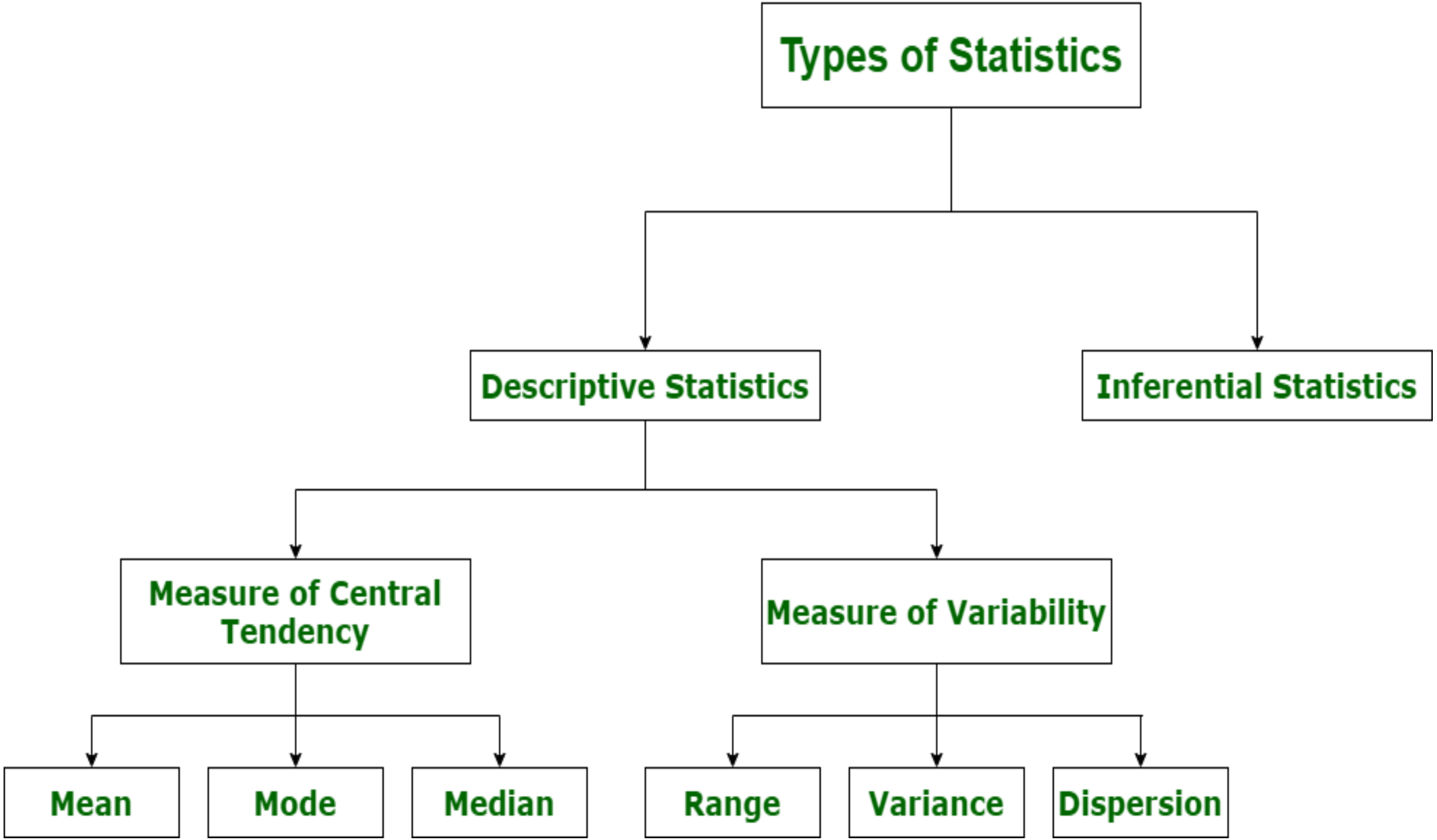
Introduction to Statistics

Statistics

Statistics is the study of the collection, analysis, interpretation, presentation, and organization of ***data***. In other words, it is a mathematical discipline to collect, and summarize data. It is a method of gathering and summarizing results. This has a wide range of uses, from small to large. **Stats** are used for any data collection.

Data:- *Data is a collection of facts and information, such as numbers, words, measurements, observations, or just descriptions of things.*

Statistics has a wide range of applications in many disciplines, including economics, psychology, geology, weather forecasts, and so on. The information gathered for research here may be **quantitative** (*counts or numbers where each data set has a unique numerical value*) or **qualitative** (*information that cannot be counted, measured or easily expressed using numbers*). Quantitative data can also be divided into two types: **discrete** and **continuous**. Continuous data has a spectrum rather than a single value, whereas discrete data has a fixed value.


Types of Statistics
Descriptive Statistics
Inferential Statistics
Measure of Central Tendency
Measure of Variability
Mean
Mode
Median
Range
Variance
Dispersion

Descriptive Statistics

Descriptive statistics uses data that provides a description of the population either through numerical calculation or graph or table. It provides a graphical summary of data. It is simply used for summarizing objects, etc. Descriptive statistics are broken down into measures of central tendency and measures of variability (spread). Measures of central tendency include the mean, median, and mode, while measures of variability include standard deviation, variance and range.

- Descriptive statistics summarizes or describes the characteristics of a data set. It consists of three basic categories of measures: measures of central tendency, measures of variability (or spread), and frequency distribution.
- Measures of central tendency describe the center of the data set (mean, median, mode).
- Measures of variability describe the dispersion of the data set (variance, standard deviation).
- Measures of frequency distribution describe the occurrence of data within the data set (count).

Type of Descriptive Statistic

(a). Measure of central tendency

(b). Measure of Variability

(a). Measure of central tendency

Central tendency is defined as "**the statistical measure that identifies a single value as representative of an entire distribution**". is **a summary measure that attempts to describe a whole set of data with a single value that represents the middle or centre of its distribution**. It aims to provide an accurate description of the entire data. It is the single value that is most typical/representative of the collected data.

In statistics, there are three common types to measures of central tendency.

I. **Mean**
II. **Median**
III. **Mode**

I. Mean

The mean (or average) is the most popular and well known measure of central tendency. It can be used with both discrete and continuous data, although its use is most often with continuous data. Mean are further classified into tree types.

Types of Mean

There are majorly three different types of mean value that you will be studying in statistics.

1. ***Arithmetic Mean:*** The mean is equal to the *sum of all the values in the data set divided by the number of values in the data set*.

$$\text{Mean (m)} = \frac{\text{Sum of all the terms}}{\text{Total no. of terms}}$$

2. ***Geometric Mean***

The **Geometric Mean (GM)** is the average value or mean which signifies the central tendency of the set of numbers by finding the product of their values. Basically, we multiply the numbers altogether and take the nth root of the multiplied numbers, where n is the total number of data values.

$$G.M = \sqrt[n]{x_1 \times x_2 \times \ldots x_n}$$

or

$$G.M = (x_1 \times x_2 \times \ldots x_n)^{\frac{1}{n}}$$

3. ***Harmonic Mean***

The **Harmonic Mean (HM)** is defined as the reciprocal of the average of the reciprocals of the data values. It is based on all the observations. The harmonic mean gives less weightage to the large values and large weightage to the small values to balance the values correctly. In general, the harmonic mean is used when there is a necessity to give greater weight to the smaller items.

$$H = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\ldots+\frac{1}{x_n}}$$

Important Note

Arithmetic Mean > Geometric Mean > Harmonic Mean

or

Harmonic Mean < Geometric Mean < Arithmetic Mean

II. Median

The Median represents the middle value for any group. It is the point at which half the data is more and half the data is less. Median helps to represent a large number of data points with a single data point.

The value of the middle-most observation obtained after arranging the data in ascending/descending order is called the **median** of the data. At many instances, it is difficult to consider the complete data for representation, and here median is useful.

Median

n is odd,

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

n is even,

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}$$

III. Mode

The mode is one of the values of the measures of central tendency. This value gives us a rough idea about which of the items in a data set tend to occur most frequently.

Mode means a value or a number that appears most frequently in a dataset. Sometimes we may need to find the value, which is occurring more frequently in the dataset. In such cases, we find the mode for the set of given data.

Formula of Mode Ungrouped Data

To find the mode for **ungrouped data**, it just requires the data values to be arranged either in ascending or descending order, then finding the repeated values and their frequency. The **observation with the highest frequency** is the modal value for the given data is here referred to as the modal value.

Mode = 3 Median – 2 Mean

1. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. Find the mean of entire data.
2. The mean of 10 items was 70. Later, it was found out that one item 92 was misread as 29. What was the correct mean?
3. Average marks of 26 students of BCA is 73 and average marks of 24 students of BCADS is 86. Find out the average marks of both BCA and BCADS.

$$\bar{x}_w = \frac{v_1 \times n_1 + v_2 \times n_2}{n_1 + n_2}$$

(b). ***Measure of Variability***

Measure of Variability is also known as a measure of dispersion and is used to describe variability in a sample or population. In statistics, there are three common measures of variability as shown below:

(i) Range

It is given a measure of how to spread apart values in a sample set or data set.

Range = Maximum value - Minimum value

(ii) Variance

It simply describes how much a random variable defers from the expected value and it is also computed as a square of deviation.

$$\sigma^2 = \sum_{i=0}^{n} \frac{(x_i - \bar{x})^2}{n}$$

In this formula, n represent total data points, $\bar{x}$ represent mean of data points and x_i represent individual data points.

(iii) Standard Deviation

Standard deviation is a statistic that measures the dispersion of a dataset relative to its mean and is calculated as the square root of the variance. The standard deviation is calculated as the square root of variance by determining each data point's deviation relative to the mean.

$$\sigma = \sqrt{\sum_{i=0}^{n} \frac{(x_i - \bar{x})^2}{n}}$$

Where:

σ = Standard Deviation

x_i = value of i^{th} term in data set

$\bar{x}$ = Mean of the Entire data

n = Number of data points in the data set.

Population and sampling

Population

The population includes all the elements from a set of data. The population is the whole set of values or individuals, you are interested in.

Or

In statistics, the population is the entire set of items from which data is drawn in the statistical study.

The population is usually denoted with **”N”**.

Some Examples of population.

a. The number of citizens living in the India represents a population of the country.
b. The number of planets in the entire universe represents the planet population of the entire universe.
c. The types of candies and chocolates are made in India.

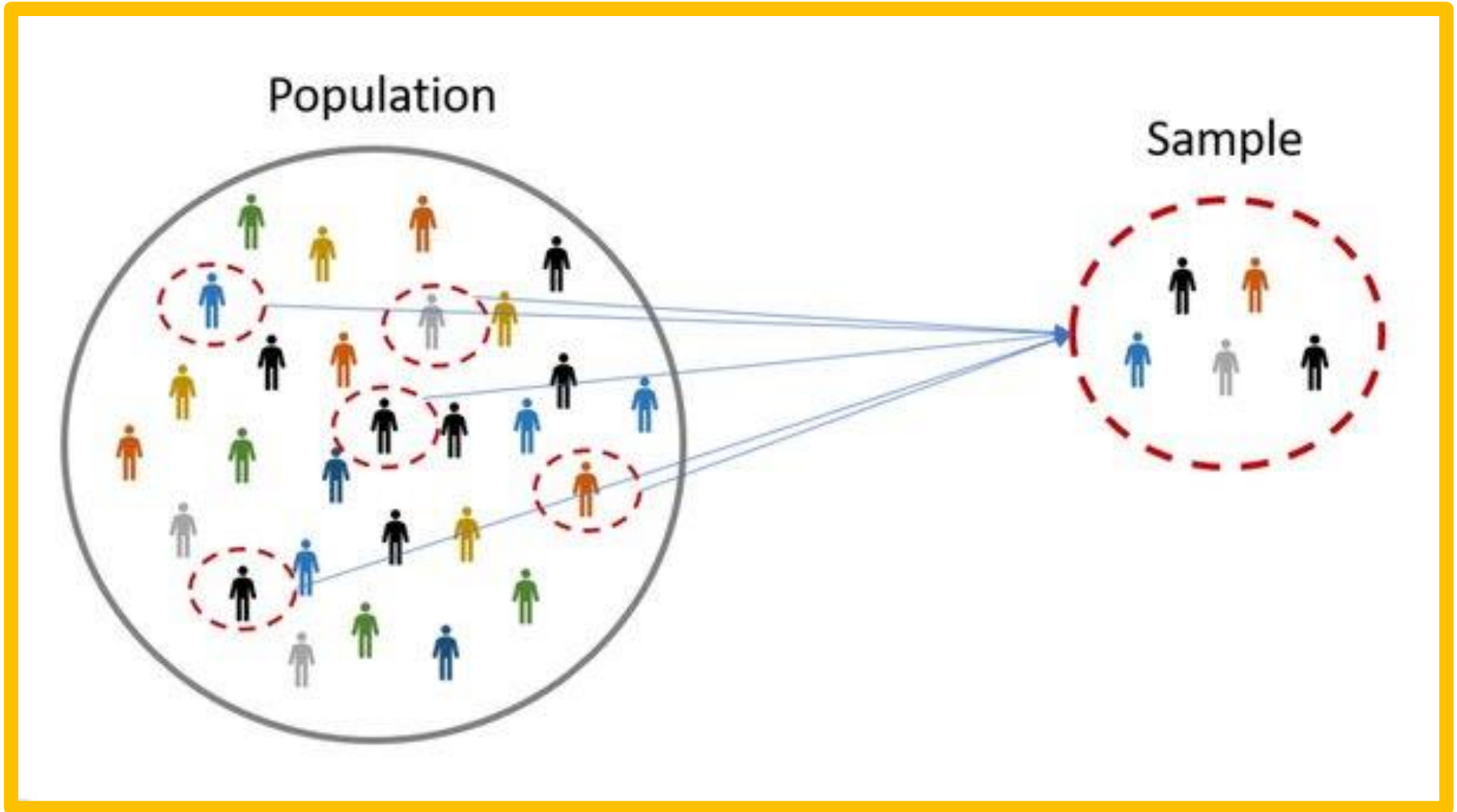
It is impossible to observe the entire statistical population due to time constraints, constraints on geographical accessibility and constraints on the researcher's resource a researcher would instead observe a sample of the same population in order to attempt to learn something about the population as a whole.

Sample

When you conduct research about a group of people, it's rarely possible to collect data from every person in that group. Instead, you select a **sample**. The sample is the group of individuals who will actually participate in the research. A sample represents a group of the interest of the population which we will use to represent the data. The sample is an unbiased subset of the population in which we represent the whole data.

A sample is the representation of a manageable size. samples are collected and stats are calculated from the sample so one can make interferences or extrapolations from the sample. This process of collecting info from the sample is called **sampling.**

- Things to be careful about taking a sample from population.
  1. Sample Size
  2. Unbiased /Random
  3. Should represent the Population.



Sampling method/techniques

To draw valid conclusions from your results, you have to carefully decide how you will select a sample that is representative of the group as a whole. This is called a **sampling method**. There are two primary types of sampling methods that you can use in your research:

A. **Probability sampling** involves random selection, allowing you to make strong statistical inferences about the whole group.

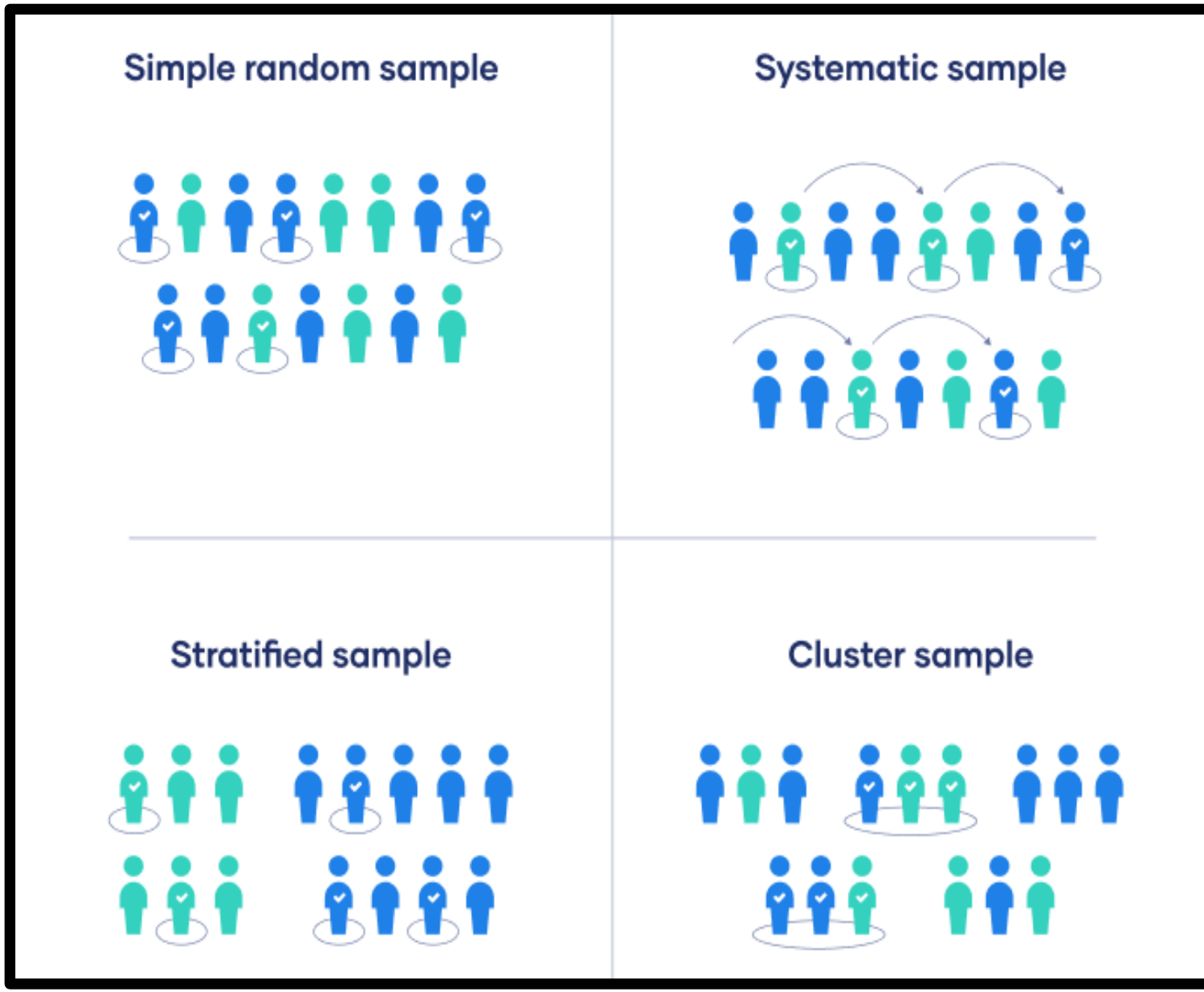
B. **Non-probability sampling** involves non-random selection based on convenience or other criteria, allowing you to easily collect data.

A.) ***Probability sampling methods***

- Probability sampling means that every member of the population has a chance of being selected. It is mainly used in quantitative research. If you want to produce results that are representative of the whole population, probability sampling techniques are the most valid choice.

There are four main types of probability sample.

1. **Simple random Sampling**
2. **Systematic Sampling**
3. **Stratified Sampling**
4. **Cluster Sampling**



1. Simple random sampling

In a simple random sample, every member of the population has an equal chance of being selected. Your sampling frame should include the whole population.

To conduct this type of sampling, you can use tools like random number generators or other techniques that are based entirely on chance.

> ***Example:*** You want to select a simple random sample of 1000 employees of a social media marketing company. You assign a number to every employee in the company database from 1 to 1000 and use a random number generator to select 100 numbers.

2. Systematic sampling

Systematic sampling is like simple random sampling, but it is usually slightly easier to conduct. Every member of the population is listed with a number, but instead of randomly generating numbers, individuals are chosen at regular intervals.

> ***Example***: Employees of the company are listed in alphabetical order. From the first 10 numbers, you randomly select a starting point: number 6. From number 6 onwards, every 10th person on the list is selected (6, 16, 26, 36, and so on), and you end up with a sample of 100 people.

3. Stratified sampling

Stratified sampling involves dividing the population into subpopulations that may differ in important ways. It allows you draw more precise conclusions by ensuring that every subgroup is properly represented in the sample.

To use this sampling method, you divide the population into subgroups (called strata) based on the relevant characteristic (e.g., gender identity, age range, income bracket, job role).

Based on the overall proportions of the population, you calculate how many people should be sampled from each subgroup. Then you use random or systematic sampling to select a sample from each subgroup.

Example: The company has 800 female employees and 200 male employees. You want to ensure that the sample reflects the gender balance of the company, so you sort the population into two strata based on gender. Then you use random sampling on each group, selecting 80 women and 20 men, which gives you a representative sample of 100 people.

4. Cluster sampling

Cluster sampling also involves dividing the population into subgroups, but each subgroup should have similar characteristics to the whole sample. Instead of sampling individuals from each subgroup, you randomly select entire subgroups.

If it is practically possible, you might include every individual from each sampled cluster. If the clusters themselves are large, you can also sample individuals from within each cluster using one of the techniques above. This is called multistage sampling.

This method is good for dealing with large and dispersed populations, but there is more risk of error in the sample, as there could be substantial differences between clusters. It's difficult to guarantee that the sampled clusters are really representative of the whole population.

> ***Example***: Cluster samplingThe company has offices in 10 cities across the country (all with roughly the same number of employees in similar roles). You don't have the capacity to travel to every office to collect your data, so you use random sampling to select 3 offices – these are your clusters.

B.) Non-probability sampling methods

In a non-probability sample, individuals are selected based on ***non-random criteria***, and ***not every individual has a chance of being included***.

This type of sample is ***easier and cheaper*** to access, ***but it has a higher risk of sampling bias***. That means the inferences you can make about the population are weaker than with probability samples, and your conclusions may be more limited. If you use a non-probability sample, you should still aim to make it as representative of the population as possible.

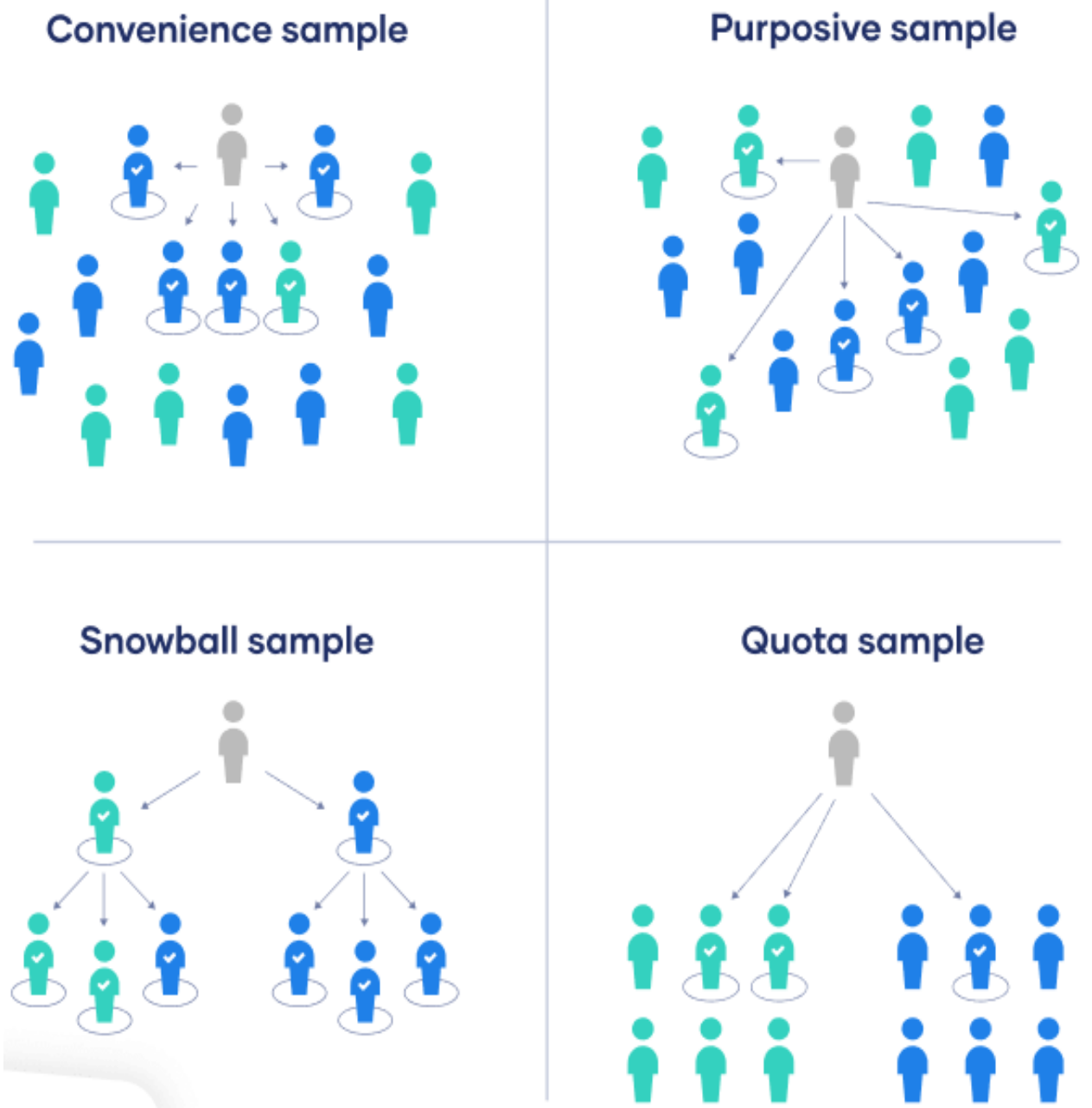
There are four main types of Non-probability sample

1. **Convenience sampling,**
2. **Purposive sampling,**
3. **Snowball sampling and**
4. **Quota Sampling.**

1. Convenience sampling

A convenience sample simply includes the individuals who happen to be most accessible to the researcher. This is an easy and inexpensive way to gather initial data, but there is no way to tell if the sample is representative of the population, so it can't produce generalizable results. Convenience samples are at risk for both sampling bias and selection bias.

Example: You are researching opinions about student support services in your university, you ask your fellow students to complete a survey on the topic. This is a convenient way to gather data, the sample is not representative of all the students at your university.



2. ***Purposive sampling***

This type of sampling, also known as judgement sampling, involves the researcher using their expertise to select a sample that is most useful to the purposes of the research.

It is often used in qualitative research, where the researcher wants to gain detailed knowledge about a specific phenomenon rather than make statistical inferences, or where the population is very small and specific. An effective purposive sample must have clear criteria and rationale for inclusion. Always make sure to describe your inclusion and exclusion criteria and beware of observer bias affecting your arguments.

Example: You want to know more about the opinions and experiences of disabled students at your university, so you purposefully select a number of students with different support needs in order to gather a varied range of data on their experiences with student services.

3. ***Snowball sampling***

If the population is hard to access, snowball sampling can be used to recruit participants via other participants. The number of people you have access to “snowballs” as you contact more people. The downside here is also representativeness, as you have no way of knowing how representative your sample is due to the reliance on participants recruiting others. This can lead to sampling bias.

4. Quota sampling

Quota sampling is a non-probability sampling technique similar to stratified sampling. This is also know as also known as dimension sampling In this method, the population is split into segments (strata) and you have to fill a quota based on people who match the characteristics of each stratum.

There are two types of quota sampling

1. Proportional quota sampling gives proportional numbers that represent segments in the wider population. For this, the population frame must be known.
2. Non-proportional quota sampling uses stratum to divide a population, though only the minimum sample size per stratum is decided.

BASIS	POPULATION	SAMPLE
Meaning	Population refers to the collection of all elements possessing common characteristics, that comprises universe.	Sample means a subgroup of the members of population chosen for participation in the study.
Includes	Each and every unit of the group.	Only a handful of units of population.
Characteristic	Parameter	Statistic
Data collection	Complete enumeration or census	Sample survey or sampling
Focus on	Identifying the characteristics.	Making inferences about population.

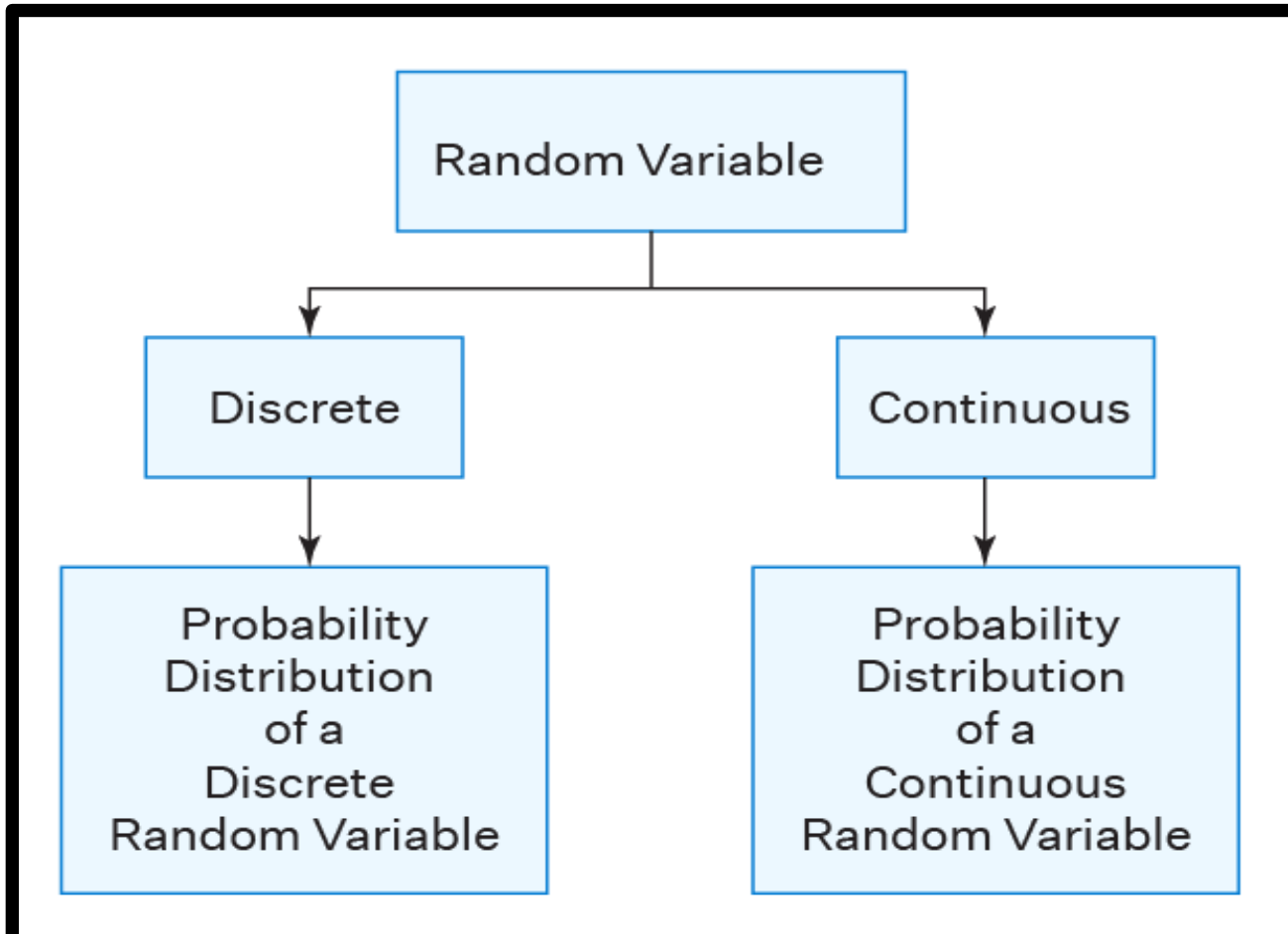
Random Variable

A random variable can be defined as a type of variable whose value depends upon the numerical outcomes of a certain random phenomenon. A random variable is a variable that is used to denote the numerical outcome of a random experiment. Random variables are always real numbers as they are required to be measurable.

Types of Random Variables

Random Variables can be divided into two broad categories depending upon the type of data available. These are given as follows:

1. **Discrete random variable**
2. **Continuous random variable**



1. ***Discrete Random Variable***

A discrete random variable is a variable that can take any whole number of values as outcomes of a random experiment. The discrete random variable takes a countable number of possible outcomes, and it can be counted as 0, 1, 2, 3, 4, ... Probability distributions are used to show the values of discrete random variables.

"A discrete random variable can be defined as a type of variable whose value depends upon the numerical outcomes of a certain random phenomenon."

Example:

Suppose 2 dice are rolled and the random variable, X, is used to represent the sum of the numbers. Then, the smallest value of X will be equal to 2, which is a result of the outcomes 1 + 1 = 2, and the highest value would be 12, which is resulting from the outcomes 6 + 6 = 12. Thus, X could take on any value between 2 to 12 (inclusive).

2. ***Continuous Random Variable***

The continuous random variable is a random variable that can take on a continuum of values. In other words, a random variable is said to be continuous if it assumes a value that falls between a particular interval.

Example:

i. A runner might complete the marathon in 3 hours 20 minutes 12.0003433 seconds. Or they may complete the marathon in 4 hours 6 minutes 2.28889 seconds.

ii. The **interest rate** of loans in a certain country. This is a continuous random variable because it can take on an infinite number of values. For example, a loan could have an interest rate of 3.5%, 3.765555%, 4.00095%, etc.

iii. Your CGPA.

Continuous Random Variable	Discrete Random Variable
The value of a continuous random variable falls between a range of values.	The value of a discrete random variable is an exact value.
The ***probability density function*** is associated with a continuous random variable.	The ***probability mass function*** is used to describe a discrete random variable
A continuous random variable can take on an infinite number of values.	Such a variable can take on a finite number of distinct values.
Mean of a continuous random variable is $E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx$.	The mean of a discrete random variable is $E[X] = \sum x\, P(X = x)$, where $P(X = x)$ is the probability mass function.
The variance of a continuous random variable is $Var(X) = \int_{-\infty}^{\infty} (x-\mu)2\, f(x)dx$	The variance of a discrete random variable is $Var[X] = \sum(x - \mu)^2\, P(X = x)$
The examples of a continuous random variable are uniform, exponential, normal, and standard normal random variables.	Examples of a discrete random variables are binomial, geometric, Bernoulli, and Poisson random variables.

Probability Distribution

The probability distribution is a ***function*** that gives the ***relative likelihood of occurrence of all possible outcomes of an experiment***. There are two important functions that are used to describe a probability distribution. These are the ***probability density function*** or ***probability mass function*** and the ***cumulative distribution function***.

Normal Distribution

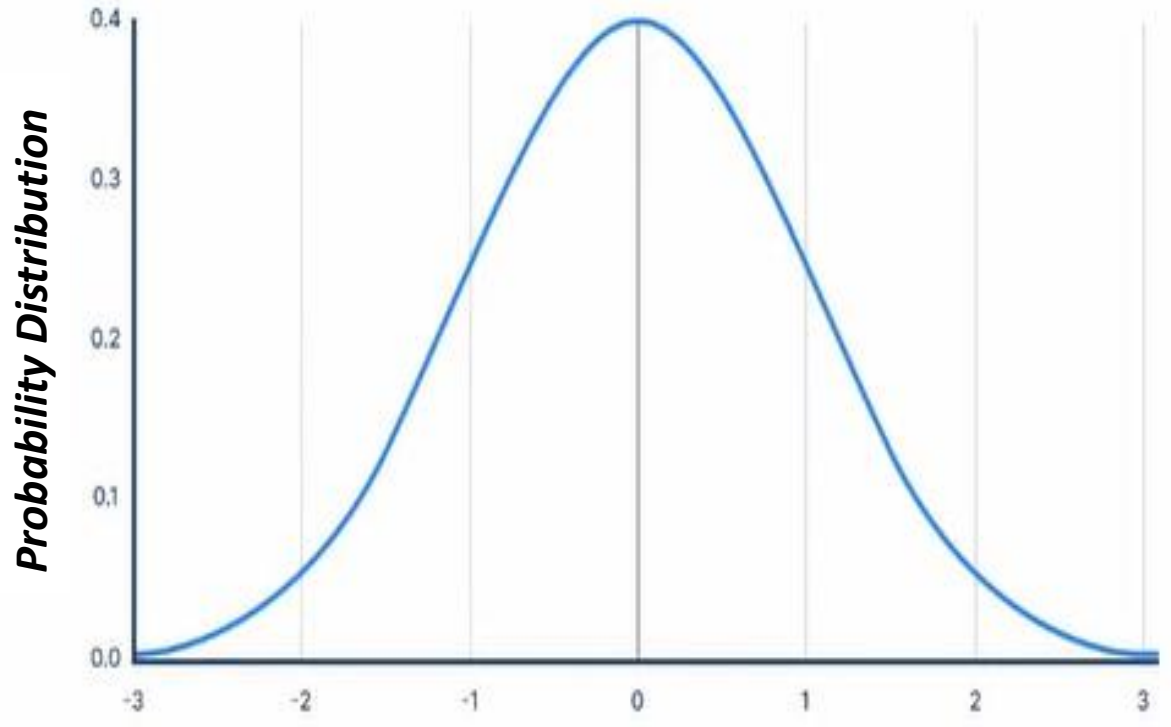
In statistics, the **Normal Distribution**, also called the **Gaussian Distribution**, is the most significant ***continuous probability distribution***. Sometimes it is also called a ***bell curve***. *Many random variables are either nearly or exactly represented by the normal distribution*. Early mathematicians and statisticians noticed the same shape for various distributions—so they named it the normal distribution, i.e., normally occurring distribution.

The Normal Distribution can be defined by the probability density function for a continuous random variable in a system. If f(x) is the probability density function, X is the random variable; then it defines a function that is integrated between the range or interval (x to x + dx). Thus, the probability of random variable X is given by considering the values between x and x+dx.

Some Properties of Normal Distributions

The properties of Normal Distributions are as follows:

- Mean = Median = Mode
- The total area under the Gaussian distribution curve equals 1.
- The normal distributions curve is unimodal (has one peak)
- The curve approaches the x-axis but does not touch it (*Asymptotic in Nature*).
- It has symmetry about the center.
- 50% of values are less than the mean, and 50% are greater than the mean
- A normal distribution curve is a bell-shaped curve.



Tail

Asymptotic in Nature

The majority of data is near to mean

- The normal distribution is characterized by two parameters which are ***mean and Standard Deviation***, where mean shows the centre of distribution and Standard deviation shows the spread of the distribution.

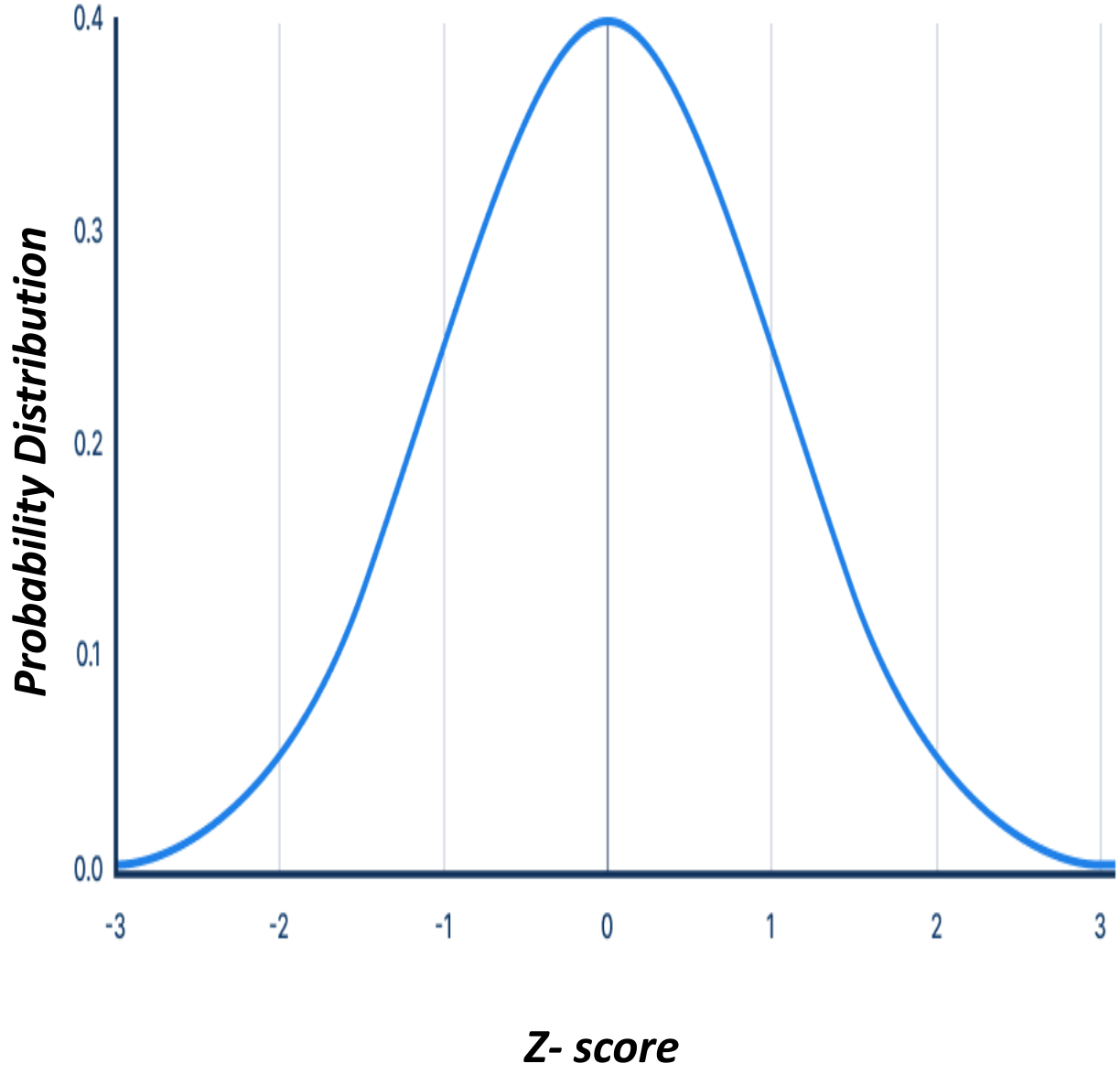
Why Normal Distribution is so important?

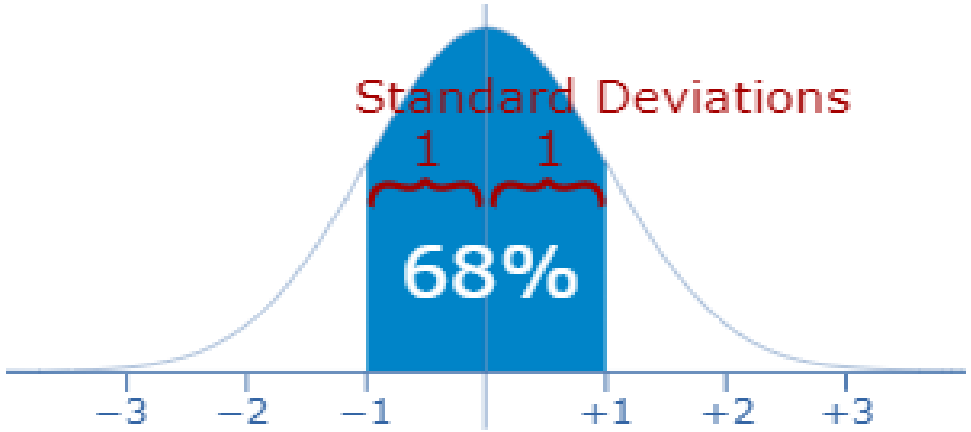
The commonality in Nature: Many natural phenomena follow a normal distribution, such as the heights of people, the weights of objects, the IQ scores of a population, and many more. Thus, the normal distribution provides a convenient way to model and analyze such data.

The ***standard normal distribution***, also called the ***z-distribution***, is a special normal distribution where the ***mean is 0*** and the ***standard deviation is 1***.

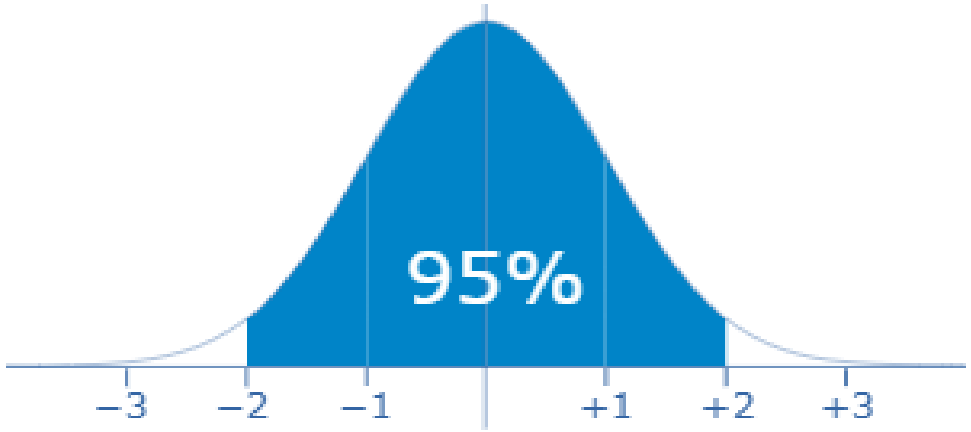
Any normal distribution can be ***standardized*** by converting its values into *z* scores. Z scores tell you how many standard deviations from the mean each value lies.

Z-score formula	Explanation
$z = \frac{x - \mu}{\sigma}$	• x = individual value • μ = mean • σ = standard deviation

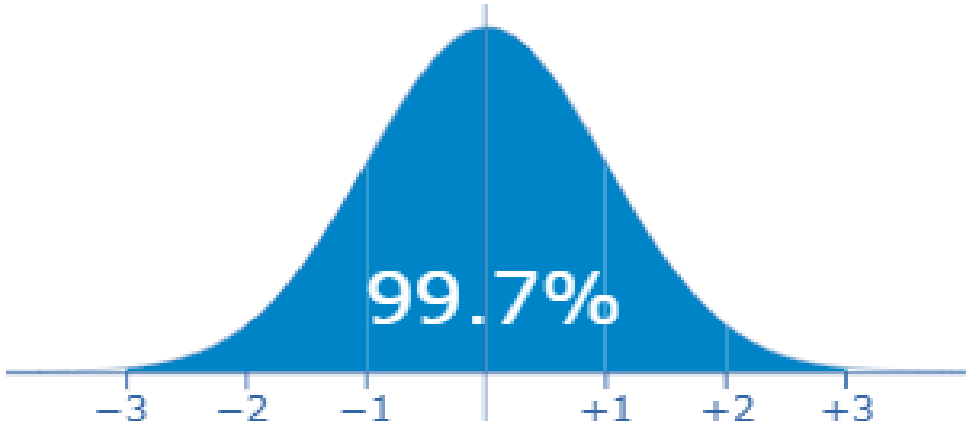




68% of values are within
1 standard deviation of the mean



95% of values are within
2 standard deviations of the mean



99.7% of values are within
3 standard deviations of the mean

Inferential Statistics

Inferential statistics can be defined as a *field of statistics that uses analytical tools for drawing conclusions about a population by examining random samples.* The goal of inferential statistics is to generalize about a population. In inferential statistics, a statistic is taken from the sample data (e.g., the sample mean) that used to make inferences about the population parameter (e.g., the population mean).

There are two main types of inferential statistics - ***hypothesis testing and regression analysis***. The samples chosen in inferential statistics need to be representative of the entire population. Hypothesis testing also includes the use of confidence intervals to test the parameters of a population.

Hypothesis testing

Hypothesis testing can be defined as a statistical tool that is used to identify if the results of an experiment are meaningful or not. It involves setting up a ***null hypothesis*** and an ***alternative hypothesis***. These two hypotheses will always be ***mutually exclusive***. This means that if the null hypothesis is true then the alternative hypothesis is false and vice versa. A conclusion is drawn based on the value of the test statistic, the critical value, and the confidence intervals.

An example of hypothesis testing is setting up a test to check if a new medicine works on a disease in a more efficient manner.

- ***Null hypothesis (H_0):*** There's no effect on the population.
- ***Alternative hypothesis (H_a or H_1):*** There's an effect on the population.

Null hypothesis

The null hypothesis is a concise mathematical statement that is used to indicate that there is ***no difference between two possibilities***. In other words, there is no difference between certain characteristics of data. This hypothesis assumes that the outcomes of an experiment are based on chance alone. It is denoted as H_0. Hypothesis testing is used to conclude if the null hypothesis can be rejected or not.

Alternative Hypothesis

The alternative hypothesis is an alternative to the null hypothesis. It is used to show that the observations of an experiment are due to some real effect. It indicates that there is a statistical significance between two possible outcomes and can be denoted as H_1 or H_a.

Important Points

1. How to decide what will be the Null hypothesis and what will be the Alternate Hypothesis (Typically the Null hypothesis says nothing new is happening)
2. We try to gather evidence to reject the null hypothesis
3. It's important to note that failing to reject the null hypothesis doesn't necessarily mean that the null hypothesis is true; it just means that there isn't enough evidence to support the alternative hypothesis.

Hypothesis tests are similar to jury trials, in a sense. In a jury trial, HO is similar to the not-guilty verdict, and Ha is the guilty verdict. You assume in a jury trial that the defendant isn't guilty unless the prosecution can show beyond a reasonable doubt that he or she is guilty. If the jury says the evidence is beyond a reasonable doubt, they reject HO, not guilty, in favour of Ha, guilty.

Types of Hypothesis Testing

Selecting the correct test for performing hypothesis testing can be confusing. These tests are used to determine a test statistic on the basis of which the null hypothesis can either be rejected or not rejected. Some of the important tests used for hypothesis testing are given below.

1. Hypothesis Testing Z Test

A z test is a way of hypothesis testing that is used for a large sample size (n ≥ 30). It is used to determine whether there is a difference between the population mean and the sample mean when the population standard deviation is known. It can also be used to compare the mean of two samples. It is used to compute the z test statistic. The formulas are given as follows:

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$\overline{x}$ is the sample mean,
μ is the population mean,
σ is the population standard deviation and
n is the sample size.

2. ***Hypothesis Testing t-Test***

The t test is another method of hypothesis testing that is used for a small sample size (n < 30). It is also used to compare the sample mean and population mean. However, the ***population standard deviation is not known***. Instead, the sample standard deviation is known. The mean of the two samples can also be compared using the t-test.

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$\bar{x}$ is the sample mean,

μ is the population mean,

s is the sample standard deviation and

n is the sample size.

One-Tailed Hypothesis Testing

One-tailed hypothesis testing is done when the rejection region is only in one direction. It can also be known as directional hypothesis testing because the effects can be tested in one direction only. This type of testing is further classified into the right tailed test and left-tailed test.

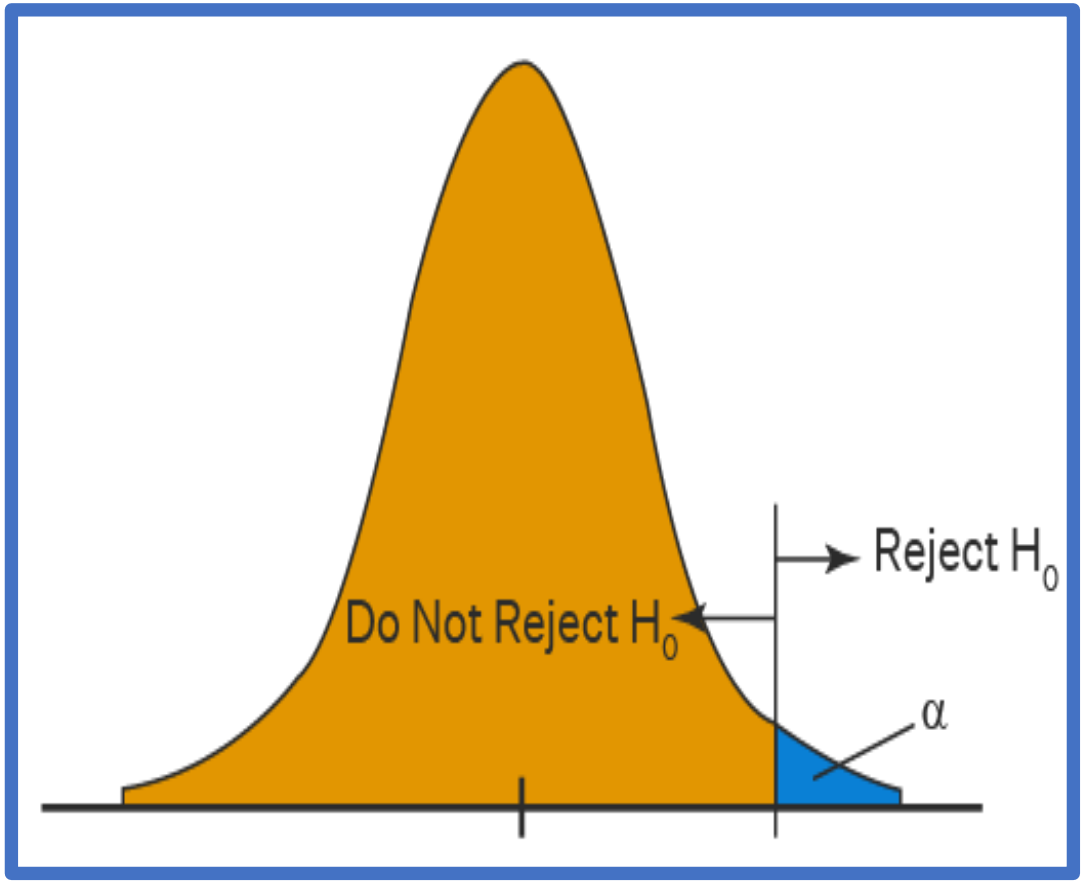
1. Right-Tailed Hypothesis Testing

The right tail test is also known as the upper tail test. This test is used to check whether the population parameter is greater than some value. The null and alternative hypotheses for this test are given as follows:

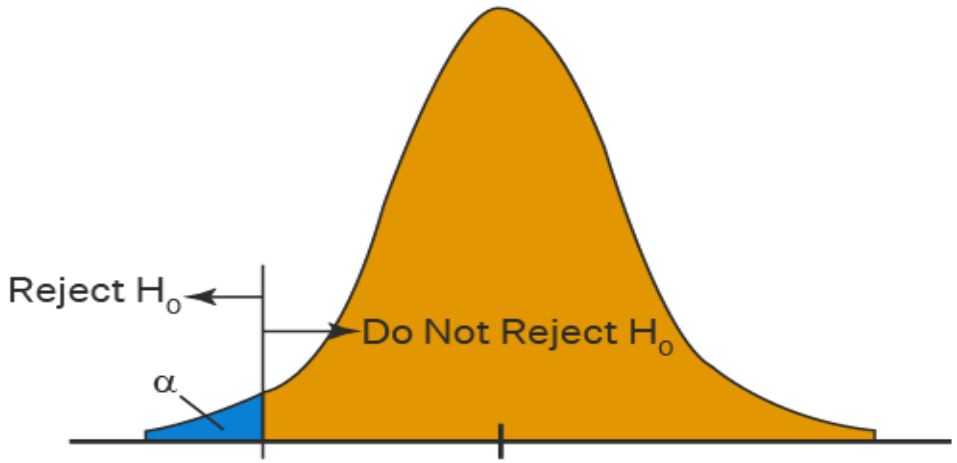
H0: The population parameter is ≤ some value

H1: The population parameter is > some value.

If the test statistic has a greater value than the critical value then the null hypothesis is rejected.



Left Tail Hypothesis Testing

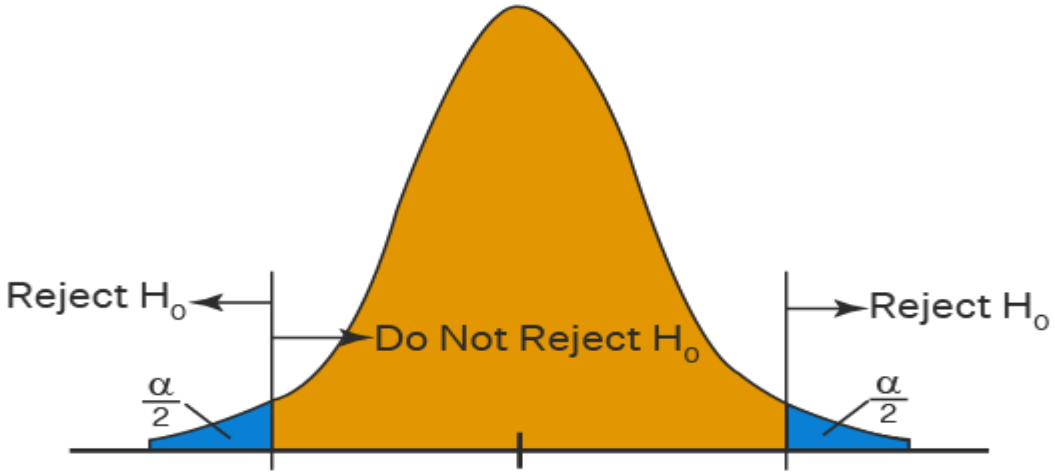


H0: The population parameter is ≥ some value

H1: The population parameter is < some value.

The null hypothesis is rejected if the test statistic has a value lesser than the critical value.

Two Tail Hypothesis Testing



In this hypothesis testing method, the critical region lies on both sides of the sampling distribution.

H0: the population parameter = some value

H1: the population parameter ≠ some value

The null hypothesis is rejected if the test statistic has a value that is not equal to the critical value.

P-value

The P-value formula is short for **probability value**. *P-value defines the probability of getting a result that is either the same or more extreme than the other actual observations*. The P-value represents the probability of occurrence of the given event. The P-value formula is used as an alternative to the rejection point to provide the least significance for which the null hypothesis would be rejected. ***The smaller the P-value, the stronger the evidence in favour of the alternative hypothesis*** given observed frequency and expected frequency.

P-value is an important statistical measure, that helps to determine whether the hypothesis is correct or not. **P-value always only lies between 0 and 1**. The level of significance(α) is a predefined threshold that should be set by the researcher. **It is generally 0.05 or 0.01**.

Hypothesis Testing Steps

Hypothesis testing can be easily performed in five simple steps. The most important step is to correctly set up the hypotheses and identify the right method for hypothesis testing. The basic steps to perform hypothesis testing are as follows:

- ***Step 1***: Set up the null hypothesis by correctly identifying whether it is the left-tailed, right-tailed, or two-tailed hypothesis testing.
- ***Step 2***: Set up the alternative hypothesis.
- ***Step 3***: Choose the correct ***significance level (α)***, and find the critical value.

  significance level (α) is *a measure of the strength of the evidence that must be present in your sample before you will reject the null hypothesis and conclude that the effect is statistically significant.*
- ***Step 4***: Calculate the correct test statistic (z, t) and p-value.
- ***Step 5***: Compare the test statistic with the critical value or compare the p-value with α to arrive at a conclusion. In other words, decide if the null hypothesis is to be rejected or not.

Example : The average score on a test is 80 with a standard deviation of 10. With a new teaching curriculum introduced it is believed that this score will change. On random testing, the score of 38 students, the mean was found to be 88. With a 0.05 significance level, is there any evidence to support this claim?

Solution: This is an example of two-tail hypothesis testing. The z test will be used.

$H_0: \mu = 80$, $H_1: \mu \neq 80$

$\bar{x} = 88$, $\mu = 80$, $n = 36$, $\sigma = 10$.

$\alpha = 0.05 / 2 = 0.025$

The critical value using the normal distribution table is 1.96

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{88 - 80}{\frac{10}{\sqrt{36}}} = 4.8$$

As 4.8 > 1.96, the null hypothesis is rejected.

Answer: There is a difference in the scores after the new curriculum was introduced.