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Deep Reinforcement Learning for Multi-Truck Vehicle Routing Problems with Multi-Leg Demand Routes

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Deep reinforcement learning (RL) has been shown to be effective in producing approximate solutions to some vehicle routing problems (VRPs), especially when using policies generated by encoder-decoder attention mechanisms. While these techniques have been quite successful for relatively simple problem instances, there are still under-researched and highly complex VRP variants for which no effective RL method has been demonstrated. In this work we focus on one such VRP variant, which contains multiple trucks and multi-leg routing requirements. In these problems, demand is required to move along sequences of nodes, instead of just from a start node to an end node. With the goal of making deep RL a viable strategy for real-world industrial-scale supply chain logistics, we develop new extensions to existing encoder-decoder attention models which allow them to handle multiple trucks and multi-leg routing requirements. Our models have the advantage that they can be trained for a small number of trucks and nodes, and then embedded into a large supply chain to yield solutions for larger numbers of trucks and nodes. We test our approach on a real supply chain environment arising in the operations of Japanese automotive parts manufacturer Aisin Corporation, and find that our algorithm outperforms Aisin's previous best solution.

I. INTRODUCTION

Vehicle Routing Problems (VRPs) [1–3] are NP-hard combinatorial optimization problems in which one or more trucks must deliver material between several different locations. VRPs arise frequently in the context of supply chain logistics. Often, the complexity of a vehicle routing problem acts as a computational bottleneck limiting the efficiency of supply chain operations. The nature of such computational bottlenecks is that even a modest improvement in the quality of solutions can lead to significant benefits, both financial and environmental.

VRPs have been thoroughly studied in operations research, which has led to many different VRP variations. Examples include the capacitated VRP (CVRP) in which the trucks have fixed carrying capacities, the VRP with time windows (VRPTW) in which demand must arrive at its destination within a specified time interval, the split-delivery VRP (SDVRP) in which trucks are allowed to load a subset of the demand at a single location and leave the rest to be picked up later, and many others. But there are some variations of the VRP that arise in real-world supply chains which have not been thoroughly studied. One example is VRPs where the demand has multi-leg routing requirements. In these problems, demand is required to move along sequences of locations, and not just from one location to another. This type of problem arises in the supply chain operations of Aisin Corporation, a Japanese automotive manufacturing company, and is the focus of this work.

Recently, there has been a growing interest in solving combinatorial optimization problems using reinforcement learning (RL) [4, 5]. More specifically, there has been a plethora of research on solving VRPs using RL approaches [6–29] (see [30] for a review of recent advances). Many of these models use attention layers, which has proven to be quite effective for solving many simple VRP variations. The use of RL for such problems is very natural. RL is most useful in very complex decision processes, where a decision's consequences can only be learned through trial-and-error. Many games such as Chess and Go fit this description, and indeed the best known solutions for such games use RL methodology [31, 32]. A VRP can be thought of as a one-player game in which the player controls the trucks and decides what routes they drive and what they pickup and dropoff at each stop. In this context, VRPs present Markov decision processes for which the consequences of decisions are best learned through experience.

However, state-of-the-art RL approaches for VRPs are not yet deployable in commercial settings since they typically only address very simple VRP variations. Past work has almost exclusively focused on the case of a single truck whose task is to simply deliver all demand to one special location. Realistic supply chain environments, like the one we consider here, involve many trucks and much more complicated delivery requirements.

In this work, we take steps towards developing RL models that are capable of obtaining good solutions to a multi-truck VRP with multi-leg routing requirements. As our testing ground, we use a real supply chain environment of Aisin Corporation and find that our method outperforms Aisin's solution for a slightly simplified version of this supply chain. We build upon the method of [8] (which was designed for single truck VRPs with simple routing requirements) by developing and incorporating new techniques which allow for multiple trucks and multi-leg routing. Importantly, our approach breaks the problem into many smaller problems of fixed size. This allows us to scale up the size of the problem without having to scale the size of our model, by just solving more sub-problems.

The rest of this paper is organized as follows. In Sec. II, we introduce the realistic VRP formulation which is the focus of this work. In Sec. III, we describe the complete workflow of our algorithm, treating our neural networks as black boxes. In Sec. IV, we describe in detail the architecture of our neural networks. In Sec. V, we outline the methods we use to train our neural networks. Finally, in Sec. VI, we present the results of deploying our algorithm on the realistic Aisin test case.

This work is an extension of the work done in [33], and was completed after SW and FS left QC Ware Corp.

II. VEHICLE ROUTING PROBLEMS

In this section, to highlight the complexity of the realistic VRP on which we will test our method, we will first review a basic version of the VRP. Then we will introduce a generalized and more complex VRP that models an actual supply chain environment that arises in the daily operations of Aisin Corporation.

A. Basic Vehicle Routing Problem

First, we review the capacitated vehicle routing problem with split deliveries (SDVRP). An instance of the SDVRP is specified by the following data:

1. A graph G with $n+1$ nodes $z_0, z_1, \ldots, z_n$.
2. An $(n+1)\times(n+1)$ matrix T with non-negative entries and $T_{ii}=0$ for all $i \in \{0,1,\ldots,n\}$ called the *drive-time matrix.*
3. n non-negative numbers $\{d_i\}_{i\in[n]}$ assigned to each node z_i with $i \neq 0$. These numbers are called *initial demands.*
4. A positive real number C, which is the capacity of the truck.

The nodes of the graph G represent a set of locations that a delivery truck may drive to. The node z_0 plays a special role, described below, and is called the *depot*. In general, G need not be complete, *i.e.* driving from z_i to z_j may be prohibited for certain pairs (i,j).

The matrix element T_{ij} of the drive-time matrix T is the time required for a truck to drive from z_i to z_j (this is why $T_{ii}=0$). In a realistic scenario, the drive-time from z_i to z_j may be slightly different from the drive-time from z_j to z_i, so T is not required to be symmetric.

The initial demand d_i is the volume of material that starts at z_i. The units of demand need not be volume, but could instead be weight, monetary value, or any other positive real number associated with the material to be delivered. Volume is chosen here as the most appropriate for the problem at hand. All initial demand must be delivered to the depot, z_0, by a single truck with capacity C. When the truck stops at z_i ($i \neq 0$), it picks up as much demand as possible before departing to another node (either filling the truck to capacity or picking up all demand at z_i). This can result in part of the demand d_i being left at z_i when the truck first stops there, thus making it necessary for the truck to stop at z_i again later to finish the delivery of d_i (this is why the problem is said to allow "split deliveries"). When the truck stops at the depot, it unloads all demand it is currently carrying before departing to another node. Demand that is not currently on the truck is called *offboard demand*, and demand currently on the truck is called *onboard demand.*

A truck route is a sequence of nodes $(z_{k_0}, z_{k_1}, \ldots, z_{k_l})$ with $k_0 = k_l = 0$ (*i.e.* routes must start and end at the depot). The total drive-time of a route can be written as

$$\mathrm{time}(z_{k_0}, z_{k_1}, \ldots, z_{k_l}) = \sum_{i=0}^{l-1} T_{k_i k_{i+1}}. \tag{1}$$

A demand-satisfying route is a route which results in 100% of the initial demand being delivered to the depot. The solution to the SDVRP is the demand-satisfying route with minimum total drive-time.

The SDVRP is a very commonly studied VRP in operations research, for which the usage of deep reinforcement learning is well-established in the literature. However, the problem we study in this work is significantly more complex than the SDVRP, as we will see in the next section. The model we use in this work is a generalized version of the model of [8], built to address the generalized VRP described below.

B. Generalized Vehicle Routing Problem

Now we will define a more complex vehicle routing problem that generalizes many features of the SDVRP. We call this problem the generalized vehicle routing problem (GVRP). The GVRP more realistically models real world supply chain logistics, and in particular, the Aisin Corporation VRP is an instance of the GVRP.

An instance of the GVRP is specified by the following data:

1. A graph G with n nodes $z_0, z_1, \ldots, z_{n-1}$.
2. An $n \times n$ matrix T with non-negative entries and $T_{ii}=0$ for all $i \in \{0,1,\ldots,n-1\}$ called the *drive-time matrix.*
3. A finite set of indivisible boxes which serves as the demand for the problem. Each box has a volume and a required route.
4. A positive real number C, which is the capacity of each truck.

5. A positive real number $T_{\max}$, which is the time-limit for daily supply chain operations.

Just like in the SDVRP, G represents the locations in the supply chain, and the entries of T are times required for a truck to drive between pairs of nodes. But a crucial difference between the SDVRP and the GVRP is in the definition of demand. In the SDVRP, the initial demand is given as a set of non-negative real numbers d_i representing the volume of demand that starts at z_i. In the GVRP, the demand is discrete; it is defined as a finite set of boxes, which cannot be further subdivided. Each box has two attributes:

1. A volume, which is a positive real number.
2. A required route, which is a finite sequence of nodes.

Each box starts at the first node of its required route, and must travel to each node of its required route (in order) within the daily time limit. Such multi-leg delivery requirements can arise for several practical reasons in a real world supply chain environment. There may be capacity limitations at a box's destination node, causing that box to have to stop at a storage warehouse first before moving on to its destination. Also, the contents of some boxes may need to have an operation performed on them, such as an assembly procedure, before its final delivery. Another important reason for multi-leg delivery requirements is that a box may need to be sent back to its starting location empty, after the parts it was designed to carry have been delivered, so that the box can be used again the next day.

The daily time limit is another feature of the GVRP which is not present in the SDVRP. As a result, the optimization goal of the GVRP is different from that of the SDVRP. Instead of fixing the number of trucks and finding the demand-satisfying route with minimum total time, we impose a time limit and the goal is to find the demand-satisfying route that uses the minimum number of trucks and finishes within the time limit.

The use of multiple trucks in the GVRP is another feature which distinguishes it from the SDVRP. This adds complexity to the problem in an obvious way, as optimal routes should now involve some form of cooperation between trucks.

III. SUPPLY CHAIN MANAGEMENT WORKFLOW

The goal of this work is to apply reinforcement learning (RL) techniques to find commercially valuable solutions to GVRP instances. However, training an RL agent to make *all* of the decisions that comprise a solution (both the truck routing decisions and the demand pickup and dropoff decisions) would be an extremely difficult task, as RL agents typically perform worse as the decision space grows larger [34]. Due to the highly complex nature of the demand in the GVRP, the pickup and dropoff decisions generally come from a very large decision space. There can be thousands of boxes waiting at a node or on a truck, with many different individual routing requirements, so the number of options for which boxes to pickup or dropoff at a given stop can be astronomically large. The routing decisions on the other hand (which node a truck should drive to next), come from a much smaller decision space. If the graph G contains n nodes, then there are at most $n-1$ options for each routing decision. For this reason, our approach uses a trained RL agent to make the routing decisions, and sensible heuristic methods to make the pickup and dropoff decisions.

The basic workflow of our algorithm is

1. Convert the discrete demand into a continuous *tensor demand structure*,
2. Extract a "good" subenvironment S from the full supply chain environment,
3. Use our trained RL agent to compute truck routes for the trucks operating in S,
4. Use a heuristic method to compute pickups/dropoffs for the truck routes from step 3,
5. Update the tensor demand of the full supply chain to account for the part of the demand that was delivered to its final destination in the solution for S,
6. Repeat steps 2-5 until the demand of the full supply chain reaches 0.

We refer to steps 2-5 as an *iteration.* Each iteration produces a solution for one subenvironment. This solution consists of a set of truck routes, and instructions indicating which boxes each truck will pickup and dropoff at each stop along its route. To construct the full solution from these subenvironment solutions, we simply run all of the subenvironment solutions simultaneously. This iterative structure allows us to easily scale our method to solve problems with larger initial demand. Problems with more demand will simply require more iterations (*i.e.* more subennvironments).

Each iteration consists of two phases: route-finding in Phase 1, pickup/dropoff-finding in Phase 2 using the routes from Phase 1 as input. There are two reasons for this two-phase approach, which will become clearer by the end of this section. The first reason is the split in how different decisions are made, described in the first paragraph of this section. The second reason is that we make the routing decisions using a simplified model of the demand. Good routing decisions cannot be made without at least some information about the demand, but using the true demand with all of its individual boxes complicates things significantly. We simplify the demand by combining all boxes that have the same required route into a "box soup" which is infinitely divisible. This continuous approximation of the true discrete demand saves us the trouble of keeping track of thousands of individual boxes while making the truck routing decisions. But Phase 2 must then return to the true demand of the problem, and make pickup/dropoff decisions for each individual box at each stop for each truck.

The rest of this section describes each step of our algorithm in detail, assuming we have a trained RL agent to make the truck routing decisions in Step 3. The next section will describe the exact nature of the RL agent and its training process.

A. Tensor Demand Structure

The first step of our algorithm is to convert the discrete demand into continuous "box soup". This naturally leads us to organize the demand into a set of higher rank tensors, or a *tensor demand structure.*

In the SDVRP, all demand must be delivered to the same node, z_0. This gives the problem a *vector demand structure*, *i.e.* the offboard demand at time t is given by a vector

$$D^t = (d_1^t, d_2^t, \ldots, d_n^t). \tag{2}$$

In the GVRP, the offboard demand at time t naturally takes the form of one or more higher rank tensors. This is easiest to see in the case of rank-2 demand. Rank-2 demand is any demand that is required to move along a sequence of nodes of length 2. For example, a box currently at z_3 that must move to z_5 contributes to the $(3,5)$ component of a rank-2 offboard demand tensor. More generally, the (i,j) component of the rank-2 offboard demand tensor at time t, D_{ij}^t (a matrix in the rank-2 case), is a non-negative real number whose value is the total volume of demand that is currently at z_i and must to move to z_j to complete its required route. This idea naturally generalizes to higher ranks. A box currently at z_2, which must first move to z_4 and then to z_6 to complete its required route contributes to the $(2,4,6)$ component of a rank-3 offboard demand tensor, D_{246}^t. For any instance of the GVRP, the initial demand can be organized into a set of offboard demand tensors of different ranks, whose components are the total volumes of demand that must move along specific sequences of nodes.

Each truck's onboard demand in the GVRP also takes the form of one or more higher rank tensors. Suppose truck T arrives at z_i and picks up one box of volume V, whose required route is (i,j,k). Before being loaded onto the truck, this box contributes volume V to the (i,j,k) component of the rank-3 offboard demand tensor, D_{ijk}. After being loaded onto the truck, the box now contributes volume V to the (j,k) component of T's rank-2 onboard demand tensor, E_{jk}^T. This allows us to keep track of the specific routing requirements of all demand on each truck. Note that all onboard demand tensors are 0 at the start of the day, before supply chain operations have commenced.

There is one more level of complexity in the demand structure. Often in real-world supply chains where the demand consists of small, intricately shaped, fragile parts, specially designed boxes are used to transport these parts from node to node. In the very common scenario where the supply chain has repeated operations, such as daily or weekly shipments of identical parts, these boxes get reused for each shipment. This means that those boxes need to return to their original locations after completing their required routes, thus forming *cyclic* box routes. The simplest way to enforce this constraint is to add one more node (the start node) to the required route of any box that must return to its original location. This has the effect of increasing the ranks of some demand tensors by 1. But a more memory-efficient way is to distinguish between *cyclic demand* and *direct demand* (boxes which must be returned to their original locations, and those which do not), by defining a cyclic offboard demand D^{cyclic} and a direct offboard demand D^{direct}. To account for the different routing requirements, there is a slight difference in how these tensors evolve as the demand gets delivered throughout the day, which will be described when we get to Step 3 of Phase 1.

B. Subenvironment Search

The second step of our algorithm is to extract a subenvironment from the full supply chain environment. A subenvironment is a subset S of the nodes of G, together with a subset D_S of the total demand, and a set of N trucks which are assigned to operate only within S (stopping only at nodes in S). D_S is fully determined by S: it consists of all demand whose required route lies fully inside S. The daily time limit for S is equal to the daily time limit $T_{\max}$ for the full supply chain. The reason we operate on subenvironments, instead of on the whole supply chain at once, is to make the routing decisions spaces smaller. When there are fewer nodes to drive to, the routing decisions become easier.

Here, we describe how "good" subenvironments are chosen in Step 2. The goal is to choose a subenvironment that results in a large volume of demand being delivered. We estimate how much demand will be delivered in a subenvironment S by N trucks within time $T_{\max}$ by using our trained RL agent to produce a solution for S, and looking at how much total volume is delivered by this solution. To find a solution for a subenvironment S, we run an *episode* on S (episodes are described below in Step 3). To find a good subenvironment, we test many subenvironments, running multiple episodes on each subenvironment and then choosing the subenvironment with the highest mean volume of delivered demand.

How do we decide which subenvironments to test? First we choose how many nodes we want in our subenvironment, n'. Now we consider the initial offboard demand D^0. Assume for this example that D^0 consists only of rank-3 demand (this method readily generalizes to other ranks). Consider only the nonzero components of D^0. These are indexed by tuples of nodes

$$\begin{gathered}(i_1, j_1, k_1)\\(i_2, j_2, k_2)\\\vdots\\(i_u, j_u, k_u)\end{gathered}$$

where u is the number of distinct box routes in the initial demand. We start by uniformly randomly selecting one tuple of nodes from the list, and then removing it from the list.

Suppose that we select the tuple (i_5, j_5, k_5). We then add these 3 nodes to our (initially empty) node subsets, to form $A = \{i_5, j_5, k_5\}$. If $3 < n'$, we continue to randomly select node tuples. Suppose that we select (i_4, j_4, k_4) next. We now consider the set $A = \{i_5, j_5, k_5, i_4, j_4, k_4\}$. We do not allow repeated elements (*i.e.* A is a set and not a multiset), so A now contains *at most* 6 elements. We continue this process of randomly selecting tuples of nodes and adding them to A via set union, until we either run out of tuples or $|A| \geq n'$. At this point, if $|A| = n'$, we use A as our test subenvironment. If $|A| > n'$, we remove the subset added last. At this point, we have $|A| < n'$, so we randomly add $n' - |A|$ nodes.

This process gives us a node subset A with $|A| = n'$. We repeat this process $k_{\text{num subsets}}$ times to produce $k_{\text{num subsets}}$ different subenvironments, and we run $k_{\text{subset attempts}}$ episodes on each subenvironment[1]. We then compute the mean volume of delivered demand for each subenvironment's $k_{\text{subset attempts}}$ episodes, and select the subenvironment with the highest mean volume of delivered demand.

C. Phase 1: Route-finding

In the next step, we obtain a solution for the chosen subenvironment of step 2 by running a large number of episodes on this subenvironment, each producing a solution. We then choose the solution which leads to the largest volume of delivered demand.

First we describe what an episode is. An episode is one full day ($T_{\max}$) of supply chain operations, with the continuous tensor demand structure described above. An episode consists of a series of time-steps, with each time-step corresponding to a truck stopping at a node. Since there are multiple trucks operating in a single episode, each time-step has an associated *active truck* and *active node*. The active truck is the one making the stop, and the active node is the location of the active truck.

At each time step, 3 things happen:

1. The active truck drops off any demand whose next required stop is the active node
2. We invoke our trained RL agent to determine which node the active truck will drive to next
3. The active truck picks up offboard demand from the active node according to a heuristic, and departs for its next node (chosen in 2)

This continues until there is no more demand to deliver, or the daily time limit is reached. The primary complexity of this process is the bookkeeping associated with steps 1 and 3. Keeping track of where all of the demand is at all times is crucial for the accuracy of our computation.

To clarify, we provide an example in the following table of the demand flow for material with rank-3 required route, and a cyclic box-return constraint. The first column says which demand tensor component the material's volume contributes to after the event in the second column.

Component	Most recent event
$D^{\text{cyclic}}_{2\,4\,6}$	Material initially at node 2; must go to node 4, then node 6, then node 2
$E^{m=1}_{4\,6\,2}$	Picked up from node 2 by truck 1, next stop node 4
$D^{\text{direct}}_{4\,6\,2}$	Dropped off at node 4 by truck 1; must go to node 6, then node 2
$E^{m=2}_{6\,2}$	Picked up from node 4 by truck 2, next stop node 6
$D^{\text{direct}}_{6\,2}$	Dropped off at node 6 by truck 2; must go to node 2
$E^{m=3}_{2}$	Picked up from node 6 by truck 3, next stop node 2
0	Dropped off at node 2 by truck 3, requirements fulfilled

D. Phase 2: Pickup and Dropoff Decisions

Phase 2 is very simple compared to Phase 1. The inputs are the routes from Phase 1 for each truck in each subenvironment, the initial (discrete) demand, the daily time limit $T_{\max}$, and the truck capacity C.

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[1] This is small compared to the number of episodes we run on the chosen subenvironment in Step 3

In Phase 2, we make pickup/dropoff decisions for each stop of each truck using the simple heuristics explained below. Crucially, these decisions are made at the individual box level, as opposed to at the "box soup" level of Phase 1.

Each time a truck stops at a node, two things happen:

1. Certain boxes on the truck are dropped off at the node.
2. Certain boxes at the node are picked up by the truck.

The heuristic we use to determine which boxes to drop off is just a discrete version of the same heuristic used in Phase 1: we drop off any box whose next required stop is the current node.

The heuristic we use to determine which boxes to pick up is a bit more complicated, but still quite simple. Suppose the truck is at its pth stop, z_{k_p}, and the truck's remaining route is $(z_{k_{p+1}}, z_{k_{p+2}}, \ldots, z_{k_l})$. We first pick up as many boxes as possible whose next destination is $z_{k_{p+1}}$. This means either fill the truck to capacity with these boxes, or continue loading these boxes until none remain. Then, if there is space on the truck to do so, load as many boxes as possible whose next destination is $z_{k_{p+2}}$. Continue in this fashion until either the truck is full, or there are no boxes remaining at z_{k_p} whose next required stop is on the truck's remaining route.

There is one important subtlety to keep in mind here. We cannot allow any partial deliveries in Phase 2. In other words, we cannot allow any box to be moved from its starting location without fully completing its required route. This is because the subenvironment solutions generated by each iteration will all run simultaneously in the full solution. Therefore, a partial delivery in one iteration cannot be completed in another iteration without substantially complicating our algorithm.

To eliminate partial deliveries in Phase 2, we simply add a subroutine at the end of Phase 2 which resets any partially delivered boxes to their starting locations, and removes them from the pickup/dropoff schedules, as if they were never moved.

1. *Total Demand Update*

Finally, in preparation for the next iteration, we must update the demand tensors that we constructed in Step 1, by subtracting any demand that was delivered to its final destination in the previous iteration.

IV. POLICY NEURAL NETWORK FOR ROUTING DECISIONS

The model we use for our neural networks is an encoder and decoder model directly inspired by that of [8]. Their model is quite general: after training with REINFORCE [35], it performs well for numerous routing problems including the traveling salesman problem, basic vehicle routing problems, and the orienteering problem. However, it does not account for multiple trucks. Moreover, there is no obvious way to incorporate a tensor demand structure into their model without a substantially new approach.

A. Reinforcement Learning for Vehicle Routing Problems

Before describing our neural networks, we first explain how VRPs fit into the framework of reinforcement learning, and also how an encoder and decoder furnish a policy.

RL is often used in the context of a Markov decision process (MDP). In this setting, there is an environment that is in some state, and an agent who can perform certain actions which change the state of the environment. Given the environment state, the agent must choose an action from a set of allowed actions. This action results in a new environment state, and a reward for the agent. This process is repeated until some halting condition is met. The agent's goal is not to maximize the reward for a single action, but to maximize the long term reward, or *return*.

The RL agent achieves its goal by learning a *policy*, which is a conditional probability distribution over possible actions to take given an environment state. Given a state s and an action a from a set A of potential actions, the agent learns to compute a quantity $\pi(a|s) \in [0,1]$ such that $\sum_{a\in A} \pi(a|s) = 1$. Given a state s, the agent can then sample from the policy π to choose its next action. The agent's goal is to learn the optimal policy, *i.e.* the one which maximizes the expectation value of the return.

In the context of the GVRP, The environment state s consists of demand data (locations and required routes of all demand) and truck data (location and remaining capacity of each truck, index of the active truck). The action set A is just the set of nodes that the active truck can drive to next.

B. Encoder

The encoder is used once at the beginning of each episode. Its purpose is to encode the data defining the problem into a set of high dimensional vectors, one for each node. The encoder we use is almost identical to the encoder of [8], but with an adjustment to account for the higher rank demand.

The input data for the encoder contains information about the locations of the nodes, as well as information about the initial demand structure. The node locations can be fully captured by simple 2-dimensional coordinate vectors. The demand structure, however, is too complex to encode in its entirety, so we construct an abbreviated version as follows.

We define outgoing and incoming offboard demands at each node as

$$\delta_i^{\text{out}} = \sum_j D_{ij} + \sum_{jk} D_{ijk} + \sum_{jkl} D_{ijkl} + \dots, \quad (3)$$

$$\delta_i^{\text{in}} = \sum_j D_{ji} + \sum_{jk} D_{jik} + \sum_{jkl} D_{jikl} + \dots, \quad (4)$$

where D is defined to be the sum of both the cyclic and direct demands. Now we can define the input vector for node z_i as the 4-dimensional vector

$$\overline{\mathbf{x}}_i = \mathbf{x}_i \oplus \left(\delta_i^{\text{in}}, \delta_i^{\text{out}}\right), \quad (5)$$

where $\mathbf{x}_i$ is the 2-dimensional coordinate vector giving the location of node z_i, and $\oplus$ denotes concatenation. The set of vectors $\{\overline{\mathbf{x}}_1, \dots, \overline{\mathbf{x}}_n\}$ forms the input for the encoder. The first encoding layer is a linear map with bias from $\mathbb{R}^4$ to an encoding space $\mathbb{R}^d$ (we typically use $d = 64$),

$$\mathbf{h}_i^0 = W^{\text{init}} \overline{\mathbf{x}}_i + \mathbf{b}^{\text{init}}, \quad (6)$$

where W^{init} is a $d \times 4$ matrix and $\mathbf{b}^{\text{init}}$ is a d-dimensional vector. Note that the same encoding map is applied to every every vector in the input set, *i.e.* W^{init} and $\mathbf{b}^{\text{init}}$ do not depend on i.

In addition to the initial layer, our encoder consists of two more layers: an attention layer (7) followed by a feedforward layer (8), given by

$$\tilde{\mathbf{h}}_i^{l-1} = \text{BN}\left(\mathbf{h}_i^{l-1} + \text{MHA}\left(\mathbf{h}_i^{l-1}\right)\right), \quad (7)$$

$$\mathbf{h}_i^{l} = \text{BN}\left(\tilde{\mathbf{h}}_i^{l-1} + \text{FF}\left(\tilde{\mathbf{h}}_i^{l-1}\right)\right). \quad (8)$$

In these equations, BN is a batch normalization layer [36], FF is a feedforward network, and MHA is a multi-head attention layer, which we describe in detail below. The feedforward layer has a single hidden dimension d_{ff} and consists of a linear layer with bias mapping $\mathbb{R}^d \to \mathbb{R}^{d_{\text{ff}}}$, followed by a ReLU activation function, a dropout layer, and finally a linear map with bias back to $\mathbb{R}^d$.

1. *Multi-head Attention Mechanism*

The function MHA appearing in (7) is a multi-head attention mechanism which is identical to that of [8]. Variations of the MHA layer are used in both the encoder and the decoder, with a modification described below for dealing with a tensor demand structure.

We start with the set of output vectors $\mathbf{h}_1, \dots, \mathbf{h}_n \in \mathbb{R}^d$ from the previous layer. For each of these vectors, we compute vectors called queries, keys, and values. In a single-head attention mechanism we compute one query, key, and value vector for each input vector $\mathbf{h}_i$,

$$\mathbf{q}_i = M^{\text{query}}\, \mathbf{h}_i \in \mathbb{R}^\alpha, \quad (9)$$

$$\mathbf{k}_i = M^{\text{key}}\, \mathbf{h}_i \in \mathbb{R}^\alpha, \quad (10)$$

$$\mathbf{v}_i = M^{\text{value}}\, \mathbf{h}_i \in \mathbb{R}^d. \quad (11)$$

Here, each M is a matrix mapping $\mathbb{R}^d$ to either $\mathbb{R}^d$ or $\mathbb{R}^\alpha$ where α is any positive integer.

For a multi-head attention mechanism, we have a positive integer n_{heads} which we typically take to be 8. Each attention head, labeled by $s \in [n_{\text{heads}}]$, now gets its own query, key, and value map,

$$\mathbf{q}_{s\,i} = M_s^{\text{query}}\, \mathbf{h}_i \in \mathbb{R}^\alpha, \quad (12)$$

$$\mathbf{k}_{s\,i} = M_s^{\text{key}}\, \mathbf{h}_i \in \mathbb{R}^\alpha, \quad (13)$$

$$\mathbf{v}_{s\,i} = M_s^{\text{value}}\, \mathbf{h}_i \in \mathbb{R}^\beta, \quad (14)$$

where β can be any positive integer. The linear maps M are learned during training.

Next, a *compatibility* is computed for every query-key pair (for each head), via the standard dot product,

$$u_{s\,ia} = \frac{1}{\sqrt{\alpha}} \mathbf{q}_{s\,i} \cdot \mathbf{k}_{s\,a}. \quad (15)$$

Then, for each head s and node i, we pass the compatibilities $\{u_{s\,ia}\}_{a \in [n]}$ through a softmax to obtain

$$\rho_{s\,ia} = \frac{\exp(u_{s\,ia})}{\sum_b \exp(u_{s\,ib})} \in \mathbb{R}. \quad (16)$$

The softmax compatibilities are then used as coefficients in a linear combination of value vectors,

$$\mathbf{g}_{s\,i} = \sum_a \rho_{s\,ia} \mathbf{v}_{s\,a}. \quad (17)$$

The next step is to merge the data from the n_{heads} attention heads, by simply concatenating the outputs from each head,

$$\mathbf{g}_i = \mathbf{g}_{1\,i} \oplus \dots \oplus \mathbf{g}_{n_{\text{heads}}\,i}. \quad (18)$$

Finally, we use a learned linear map with bias to map these vectors from dimension $n_{\text{heads}}\beta$ to dimension d to produce vectors $\mathbf{h}'_i \in \mathbb{R}^d$, one for each node $i \in [n]$.

C. Decoder

After running the encoder once at the start of an episode, the decoder is then used once at every time step (after dropping off demand from the active truck) to decide which node the active truck will drive to next. Unlike the encoder, we make significant modifications to the decoder of [8], in order to allow for multiple trucks.

The output from the encoder is a set of n vectors $\{\mathbf{h}_i^l\}_{i \in [n]}$, one for each node. To compute a probability distribution over the other nodes (which we then sample to decide the next node), we “decode” these encoded nodes along with some information about the current state of the environment.

The state of the environment, at the time we use the decoder to make a routing decision, consists of the following data:

- the current on-board demands E^m,

- the current off-board demands D^{direct}, D^{cyclic},
- the encoded nodes $\{\mathbf{h}_i\}_{i\in[n]}$, which were encoded once at the start of the episode,
- the remaining capacity of each truck,
- the active truck $m_\star$,
- the active node $z_\star$,
- the next expected nodes for each passive truck and the time until those trucks arrive.

Ideally, we would pass all of this data to the decoder. The difficulty is that there is too much demand data, so we must abbreviate the demand data to include only its most immediately relevant features, much like for the encoder.

Demand Information

Since the demand tensor indices refer to nodes, a natural way to condense the demand data and pass it to the decoder is to reduce each demand tensor to rank 1 by summing over all but one index, and then concatenate its components with the corresponding encoded node.

More precisely, we use the outgoing offboard demand δ_i^{out} defined in (3), and define a consolidated version of each truck's onboard demand as

$$\epsilon_i^m = E_i^m + \sum_j E_{ij}^m + \sum_{jk} E_{ijk}^m + \dots \tag{19}$$

In words, ϵ_i^m is the total volume of demand on truck m whose next required stop is node i.

Now, let $m_\star$ be the active truck index. We then modify the encoded nodes as follows

$$\overline{\mathbf{h}}_i = \mathbf{h}_i \oplus \left(\delta_i^{\text{out}}, \epsilon_i^{m_\star}, \epsilon_i^1, \dots, \epsilon_i^{m_\star-1}, \epsilon_i^{m_\star+1}, \dots, \epsilon_i^N\right), \tag{20}$$

where $\oplus$ is concatenation.

Additional Contextual Information

In the previous subsection, we explained how we pass the encoded nodes and demand information to the decoder: we consolidate the demand data into a vector for each node, and then concatenate these vectors with the corresponding encoded node. But the remaining pieces of environment data – the remaining capacity of each truck, the active truck index, the active node index, and the next node and arrival time for each truck – are associated with trucks. Therefore, we need a different way to pass this information to the decoder.

To do this, we follow the model of [8] and introduce an extra node called the *context node*. The context node will contain relevant information about the status of each truck. Let $m_\star$ be the active truck index and define

$$\mathbf{C} = (C_{m_\star}, C_1, C_2, \dots, C_{m_\star-1}, C_{m_\star+1}, \dots, C_N), \tag{21}$$

where C_k is the remaining capacity for truck k. $\mathbf{C}$ will form the first part of our context node.

The second part of the context node will contain information about which node each truck will arrive at next. Let $k_{m_\star}$ be the active node index, and let $(k_1, \dots, k_{m_\star-1}, k_{m_\star+1}, \dots, k_N)$ be the next node indices for all passive trucks. Ideally, we would like to append to the context node

$$\left(h_{k_{m_\star}}, h_{k_1}, h_{k_2}, \dots, h_{k_{m_\star-1}}, h_{k_{m_\star+1}}, \dots, h_{k_N}\right),$$

but this would be extremely expensive: the context vector would pick up dN dimensions just from these components. To avoid this, we take the view that the passive trucks are less important than the active truck, and we use a feedforward neural network f, consisting of a linear layer, ReLU nonlinearity, and then another linear layer, to reduce the dimension of the passive nodes (typically to 4 dimensions). We therefore define

$$\begin{aligned}\mathbf{H} = &\left(h_{z_{m_\star}}, f\left(h_{z_1}\right), f\left(h_{z_2}\right), \dots, \right) \\ &\left(f\left(h_{z_{m_\star-1}}\right), f\left(h_{z_{m_\star+1}}\right), \dots, f\left(h_{z_N}\right)\right),\end{aligned} \tag{22}$$

and $\mathbf{H}$ will form the second part of our context node.

The third and final piece of the context node will contain information about how much time until each passive truck arrives at its next scheduled destination. Let t_m denote the time until truck m arrives at its next stop. Note that we can have $t_m = 0$ if truck m has the same arrival time as the active truck. Now we define

$$\mathbf{T} = (t_1, \dots t_{m_\star-1}, t_{m\star+1}, \dots, t_N). \tag{23}$$

Note that the last $N-1$ components of $\mathbf{H}$ (equation (22)) correspond to the same trucks with the same order as the components of $\mathbf{T}$. This consistency is very important. With different events, the same components may correspond to different trucks, but for any given event, the components of $\mathbf{C}, \mathbf{H}$, and $\mathbf{T}$ line up in the same way.

We finally define the *context node*:

$$\mathbf{h}_{\text{ctx}} = \mathbf{H} \oplus \mathbf{T} \oplus \mathbf{C}. \tag{24}$$

Decoder Structure

The first layer of the decoder has exactly the same structure as the encoder layer of (7) and (8), and takes as input the modified nodes of equation (20). The only differences between this layer and the encoder layer are that the nodes have a greater dimension due to the modifications in equation (20), and one additional modification explained below. We denote the output as $\{\overline{\mathbf{h}}_i^1\}_{i\in[n]}$.

The additional modification is that, following [8], we adjust the computation of keys and values in equations (13) and (14) by adding *source terms*:

$$k_{s\,i} = M_s^{\text{key}}\,\mathbf{h}_i + \mathbf{u}_s^{\text{key,out}}\,\delta_i^{\text{out}} + \mathbf{u}_s^{\text{key,in}}\,\delta_i^{\text{in}}, \tag{25}$$

$$v_{s\,i} = M_s^{\text{value}}\,\mathbf{h}_i + \mathbf{u}_s^{\text{val,out}}\,\delta_i^{\text{out}} + \mathbf{u}_s^{\text{val,in}}\,\delta_i^{\text{in}}. \tag{26}$$

Here, δ_i^{out} and δ_i^{in} are the outgoing and incoming demands at node i defined in (3) and (4), respectively, evaluated at the time of decoding. s indexes the attention heads, i indexes the nodes, and the various $\mathbf{u}$'s are learned vectors with the same dimension as the object on the left hand side of the equations they appear in; for example, $\mathbf{u}_s^{\text{key,out}}$ is a vector with the same dimension as the keys which is α as specified in equation (13). This modification of keys and values helps to convey the current state of the demand directly to the decoder.

After the initial decoder layer, we do a second attention layer with the same number of heads n_{heads}, but acting on $n+1$ nodes: the n nodes $\{\overline{\mathbf{h}}_i^1\}_{i\in[n]}$ from the first decoder layer, and the one context node (24). The purpose of this layer is only to produce a new transformed context vector. For this attention layer, following [8], the only node we construct query vectors for is the context node, and we do not construct key and value vectors for the context node. In other words, for each head we compute one query (for the context node), n keys, and n values (for all other nodes). In equations (15)-(18), the index i only takes on a single value, the value which labels the context node. We use sources $\delta_i^{\text{out}}, \delta_i^{\text{in}}$, just as in equations (25) and (26). Moreover, for this layer we only compute MHA as described at the end of section IV B. This is a pure attention layer (as opposed to and encoder layer which uses equations (7) and (8)).

The output of the second decoder layer is one new context vector $\mathbf{h}'_{\text{ctx}}$ with dimension d_{ctx}. This new context vector is then used for a third and final layer which only uses one attention head. Once again, a query vector q_{ctx} is computed only for $\mathbf{h}'_{\text{ctx}}$, and keys are computed for the encoded nodes $\overline{\mathbf{h}}_i^1$. There is no need to compute value vectors in the final layer, since we do not need to generate a new set of vectors to go into another layer (we want this layer to return a probability distribution over nodes). We will use the compatibilities directly to generate a distribution over nodes.

For this last layer, we compute compatibilities in the usual way except that we regulate with tanh and we allow for *masking*:

$$u_i = \begin{cases} A\tanh(q_{\text{ctx}}\cdot k_i) & \text{if node } i \text{ is allowed} \\ -\infty & \text{otherwise} \end{cases}, \tag{27}$$

where A is a hyperparameter that we take to be 10. The idea is that we can block certain nodes for the active truck to drive to if we know, for some reason, that doing so is a poor choice.

Finally, the u_i are converted to probabilities with a softmax layer, and these probabilities are interpreted as the values of the policy: the probability of selecting node i for the active truck's next destination:

$$\pi(i) = \frac{e^{u_i}}{\sum_{j=1}^{n} e^{u_j}}. \tag{28}$$

D. Incorporating Tensor Demand Structure

Thus far, all demand information passed to the attention mechanism has been consolidated into rank 1 objects. None of the higher rank demand data is accessible to our RL agent at this point. However, for any realistic problem instance, attempting to convey the entirety of the demand data in its exact form would consume far too much memory. Therefore, we have implemented a method for passing rank 2 demand data to the attention mechanism. This still falls short of giving the RL agent complete information about the state of the environment, but is an improvement over only using rank 1 data. The method we use is called *dynamical masking.*

1. Dynamical Masking

A natural way to pass a rank 2 consolidated demand tensor D_{ij} to the attention mechanism is to identify an object in the attention mechanism pipeline that makes use of two different node indices i and j, and then multiply this object by some function of D_{ij}. The part of the attention mechanism that involves two nodes is the dot product evaluation between keys and queries. To incorporate a demand tensor D_{ij}, we can replace the dot product with

$$\frac{1}{\sqrt{\alpha}} G_{ij}\,\mathbf{q}_i\cdot\mathbf{k}_j, \tag{29}$$

where G is some function of D. This approach can exaggerate query-key compatibility in cases where D_{ij} is large and suppress compatibility when the demand is small.

There are a few reasonable choices for G. The first is $G_{ij} = 1$ which reduces to a basic dot product compatibility. Next is $G_{ij} = M_{ij}$ where M is the mask defined as $M_{ij} = 1$ when $D_{ij} > 0$ and $M_{ij} = -\infty$ otherwise. Both of these are within the methodology of [8]. A third and more novel choice of G is $G_{ij} = \log D_{ij}$. This last form has several virtues: it reduces to a mask in the case in the sense that it approaches $-\infty$ as $Dij \to 0+$. Moreover, it can exaggerate compatibility when D_{ij} is large. A simple additional adjustment is to use

$$G_{ij} = AD_{ij} + B\log D_{ij},$$

which is more sensitive to changes in D_{ij} for larger values.

Rather than having to pick from these various choices, we can in fact choose all of them by taking advantage of the multiple heads. In other words, for a given head $s \in \{1, \ldots, n_{\text{heads}}\}$, we can put

$$G^s_{ij} = A^s_{\text{basic}} + A^s_{\text{mask}} M_{ij} + A^s_{\log} \log D_{ij} + A^s_{\text{lin}} D_{ij}. \quad (30)$$

In principle, even more terms can be used, and a more thorough investigation of various models would be sensible.

V. TRAINING METHODOLOGY

The method we use to train our RL agent includes two parts:

1. an RL algorithm which adjusts the model weights to optimize the value of a cost function
2. a method for generating synthetic training data

In this section, we describe both parts of the training process.

REINFORCE Implementation

Following [8], we use a variant of REINFORCE [35] to train our agent. REINFORCE is a policy-gradient RL algorithm. While many RL algorithms first try to estimate the "value" of various actions in a given state and then learn to take actions with higher estimated value, policy-gradient algorithms circumvent the intermediate step of estimating values. Instead, we work directly with a parameterized policy, varying parameters to optimize the return from an episode.

The REINFORCE algorithm, following [37], is given as follows

Algorithm1 REINFORCE

```
Inputs:
  Parameterized policy $\pi$
  Initial parameter $\theta$
while desired performance not achieved do
  Using $\pi(\theta)$, generate episode
  $(s_0, a_0, r_1, \ldots, r_T) \leftarrow$ episode
  for $t = 0, 1, 2, \ldots, T-1$ do
    $G \leftarrow r_{t+1} + \gamma r_{t+2} + \ldots \gamma^{T-t-1} r_T$
    $\nabla J \leftarrow \gamma^t G \, \nabla_\theta \log(\pi(a_t \mid s_t, \theta))$
    $\theta \leftarrow \text{Ascent}(\theta, \nabla J)$
  end for
end while
```

In this algorithm, $\gamma \in (0, 1)$ is a fixed discount factor. "Ascent" refers to any gradient-based ascent optimization step. We use Adam optimization [38], but any gradient ascent on $J(\theta)$ could be used here.

Our REINFORCE variant defines actions differently from most typical implementations. Since it is very difficult to evaluate how good or bad any single routing decision is, we take the entire episode to be defined by a single action. In other words, the episode is simply a_0, r_1. The action a_0 is the entire route $a_0 = (\xi_1, \xi_2, \ldots \xi_k)$ where each ξ_i is a node. Since REINFORCE updates policy weights using the log probabilities of actions, and we define an action as the complete sequence of routing decisions in an episode, we simply use the sum of the log probabilities of each individual routing decision for the policy update step. The reward r_1 for the action a_0 is simply the demand coverage $\eta(\xi)$ at the end of the episode.

REINFORCE can learn much faster when we use a *baseline*. A baseline is some function b of states (but not of actions) which is constructed with each episode. The return G in the algorithm is then replaced by $G - b(s)$. This algorithm still converges to the optimal policy theoretically and, with a well-chosen baseline, does so much faster. Typically $b(s)$ is taken to be an estimate of the return after state s based on data from recent previous episodes. In this case, $G - b(s) > 0$ for an episode indicates that the policy performed better than expected in that episode, and thus we should increase the probability of taking that sequence of actions. On the other hand, if $G - b(s) < 0$, the policy performed worse than expected, and we should adjust the model weights to decrease the probability of taking that sequence of actions. REINFORCE works with or without a baseline, but the learning time can be dramatically reduced with a good baseline.

In light of the discussion in the previous paragraph, another important aspect of our REINFORCE variant is how we define our baseline. This method is essentially adapted directly from [8]. We maintain a "baseline agent" which uses the same model as the primary agent, and executes an episode for each synthetic training environment. Our baseline $b(s)$ is simply the value of the return when an episode beginning in state s is executed by the baseline agent. The baseline agent uses a parameter θ_{BL} which is occasionally updated to match the primary agent's θ, but only when the agent substantially and consistently outperforms the baseline agent.

Algorithm2 REINFORCE variant for GVRP

```
Input: Parameterized policy $\pi$
Input: Integers num_epochs, batch_size, batches_per_epoch
Input: Initial parameter $\theta$
$\theta_{\text{BL}} \leftarrow \theta$
for $e = 1, \ldots,$ num_epochs do
  for $b = 1, \ldots,$ batches_per_epoch do
    $\xi \leftarrow$ (batch_size many episodes from $\pi(\theta)$)
    $\xi_{\text{BL}} \leftarrow$ (batch_size many episodes from $\pi(\theta_{\text{BL}})$)
    $\nabla J \leftarrow$ batch_mean
        $\left[(F(\xi) - F(\xi_{\text{BL}}))\nabla_\theta \left(\sum_{i=1}^k \log \pi(\xi^i, \theta)\right)\right]$
    $\theta \leftarrow \text{descent}(\theta, \nabla J(\theta))$
  end for
  if baseline_test() then
    $\theta_{\text{BL}} \leftarrow \theta$
  end if
end for
```

Our REINFORCE variant is given in algorithm 2.

Note that this algorithm is broken up into epochs and batches. To reiterate the definition of actions discussed above, notice the summation $\sum_{i=1}^{k} \log \pi(\xi^i, \theta)$ appearing in algorithm 2. To clarify, this is a sum over the log probabilities computed by the encoder/decoder network at each stage of the route. As explained above, we use this form because it is equal to the log of the product of the probabilities, and the product of probabilities gives the probability of the whole route. k refers to the number of steps in the route and the index i runs over steps in the route, not over batch entries. The entire computation is performed for each batch entry and averaged over the batch.

The `baseline_test()` subroutine returns `true` when the policy $\pi(\theta)$ substantially outperforms the baseline policy $\pi(\theta_{\text{BL}})$. More specifically, after each epoch we compute the percentage of episodes in which the policy outperforms the baseline policy. If this percentage exceeds 50% for 10 *consecutive epochs* then we update the baseline parameters. Moreover, if the percentage exceeds 70% for any epoch, we update the parameters. There is certainly room for experimentation with different methods here (like the one-sided T Test used in [8] but our methods were satisfactory).

Cost Function

Reinforcement learning requires a reward definition. Since the entire episode can be regarded as a single action, we only need to define a reward $R(\xi)$ for a full route ξ. Equivalently, we can define a cost function $F(\xi) = -R(\xi)$, which we minimize in training.

Typical VRPs can use total driving time or distance as a cost function to minimize. However, our routing problem has a time constraint T_{max} and it is not guaranteed that all demand will be fulfilled. To address this, we define *demand coverage* as the percentage $\eta(\xi)$ of initial demand volume that is eventually fulfilled by route ξ. We then define the cost function as simply

$$F(\xi) = -\eta(\xi), \tag{31}$$

which is then minimized in training. An example training curve is shown in Fig. 1.

Environment Generation

In order to train a model that can solve a wide range of problem instances, we randomly generate batches of synthetic supply chain environments and use these as training data. A supply chain environment consists of a graph representing the set of locations, and a set of initial demand tensors. We must randomly generate both of these for each synthetic training environment.

We randomly generate graphs using a single parameter, a timescale τ, that represents the scale of drive times we want in our environment. To generate an n-node graph, we simply choose n random points in the unit square, and then multiply the coordinates of all n points by τ. The drive times are then computed as the pairwise distances between points.

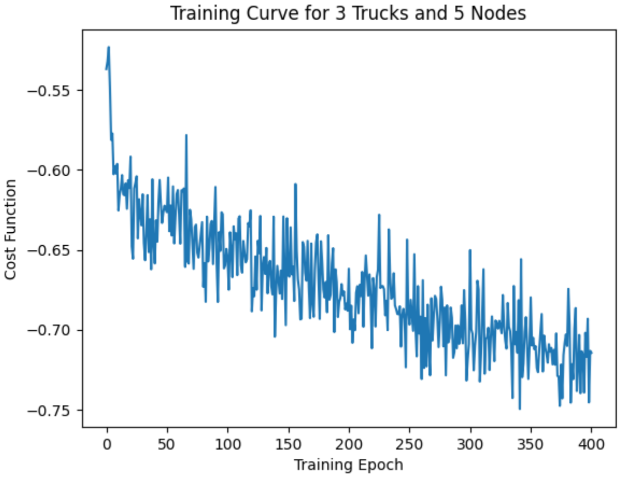

FIG. 1. An example training curve showing the cost function, Eq. (31), over 400 training epochs on subenvironments containing 3 trucks and 5 nodes.

Randomly generating initial demand tensors is a bit more involved. An arbitrary tensor of the correct rank and dimension will, in general, not be acceptable. For example, a component like D^{cyclic}_{232} must be excluded, since this would imply a required box route (z_2, z_3, z_2, z_2). For a rank r demand tensor, we start by randomly choosing a subset of nodes which are allowed to be the first node of a box route, a subset which are allowed to be the second, and so on up to a subset of nodes which are allowed to be the final (rth) node of a box route. We then generate a mask tensor whose components are 1 only for those box routes which are allowed according to these node subsets, and contain no repeated nodes. We then further mask components randomly with some given probability. This is meant to create instances which are more representative of real problem instances, where the demand structure does not have all-to-all connectivity. We then randomly assign the unmasked components a value in $(0, 1]$, and multiply the entire tensor by a positive number which represents the scale of demand we want in our synthetic environment. We repeat this process once for each demand rank r we want our synthetic environment to contain. Our synthetic training environments contain demand up to rank 3.

VI. RESULTS

We apply the algorithm described above to solve a special case of the GVRP, which we call the Aisin VRP. The Aisin VRP arises in the daily supply chain operations of Aisin Corporation in Japan's Aichi Prefecture. In this section we describe the Aisin VRP, and then evaluate the performance of our algorithm in solving this problem.

A. Aisin VRP

In the Aisin VRP, the graph G consists of 21 nodes, representing 21 different supply chain locations. The drive-time matrix T has entries that range from 2 minutes up to about 2 hours. There are approximately 340,000 individual boxes to deliver, with a total volume of approximately 11,500m^3. Each box has one of 107 unique required routes. The trucks have volume capacity $C = 30\text{m}^3$, and the daily time limit is 16 hours. To solve this problem, Aisin Corporation relies on a team of logistics experts using intuition and experience to plan the truck routes and pickups/dropoffs.

Our goal is to produce solutions to this problem which are on par with or better than Aisin Corporation's current best solution. Aisin's previous best solution uses 142 trucks to complete this supply chain task in 16 hours. We use this number as our performance benchmark.

There is an important caveat to mention here. There are several constraints that appear in the real-world version of this problem, which we have not imposed in our solution. First, our algorithm does not include a minimum time for which a box must stay at an intermediate node along its required route. This means that boxes in our solution can be picked up for the next leg of their journey immediately after they are dropped off, which is somewhat unrealistic. Second, the trucks have a weight capacity in addition to a volume capacity, but our algorithm does not impose a weight constraint. Finally, realistic truck routes must start and end at the same node so that the truck driver can park their car at the start node at the beginning of the shift, and get back in their car to drive home at the end of the shift. We have not imposed a cyclic truck route constraint. Even without these constraints, the solutions we find still represent a big step towards a commercially viable RL approach for realistic VRPs.

B. Performance

The results presented in this section were obtained using the algorithm described above, with teams of three trucks operating in 5-node subenvironments. The initial encoding dimension is 64, and both the encoder and decoder MHA layers have 8 attention heads. We trained our model for 400 epochs using Adam optimization with an exponentially decaying learning rate. The learning rate started at 0.05 and decreased by a factor of 0.9 after each epoch until reaching its minimum value of 2^{-14}. The subenvironment search routine described in Sec. III B tested 20 different subenvironments in each iteration, and ran 20 test episodes on each test subenvironment. After choosing a subenvironment, we ran a batch of 500 episodes on the chosen subenvironment.

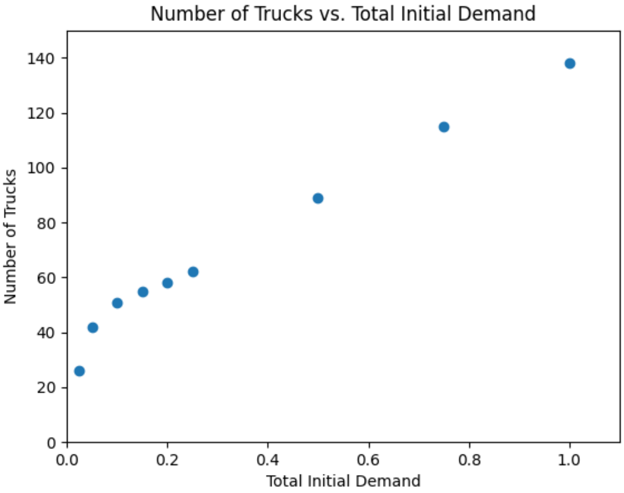

FIG. 2. Number of trucks required as a function of total initial demand. Total initial demand is given as a fraction of the total demand of the Aisin VRP. Note for the full-scale problem, our algorithm finds a solution using 138 trucks, thus outperforming Aisin's 142-truck solution. However, as seen on the left side of the plot, the algorithm is less effective with a small amount of demand spread over many nodes.

To get a sense for how our algorithm performs as a function of the total amount of demand in the supply chain, we consider nine problem instances each with a different demand scale. Each problem instance was extracted from the full-scale Aisin VRP by considering only a subset of the total demand of that problem. As fractions of the total demand of the Aisin VRP, these problem instances contain 2.5%, 5%, 10%, 15%, 20%, 25%, 50%, 75%, and 100% of the total demand volume. What is clearly visible in Fig. 2 is that this algorithm delivers significantly less demand volume per truck when faced with a smaller amount of demand spread across the same number of nodes. This is not surprising, since a small amount of demand spread across many nodes means that trucks will be less full on average, and therefore must do more driving to deliver the same amount of demand. Let us focus now on the full-scale Aisin VRP, represented in Fig. 2 by the rightmost data point. Our algorithm was able to solve this problem using only 138 trucks, thus outperforming the 142-truck solution currently used by Aisin. The graph edges used in this solution are shown in Fig. 3.

Fig. 4 shows the volume of demand delivered by each truck team in the 138-truck solution. Here we can see the same effect that is visible in Fig. 2. The truck teams that are deployed in later iterations deliver less volume per truck. This is because the later truck teams operate in an environment with small amounts of demand spread over all 21 nodes, so it is much harder for them to be efficient.

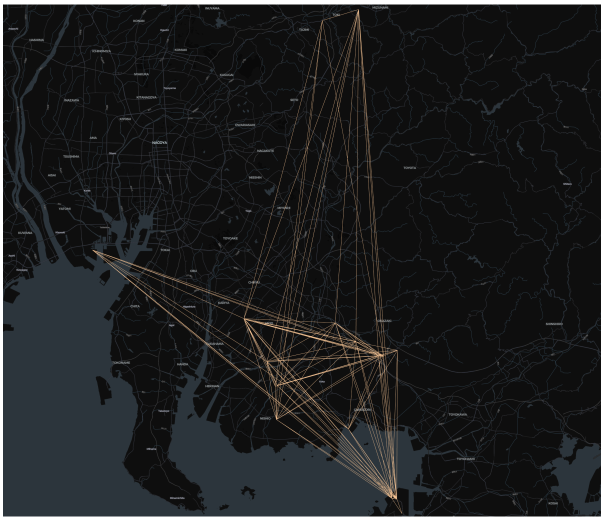

FIG. 3. Truck-routing connectivity graph for the 138-truck solution obtained using the algorithm described in this paper. The underlying map shows an area of approximately 85km by 85km of Nagoya, Japan. Note that this figure only shows which edges are used in the solution, and does not show the demand flow along each edge, which direction trucks are driving, or timing details.

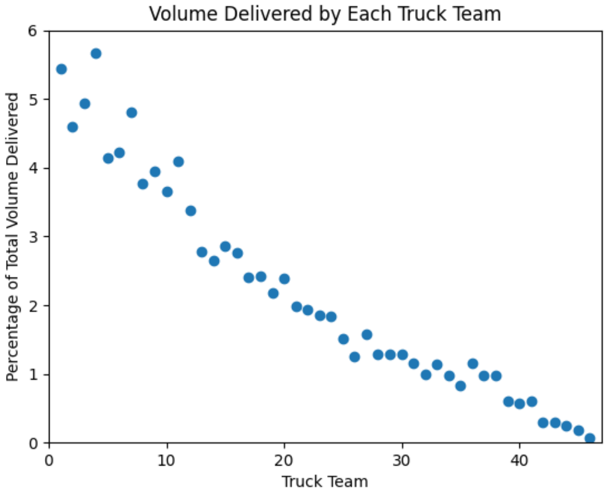

FIG. 4. The percentage of the total initial demand volume delivered by each 3-truck team. The first teams deployed are the most efficient due to the abundance of demand available to them.

VII. CONCLUSION

In this work, we have developed an algorithm that uses deep reinforcement learning with an attention-based model to solve realistic VRPs with multiple trucks and multi-leg routing requirements. These problems are made even more complex by the presence of a box-return constraint making all box routes cyclic. An important feature of our algorithm is that it can produce solutions to problems of arbitrary size without having to scale up the size of the model. It does this by using an iterative procedure in which each iteration produces a solution to one small sub-problem, so larger problems can be solved by simply doing more iterations of the same algorithm using the same model. We tested our algorithm on a real supply chain environment of Aisin Corporation, and found that our algorithm outperformed Aisin's solution for a slightly simplified version of this problem.

As mentioned in the previous section, it is important to point out that the solutions we obtain using the algorithm described in this paper cannot be immediately deployed. Our algorithm, despite incorporating many realistic aspects of Aisin supply chain environments, still does not include every constraint that Aisin must contend with. It is likely that our algorithm will need to further improve in order to outperform Aisin's solutions after adding all necessary constraints. Future work will involve incorporating more real-world constraints into our approach, and adjusting our algorithm to handle the added difficulty.

While the results presented here are promising, there is still quite a bit of room for improvement. The most noticeable weakness of our algorithm is that we deploy truck teams into the supply chain environment in series, updating the total demand each time by subtracting any demand that was delivered by the previous truck team. The result of this, which is apparent from Figs. 2 and 4, is that the trucks deployed later in the workflow are very inefficient, since they only have small bits of demand available to pick up but still must drive the same distances to make deliveries. This problem could be mitigated by using an approach that deploys all trucks simultaneously. This way, all trucks would have access to the full scale of the problem demand at the moment they are deployed. Such an approach should be a goal of future work on this problem.

Another part of our algorithm that can be improved is the pickup heuristic used in Step 3 of the route-finding routine described in Sec. III C. During training, we run episodes that use the updated model to make the routing decisions, and this heuristic to make the pickup decisions. This means that the learning process of our model is highly dependent on the details of the pickup heuristic. A suboptimal pickup heuristic will lead to suboptimal route-finding. An advanced approach would use a trained neural net to make all decisions, both routing and pickup, since ML tends to work best when hand engineering is kept to a minimum. However, this may be very difficult due to the very large number of boxes in the problems we consider. Another option is to experiment with different pickup heuristics to find one that is more compatible with the training of the route-finding model.

Finally, as described in Sec. III, our algorithm uses two different pickup heuristics. We use a continuous volume pickup heuristic during the route-finding phase. We then forget these pickup decisions before going into the pickup-finding phase where we decide the final pickups with a discrete heuristic. The reasons for this approach are described at the beginning of Sec. III. It is likely that the discrepancy between the final pickup decisions the pickup decisions made during route-finding results in some loss of efficiency. This issue can be addressed in future work with an approach that makes the routing decisions and final pickup decisions at the same time, so that we do not have to go back and make new pickup decisions once the full routes are known.

As always in ML, there is still plenty of exploration that can be done on what hyperparameters give the best performance. These include subenvironment parameters such as the number of nodes and trucks in each subenvironment, model parameters like the encoding dimension and the number of attention heads in each layer, training parameters like the start and minimum learning rate, and many others. The hyperparameters chosen for the results presented here were determined via a combination of Ray Tune [39] (an open-source hyperparameter tuning software) and trial-and-error. However, due to the vastness of the space of hyperparameter combinations, there surely exist configurations which lead to better results. Future work may include further hyperparameter tuning on the model presented in this paper.

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