EE386 Digital Signal Processing Lab **Jul-Dec 2021**

Experiment : 8

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This lab experiment uses all the skills that we've acquired through the previous labs Digital Signal Processing. The code to my entire work in this lab experiment is here. The GitHub repo link to my entire work regarding this experiment can be found here.

Question 1 - First-Order Model

The propagation mechanism of an epidemic, such as the one caused by the SARS-CoV-2 virus, can be modelled, at least in its initial phase, as a process in which each infected individual will eventually transmit the disease to an average of R0 healthy people; these newly infected patients will, in turn, infect R0 healthy individuals each, and so on, creating a pernicious positive feedback in the system. The constant R0 is called the basic reproduction number for a virus. In signal processing terms, the infection mechanism is equivalent to a first-order recursive filter.
Assumptions :

- Each infected person spreads the virus over a single day and then recovers
- An initial patient zero appears at day n = 0

Based on all this info, the number of newly infected people per day is described by the difference equation :

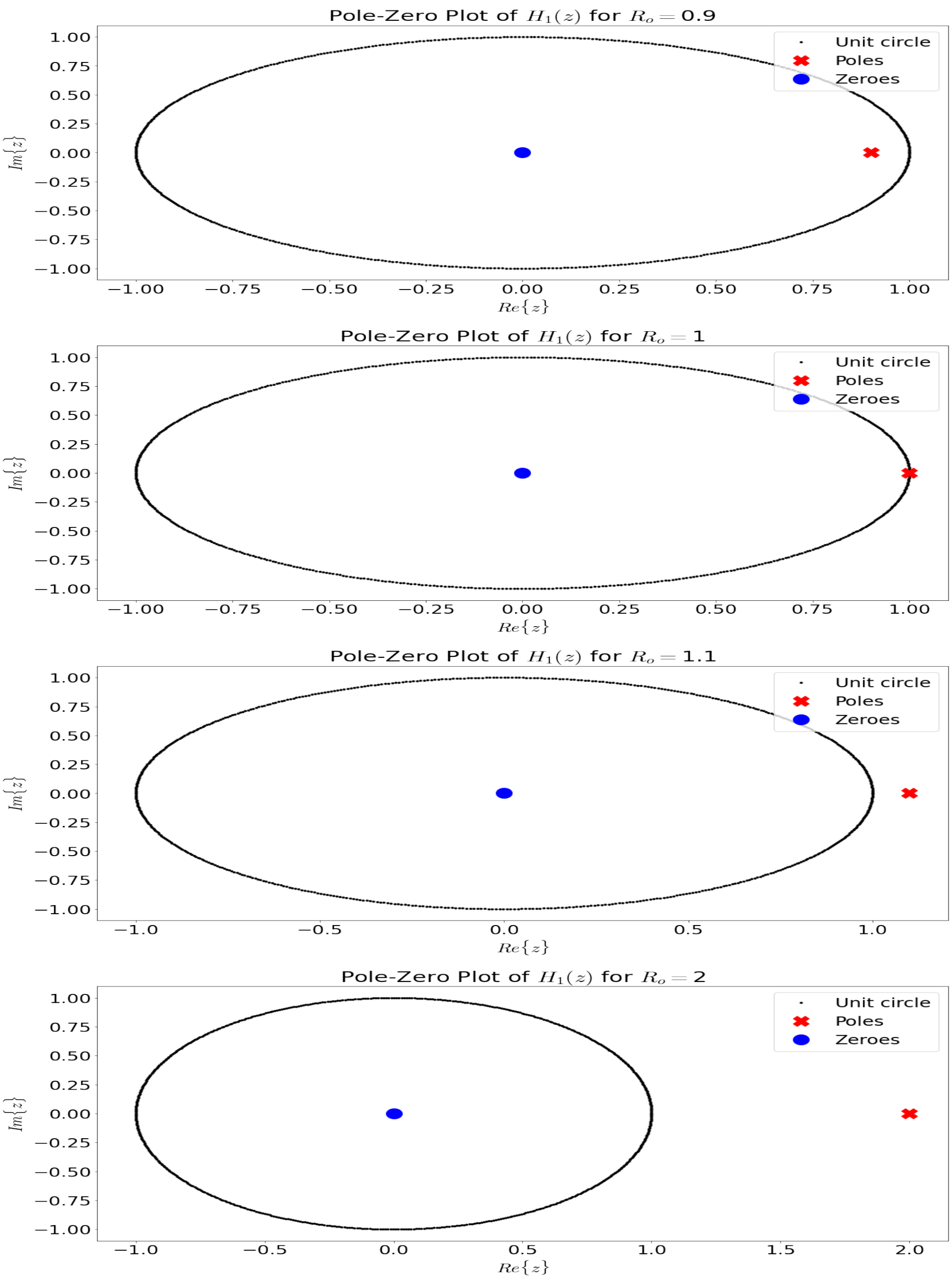
$$y[n] = \delta[n] + R_o y[n-1] \tag{0.1}$$

The transfer function of the system mentioned above is :

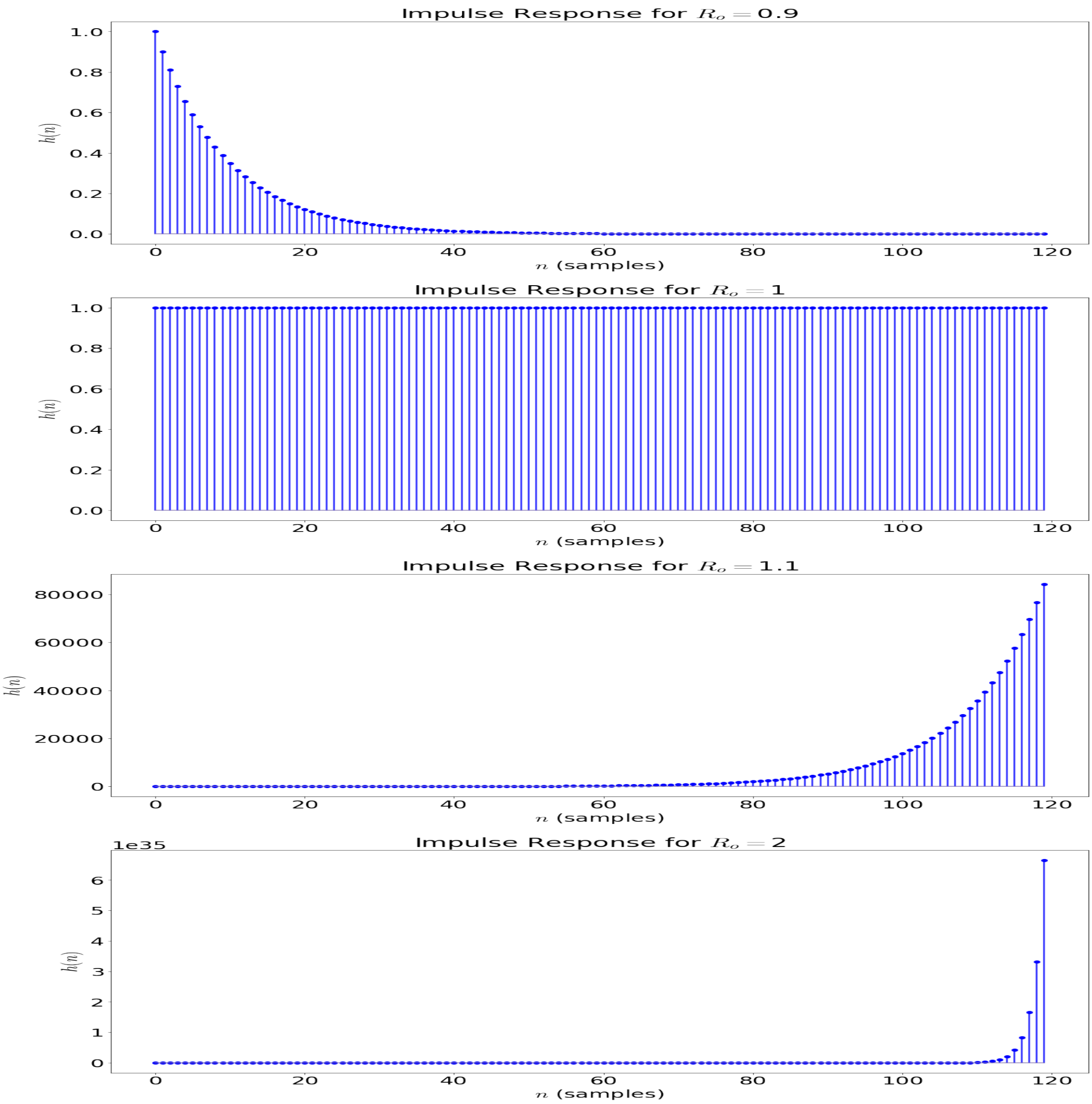
$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z - R_o} \tag{0.2}$$

R_O	H(Z)
0.9	$\frac{z}{z-0.9}$
1	$\frac{z}{z-1}$
1.1	$\frac{z}{z-1.1}$
2	$\frac{z}{z-2}$

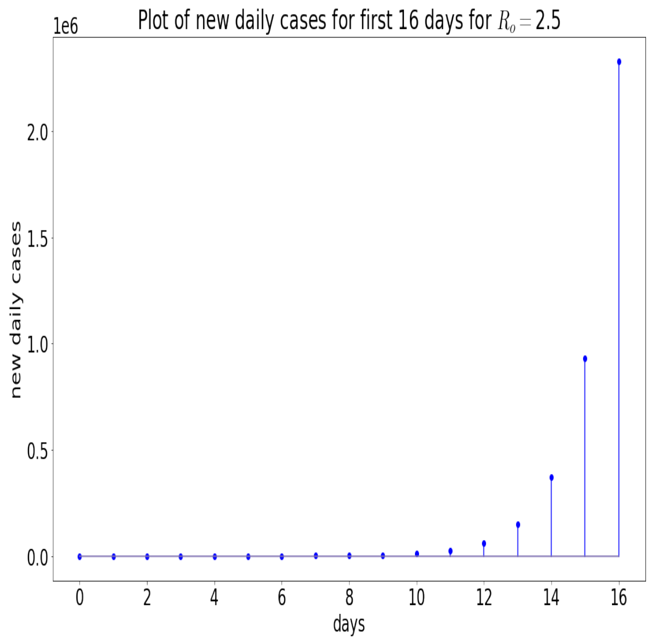
The pole-zero plots are as follows :



The impulse responses are as follows :



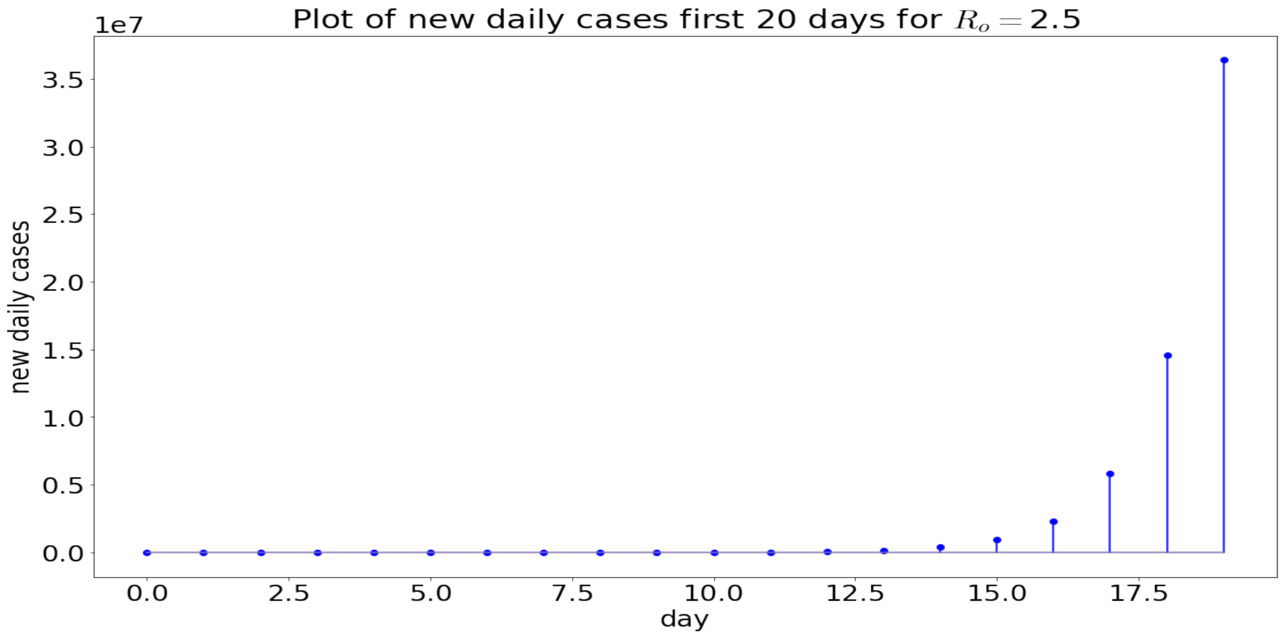
For $R_o = 2.5$, it takes around 16 days to reach 1 Million new daily infections, as we can see from the below curve.

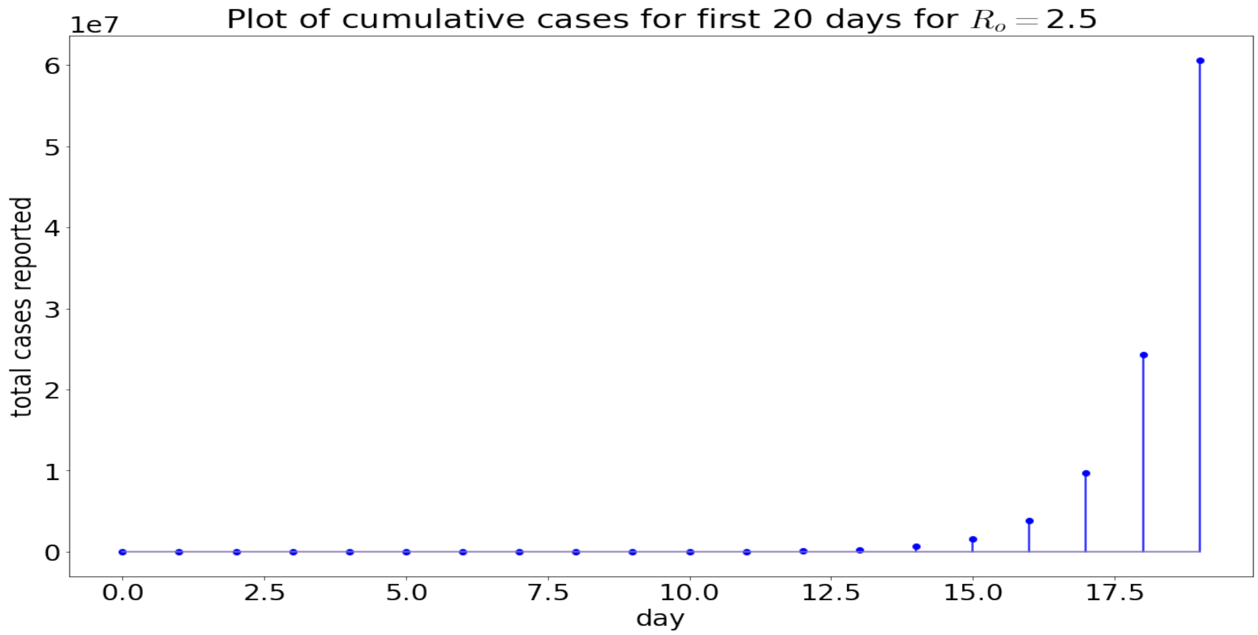


With $R_o = 2.5$, plotting the new daily infections for the first n = 20 days and designing an integrator filter to obtain the total number of infections for the first n = 20 days, gives us the transfer function as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-1} \tag{0.3}$$

Total Number of Cases can be plotted :





Question 2 - Increasing the Complexity

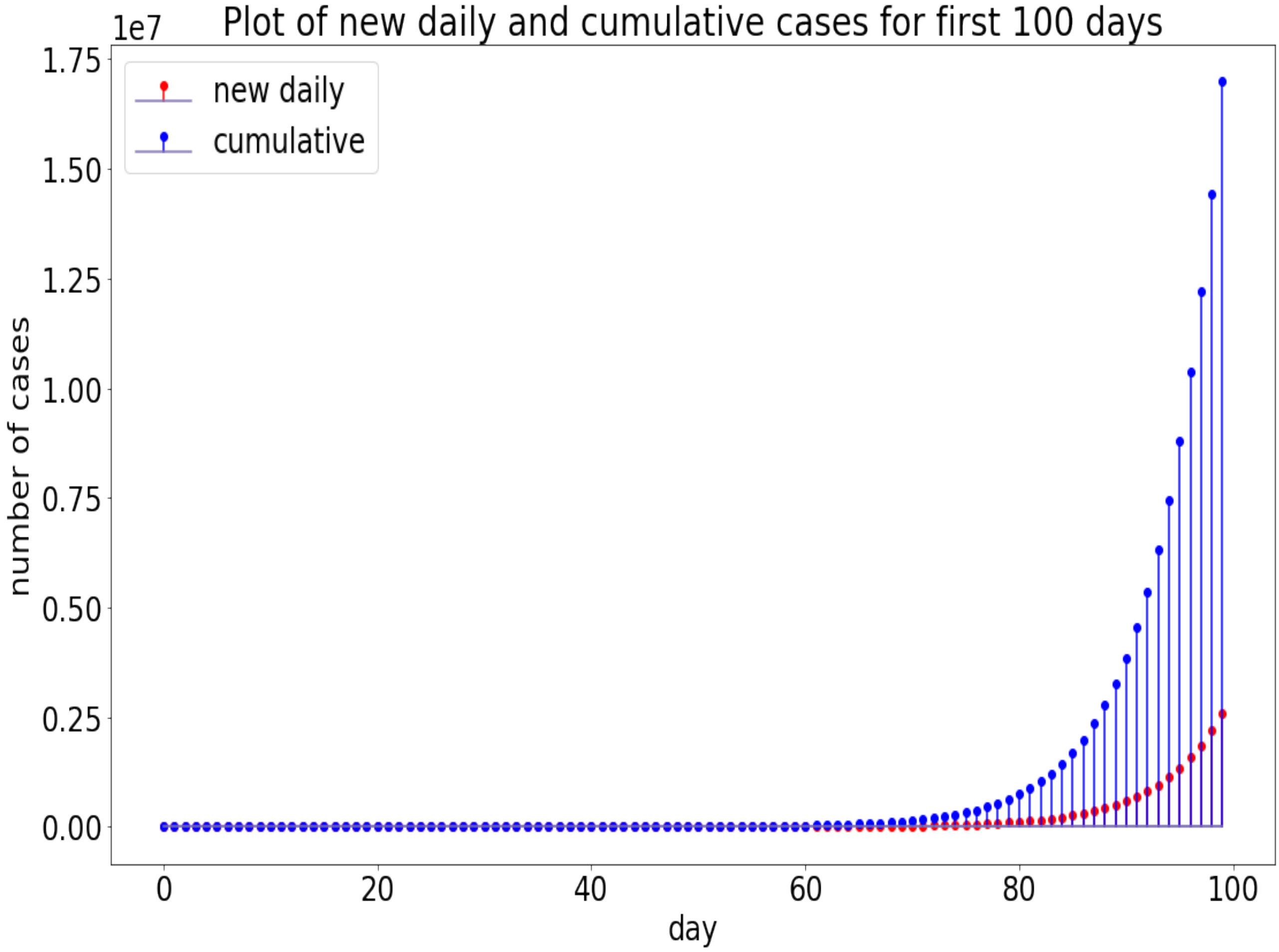
Given transfer function is :

$$H(z) = \frac{1}{1 - \sum_{k=1}^{M} a_k z^{-k}} \tag{0.4}$$

and with the given data, we can model the system as :

$$y[n] = \delta[n] + \sum_{k=1}^{M} a_k y(n-k) \tag{0.5}$$

where a_k is the number of people, a person who was affected k days ago, is going to infect. Plotting this infection's cases for first 100 days, we can realise :



We can see that the first system ($R_o = 2.5$) took 16 days to reach 1 million new daily infections whereas this system takes around 94 days for the same.

Question 3 - Effects of Social Distancing

The earlier systems did not have this counter-active measure of social distancing but this new system does and the given transfer function is

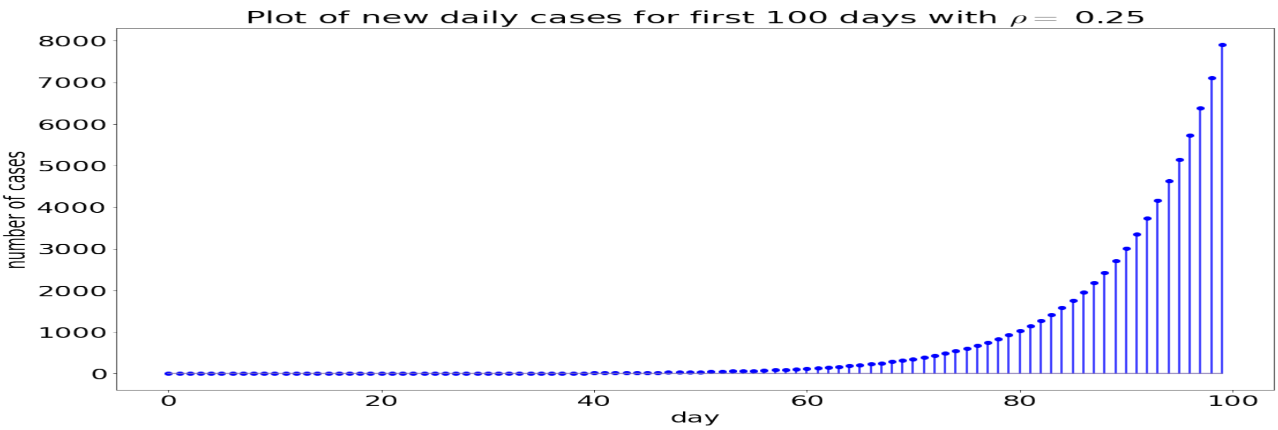
$$H(z) = \frac{1}{1 - \sum_{k=1}^{M}(1-\rho)a_k z^{-k}} \tag{0.6}$$

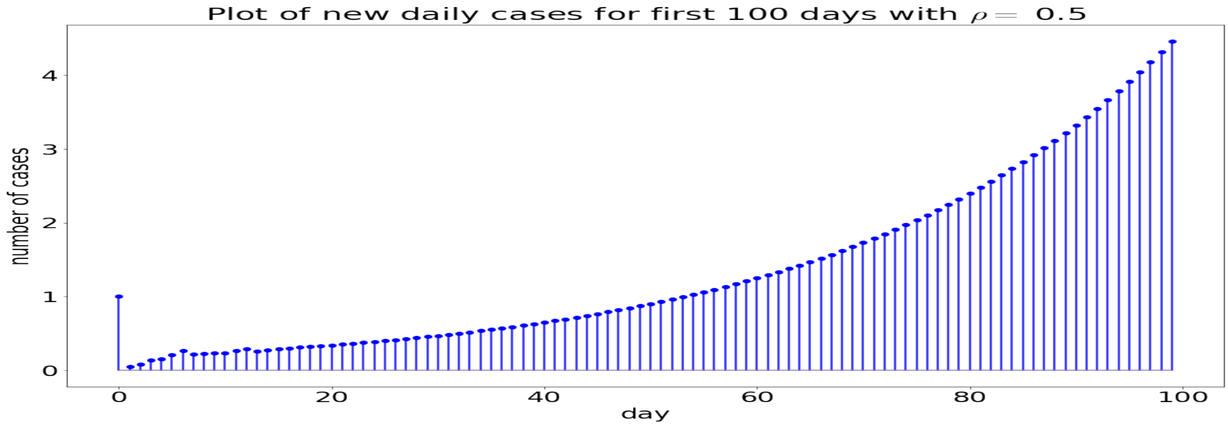
So we can model the system as :

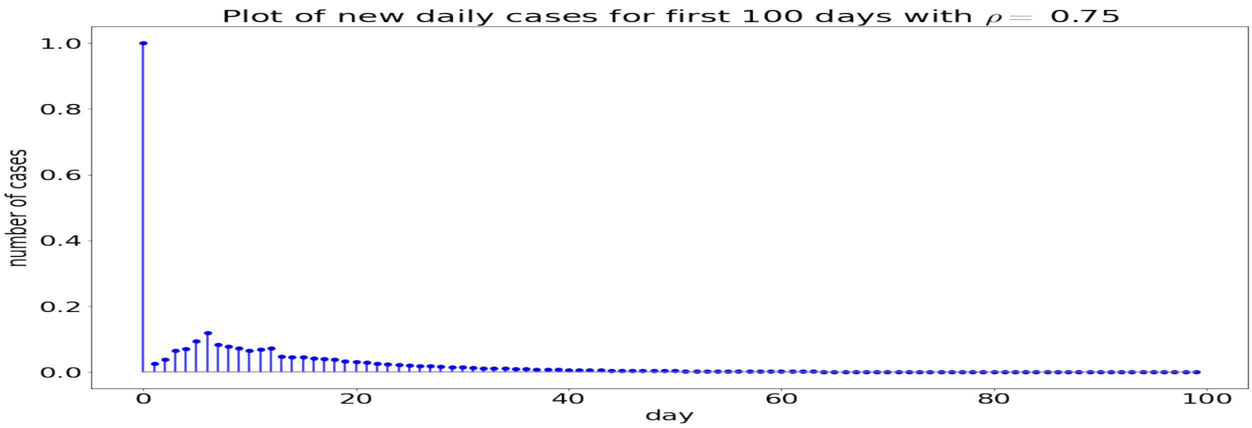
$$y[n] = \delta[n] + \sum_{k=1}^{M}(1-\rho)a_k y(n-k) \tag{0.7}$$

Where ρ is the rate at which spreading reduces.

The plots for the daily new cases for 100 days for $\rho = \{0.25, 0.5, 0.75\}$ are as follows :







The total number of cases for $\rho = \{0.25, 0.5, 0.75\}$ after 100 days is {77744.79,133.89,2.53} respectively.

Question 4 - Saturation and Towards Normality

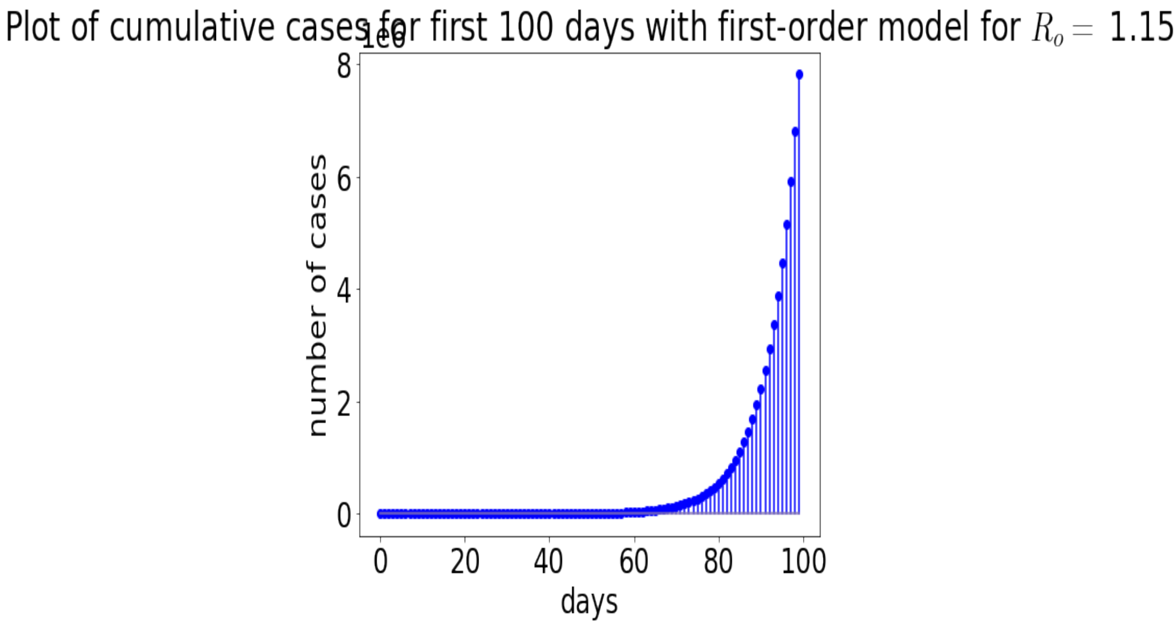
No natural system can support a purely exponential growth. In the case of a viral epidemic, as more and more people contract the disease and achieve immunity, the rate of transmission for the infection progressively decreases. If the rate of diffusion is assumed to be inversely proportional to the fraction of healthy people in a population, the evolution of the cumulative number of infections :

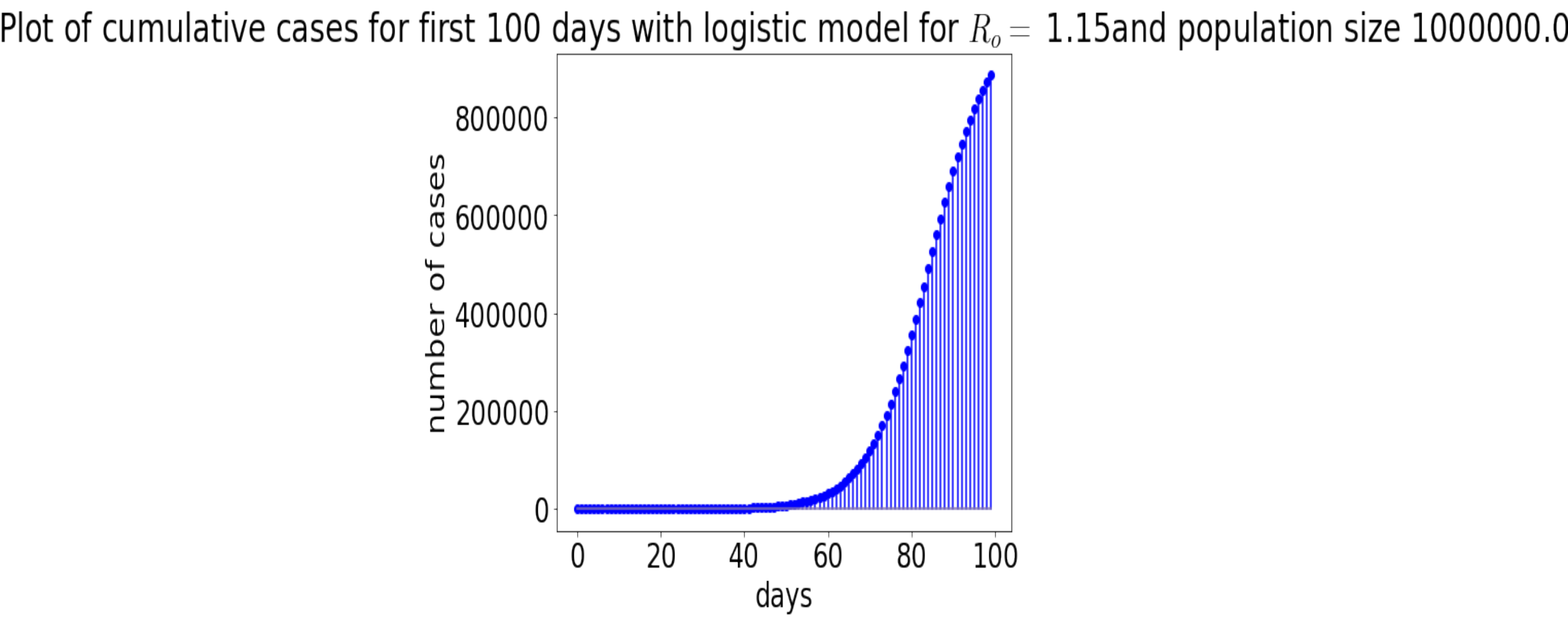
$$y_t(n) = \frac{K}{1 + (K(R_o - 1) - R_o)R_o^{-(n+1)}} - \frac{1}{R_o - 1} \tag{0.8}$$

where K is the population size. The notable thing about the logistic function is that it has a clear inflection point, after which the epidemic starts to level out; this corresponds to the moment in which the implicit reproduction number becomes less than one. It would be useful to detect the inflexion point because in that case some of the more restrictive measures could start to be relaxed gradually.

The inflection point corresponds to the global maximum of the first derivative of the logistic function. We can approximate the derivative with a simple two-tap FIR filter of the form :
$D_1(\mathrm{Z}) = 1 - z^{-}1$ and we can also look directly at the zero-crossing of the second derivative, approximated with the FIR filter :
$D_2(\mathrm{Z}) = 1 - 2z^{-}1 + z^2$

Realising the cases for the first 100 days of both the models would look as follows :





1 Appendix

- The link to all the code is here
- The Github repo to all code and previous experiments can be found here