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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FOURTH SEMESTER B.Tech. DEGREE END SEMESTER EXAMINATION- May 2015

SUB: ENGINEERING MATHEMATICS IV (MAT 212) CS-ICT-CC
(REVISED CREDIT SYSTEM -2011)

Time: 3 Hrs. **Max.Marks: 50**

Note: a) Answer any FIVE full questions. b) All questions carry equal marks (4+3+3)

1A. If A and B are two events in a sample space such that $P(A)=3/4$ and $P(B)=3/8$, show that a) $P(A\cup B)\geq 3/4$ b) $P(\bar{A}\cup\bar{B})\geq 5/8$ c) $1/8\leq P(A\cap B)\leq 3/8$

1B. A random variable X is uniformly distributed over the interval $(-1,1)$. Find the pdf of $Y=X^4$.

1C. Show that the sample variance S^2 is not an unbiased estimator for σ^2.

2A. Box 1 contains 4 black and 5 green balls; Box 2 contains 5 black and 4 green balls. Three balls are drawn at random form Box 1 and transferred to Box 2. Then a ball is drawn from Box 2. What is the probability that it is green? If it is green then what is the probability that 2 green and 1 black balls are transferred from Box 1 to Box 2?

2B. Let X and Y represent the life lengths of two light bulbs manufactured by different processes. Assume that X and Y are independent random variables with the pdf's $f(x)=e^{-x}, x\geq 0$ and $g(y)=2e^{-2y},\ y\geq 0$ respectively. Find the pdf of the random variable $Z=\frac{X}{Y}$.

2C. A student takes a multiple choice test consisting of three problems. The first question has 3 possible answers, second has 5 and the third question has 4 possible answers. The student chooses at random one answer as the right one from each of the three problems. Let X be the right answers. Find E(X) and V(X).

3A. Suppose that the scores of an examination are normally distributed with mean 76 and standard deviation 15. The top 15% of the students receive A grade and bottom 10% receive F grade. Find (i) the minimum score to receive A grade, (ii) the minimum score not to score an F grade.

3B. The joint pdf of (X, Y) is given by $f(x,y)=\begin{cases} ke^{-(2x+3y)}, & x>0, y>0 \\ 0, & \text{elsewhere} \end{cases}$

i) Determine the value of k.
ii) Find the marginal pdf of X and Y.
iii) Evaluate P(X>Y/ X>2)

3C. If the random variable K is uniformly distributed over the interval (0,5) then what is the probability that the roots of the equation $4x^2+4xk+k+2=0$ are real?

4A. Let (X_1, X_2) be a random sample of size $n=2$ from the distribution having probability density function $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}$ for $0 < x < \infty$ and $\theta > 0$. $H_0: \theta = 2$ and $H_1: \theta = 1$. If the observed values of (X_1, X_2) are such that $\frac{f(x_1,2).f(x_2,2)}{f(x_1,1).f(x_2,1)} \le \frac{1}{2}$ then find the significance level of the test and the power of the test.

4B. Let $X_1, X_2, ..., X_n$ denote the random sample of size n from a distribution having the pdf $f(x, \theta) = \theta^x (1-\theta)^{1-x}$, $x= 0,1,2,....; 0 \le \theta \le 1$. Find the maximum likelihood estimator for θ.

4C. If X,Y and Z are uncorrelated random variables with mean zero and standard deviation 5, 12 and 7 respectively. If $U = X+Y$ and $V = Y+Z$ then find the correlation coefficient between U and V.

5A. If $M_X(t) = e^{2t(1+t)}$ what is the pdf of $Y = \frac{(X-2)^2}{4}$. Hence obtain the m.g.f of Y.

5B. Let $\bar{X}$ and S^2 be the mean and variance of a random sample of size 25 from a distribution which is N (3,100). Evaluate P ($0 < \bar{X} < 6$, $55.2 < S^2 < 145.6$).

5C. A random sample of size 15 from a normal distribution $N(\mu, \sigma^2)$ yields $\bar{X} = 3.2$ and $S^2 = 4.24$. Determine a 95% confidence interval for σ^2 .

6A. The Mendelian theory states that the probabilities of classifications a) round and yellow, b) wrinkled and yellow, c) round and green and d) wrinkled and green are respectively $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}$ and $\frac{1}{16}$. From a sample of 160 the actual numbers observed were 86, 35, 26 and 13. Is this data consistent with the theory at 0.01 significance level?

6B. Find the mean and variance of Binomial distribution.

6C. Let Y_1 and Y_2 be two independent unbiased statistics for θ. The variance of Y_1 is twice the variance of Y_2 . Find the constants K_1 and K_2 such that $K_1 Y_1 + K_2 Y_2$ is an unbiased statistic for θ with the smallest possible variance for such a linear combination.

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