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MANIPAL INSTITUTE OF TECHNOLOGY
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IV SEMESTER B.E DEGREE END SEMESTER MAKE UP EXAMINATION – July, 2014

SUB: PROBABILITY, STATISTICS AND STOCHASTIC PROCESS – IV
(MAT –CSE/IT – 212)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions. b) All questions carry equal marks.**

1A. The odds that a book will be reviewed favorably by 3 independent critics are 5 to 2, 4 to 3, 3 to 4. What is the probability that of the 3 reviews, a majority will be favorable?.

1B. If the random variable 'K' is uniformly distributed over [0, 5], what is the probability that the roots of the equation $4x^2 + 4xK + K + 2 = 0$ are real ?

1C. In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of total. Of these output 5, 4, 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured from B?

2A. A continuous random variable X has the p.d.f. $f(x) = 6x(1-x), 0 \le x \le 1$. Determine the mean and variance of this distribution.

2B. Suppose that 15% of the families in a certain community have no children, 20% have 1, 35% have 2 and 30% have 3 and suppose further that in each family each child is equally likely to be a boy or girl. If a family is chosen at random from this community, find the joint p .d. f of the number of boys and number of girls.

2C. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

3A. Suppose that the random variable X is uniformly distributed over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Find the p.d.f of $Y = \tan X$

3B. If the random variable X has $N(\mu, \sigma^2)$ distribution, then show that the random variable

$$Z = \frac{X-\mu}{\sigma} \text{ has } N(0, 1) \text{ and that } V = \frac{(X-\mu)^2}{\sigma^2} \text{ has } \chi^2(1).$$

3C. Find the m.g.f of binomial distribution. Hence find its mean and variance.

4A. If X is a random variable and $P(x) = ab^x$, where a and b are positive such that $a + b = 1$ and $X = 0,1,2,\ldots$. Find the moment generating function of X. Hence, show that $m_2 = m_1(2m_1 + 1)$ where m_1 and m_2 being the first two moments.

4B. Two independent random variables X_1 and X_2 have means 5, 10 and variance 4, 9. Find covariance between $U = 3X_1 + 4X_2$ and $V = 3X_1 - X_2$.

4C. Compute an approximate probability that mean of a random sample of size 15 from a distribution having pdf $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ is between $\frac{3}{5}$ & $\frac{4}{5}$.

5A. Let $(X_1, X_2, \ldots\ldots X_n)$ denote a random sample of size *n* from the distribution with pdf $f(x,\theta) = \begin{cases} \dfrac{\theta^x e^{-\theta}}{x!}, & x = 0,1,2,\ldots\ldots, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$. Find MLE for θ.

5B. Show that the sample mean $\overline{X}$ is both unbiased and consistent estimator for the population mean.

5C. Let $\overline{X}$ be the sample mean of a random sample of size 20 from a normal distribution which is $N(\mu, 100)$. Find a 95% confidence interval for μ.

6A. Let us assume that the life length of a tyre in miles, say X is normally distributed with mean θ and standard deviation 5000. Past experience indicates that θ = 30,000 the manufacturer claims that the tyres made by a new procedure have mean θ > 30,000 and it is very possible that θ = 35, 000. Let us check this claim by testing $H_0 : \theta < 30{,}000$ against $H_1 : \theta > 30{,}000$. We shall observe n independent values of X say $X_1, X_2, \ldots, X_n$ and we shall reject H_0 if and only if $\overline{x} \geq c$. Determine n and c so that the power function K(θ) of the test has values K (30,000) = 0.01 and K (35,000) = 0.98.

6B. A two dimensional random variable (X, Y) is uniformly distributed over a rectangle with vertices (–1 ,0), (1,0), (0, – 1) and (0,1). Find E(x) and E(Y).

6C. The Mendelian theory states that the probabilities of classification A, B, C, D are respectively $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16}$. From a sample of 160 the actual numbers observed were 86, 35, 26 and 13. Is this data consistent with the theory at 0.01 significance level ?.

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