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Robotics

HW1

Problem 1

1) Moon in Wall's Reference Frame (6, 0, 4)
 Eve's position in Wall's Reference Frame (6, 10, 4)

2a)

-90° along axis y
 -90° along axis z

$$\begin{aligned}
 R &= \begin{pmatrix} \cos -90 & 0 & \sin -90 \\ 0 & 1 & 0 \\ -\sin -90 & 0 & \cos -90 \end{pmatrix} \begin{pmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}
 \end{aligned}$$

$${}^m T_w = \begin{pmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2b) eT_m

rotate 90° , y axis

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$eT_m = \begin{pmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$2c) e_{Tw}$$

rotale z by -90°

$$R \begin{pmatrix} C & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$e_{Tw} = \begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3) $\begin{pmatrix} -10 \\ 6 \\ -4 \end{pmatrix}$ represents walle's position
in the reference frame of Eve

$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the orientation of
Wall'e in the reference frame of Eve

$$4 e_{T_m} \times {}^m T_w =$$

$$\begin{pmatrix} 0 & 0 & 1 & -10 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & -1 & 4 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \times 1 & 0 & -10 \times 1 \\ 1 \times -1 & 0 & 0 & 1 \times 6 \\ 0 & 0 & -1 \times -1 & -1 \times 4 \\ 0 & 0 & 0 & 1 \times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$5) {}^wT_E =$$

translate by $\begin{pmatrix} 6 \\ 10 \\ 4 \end{pmatrix}$

rotate by 90° in z axis

$$\begin{pmatrix} 0 & -1 & 0 & 6 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$({}^wT_E)^{-1} =$$

$$\begin{pmatrix} 0 & 1 & 0 & -10 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$: {}^E T_w$$

6) e_{T_w} represents the reference frame
pose of Walle in reference frame
of Eve, and thus changes in the
reference frame of the moon does
not affect Eve and Walle

Assume new location of moon = m'

$$m'_{T_w} = \begin{pmatrix} 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e_{T_{m'}} = \begin{pmatrix} 0 & 0 & 1 & -8 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$e_{T_w} = e_{T_{m'}} \times m'_{T_w}$$

$$= \begin{pmatrix} 0 & 1 & 0 & -6 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$