Question Paper

Exam Date & Time: 03-May-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH. (INFORMATION TECHNOLOGY) DEGREE EXAMINATIONS - APRIL / MAY 2024
SUBJECT: MAT 2226/MAT_2226 - ENGINEERING MATHEMATICS - IV

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) In a shooting test, probability of hitting the target is 0.5 for A_1, $\frac{1}{3}$ for A_2, and 0.25 for A_3. If all of them fire at the target, find the probability that (4)

i) At least two of them hit the target.
ii) At most one of them hits the target.

1B) Two absent minded roommates A and B, forget their umbrellas in some way or another. A always takes his umbrella when he goes out, while B forgets to take his umbrella with probability 0.5. Each of them forgets his umbrella at a shop with probability 0.25. After visiting 3 shops they return home. Find the probability that (3)

i) They have only one umbrella
ii) B lost his umbrella given that there is only one umbrella after their return.

1C) A random variable X has the distribution function (cdf), $F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ (3)

Find

i) The probability density function (pdf)
ii) $P(X > 2)$
iii) $P(-3 < X < 4)$

2A) i) Suppose that $X \sim N(8, \sigma^2)$, and the observed values of a sample of size 9 from this population are 8.6, 7.9, 8.3, 6.4, 8.4, 9.8, 7.2, 7.8, and 7.5. Construct a 95 percent confidence interval for σ^2. (4)

ii) A random sample of size 9 from the distribution $N(\mu, \sigma^2)$ yields $s^2 = 7.63$. Determine a 95 percent confidence interval for μ.

2B) In playing with an opponent of equal ability, which is more probable in each of the following: (3)

i) Winning 2 games out of 4 OR 5 games out of 8
ii) Winning at least 2 out of 4 OR at least 5 out of 8

2C) Let X have a pdf of the form $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1$, where $\theta = 1, 2$. To test the simple hypothesis $H_0: \theta = 1$ against the alternative hypothesis $H_1: \theta = 2$ use a random sample X_1, X_2 of size 2 and define the critical region to be C: $\left\{(x_1, x_2) \middle| \frac{3}{4} \leq x_1 x_2\right\}$. Find the power function and significance level of the test. (3)

3A) Let X and Y have the joint pdf $f(x,y) = \begin{cases} 15x^2y, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal pdfs of X and Y and compute $P(X + Y \leq 1)$. (4)

3B) In a courier company, three office assistants are assigned to process incoming mails. The first assistant A_1 processes 40%, the second assistant A_2 processes 35%, and the third assistant A_3 processes 25% of the mails. The first assistant has an error rate of 0.04, the second has an error rate of 0.06 and the third has an error rate of 0.03. A mail selected at random is found to have an error. The manager of the company wishes to know the probability that the mail was (3)

processed by first assistant.

3C) If the random variable X has a Gamma distribution with parameters r and α then derive the mean and variance of X. (3)

4A) A fair coin is tossed 3 times. Let X be 0 or 1 according as H or T is obtained on the first toss. Let Y be the number of heads. Find the joint probability distribution of X and Y, expectation of X, and expecation of Y. (4)

4B) A pot has 10% of defective items. What should be the number of items selected from the pot, such that the probability of finding at least 1 defective item in the selection is at least 0.95. (3)

4C) The heights of 500 soldiers are found to have normal distribution. Of them 258 are found to be within 2cm of the mean height of 170cm. Find the standard deviation of the distribution. (3)

5A) The Mendelian theory of genetics of crossing two types of peas states that the probabilities of classification of the four resulting types are $\frac{1}{16}, \frac{3}{16}, \frac{3}{16}, \frac{9}{16}$ respectively. If, from 160 independent observations, the observed (4)

frequencies of these classifications are 14, 35, 26, 85 respectively, test whether the data is consistent with the theory with level of significance 0.01.

5B) If X and Y both follow an exponential distribution with parameter 1, then find the pdf of $U = X - Y$. (3)

5C) The mean life length of a certain cutting tool is 41.5 hours with a standard deviation of 2.5 hours. What is the probability that a random sample of size 50 drawn from this population will have a sample mean between 40.5 and 42 hours. (3)

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