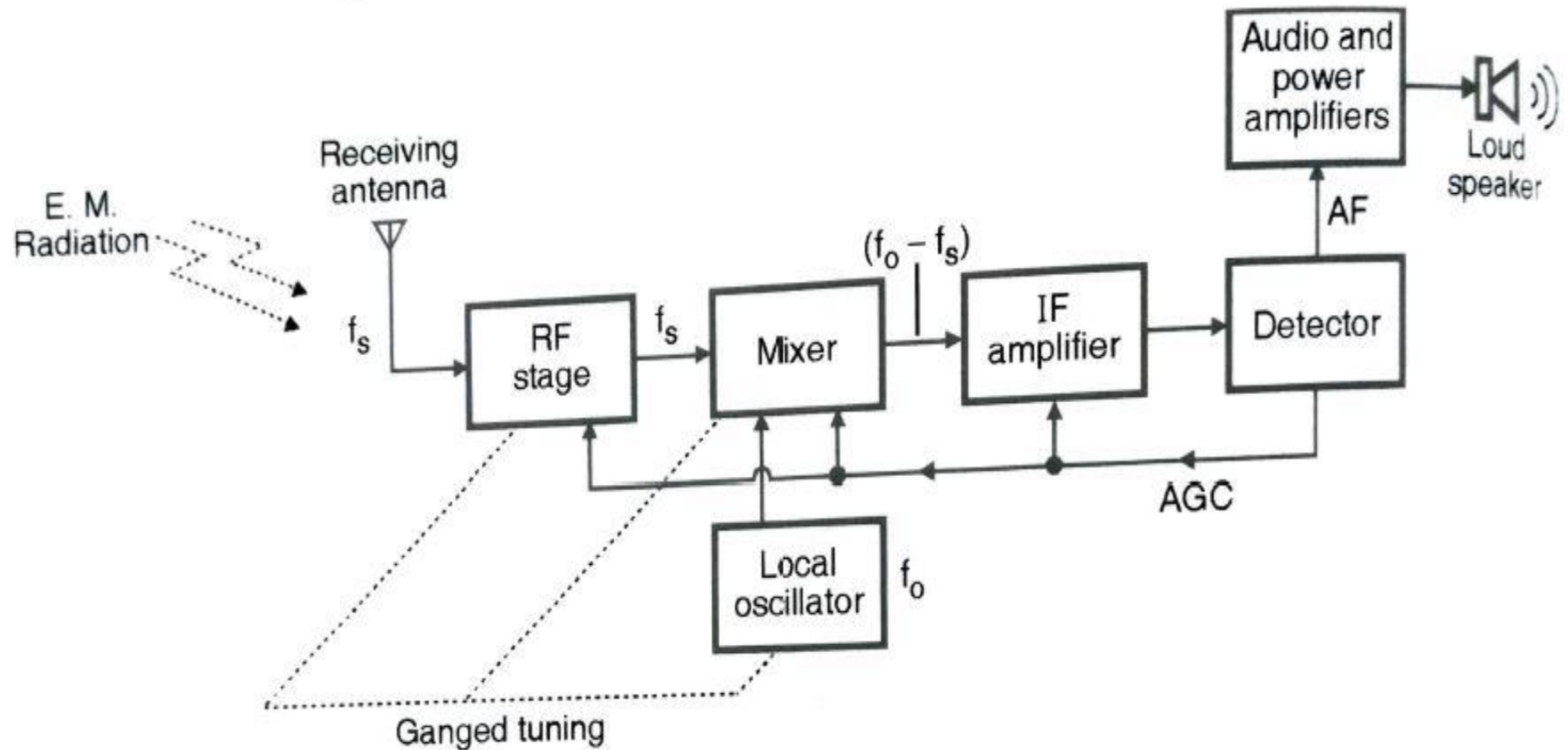
$$BW = \frac{f_r}{Q} = \frac{1640 \text{ kHz}}{120} = 13.7 \text{ kHz}$$

- But the required BW is 10 kHz. Due to increased bandwidth the receiver will pick the adjacent channel along with the desired one.

Insufficient selectivity

- Due to increased BW at higher frequencies, the ability of the TRF receiver to select the desired signal and reject all others is seriously affected. This is called loss of selectivity.
- Due to these problems of instability, and poor adjacent channel rejection, the TRF receivers are not used. They are replaced by the superheterodyne receivers.

Superheterodyne receivers



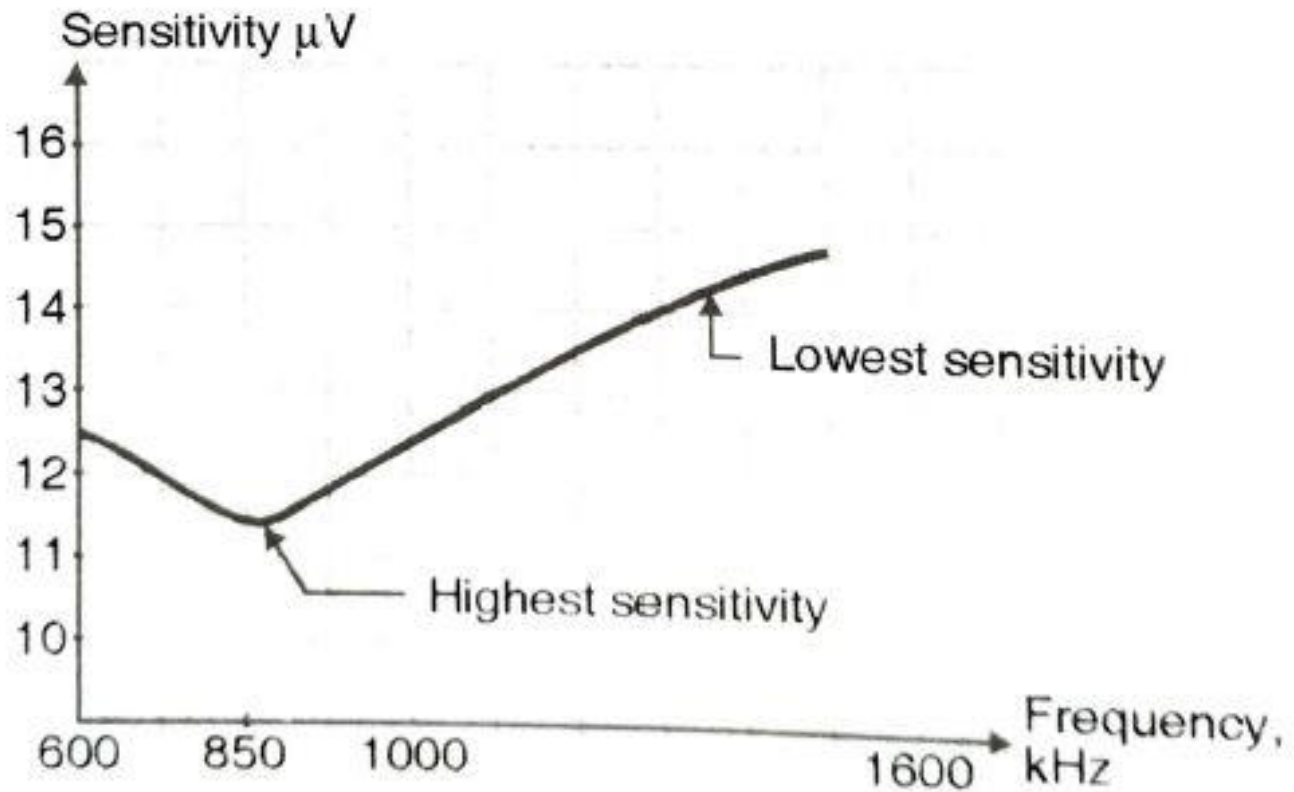
CHARACTERISTICS OF RADIO RECEIVER

- Sensitivity
- Selectivity
- Fidelity
- Double Spotting
- Image frequency and its rejection

Sensitivity

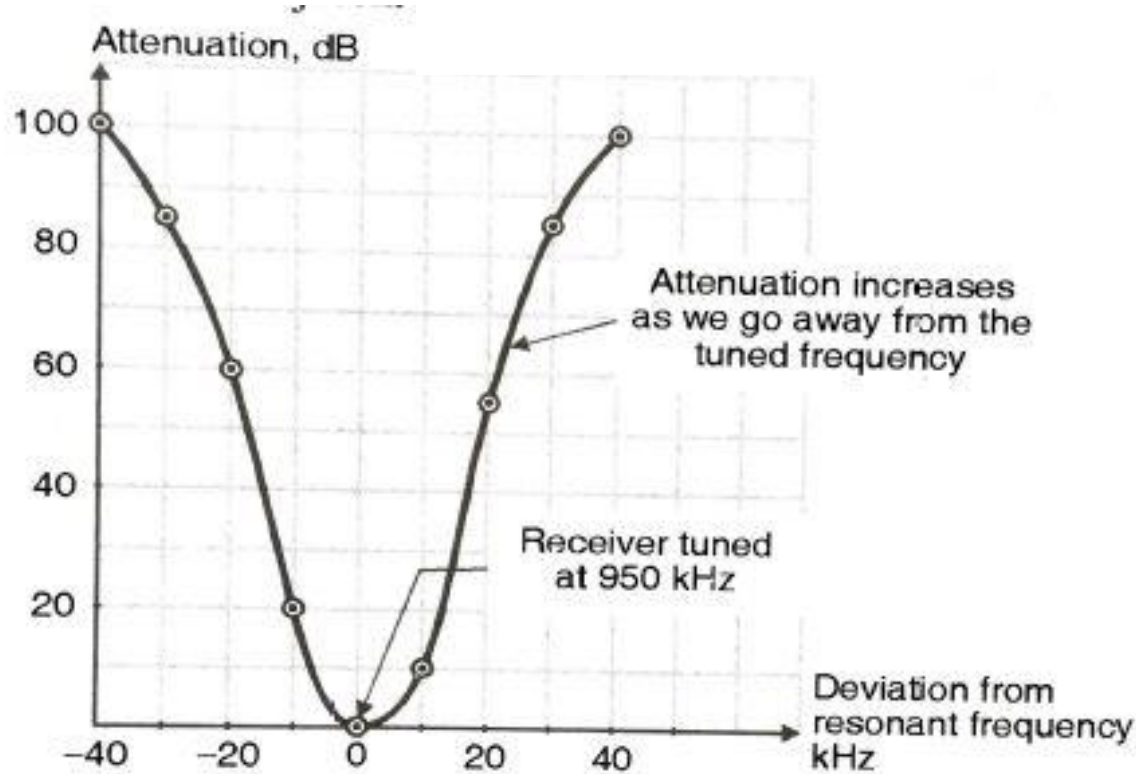
- Ability to amplify weak signals.
- Minimum RF signal level that can be detected at the input to produce a usable demodulated information signal.
- Receiver should have high sensitivity so that it gives good response to the desired signal.
- But should not have excessively high sensitivity otherwise it will pick up all undesired noise signals.
- It is a function of receiver gain .

Sensitivity



Selectivity

- Selectivity of radio receiver is its ability to reject unwanted signals.



Fidelity

- Ability of a communication system to produce an exact replica of the original source information at the output of the receiver.
- Radio receiver should have high fidelity or accuracy.
- For high fidelity, it is essential to have a flat frequency response over a wide range of audio frequencies when amplified.

Fidelity

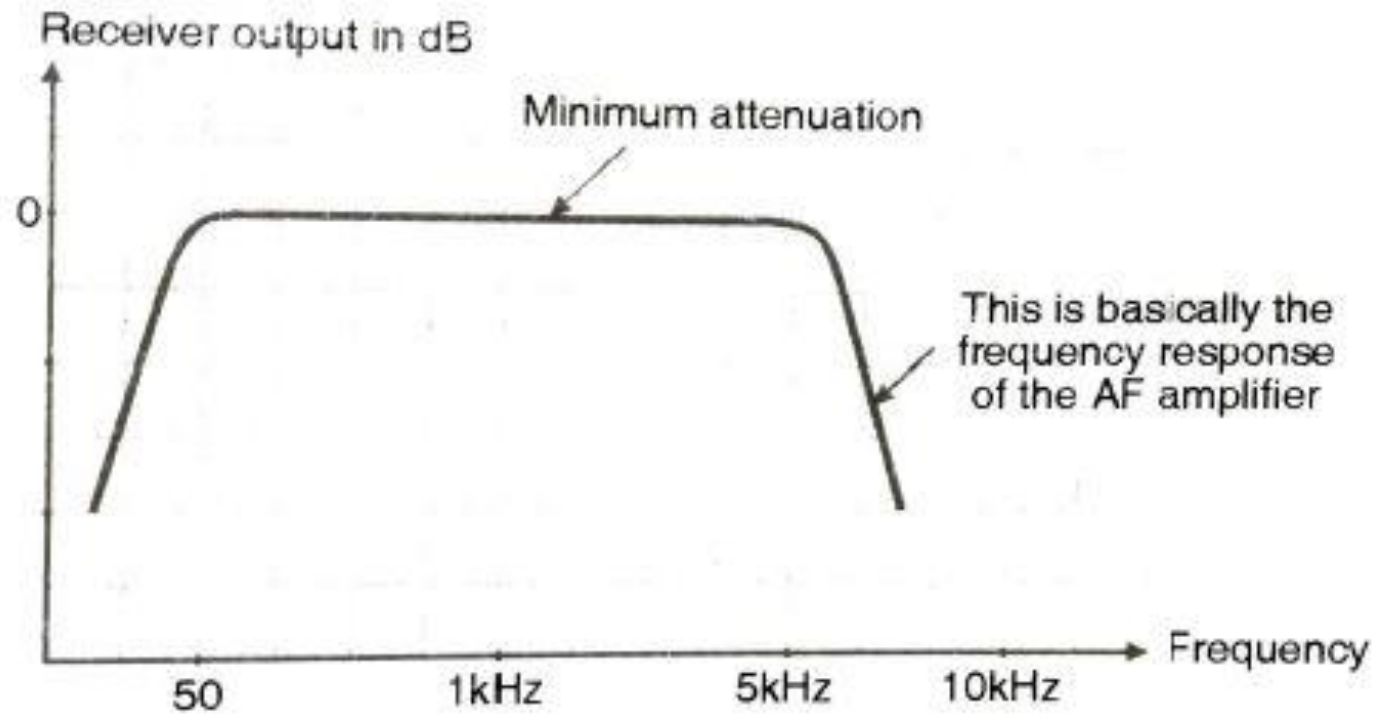


Image Frequency

- Generally, local oscillator frequency is equal to the sum of signal frequency and intermediate frequency.

$$f_o = f_s + f_i$$

- When f_s and f_o are mixed, the difference frequency is equal to f_i which is the only one passed and amplified by the IF stage.

Image Frequency

- Generally, local oscillator frequency is equal to the sum of signal frequency and intermediate frequency.

$$f_o = f_s + f_i$$

- When f_s and f_o are mixed, the difference frequency is equal to f_i which is the only one passed and amplified by the IF stage.

Image Frequency

- Suppose an undesired frequency $f_{si} = f_o + f_i$ reaches the mixer.
- The two frequency components will now be f_o (local oscillator) and f_{si} (undesired freq)
- And the harmonics will be $f_o, f_{si}, f_o + f_{si}, f_o - f_{si}$
- Substituting the value of f_{si} , the difference frequency is again f_i .

Image Frequency

- This IF signal will also be amplified by the IF stage and provide interference.
- This has the effect of two stations being received simultaneously.
- The term f_{si} is called the image frequency and is defined as the signal frequency plus twice the intermediate frequency.

$$f_{si} = f_s + 2f_i$$

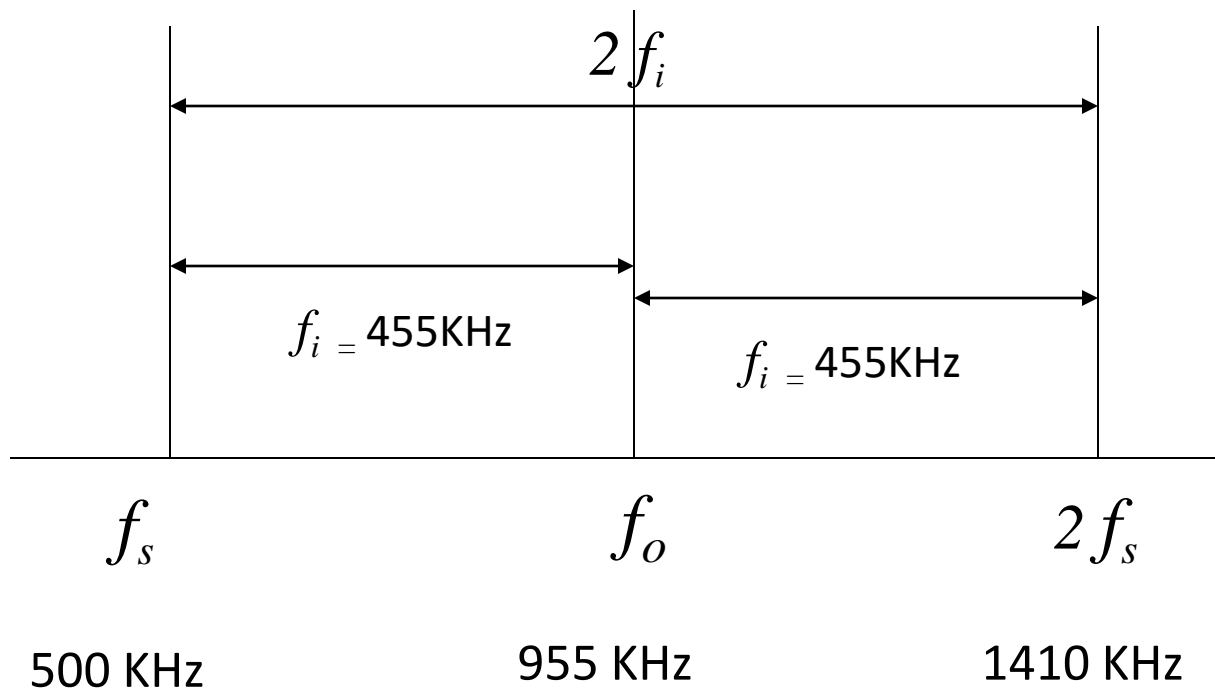


Image Frequency Rejection Ratio (IFRR)

- Ability of the preselector to reject the image frequency.
- Mathematically, it is given as

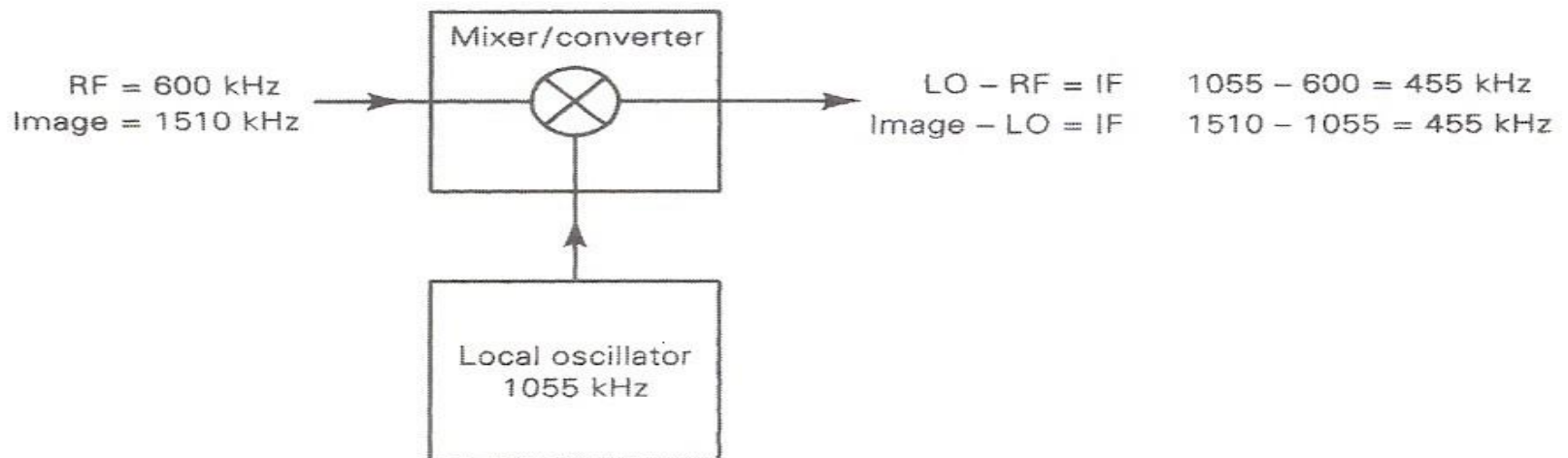
- $\alpha = \sqrt{1 + Q^2 \rho^2}$

where $\rho = \frac{f_{im}}{f_s} - \frac{f_s}{f_{im}}$

$$\alpha (dB) = 10 \log \sqrt{1 + Q^2 \rho^2}$$

IMAGE FREQUENCY

- Once an image frequency is down-converted to IF, it cannot be removed. In order to reject the image frequency, it has to be blocked prior to the mixer stage.



Choice of Intermediate Frequency

- High IF – Better image frequency rejection
Difficult to build stable amplifiers.
- Low IF – Stability since low frequency is used
Poor image frequency rejection

Trade off between image frequency rejection and stability.

Double Spotting

- Same stations get picked up at two different nearby points, on the receiver dial.
- Due to inadequate image frequency rejection.
- Harmful, since a weak station can be masked by the reception of a strong station at the same point.
- Can be reduced by increasing front end selectivity of the receiver.
- Inclusion the RF amplifier stage will help in avoiding double spotting.

For an AM broadcast-band Superheterodyne Receiver with RF and IF frequencies of 600 kHz and 455 kHz respectively. Determine

1. Local oscillator frequency
2. Image frequency
3. Image frequency rejection ratio for a preselector Q of 100.

Given : IF = 455 kHz and f_s = signal frequency = 600 kHz

Local oscillator frequency :

$$\begin{aligned}f_o &= f_s + \text{IF} \\&= (600 + 455) \text{ kHz} \\ \therefore f_o &= 1055 \text{ kHz}\end{aligned}$$

Image frequency :

$$\begin{aligned}f_{si} &= f_s + 2 \text{ IF} \\&= 600 + (2 \times 455) \\f_{si} &= 1510 \text{ kHz}\end{aligned}$$

Image frequency rejection ratio :

$$\text{Given } Q = 100$$

$$\alpha_1 = \sqrt{1 + Q^2 \rho^2}$$

$$\text{where } \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1510}{600} - \frac{600}{1510}$$

$$\therefore \rho = 2.119$$

$$\alpha = \sqrt{1 + [(100)^2 \times (2.119)^2]}$$

$$\alpha = 211.90$$

In an AM radio receiver the loaded Q of the antenna circuit at the input to the mixer is 100. If the intermediate frequency is 455 kHz, calculate the image frequency and its rejection at 1 MHz.

Loaded $Q = 100$, $IF = 455 \text{ kHz}$, $f_s = 1 \text{ MHz}$

Calculate image frequency :

$$\begin{aligned}f_{si} &= f_s + 2 \text{ IF} \\&= 1 \text{ MHz} + (2 \times 455 \text{ kHz}) \\&= 1910 \text{ kHz}\end{aligned}$$

Calculate image rejection ratio :

$$\alpha = \sqrt{1 + Q^2 \rho^2}$$

$$\text{But } \rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910}{1000} - \frac{1000}{1910}$$

$$\rho = 1.3864 \text{ and } Q = 100$$

$$\therefore \alpha = \sqrt{1 + [100^2 \times (1.3864)^2]}$$

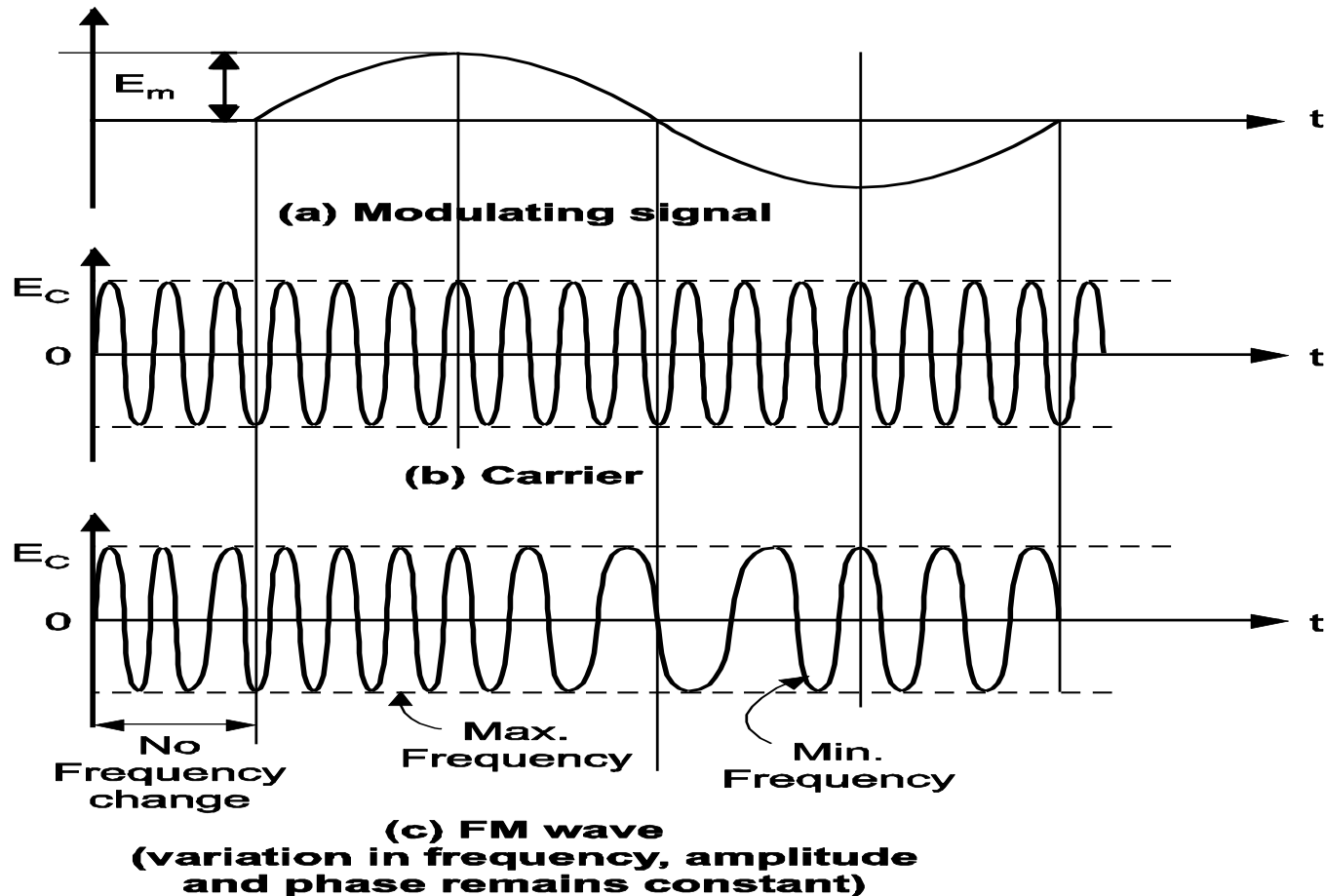
$$\therefore \alpha = 138.64$$

Frequency Modulation

Definition of FM:

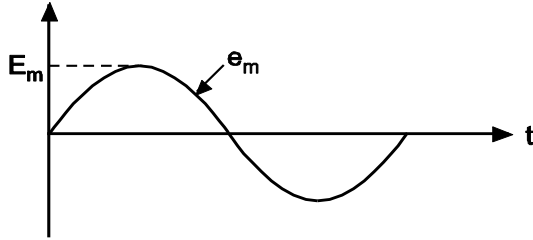
Frequency modulation is a technique of modulation in which the frequency of carrier is varied in accordance with the instantaneous amplitude of the modulating signal.

- In FM, amplitude and phase remains constant.
- Thus, the information is conveyed via. frequency changes



Mathematical Representation of FM

(i) Modulating Signal:



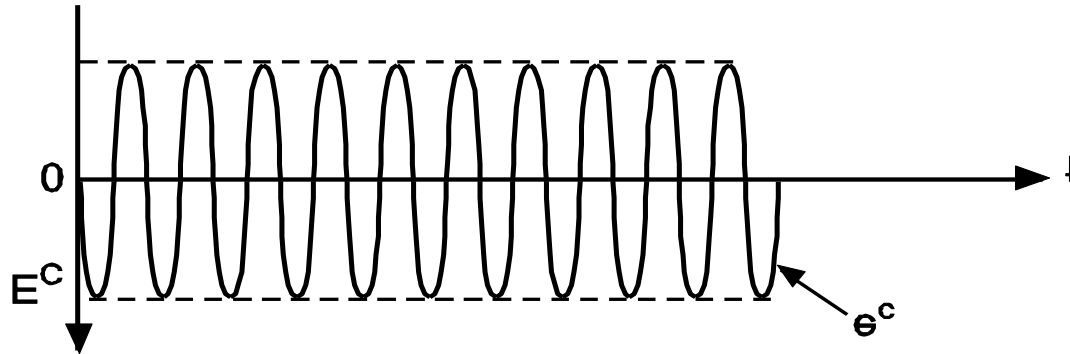
It may be represented as,

$$e_m = E_m \cos \omega_m t \quad \dots(1)$$

Here cosine term is taken for simplicity
where,

e_m	=	Instantaneous amplitude
ω_m	=	Angular velocity
	=	$2\pi f_m$
f_m	=	Modulating frequency

(ii) Carrier Signal:



Carrier may be represented as,

$$e_c = E_c \sin (\omega_c t + \phi) \quad \text{-----(2)}$$

where,

e_c	=	Instantaneous amplitude
ω_c	=	Angular velocity
	=	$2\pi f_c$
f_c	=	Carrier frequency
ϕ	=	Phase angle

(iii) FM Wave:

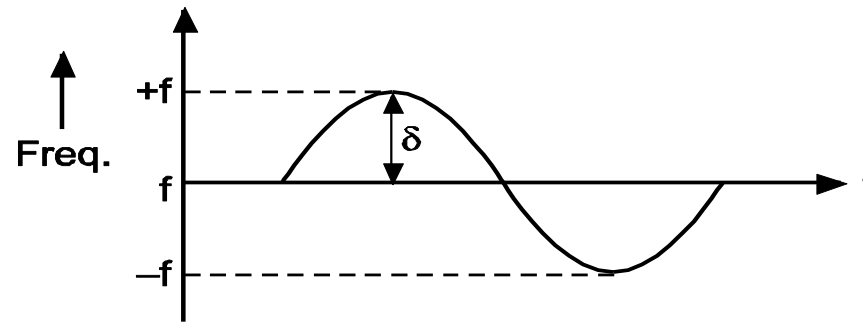


Fig. Frequency Vs. Time in FM

FM is nothing but a deviation of frequency.

From Fig. 2.25, it is seen that instantaneous frequency 'f' of the FM wave is given by,

$$\mathbf{f = f_c (1 + K E_m \cos \omega_m t) \dots (3)}$$

where,

f_c = Unmodulated carrier frequency

K = Proportionality constant

$E_m \cos \omega_m t$ = Instantaneous modulating signal

(Cosine term preferred for simplicity otherwise we can use sine term also)

The maximum deviation for this particular signal will occur, when $\cos \omega_m t = \pm 1$ i.e. maximum.

\therefore Equation (2.26) becomes,

$$\mathbf{f = f_c (1 \pm K E_m) \dots (4)}$$

$$\therefore \mathbf{f = f_c \pm K E_m f_c \dots (5)}$$

So that maximum deviation δ will be given by,

$$\delta = K E_m f_c \quad \dots (6)$$

The instantaneous amplitude of FM signal is given by,

$$\begin{aligned} e_{FM} &= A \sin [f(\omega_c, \omega_m)] \\ &= A \sin \theta \quad \dots (7) \end{aligned}$$

where,

$$f(\omega_c, \omega_m) = \text{Some function of carrier and modulating}$$

frequencies

Let us write equation (3) in terms of ω as,

$$\omega = \omega_c (1 + K E_m \cos \omega_m t)$$

To find θ , ω must be integrated with respect to time.

Thus,

$$\begin{aligned} \theta &= \int \omega \, dt \\ &= \int \omega_c (1 + K E_m \cos \omega_m t) dt \\ &= \omega_c \left(t + K E_m \frac{\sin \omega_m t}{\omega_m} \right) \\ &= \omega_c t + K E_m \omega_c \frac{\sin \omega_m t}{\omega_m} \end{aligned}$$

$$= \omega_c t + K E_m f_c \frac{\sin \omega_m t}{f_m}$$

$$= \omega_c t + \frac{\delta \sin \omega_m t}{f_m} \quad [\because \delta = K E_m f_c]$$

\therefore

Substitute value of θ in equation (7)

Thus,

$$e_{FM} = A \sin \left(\omega_c t + \frac{\delta \sin \omega_m t}{f_m} \right) \text{---(8)}$$

$$e_{FM} = A \sin (\omega_c t + m_f \sin \omega_m t) \text{---(9)}$$

This is the equation of FM.

Modulation Index

Definition:

Modulation Index is defined as the ratio of frequency deviation (δ) to the modulating frequency (f_m).

$$\text{M.I.} = \frac{\text{Frequency Deviation}}{\text{Modulating Frequency}}$$

$$mf = \frac{\delta}{f_m}$$

In FM M.I. > 1

Modulation Index of FM decides –

- (i) Bandwidth of the FM wave.
- (ii) Number of sidebands in FM wave.

Deviation Ratio

The modulation index corresponding to maximum deviation and maximum modulating frequency is called deviation ratio.

$$\begin{aligned}\text{Deviation Ratio} &= \frac{\text{Maximum Deviation}}{\text{Maximum modulating Frequency}} \\ &= \frac{\delta_{\max}}{f_{\max}}\end{aligned}$$

In FM broadcasting the maximum value of deviation is limited to 75 kHz. The maximum modulating frequency is also limited to 15 kHz.

Percentage modulation

The percentage modulation is defined as the ratio of the actual frequency deviation produced by the modulating signal to the maximum allowable frequency deviation.

$$\% \text{ M.I} = \frac{\text{Actual deviation}}{\text{Maximum allowable deviation}}$$

Frequency Spectrum of FM

Frequency spectrum is a graph of amplitude versus frequency.

The frequency spectrum of FM wave tells us about number of sideband present in the FM wave and their amplitudes.

The expression for FM wave is not simple. It is complex because it is sine of sine function.

Only solution is to use '**Bessels Function**'.

Equation (9) may be expanded as,

$$\begin{aligned} e_{FM} = & A \{ J_0(m_f) \sin \omega_c t \\ & + J_1(m_f) [\sin(\omega_c + \omega_m) t - \sin(\omega_c - \omega_m) t] \\ & + J_2(m_f) [\sin(\omega_c + 2\omega_m) t + \sin(\omega_c - 2\omega_m) t] \\ & + J_3(m_f) [\sin(\omega_c + 3\omega_m) t - \sin(\omega_c - 3\omega_m) t] \\ & + J_4(m_f) [\sin(\omega_c + 4\omega_m) t + \sin(\omega_c - 4\omega_m) t] \\ & + \dots \} \quad \dots (10) \end{aligned}$$

From this equation it is seen that the FM wave consists of:

- (i) Carrier (First term in equation).
- (ii) Infinite number of sidebands (All terms except first term are sidebands).

The amplitudes of carrier and sidebands depend on 'J' coefficient.

$$\omega_c = 2\pi f_c, \quad \omega_m = 2\pi f_m$$

So in place of ω_c and ω_m , we can use f_c and f_m .

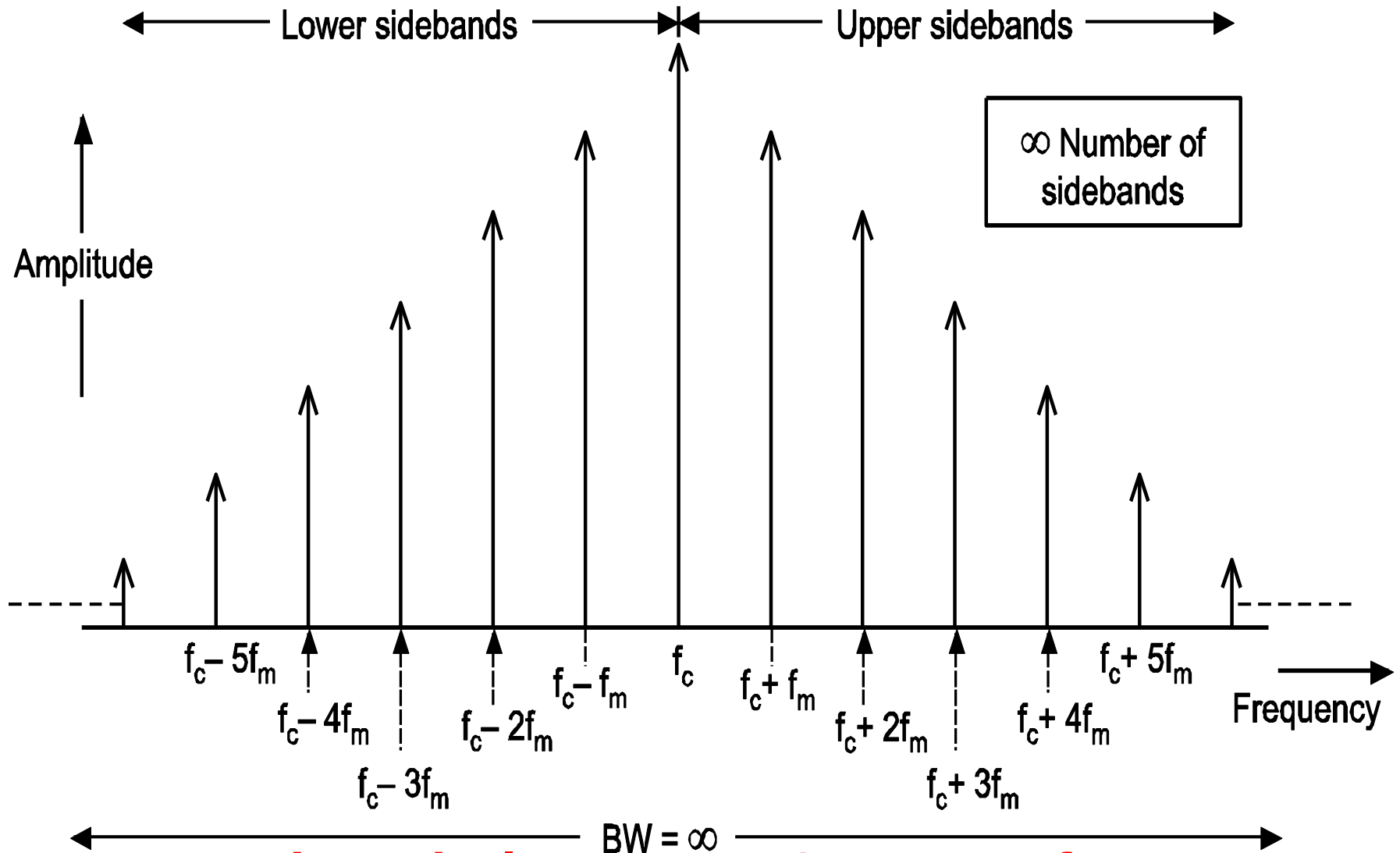


Fig. : Ideal Frequency Spectrum of FM

Bandwidth of FM

From frequency spectrum of FM wave shown in fig., we can say that the bandwidth of FM wave is infinite.

But practically, it is calculated based on how many sidebands have significant amplitudes.

(i)The Simple Method to calculate the bandwidth is –

$$BW = 2f_m \times \text{Number of significant sidebands} \text{ --(1)}$$

With increase in modulation index, the number of significant sidebands increases. So that bandwidth also increases.

(ii)The second method to calculate bandwidth is by **Carson's rule.**

Carson's rule states that, the bandwidth of FM wave is twice the sum of deviation and highest modulating frequency.

$$BW = 2(\delta + f_{m\max}) \quad \dots(2)$$

Highest order side band = To be found from table 2.1 after the calculation of modulation Index m where, $m = \delta / f_m$

e.g. If $m = 20\text{KHz} / 5\text{KHz} = 4$

From table, for modulation index 4, highest order side band is 7th.

Therefore, the bandwidth is

$$\begin{aligned} \text{B.W.} &= 2 f_m \times \text{Highest order side band} \\ &= 2 \times 5 \text{ kHz} \times 7 \\ &= 70 \text{ kHz} \end{aligned}$$

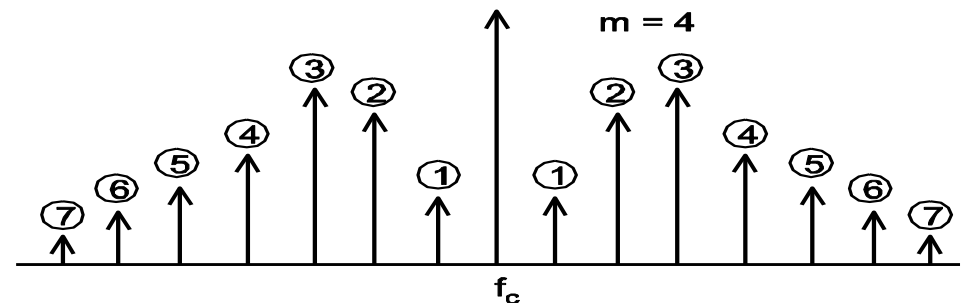
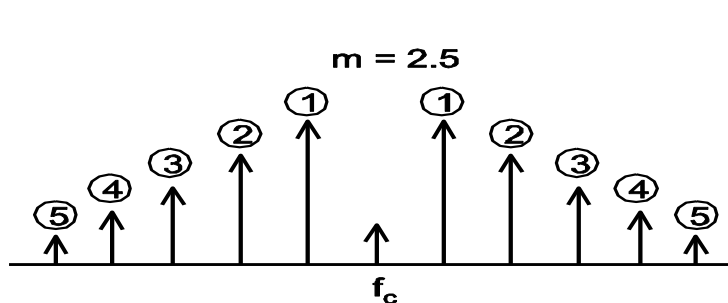
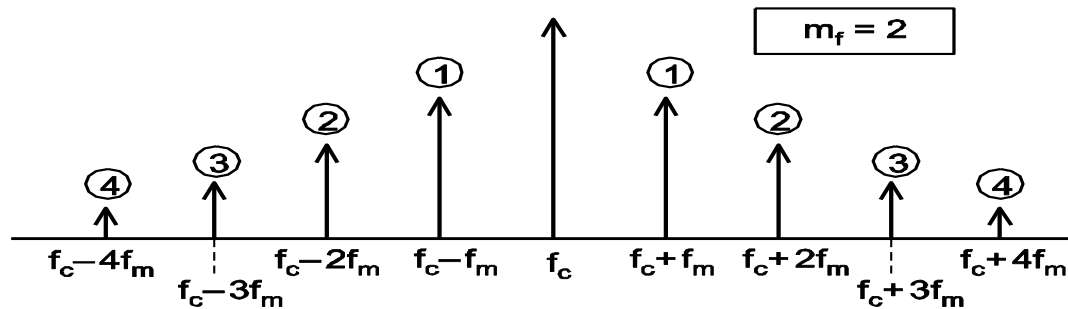
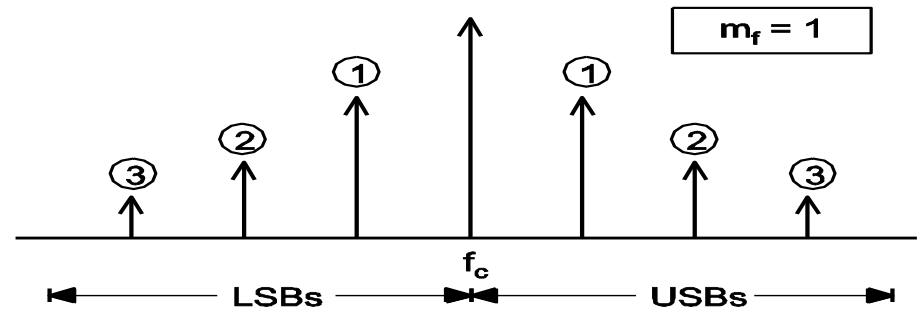
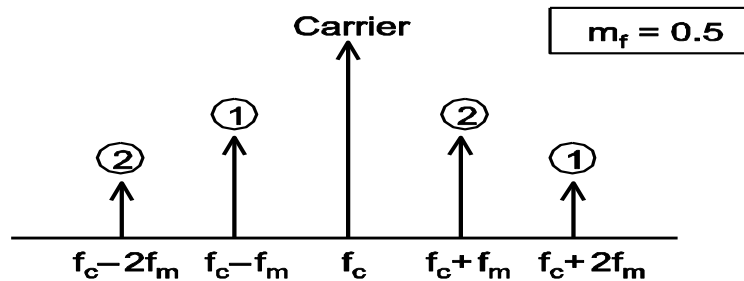
Carrier Distribution Charts:

Table 2.2: Carrier Side Band Distribution Chart for different Modulation

[illegible]

Effect of Modulation Index on Sidebands

Modulation index	0.5	1	2	2.5	4
Number of significant sideband on either side of carrier	2	3	4	5	7



Types of Frequency Modulation

FM (Frequency Modulation)



```
graph TD; FM[FM (Frequency Modulation)] --> NBFM[Narrowband FM (NBFM)]; FM --> WBFM[Wideband FM (WBFM)];
```

Narrowband FM
(NBFM)

[When modulation index is small]

Wideband FM
(WBFM)

[When modulation index is large]

Comparison between Narrowband and Wideband FM

Sr. No.	Parameter	NBFM	WBFM
1.	Modulation index	Less than or slightly greater than 1	Greater than 1
2.	Maximum deviation	5 kHz	75 kHz
3.	Range of modulating frequency	20 Hz to 3 kHz	20 Hz to 15 kHz
4.	Maximum modulation index	Slightly greater than 1	5 to 2500
5.	Bandwidth	Small approximately same as that of AM $BW = 2f_m$	Large about 15 times greater than that of NBFM. $BW = 2(\delta + f_{mmax})$
6.	Applications	FM mobile communication like police wireless, ambulance, short range ship to shore communication etc.	Entertainment broadcasting (can be used for high quality music transmission)

Representation of FM

FM can be represented by two ways:

1. Time domain.
2. Frequency domain.

1. FM in Time Domain

Time domain representation means continuous variation of voltage with respect to time as shown in Fig.

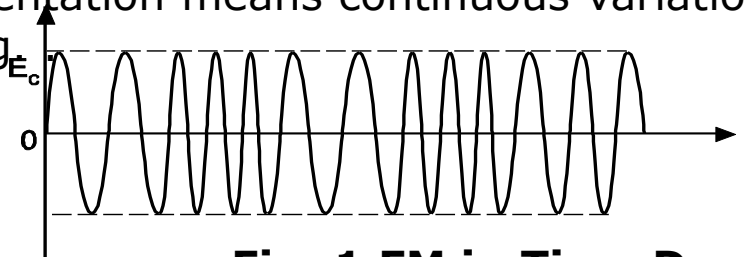


Fig. 1 FM in Time Domain

2. FM in Frequency Domain

- Frequency domain is also known as **frequency spectrum**.
- FM in frequency domain means graph or plot of amplitude versus frequency as shown in Fig. 2.29.

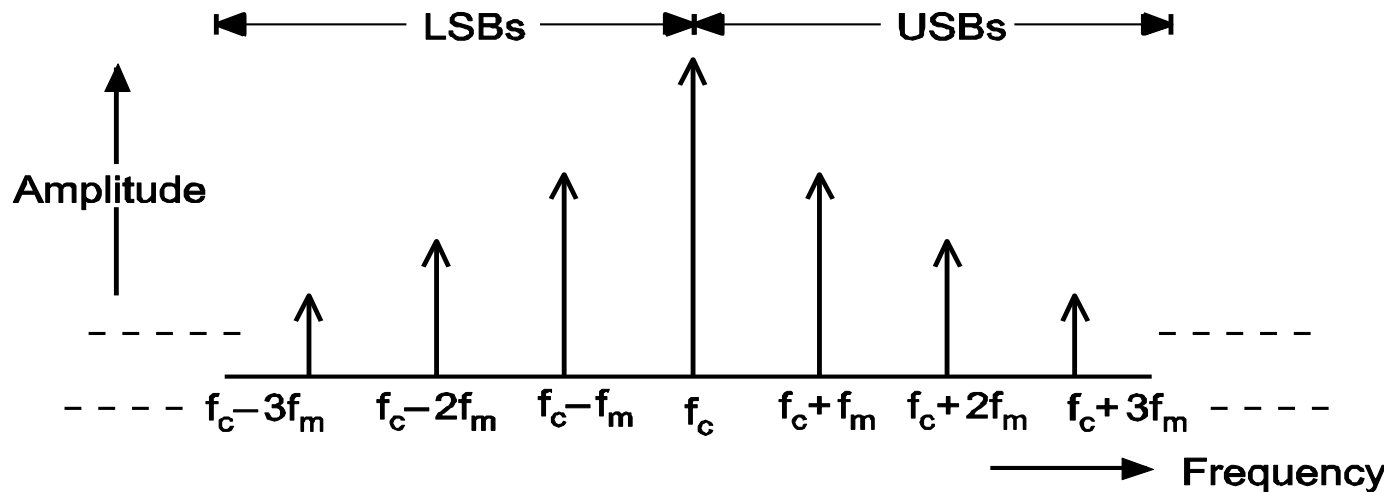


Fig. 2: FM in Frequency Domain

In an F.M. system, if the maximum value of deviation is 75 kHz and the maximum modulating frequency is 10 kHz calculate the deviation ratio and bandwidth of the system using Carson's rule.

Solution:

$$\Delta f_{\max} = 75 \text{ kHz}, f_{m(\max)} = 10 \text{ kHz}$$

$$\text{Deviation ratio } D = \frac{\Delta f_{\max}}{f_{m(\max)}} = \frac{75 \text{ kHz}}{10 \text{ kHz}} = 7.5$$

$$\begin{aligned} \text{System bandwidth } B &= 2 [\Delta f_{\max} + f_{m(\max)}] \\ &= 2 (75 + 10) = 170 \text{ kHz} \end{aligned}$$

A 20 megahertz carrier is frequency modulated by a sinusoidal signal such that the maximum frequency deviation is 100 kHz. Determine the modulation index and the approximate bandwidth of the FM signal if the frequency of the modulating signal is 50 kHz.

Solution:

$$f_c = 20 \text{ MHz}, \Delta f_{\max} = 100 \text{ kHz}, f_m = 50 \text{ kHz}$$

BW m_f ,

$$\text{Modulation index } m_f = \frac{\Delta f_{\max}}{f_m} = \frac{100 \text{ kHz}}{50 \text{ kHz}} = 2$$

$$\text{BW} = 2 [\Delta f_{\max} + f_m] = 2 [100 \text{ kHz} + 50 \text{ kHz}] = 300 \text{ kHz}$$

A 107.6 MHz carrier signal is frequency modulated by a 7 kHz sinewave. The resultant FM signal has a frequency deviation of 50 kHz. Determine the modulation index of the FM wave.

Solution:

$$f_c = 107.6 \text{ MHz},$$

$$f_m = 7 \text{ kHz},$$

$$\Delta f = 50 \text{ kHz}$$

$$m_f$$

$$m_f = \frac{\Delta f}{f_m} = \frac{50 \times 10^3}{7 \times 10^3} = 7.14$$

Consider an angle modulated signal $x_c(t) = 10 \cos(\omega_c t + 3 \sin \omega_m t)$. Assume FM and $f_m = 1$ kHz. Calculate the modulation index and find the bandwidth when 1. f_m is doubled
2. f_m is decreased by one half.

Solution:

An FM wave $x_c(t) = 10 \cos(\omega_c t + 3 \sin \omega_m t)$

$$f_m = 1 \text{ kHz}$$

Modulation index m_f and BW.

Calculate deviation Δf :

From the given expression modulation index is

$$m_f = 3$$

$$\text{But } m_f = \frac{\Delta f}{f_m}$$

$$\therefore \Delta f = m_f \times f_m = 3 \times 1 \text{ kHz} = 3 \text{ kHz}$$

We assume that the deviation remains constant

: Calculate m_f and BW for $f_m = 2$ kHz :

$$m_f = \frac{\Delta f}{f_m} = \frac{3 \text{ kHz}}{2 \text{ kHz}} = 1.5$$

$$BW = 2 [\Delta f + f_m] = 2 [3 + 2] \text{ kHz} = 10 \text{ kHz}$$

Calculate m_f and BW for $f_m = 0.5 \text{ kHz}$:

$$m_f = \frac{3 \text{ kHz}}{0.5 \text{ kHz}} = 6$$

$$BW = 2 [3 + 0.5] = 7 \text{ kHz}$$

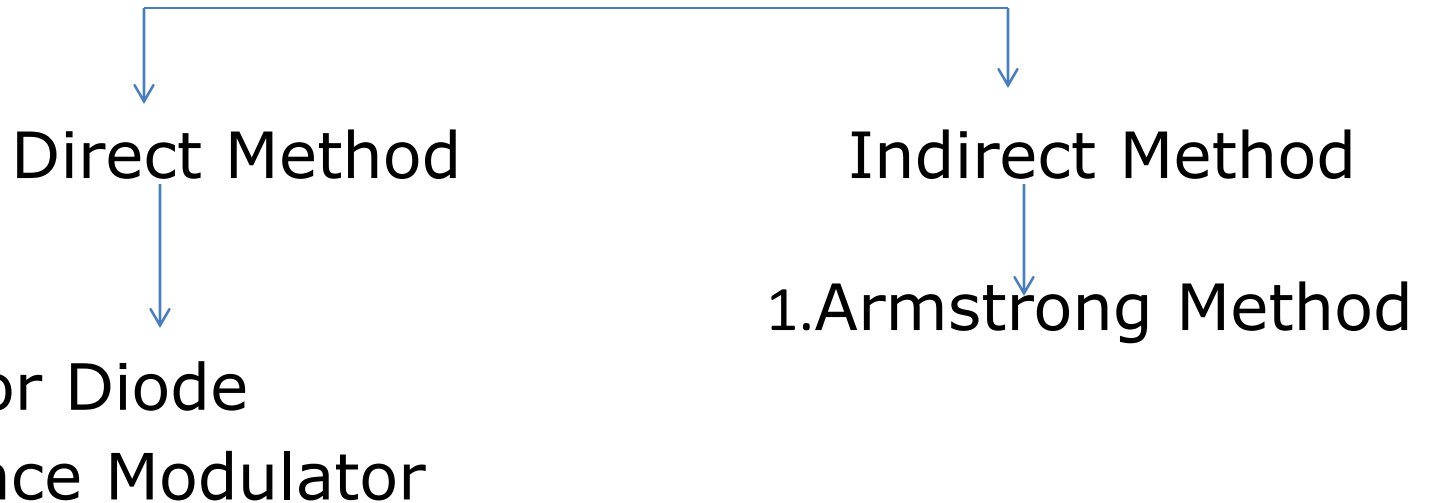
Comparison between AM and FM

Parameter	AM	FM
1. Definition	Amplitude of carrier is varied in accordance with amplitude of modulating signal keeping frequency and phase constant.	Frequency of carrier is varied in accordance with the amplitude of modulating signal keeping amplitude and phase constant.
2. Constant parameters	Frequency and phase.	Amplitude and phase.
3. Modulated signal	<p style="text-align: center;">AM Wave</p>	<p style="text-align: center;">FM Wave</p>
4. Modulation Index	$m = E_m / E_c$	$m = \delta / f_m$
5. Number of sidebands	Only two	Infinite and depends on m_f .
6. Bandwidth	$BW = 2f_m$	$BW = 2(\delta + f_{m(max)})$
7. Application	MW, SW band broadcasting, video transmission in TV.	Broadcasting FM, audio transmission in TV.

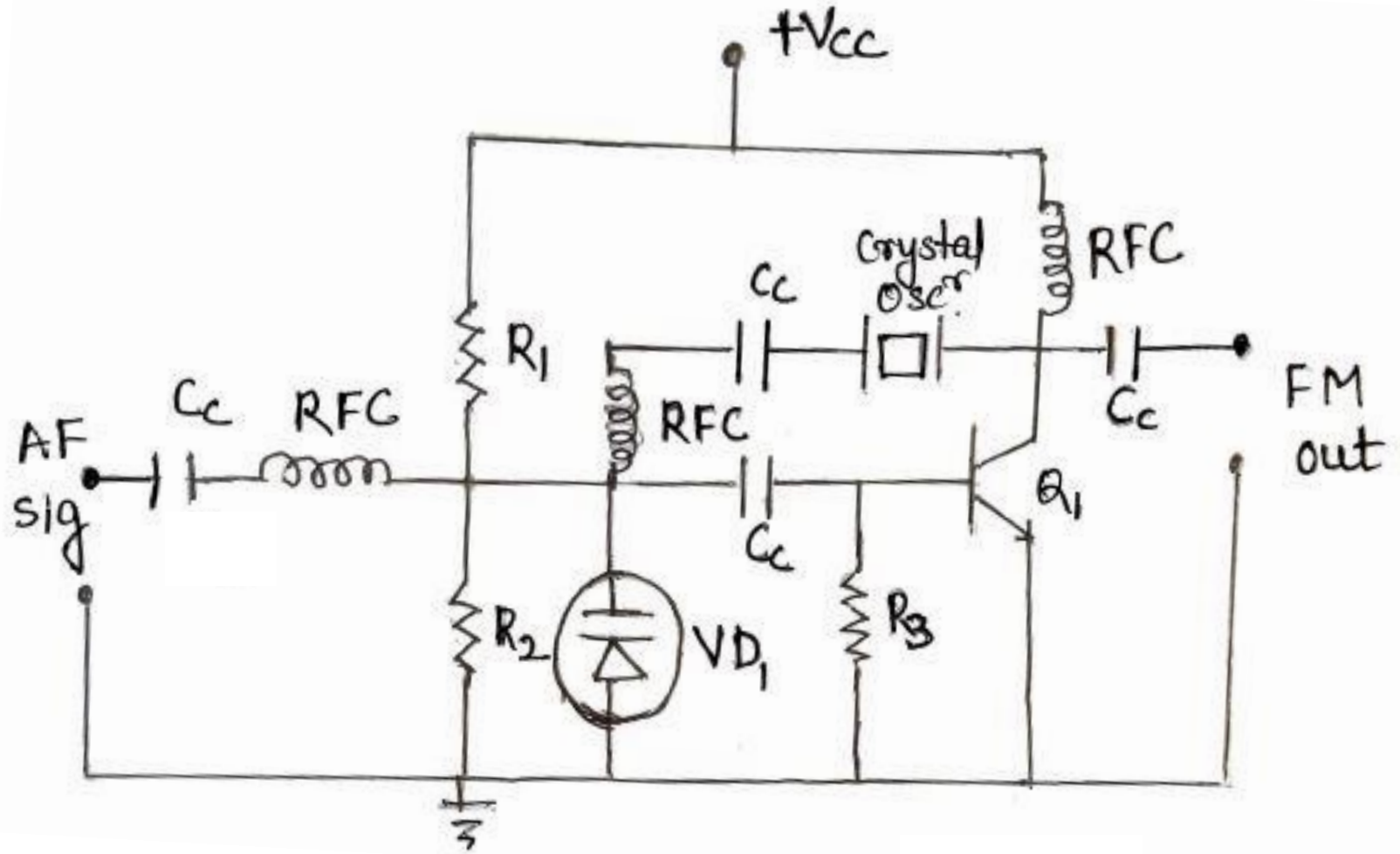
FM Generation

There are two methods for generation of FM wave.

Generation of FM



Varactor Diode Modulator

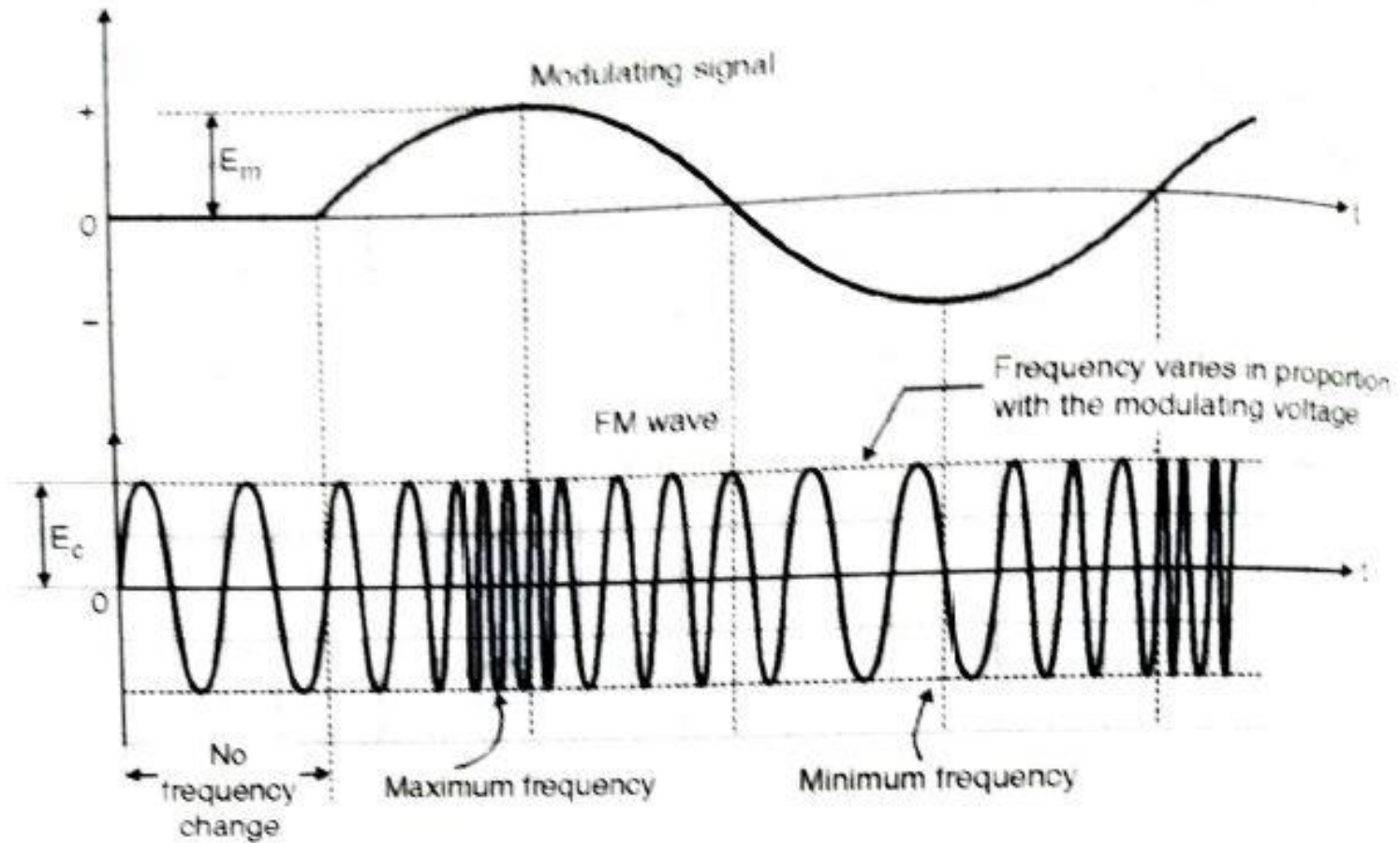


Varactor Diode Frequency Modulator

Limitations of Direct Method of FM Generation

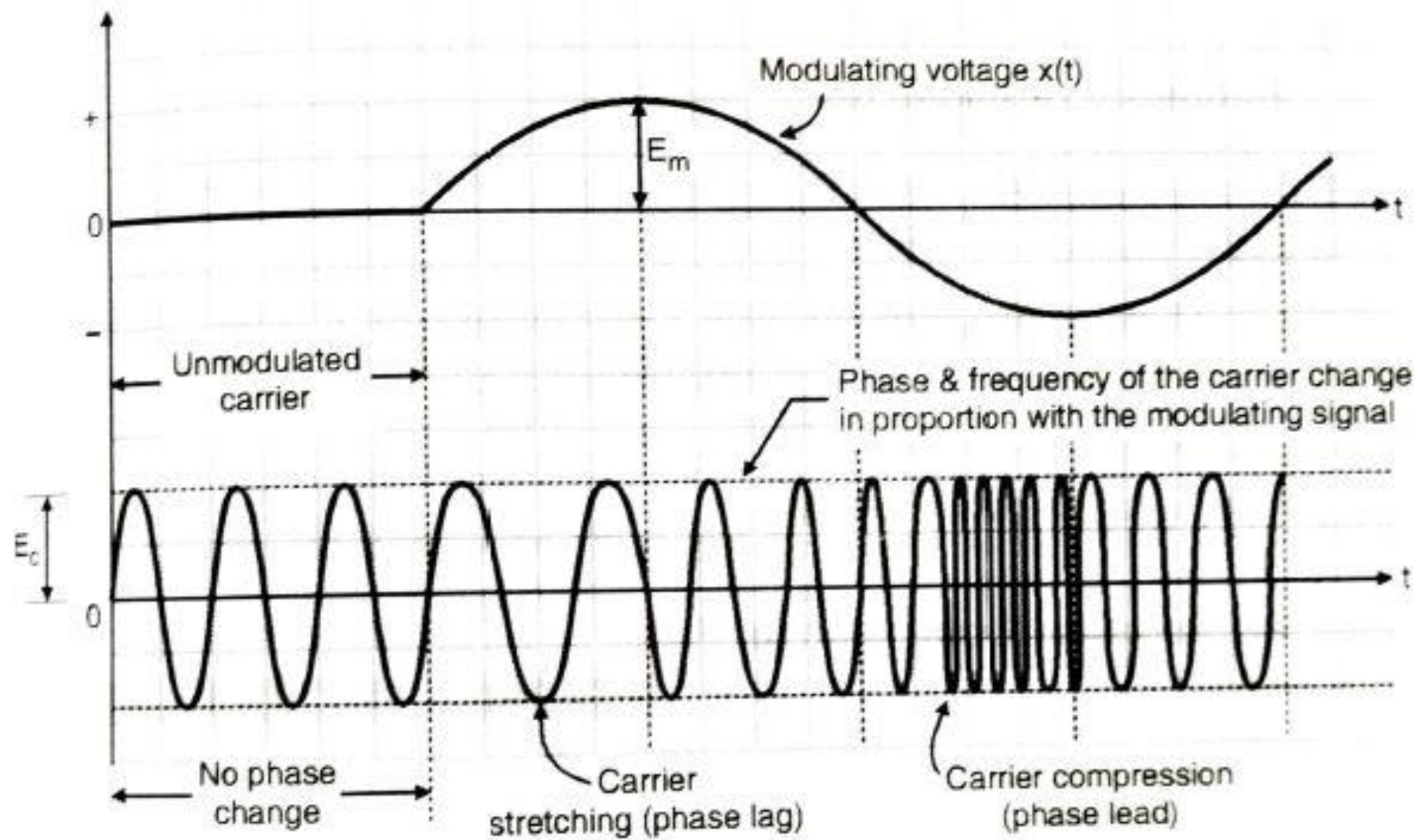
1. It is very difficult to get high order stability in carrier frequency in reactance modulator method because the LC oscillator used is not a stable oscillator.
2. Generally in this method we get distorted FM, due to non-linearity of the varactor diode.

Indirect Method of FM Generation



Time domain representation of FM wave

Indirect Method of FM Generation



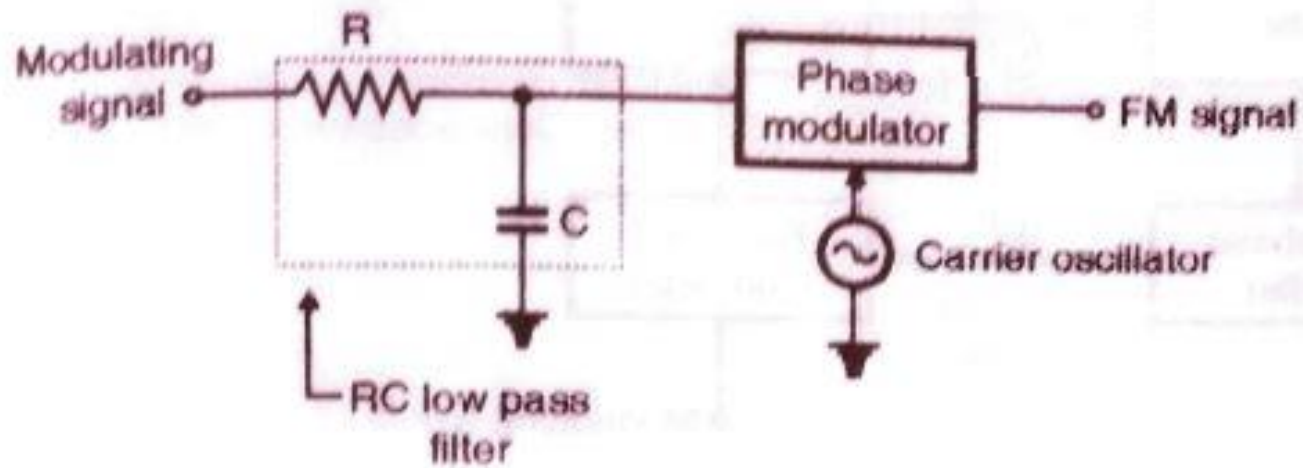
Time domain representation of PM wave

Indirect Method of FM Generation

- In PM along with phase variation, some frequency deviation also takes place.
- Higher modulating voltages produces greater phase shift which in turn produces greater frequency deviation.
- If higher is the modulating frequencies, then it produces a faster rate of change of modulating voltages and hence they also produce greater frequency deviation.
- Thus, in PM, the carrier frequency deviation is proportional to the modulating voltage as well as to the modulating frequency.
- But in FM, frequency deviation is proportional only to the modulating voltage regardless of frequency.

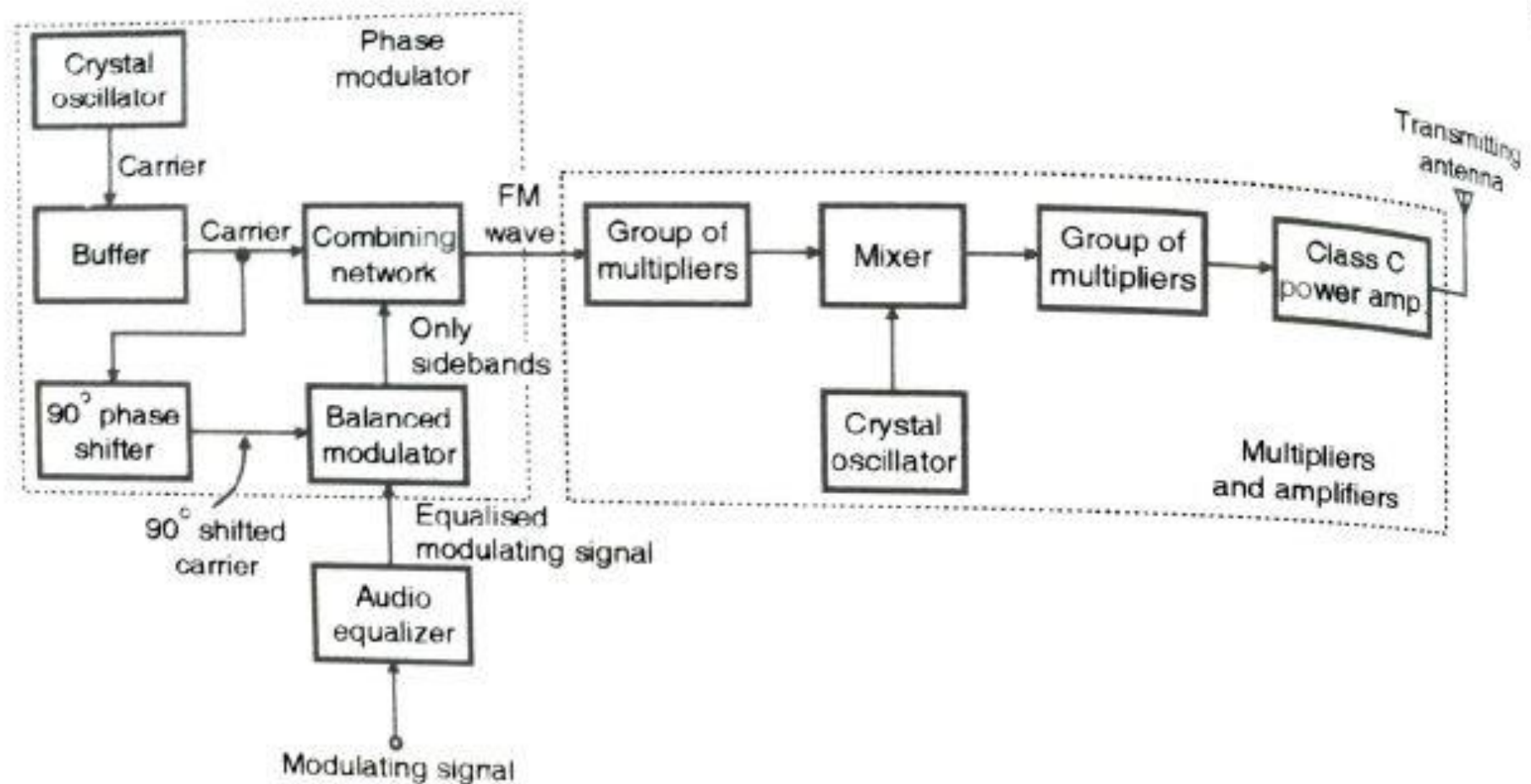
Indirect Method of FM Generation

- To correct this problem, the modulating signal is passed through LPF, due to this, high frequency modulating signals are attenuated but there is no change in the amplitudes of low frequency modulating signals.
- The filter output is then applied to a phase modulator along with the carrier.
- Hence extra deviation in carrier frequency due to higher modulating frequency is compensated by reducing the amplitude of the high frequency modulating signals.



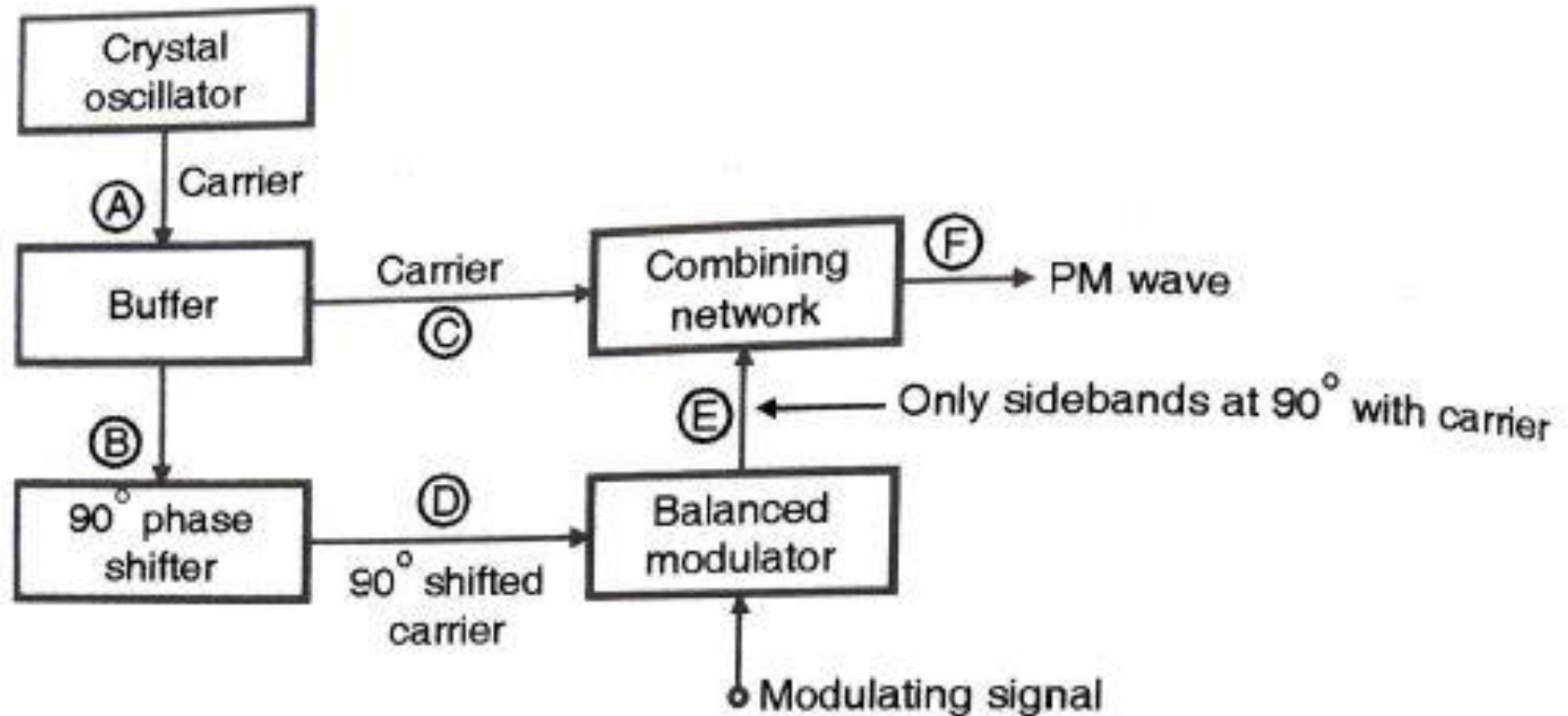
Generation of FM using PM

Armstrong method of FM generation



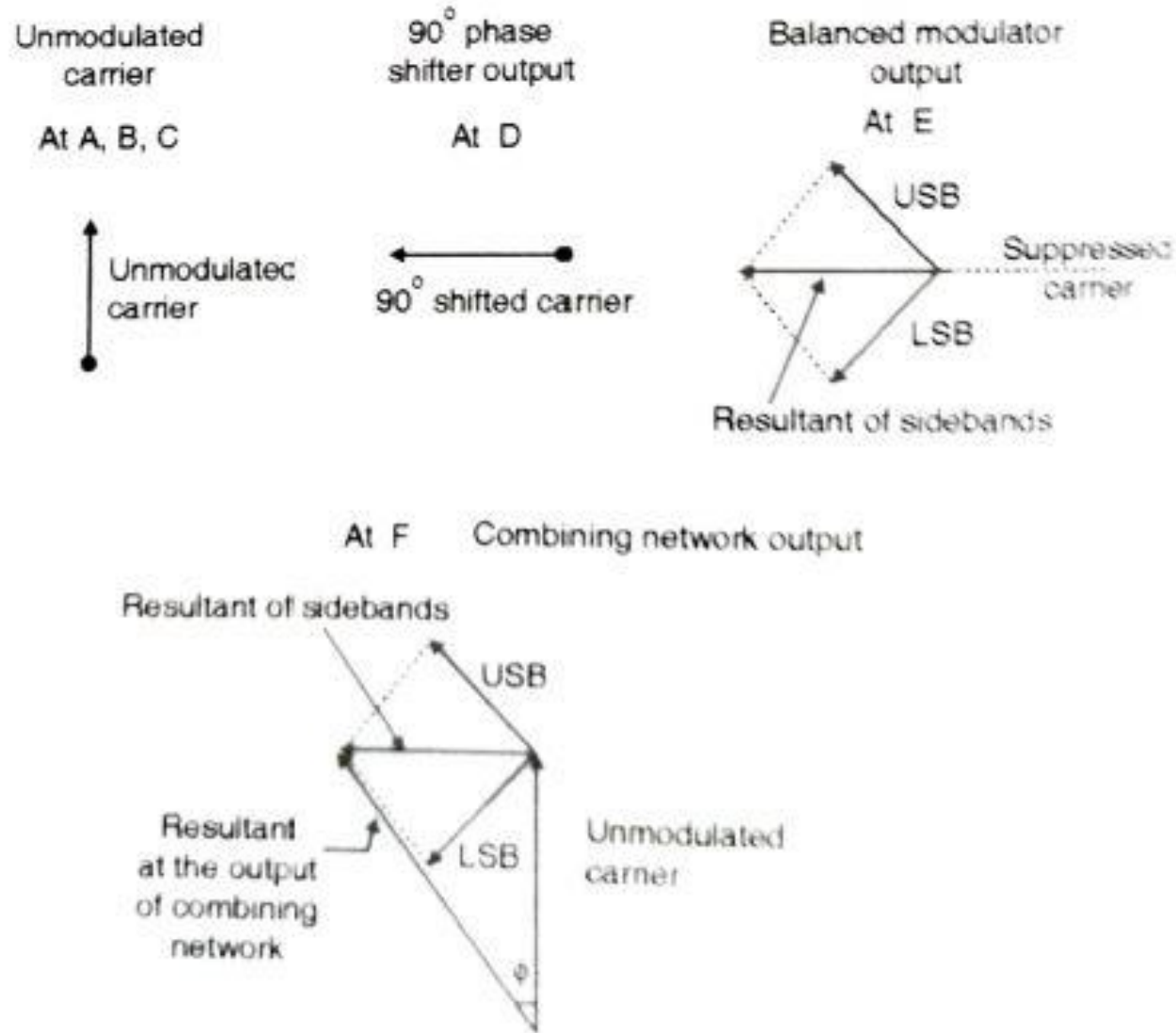
Armstrong Method (Indirect Method) of FM generation

Armstrong method of FM generation



Phase modulator circuit

Armstrong method of FM generation



Phasors explaining the generation of PM

Low modulation index

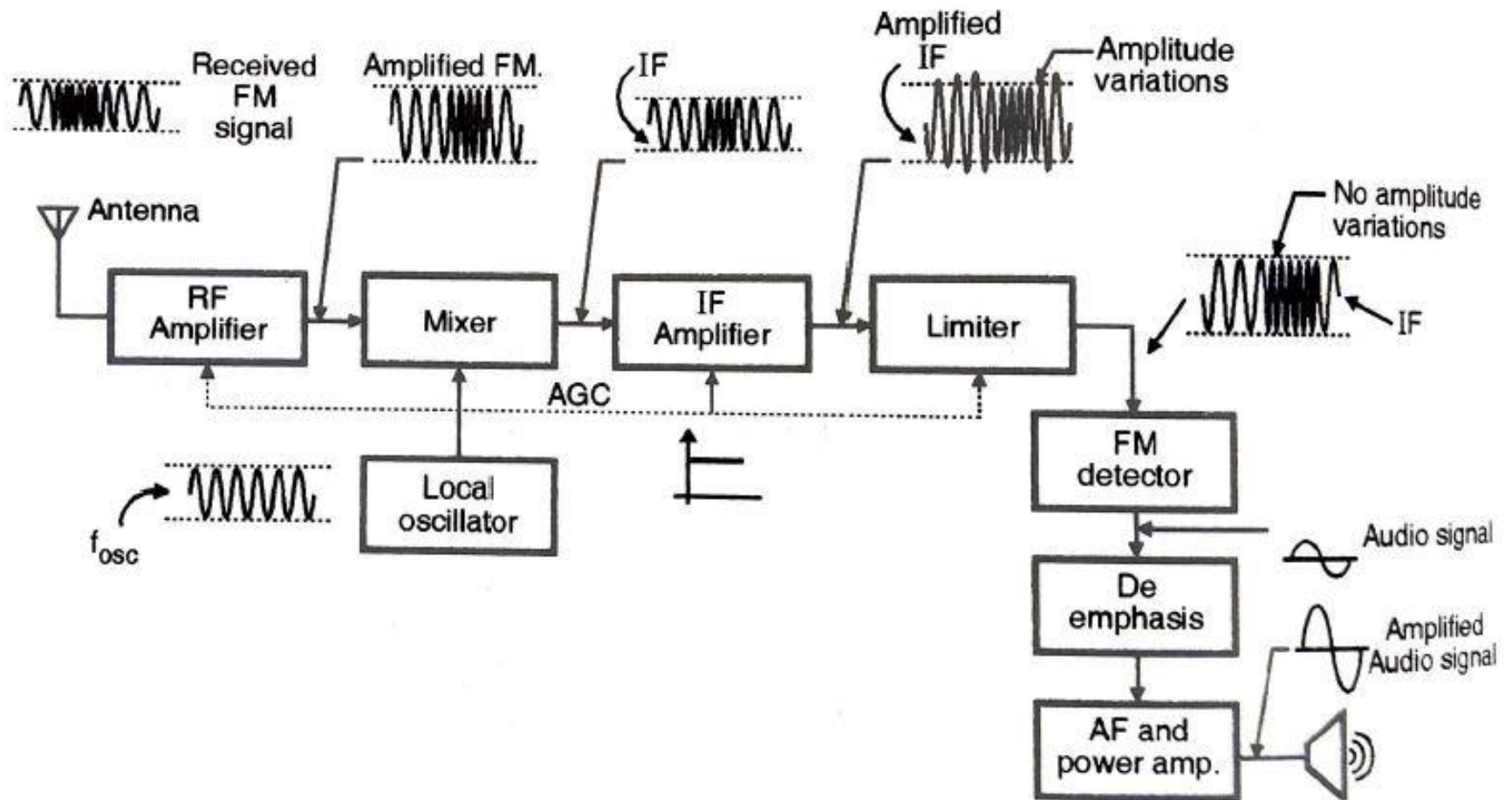


High modulation index



Effect of modulation index on phase

FM demodulators

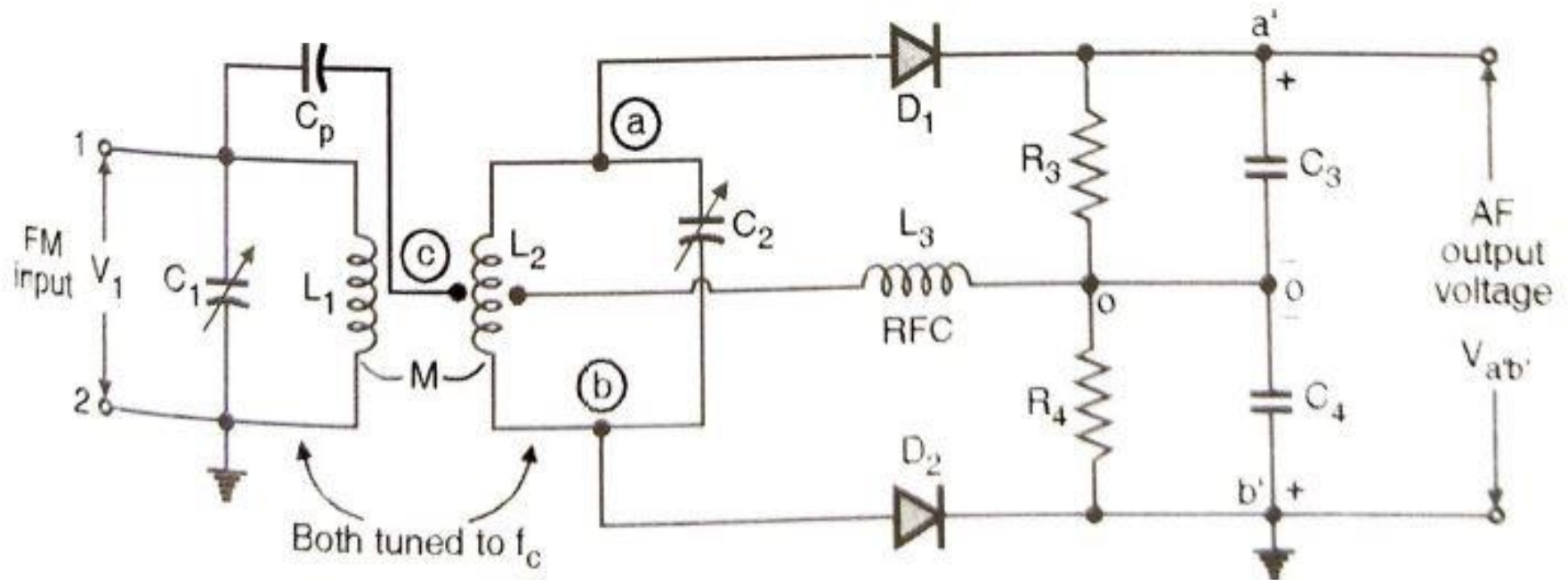


Waveforms at various points in an FM receiver

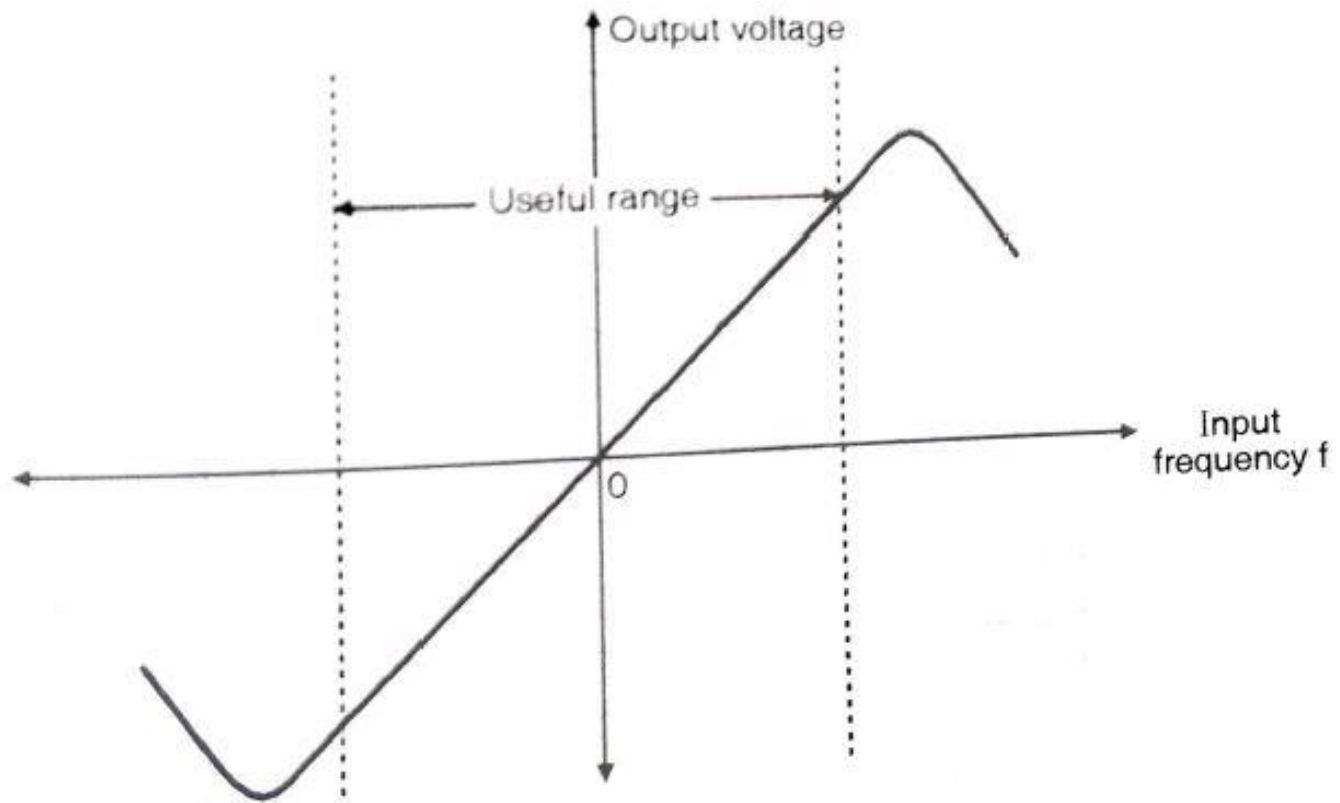
FM demodulators

- Foster-Seeley Discriminator/Phase Discriminator
- Ration Detector

- Foster-Seeley Discriminator/Phase Discriminator

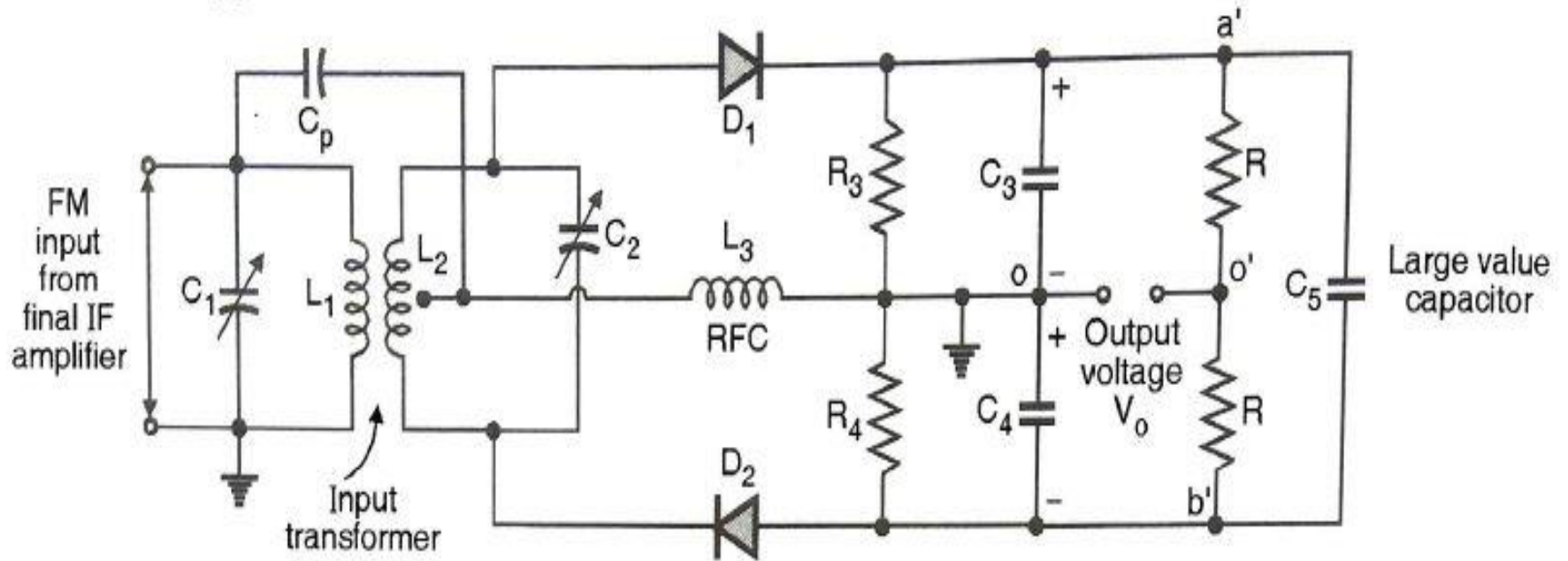


Phase Discriminator/ Foster-Seeley Discriminator



Discriminator response

- Ration Detector



Ratio Detector

Pre-emphasis and De-emphasis

Pre-emphasis and de-emphasis circuits are used only in frequency modulation. Pre-emphasis is used at transmitter and de-emphasis is used at receiver.

Pre-emphasis

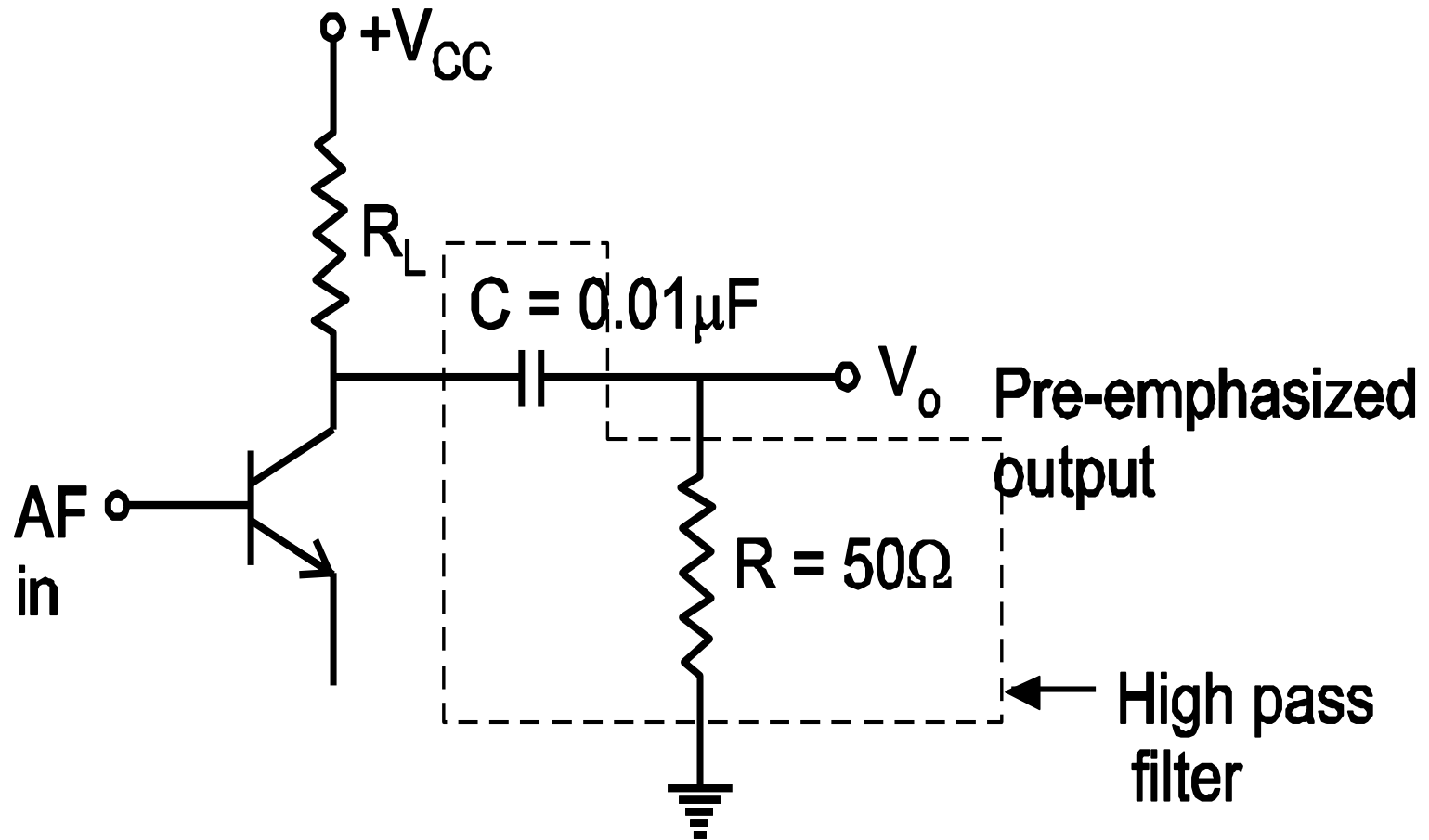
- In FM, the noise has a greater effect on the higher modulating frequencies.
- This effect can be reduced by increasing the value of modulation index (m_f), for higher modulating frequencies.
- This can be done by increasing the deviation ' δ ' and ' δ ' can be increased by increasing the amplitude of modulating signal at higher frequencies.

Definition:

The artificial boosting of higher audio modulating frequencies in accordance with prearranged response curve is called pre-emphasis.

- Pre-emphasis circuit is a high pass filter as shown in Fig. 1

Fig. 1: Pre-emphasis Circuit



As shown in Fig. 1, AF is passed through a high-pass filter, before applying to FM modulator.

As modulating frequency (f_m) increases, capacitive reactance decreases and modulating voltage goes on increasing. $f_m \propto$ Voltage of modulating signal applied to FM modulator. Boosting is done according to pre-arranged curve as shown in Fig. 2.

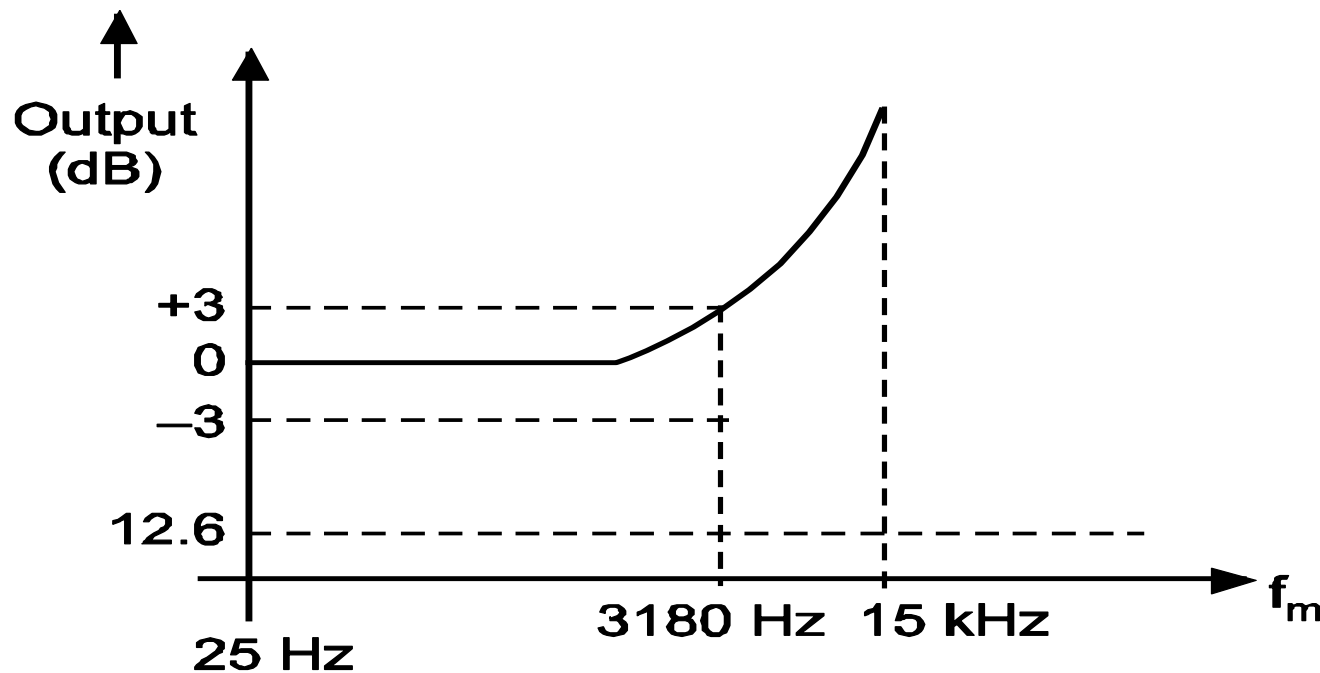


Fig. 2: Pre-emphasis Curve

The time constant of pre-emphasis is at $50\text{ }\mu\text{s}$ in all CCIR standards. In systems employing American FM and TV standards, filters having time constant of $75\text{ }\mu\text{sec}$ are used.

- **The pre-emphasis is used at FM transmitter** as shown in Fig. 3.

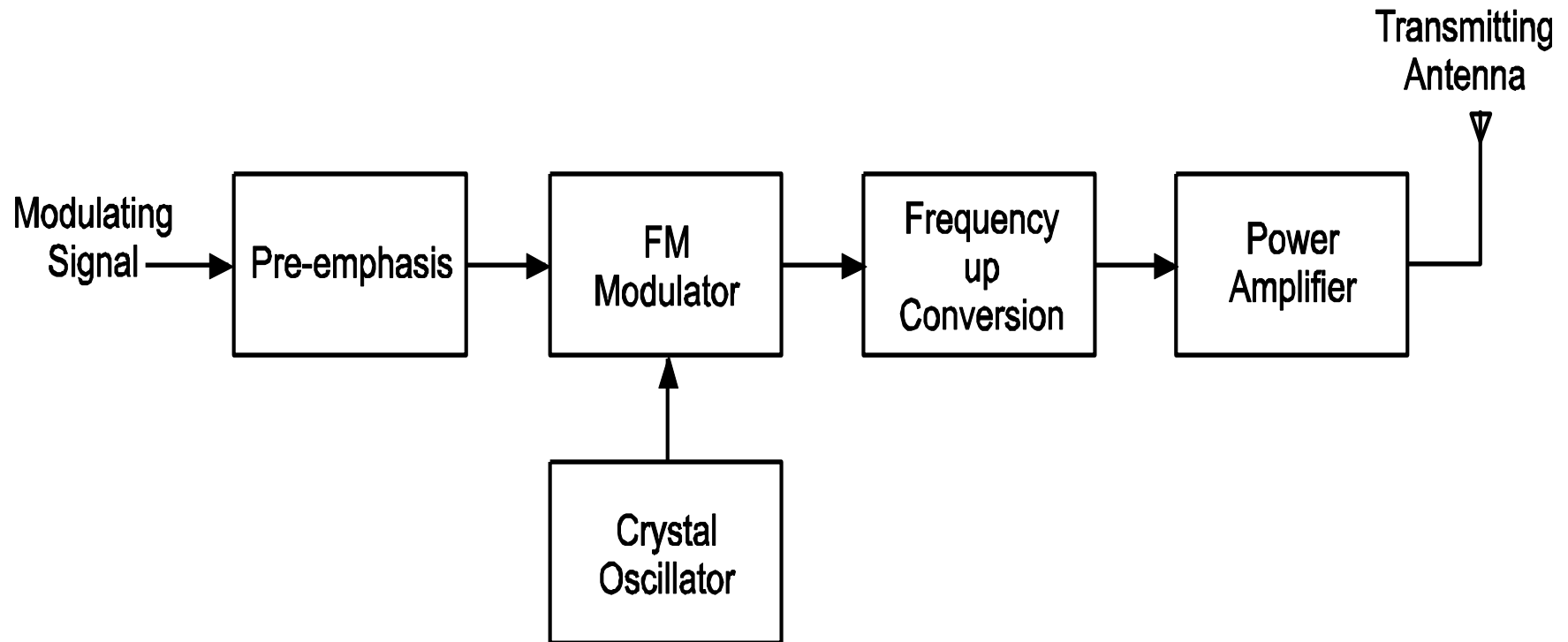


Fig. 3: FM Transmitter with Pre-emphasis

De-emphasis

- De-emphasis circuit is used at FM receiver.

Definition:

The artificial boosting of higher modulating frequencies in the process of pre-emphasis is nullified at receiver by process called de-emphasis.

- De-emphasis circuit is a low pass filter shown in Fig. 4.

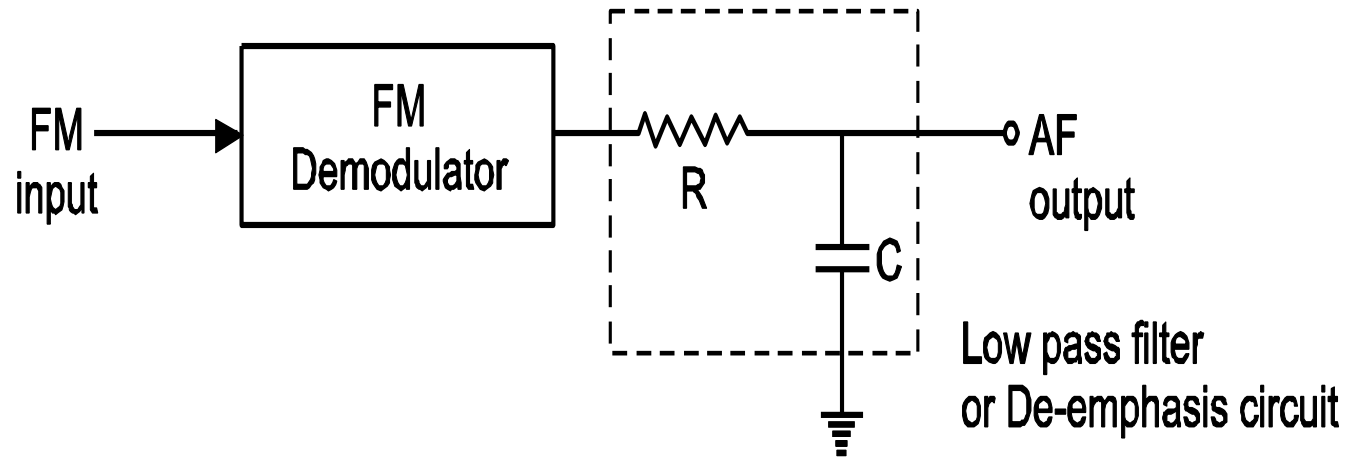
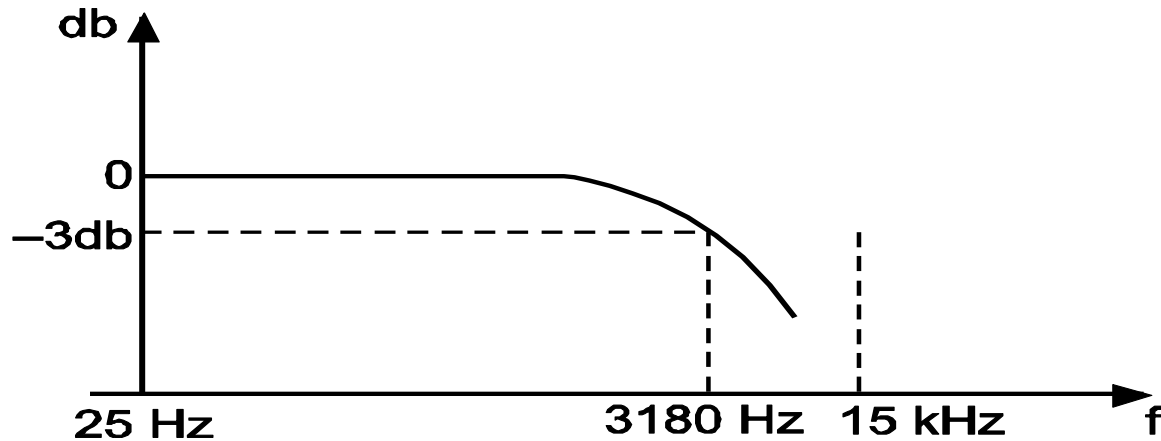


Fig. 4: De-emphasis Circuit

Fig. 5: De-emphasis Curve



As shown in Fig.5, de-modulated FM is applied to the de-emphasis circuit (low pass filter) where with increase in f_m , capacitive reactance X_c decreases. So that output of de-emphasis circuit also reduces.

Fig. 5 shows the de-emphasis curve corresponding to a time constant $50 \mu s$. A $50 \mu s$ de-emphasis corresponds to a frequency response curve that is 3 dB down at frequency given by,

$$\begin{aligned} f &= 1 / 2\pi RC \\ &= 1 / 2\pi \times 50 \times 1000 \\ &= 3180 \text{ Hz} \end{aligned}$$

The de-emphasis circuit is used after the FM demodulator at the FM receiver shown in Fig. 6.

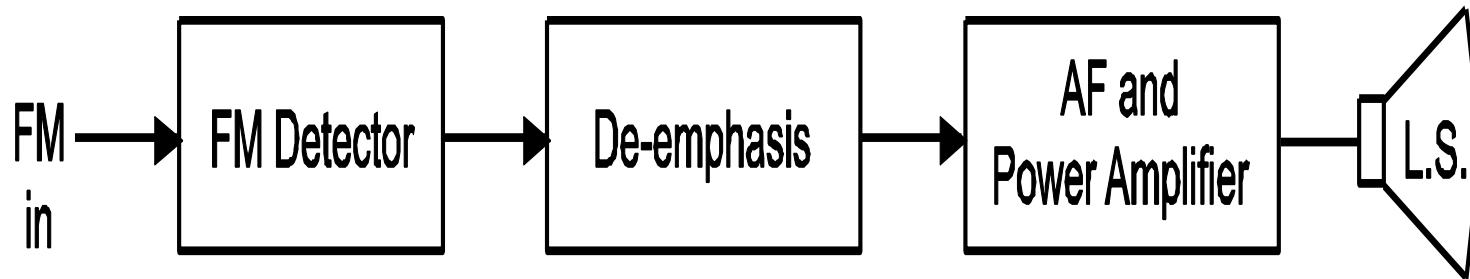
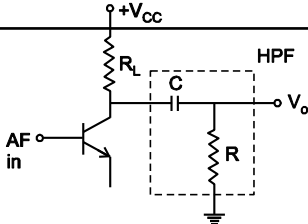
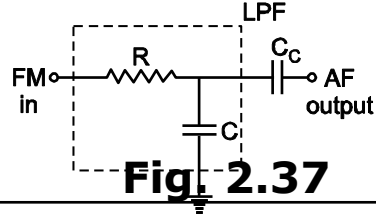
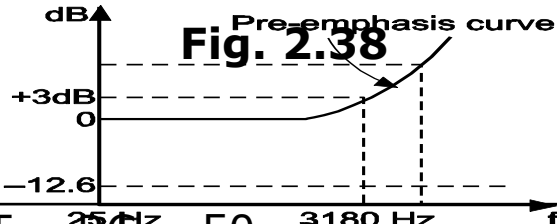
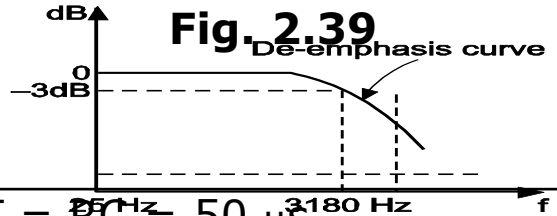


Fig. 6: De-emphasis Circuit in FM Receiver

Comparison between Pre-emphasis and De-emphasis

Parameter	Pre-emphasis	De-emphasis
1. Circuit used	High pass filter.	Low pass filter.
2. Circuit diagram	 <p>Fig. 2.36</p>	 <p>Fig. 2.37</p>
3. Response curve	 <p>Fig. 2.38</p>	 <p>Fig. 2.39</p>
4. Time constant	$T = RC = 50 \mu s$	$T = RC = 50 \mu s$
5. Definition	Boosting of higher frequencies	Removal of higher frequencies
6. Used at	FM transmitter	FM receiver.

Advantages / Disadvantages / Applications of FM

Advantages of FM

1. Transmitted power remains constant.
2. FM receivers are immune to noise.
3. Good capture effect.
4. No mixing of signals.

Disadvantages of FM

The greatest disadvantages of FM are:

1. It uses too much spectrum space.
2. The bandwidth is wider.
3. The modulation index can be kept low to minimize the bandwidth used.
4. But reduction in M.I. reduces the noise immunity.
5. Used only at very high frequencies.

Applications of FM

1. FM radio broadcasting.
2. Sound transmission in TV.
3. Police wireless.