

Question Paper

Exam Date & Time: 26-May-2023 (02:30 PM - 05:30 PM)



MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH. DEGREE EXAMINATIONS - MAY/JUNE 2023

SUBJECT: MAT 2256/ MAT-2256 ENGINEERING MATHEMATICS - IV

(SPL: COMPUTER SCIENCE AND ENGINEERING - CYBER SECURITY/COMPUTER SCIENCE/COMPUTER AND COMMUNICATION/
INFORMATION TECHNOLOGY/COMPUTER SCIENCE AND ENGINEERING - ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING)

Marks: 50

Duration: 180 mins.

Answer all the questions.

- 1A) Three group of children contain respectively 3 girls 1 boy, 2 girls 2 boys, 1 girl 3 boys. One child is selected at random from each group. Show that the chance that the 3 selected consist of 1 girl and 2 boys is $13/32$. (4)
- 1B) A bag contains 10 gold coins and 8 silver coins. Two successive drawings of 4 coins are made such that: (3)
i) The coins are replaced before the second trial.
ii) The coins are not replaced before the second trial.
Find the probability that the first drawing will give 4 gold coins and second drawing will give 4 silver coins.
- 1C) Suppose the pdf of a random variable X is given as: (3)
$$f(x) = \begin{cases} 4x - 4x^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
 Find
i) $V(X)$
ii) Mode of the distribution
- 2A) If X is normally distributed with mean μ and variance, (4)
 σ^2 then show that $E(X - \mu)^{2n} = 1.3.5 \dots (2n - 1)\sigma^{2n}$.
- 2B) An aircraft knows that 5% of the people making reservation on a certain flight will not show up. Consequently, their policy is (3)
to sell 52 tickets for the flight that can only hold 50 passengers. What is the probability that there will be a seat available for every passenger who turns up?
- 2C) If X_1, X_2, X_3 be uncorrelated random variables having same standard deviation. (3)
Find the correlation coefficient between $U = X_1 + X_2$ and $V = X_3 + X_2$.
- 3A) Let X has pdf $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$ where $\theta > 0$. To test $H_0: \theta = 1$ against $H_1: \theta = 2$, (4)
a random sample (X_1, X_2) of size 2 is used with critical region $C = \{(x_1, x_2) \mid x_1 x_2 \geq \frac{3}{4}\}$.
Compute the power function and significance level of the test.
- 3B) Four roads A, B, C and D lead away from a jail. A prisoner escaping from the jail selects a road at random. (3)
If road A is selected, the probability of escaping is $\frac{1}{8}$. Similarly, for road B it is $\frac{1}{6}$, for road C it is $\frac{1}{4}$ and for road D it is $\frac{4}{5}$. What is the probability that the prisoner will succeed in escaping?
- 3C) (3)
Suppose that an electronic device has a life length X which satisfies Gamma Distribution with mean 1 and variance 2. Then, determine $P(X \geq 0)$.
- 4A) Two production lines manufacture a certain type of item. Suppose that the capacity (on any given day) is 5 items for line I (4)
and 3 item for line II. Let (X, Y) represent the two-dimensional random variable yielding the number of items produced by line I and line II, respectively.

X \ Y	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

0	0	0.01	0.03	0.05	0.07	0.09
1	0.01	0.02	0.04	0.05	0.06	0.08
2	0.01	0.03	0.05	0.05	0.05	0.06
3	0.01	0.02	0.04	0.06	0.06	0.05

Find the probability that more items are produced by line I than line II.

- 4B) An insurance company has discovered that only about 0.1% of the population is limited in a certain type of accidents each year. If its 10000 policy holders were randomly selected from the population then what is the probability that not more than 5 of the clients are involved in such accidents each year? (3)

- 4C) The monthly income of a group of 10,000 person were found to be normally distributed with mean rupees 750 and SD rupees 50. Show that of this group about 95% had income exceeding rupees 668 and only 5% had income exceeding rupees 832. What was the lowest income among the richest 100? (3)

- 5A) A die was rolled $n = 120$ times independently and the following data was obtained: (4)

Outcome	1	2	3	4	5	6
Frequency	m	20	20	20	20	20

For what values of m would the hypothesis that the die is unbiased be rejected at 0.025 significance level in a chi-square test?

- 5B) Let (X_1, X_2, \dots, X_n) be a random sample of size n from a distribution whose pdf is given by (3)

$$f(x; \theta) = \begin{cases} \theta^x (1 - \theta)^{1-x} & x = 0, 1, 2, \dots \text{ and } 0 \leq \theta \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the maximum likelihood estimator for θ

- 5C) The mean life length of a certain cutting tool is 41.5 hours with a standard deviation of 2.5 hours. What is the probability that a random sample of size 50 drawn from this population will have a sample mean between 40.5 and 42 hours. (3)

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