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IV SEMESTER B.E DEGREE END SEMESTER EXAMINATION – May, 2014

SUB: ENGINEERING MATHEMATICS – IV
(MAT –CSE/IT – 212)
(REVISED CREDIT SYSTEM)

Time : 3 Hrs. **Max.Marks : 50**

✍ **Note : a) Answer any FIVE full questions.**
b) All questions carry equal marks (3 +3 + 3 +1).

1A. Suppose that X is a random variable with pdf given by $f(x)=\begin{cases} 2x & ; \quad 0<x<1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$

Find the pdf of $Y=e^{-X}$.

1B. A bag contains three coins, one of which is two headed and the other two coins are normal and unbiased. One coin is chosen at random and is tossed four times in succession. If each time head comes up, what is the probability that this is a two headed coin?

1C. If X and Y are two random variables having the joint density function
$f(x,y)=\begin{cases} \frac{1}{8}(6-x-y), & 0\le x<2,\ 2\le y<4 \\ 0, & \text{otherwise} \end{cases}$. Find i) $P(X+Y<3)$

ii) $P(X<1\,/Y<3)$.

1D. The moment generating function for N (μ, σ^2) distribution is …….

2A. If X ~ N $(0, \sigma^2)$, Y~ N $(0, \sigma^2)$ where X and Y are independent, find the pdf of R = $\sqrt{X^2+Y^2}$.

2B. A man alternatively tosses a coin and throws a die beginning with coin. What is the probability that he will get a head before he gets a 5 or 6 on the die ?

2C. In a certain factory, turning out of optical lenses, there is a chance of $1/500$ for any lens to be defective. The lenses are supplied in a packet of 10. Calculate the approximate number of packets containing no defective, one defective, two defective and three defective lenses in a consignment of 20,000 packets.

2D. If (X, Y) be two dimensional random variable uniformly distributed over unit circle then its joint pdf is …………..

3A. A random variable X has probability function $P(X = k) = \frac{c}{2^k}$, k = 0,1,2,3… Then find i) the value of c ii) P(X is multiple of 3)

3B. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal distribution with mean 0 and variance θ, $0<\theta<\infty$. Show that $\frac{\sum_{i=1}^{n} {X_i}^2}{n}$ is an unbiased estimator of θ and has variance $\frac{2\theta^2}{n}$.

3C. Show that for the random variable X having normal distribution with mean μ and variance σ^2, $E(X-\mu)^{2n} = 1.3.5.....(2n-1)\sigma^{2n}$.

3D. Let X be a random variable having Poission distribution with$\Pr(X = 0) = \frac{1}{e}$. Then $\Pr(X = 10) = \cdots$

4A. Let $X_1, X_2, \ldots, X_n$ be stochastically independent random variables with X_i having $N(\mu, \sigma^2)$ distribution. If $Y = \sum_{i=1}^{n} k_i X_i$ then find the distribution of Y. Hence, deduce the reproductive property of normal distribution.

4B. If X, Y, Z are uncorrelated random variables with zero means and standard deviations 5, 12, 7 respectively and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between U and V.

4C. The income of a group of 10,000 persons was found to be normally distributed with mean Rs.750 and standard deviation of Rs. 50. Find the number of persons out of 10,000 who have income exceeding Rs.668 and those who have income exceeding Rs. 832?.Also, find the lowest income among the richest 100?

4D. The statistic used for estimating confidence interval for μ when σ^2 is not known is ….

5A. Suppose that X has distribution $N(\mu, \sigma^2)$. A sample of size 15 yields $\bar{x} = 3.2$ and $s^2 = 4.24$. Obtain a 90 percent confidence interval for σ^2 and μ.

5B. A die is cast n = 120 independent times and the following resulted.

Spots up	1	2	3	4	5	6
Frequency	b	20	20	20	20	40 – b

If we use chi-square test, for what values of b would the hypothesis that the die is unbiased be rejected at 0.025 significance level.

5C. Let $X_1, X_2, \ldots, X_n$ denote the random sample of size n from a distribution having pdf $f(x,\theta) = \begin{cases} \theta^x(1-\theta)^{1-x}; & x = 0,1,\ldots; 0 \le \theta \le 1 \\ o & Otherwise \end{cases}$. Find maximum likelihood estimation for θ.

5D. The significance level of a test is probability of committing …………

6A. Let (X_1, X_2) be a sample of size 2 from the distribution having the pdf

$$f(x;\ \theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, \quad 0 < x < \infty,\ \theta > 0.$$
$$= 0, \quad \text{elsewhere}$$

We reject $H_0 : \theta = 2$ and accept $H_1 : \theta = 1$, if the observed values (x_1, x_2) are such that $\frac{f(x_1;2)f(x_2;2)}{f(x_1;1)f(x_2;1)} \le 1/2$. Find significance level of the test.

6B. A computer in adding numbers rounds each number to its nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5)

(i) If 1500 numbers are added, what is the probability that magnitude of the total error exceeds 15?

(ii) How many numbers may be added together so that magnitude of the total error is less than 10 with probability 0.9?

6C. Find the mgf of chi-square distribution. Hence find its mean and variance.

6D. By Chebyshev's inequality $\Pr\{|X - c| \ge \epsilon\} \le$ . ...

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