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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL UNIVERSITY, MANIPAL - 576 104

FOURTH SEMESTER B.Tech. DEGREE END SEMESTER MAKE-UP EXAMINATION

SUB: ENGINEERING MATHEMATICS IV (MAT 212) CS-ICT-CC
(REVISED CREDIT SYSTEM -2011)

Time: 3 Hrs. **Max.Marks: 50**

Note: a) Answer any FIVE full questions. b) All questions carry equal marks (4+3+3)

1A. In a certain factory producing blades there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Calculate the approximate number of packets containing (a) no defective (b) one defective (c) two defectives blades, in a consignment of 10000 packets.

1B. Find the mgf of Poisson distribution and hence find its mean and variance.

1C. State Kolmogorov's axioms on probability. For any two events A and B prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.

2A. Suppose that there is a chance for a newly constructed house to collapse whether the design is faulty or not. The chance that the design is faulty is10%. The chance that the house collapses if the design is faulty is 95% and otherwise it is 45%. It is seen that the house collapsed. What is the probability that it is due to faulty design?

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2B. Let X has uniform distribution over the interval $(-\pi/2, \pi/2)$. Obtain the pdf of Y where Y = tan X.

2C. A coin is known to come up heads 3 times as often as tail. This coin is tossed 3 times. Let X be the number of heads that appear. Write the probability distribution of X and also the cdf.

3A. Let X has a pdf of the form $f(x;\theta)=\begin{cases}\theta x^{\theta-1}, & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$ where $\theta \in \{\theta : \theta = 1, 2\}$.

To test the simple hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$, use a random sample (X_1, X_2) of size n = 2 and define the critical region to be $C=\left\{(x_1,x_2):\frac{3}{4}\le x_1x_2\right\}$. Find the power function of the test.

3B. If Y= a X + b then show that $\rho=\pm1$.

3C. Let (X_1, X_2) be a random sample from a distribution with the pdf $f(x) = e^{-x}, \quad 0 \le x < \infty$. Show that $Z = X_1/X_2$ has F-distribution.

4A. Let $(X_1, X_2, \ldots, X_n)$ denote a random sample from a distribution which is $N(\theta_1, \theta_2), \; -\infty < \theta_1 < \infty, \; 0 < \theta_2 < \infty$. Find a maximum likelihood estimator for θ_1 & θ_2

4B. Let $\bar{X}$ be the mean of a random sample of size 25 from a distribution which is N (75,100). Evaluate P (71 < $\bar{X}$ < 79).

4C. Suppose that X is a random variable for which E(X) = 10, V(X) = 25. For what positive values of a and b, Y = aX – b has expectation 0 and variance 1.

5A. Suppose that the joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$. Compute: (i) $P(Y \le X)$, (ii) $P(X + Y \ge 1)$

5B. Show that the mean $\bar{X}$ of a random sample of size n from a distribution having pdf $f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x > 0, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$ is an unbiased estimator of θ and has variance $\frac{\theta^2}{n}$

5C. Let $\bar{X}$ denotes the mean of a random sample of size n from $N(\mu, 10)$. Find n so that the probability is approximately 0.954 that the random interval $\left(\bar{X} - \frac{1}{2}, \; \bar{X} + \frac{1}{2}\right)$ includes μ.

6A. A die is thrown 132 times with the following results.

No:turns up	1	2	3	4	5	6
frequency	16	20	25	14	29	28

Is the die unbiased? Test with the help of χ^2 at 5% level of significance.

6B. Suppose that the life lengths of two electronic devices say D_1 and D_2 have distributions N(40,36) and N(45,9) respectively. If the electronic device is to be used for a 48 hour period which device is to be preferred?

6C. The coefficients a, b and c of a quadratic equation $ax^2 + bx + c = 0$ are obtained by tossing a die thrice. What is the probability that the roots of the equation are real?

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