Gravitational Wave Physics and LIGO

Sagar JC[1], Aditya Srivatsa[1], Ajith U Shanbhag[1], Kushal J[2], Hardik Medhi[3], Bhavya Goradia[4], Kshitija Kshirsagar[5], Tasmi Memon[6], Samrudhi R Kanjarpane[7], Ajay Atwal[8]and Tanay[9]

[1]St. Joseph's College, Bengaluru
[2]Christ Junior College, bengaluru
[3]REVA University, Bengaluru
[4]KJ Somaiya College of Engineering, Mumbai
[5]Institute of Science, Nagpur
[6]Maharaja Sayajirao University, Vadodara
[7]Poornaprajna College, Udupi
[8]University of Hyderabad
[9]Mumbai University, Mumbai

March 22, 2021

Acknowledgement

A special thanks to Naxxatra Science for giving us the opportunity to write a collaborative review paper. We would like to express our gratitude to Ms. Sitara Srinivasan, owner of Naxxatra Science, Mr. Vikranth Pulamathi and Mr. Jishnu P Das for teaching us LaTeX.

Abstract

Abstract here

Keywords— General Relativity, Space time, Tensors, indices, Field equations, Wave equation, Polarization, Energy Flux, Doppler effect, Inverse square law, Inspiral mechanism, Black holes, Neutron stars, Pulsars, Add key words here relating to ur topic

1 Introduction

Write small paragraph about general relativity Write small paragraph about Space-time write who al worked in the field of general relativity and gravitational waves write a small intro for gravitational waves

2 Linearized theory of Gravitational waves

Linearized theory of Gravitational waves is a basic understanding of gravitational waves based on an assumption that any perturbation in space-time can be approximated to a linear factor whose degree is One. This simplifies the calculations a lot. And more over since the sources of gravitational waves are very far away, the effects they produce here on earth will be very small. So we can neglect the higher degree of perturbation and linearize it to the first degree.

Einstein's field equations are a set of ten Tensor equations which describe gravity as a curvature in space-time. And one among them is:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1}$$

This is a tensor equation which describes gravity in form of Einstein's tensor, $G_{\mu\nu}$ which is directly dependent on the geometry of space-time which is altered by the stress-energy tensor $T_{\mu\nu}$. Another field equation that relates the geometry or curvature of space-time to stress-energy tensor is

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

where $R_{\mu\nu}$ is the Riemann tensor which describes the curvature of space-time, R is the scalar curvature and $g_{\mu\nu}$ is the gravitational field tensor. Any change in matter distribution will be recorded in in $T_{\mu\nu}$. So if $T_{\mu\nu}$ changes then according to equation 2, gravitational field tensor $g_{\mu\nu}$ also has to change. If $h_{\mu\nu}$ is the perturbation induced in space-time then the new gravitational field tensor $\tilde{g}_{\mu\nu}$ is given by [**Kokkotas_2008**]

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \tag{3}$$

To get the new gravitational field, field equation should be solved for $\tilde{g}_{\mu\nu}$ which gives

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\, \eta_{\mu\nu}\, h^{\alpha}_{\alpha} \tag{4}$$

where $\eta_{\mu\nu}$ is the gravity where space is flat i.e. $\eta_{\mu\nu} = g_{\mu\nu}$ and h^{α}_{α} is summed for all spatial coordinates i.e. α takes values $(1, 2, 3)$ which corresponds to (x, y, z). The admitted solutions for this variations in space time $\tilde{h}_{\mu\nu}$ has solution in the form of

$$\tilde{h}_{\mu\nu} = A^{\mu\nu}\, e^{ik_\alpha x^\alpha} \tag{5}$$

This is a 3D wave equation where $A^{\mu\nu}$ is the Amplitude tensor, $i = \sqrt{-1}$, $k_\alpha = (k_x, k_y, k_z)$ is the wave vector and $x^\alpha = (x^1, x^2, x^2) = (x, y, z)$ is the position vector.

Thus we can say that when ever a body causes disturbances in curvature of space-time, these travel through space in form of waves whose speed is equal to the speed of light.

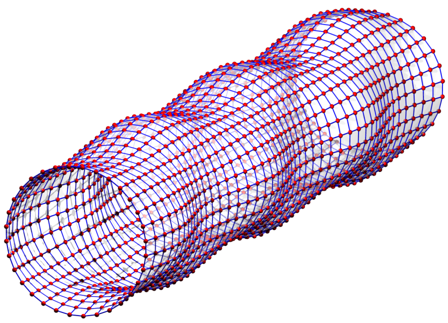

Figure 1: This is a computer simulated image that shows Gravitational wave as a 3D wave
Source:- `https://www.universetoday.com/127255/gravitational-waves-101/`

3 Properties of Gravitational waves

Now we shall see the properties of these gravitational waves. like any other waves, even gravitational waves have frequency, wavelength, speed which is speed of light, etc. These properties depend on the source of gravitational waves.

3.1 Polarization of Gravitational waves

Gravitational waves can also be. Since they are three dimensional waves their polarization can be restricted to two forms where the the amplitude tensor $A^{\mu\nu}$ has two forms $A_{+}^{\mu\nu}$ and $A_{\times}^{\mu\nu}$ which are orthogonal to each other [**Dirkes_2018**]. They can be represented as

$$A_{+}^{\mu\nu} = h_{+}\, \varepsilon_{+}^{\mu\nu} \tag{6}$$

$$A_{\times}^{\mu\nu} = h_{\times}\, \varepsilon_{\times}^{\mu\nu} \tag{7}$$

where $\varepsilon_{+}^{\mu\nu}$ and $\varepsilon_{\times}^{\mu\nu}$ are unit polarization tensors.

$$\varepsilon_{+}^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$\varepsilon_{\times}^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

In general relativity any tensor with indices $\mu\nu$ is a rank 2 tensor with 4 rows and 4 columns where each index can take values of space time coordinates which are (t, x, y, z) , and position of each element is associated with any two coordinates. Thus in such tensors, the positions of elements are associated with space-time as follows:

$$\begin{bmatrix} tt & tx & ty & tz \\ xt & xx & xy & xz \\ yt & yx & yy & yz \\ zt & zx & zy & zz \end{bmatrix}$$

So when we compare the unit polarization tensors $\varepsilon_{+}^{\mu\nu}$ and $\varepsilon_{\times}^{\mu\nu}$ with the above one, we see that in $\varepsilon_{+}^{\mu\nu}$ the non zero entries are $+1$ in 'xx' direction and -1 in 'yy' direction, hence the $A_{+}^{\mu\nu}$ amplitude is oriented only along X and Y axes, thus this gravitational wave which oscillates along X and Y axes is called as 'PLUS' polarized wave because the vibration resembles '$+$' symbol. But in $\varepsilon_{\times}^{\mu\nu}$ the non zero entries are $+1$ in 'xy' direction and -1 in 'yx' direction, hence the $A_{+}^{\mu\nu}$ amplitude is oriented in the 'XY' plane at a an angle of $45°$ to the axes, thus this gravitational wave which oscillates in the 'XY' plane at a an angle of $45°$ to the axes is called as 'CROSS' polarized wave because the vibration resembles '$\times$' symbol.

So the equation of polarized gravitational waves are:-
$(+)$ wave $\Rightarrow \tilde{h}_{\mu\nu} = h_{+}\, \varepsilon_{+}^{\mu\nu}\, e^{i(\omega t - k_z z)}$
$(\times)$ wave $\Rightarrow \tilde{h}_{\mu\nu} = h_{\times}\, \varepsilon_{\times}^{\mu\nu}\, e^{i(\omega t - k_z z)}$

To simplify things here position variable is just 'z', i.e we assume the wave is travelling in z direction and the polarized characteristics are seen in in the X-Y plane. thus it is easier to figure out the effects of these polarized gravitational waves.

3.2 Effect of Gravitational waves on objects

Gravitational waves carry the fluctuations of space along with them. So if they move through an object, since space itself will oscillate, even the object which occupies space will oscillate according to the wave. Thus the shape of object will change periodically.

3.2.1 Plus polarized effect

When a plus polarized wave passes through the object, since such gravitational wave makes space-time oscillate in X and Y axes only. So the points in space along axis will come close during compression and go far during stretching. Thus the object itself will be compressed and stretched along the axes, perpendicular to the direction of propagation of wave.

3.2.2 Cross polarized effect

When a cross polarized wave passes through the object, since such gravitational wave makes space-time oscillate along the line which makes an inclination of 45° with X and Y axes (i.e. along the line $x = y$ and $x = -y$). So the points in space along those line will come close during compression and go far during stretching. Thus the object itself will be compressed and stretched along those lines, perpendicular to the direction of propagation of wave.

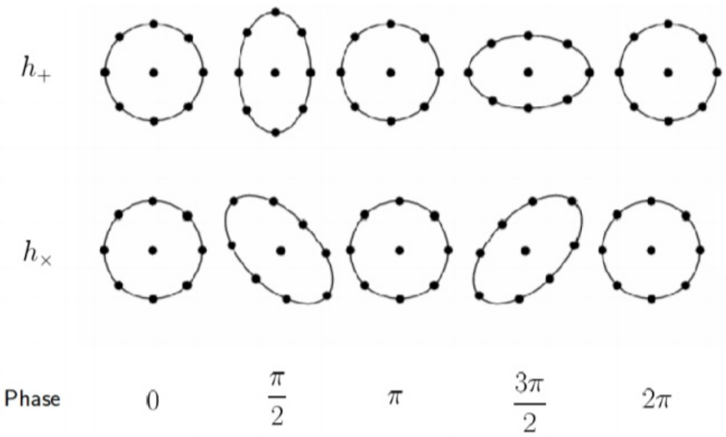

Figure 2: Shape of the object when gravitational wave passes through it when the phase difference of wave changes by $\pi/2$.
Source :- Wavelet graphs for the detection of gravitational waves : application to eccentric binary black holes by Philippe Bacon

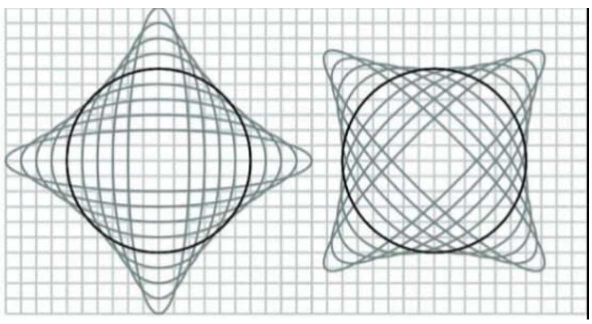

Figure 3: Pictorial representation of two Linear polarization of gravitational waves.
Source:- `https://www.researchgate.net/figure/fig4_228909324`

3.3 Energy transported by gravitational wave

When sources produce gravitational waves, their energy is converted to Gravitational waves. Since generally the sources are very massive Energy of gravitational waves will be very large. Since gravitational wave travels in the speed of light, the energy is also transported at that speed. And since Energy flux is equal to the product of Energy and speed, The average energy flux 'E' is given by [**1972ARA&A..10..335P**]

$$E = \frac{c^3}{16\pi G} \left\langle (h_+)^2 + (h_\times)^2 \right\rangle \tag{10}$$

So we see the energy flux is very huge because of the term $\frac{c^3}{16\pi G}$ which is in the order of 10^{33} Joules sec/ metre^2 and it also depends on the average of the square of the plus and cross polarized amplitudes 'h_+' and '$h_\times$'.

Due to such huge energy it carries the wave can travel unimpeded forever through space and no obstacle can damp gravitational wave because the space in which the obstacle lies itself is the medium of the wave. But the Doppler effect and decrease in amplitude due to radiation of energy causes the wave to die out after the wave travels a very long distance according to the relation $Amplitude \propto \frac{1}{r}$.

So the power or intensity of gravitational wave decreases as it moves through space according to this inverse square law i.e. as the wave moves in space through a distance 'r' The energy of wave will be spread-out in space across a sphere of radius'r' through Surface area of sphere $4\pi r^2$. Since the intensity of wave is Energy over time, Intensity reduces as r^2 [**Obukhov_2009**].

$$E_{flux} = \frac{Energy}{Area} = \frac{E}{4\pi r^2}$$

$$E_{flux} \propto \frac{1}{r^2}$$

But since $E_{flux} \propto Amplitude^2$ we get the relation that $Amplitude \propto \frac{1}{r}$. i.e Amplitude decreases as distance from source increases.

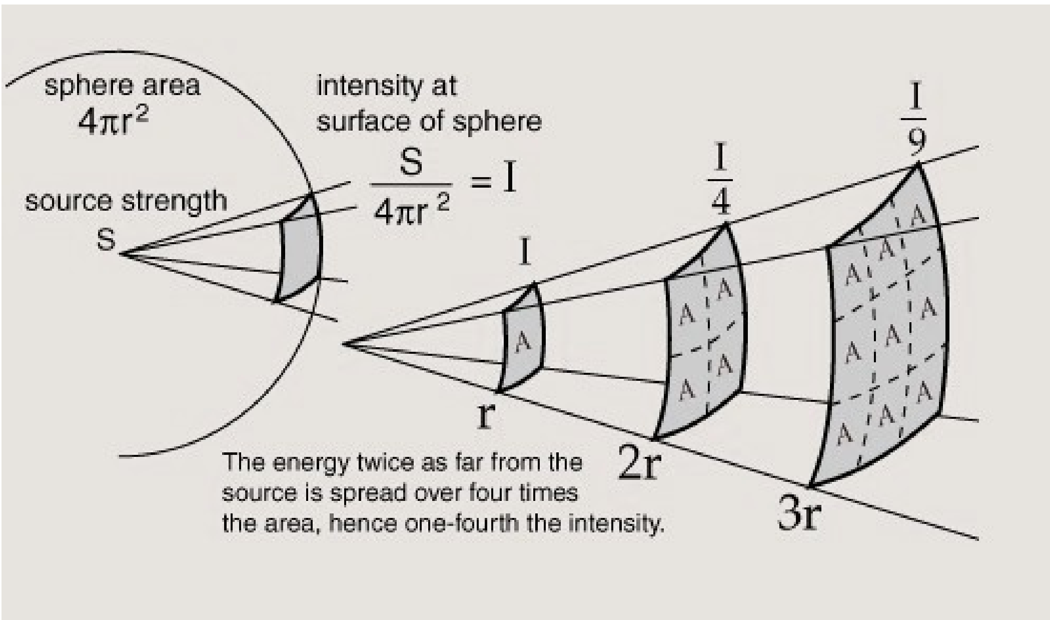

Figure 4: Here we see how the intensity of wave changes as it goes farther from the source according to the inverse square law.
Source :- `http://www.mysearch.org.uk/website1/html/339.Laws.html`

wirte similarities and differences between EM and GW

4 Sources of Gravitational waves

Theory of general relativity predicts that gravitational waves can be generated by any dynamically changing system containing moving objects by producing radiation-reaction forces in their source i.e. waves will be generated and carries the exact rate of energy which is extracted from the source.

Gravitational waves can be produced by an object which is accelerating or a Binary revolving system, merging black holes, neutron star collisions, primordial black holes, etc. But one common nature is all these objects is, they change the curvature of space-time. Thus radiate waves.

4.1 Single accelerating object in space

Accelerating objects like pulsars can create gravitational wave [**Creighton:2011zz**]. According to general relativity, mass creates stress in space-time and thus can change the geometry of it by bending and changing the curvature. Then if this object moves, then the curvature also moves along with it. But if the object accelerates in space-time in a circular manner, then the ripples will be created in space-time which is which radiates gravitational waves. This is similar to creation of water waves when we move our finger in a circular fashion in water. So higher the mass of object and it's acceleration, stronger is the gravitational wave it produces.

Such continuously spinning bodies produces continuous gravitational waves, where it's nature is sinusoidal for a longer period of time. This happens only if spin rate of this object is constant. Such gravitational waves have same frequency and amplitude. Such types of gravitational waves are not yet discovered.

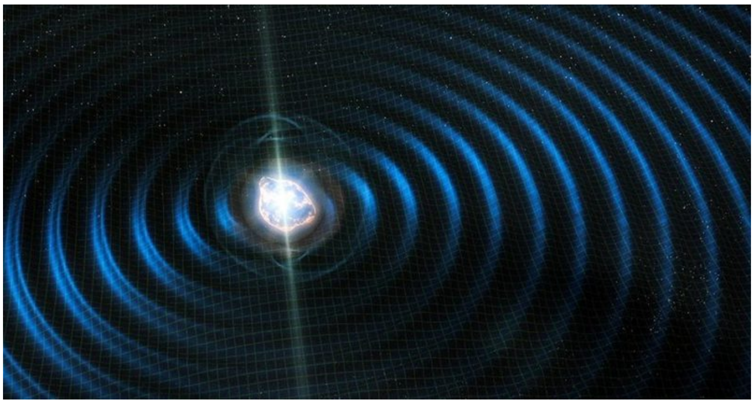

Figure 5: Computer simulation of Continuous gravitational wave by a pulsar.
Source :- `https://earthsky.org/space/`

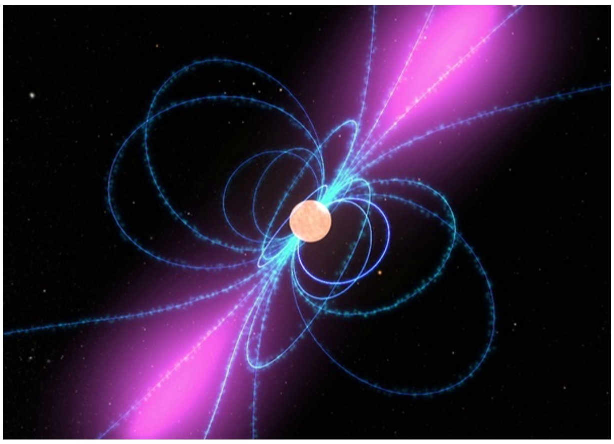

Figure 6: A rotating pulsar. **Source**:- `https://astronomy.com/news/2018/02/`

4.2 Revolving Binary Systems

Revolving binary systems are a very high energy systems which are formed when two massive objects orbit around their common centre of mass called barycenter. Average total mass of such systems are usually greater than 30 solar masses. And average loss of energy per second by such systems will be every huge which will be converted to gravitational waves. Such systems create gravitational waves by a mechanism called 'Inspiral' [**van_der_Sluys_2008**]. There are four phases in this mechanism.

Interlocking phase :- This the longest phase where the bodies come closer and get interlocked by their gravity and start to revolve each other around the barycentre.

Spiral phase :- Here the objects start getting close as they revolve. Due to the decrease in the distance between them the orbital energy is decreased and this energy is radiated as gravitational wave. But as they come closer and closer, they loose more and more energy, thus the intensity of gravitational wave increases.

Merger phase :- During this phase, the bodies collide by producing immense gravitational waves and merge [**article**].

Ring-down phase :- Finally, the merged bodies become stable and the gravitational wave intensity decreases exponentially and they stop producing gravitational waves.

Revolving binary systems produce compact binary inspiral gravitational waves. This is because the intensity of gravitational waves increases slowly during interlocking phase, and exponentially during the spiral phase, then it reaches a peak in merger phase, and finally it decreases rapidly to zero during ring-down phase. The detectors are capable of recording the signal only for a small range of frequency, So in this wave form, the frequency comes to the detecting range and rapidly goes out of range. Thus the signal strength suddenly increases and stops.

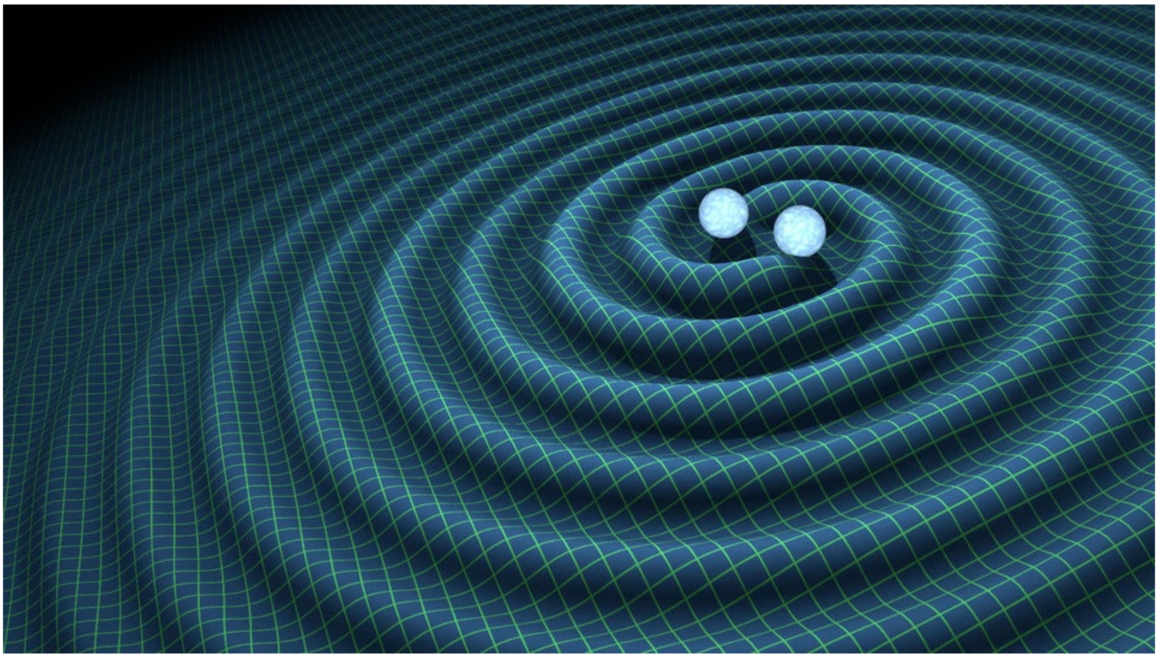

Figure 7: Two massive bodies in inspiral mechanism creating gravitational waves
Source :- https://www.scientificamerican.com/article/gravitational-waves-discovered-from-colliding-black-holes1/

4.2.1 Binary Black Holes (BBH)

Black holes are massive objects that can warp space-time extensively. If two black holes get closer and start the inspiral mechanism ,they create ripples in space-time and radiate gravitational waves. Such gravitational waves were the first ones to be detected by LIGO in 2015, September 14th. It was estimated that the collision occurred 1.3 billion years ago, thus the merger occurred 1.3 billion light-years away. This merger was named as '**GW150914**' meaning Gravitational Wave on 15/09/14. This signal lasted for about half a second.

4.2.2 Binary Neutron Stars (BNS)

Neutron stars are dense stars formed by the remnants when a massive star explodes as Supernova. So when two neutron stars merge through inspiral mechanism, they can radiate gravitational waves. First BNS merger was detected on 17th August 2017 and this was named as '**GW170817**', where the merger was analyzed both by Electromagnetic waves (Gamma ray) and gravitational waves. The signal lasted for comparatively longer duration for about 100 seconds, thus the mass was estimated to be lesser than black holes and was recognised as neutron star merge.

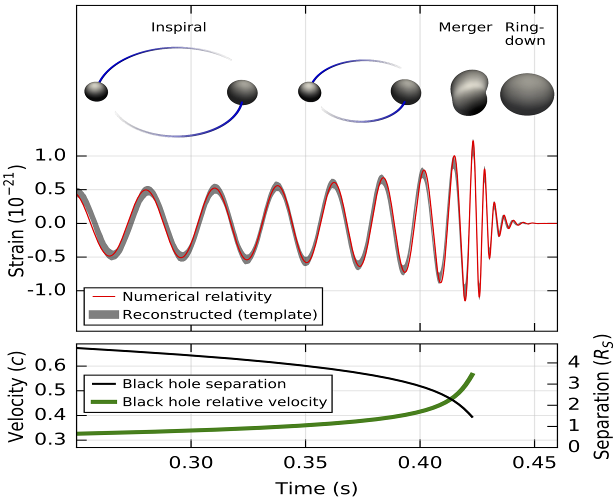

Figure 8: Characteristics of GW150914 Source:-`https://www.ligo.org/science/`

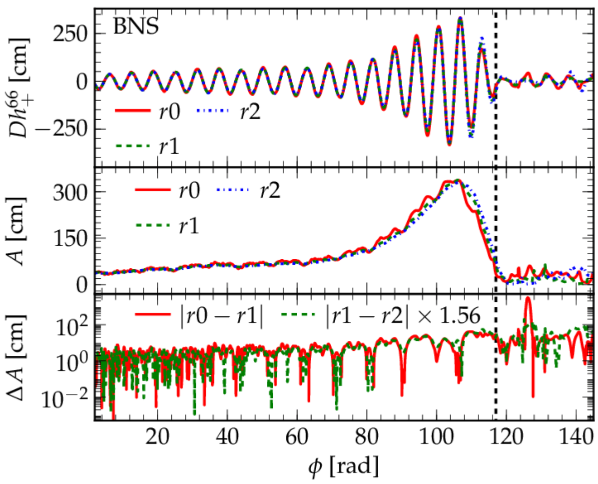

Figure 9: Characteristics of GW170817
Source:- `https://www.researchgate.net/publication/233846764`

4.3 Primordial Black Holes

Primordial Black Holes (PBH) are hypothetical black holes thought to have formed in the early universe due to the gravitational collapse of highly dense regions. Since these black holes don't have the usual star as a progenitor, their masses can be lower than the actual mass required for forming a normal black hole. They were first theorized by Yakov Barisovich Zel'dovich and Igor Dmitriyevich Novikov in 1966, and theories of their origin were studied by Stephen Hawking in 1971. [**PBH_defn**]

4.3.1 PBH and Stochastic Gravitational Waves

In the early universe, quantum fluctuations made the inflaton field highly unstable and non-uniform. In rare cases, these fluctuations might have spiked high enough to form energetic peaks which then collapse to form PBH's. This phenomenon would also result in the generation of a stochastic GW (discussed in the next section) background. But, to form such a background, a sufficient number of PBH's are required, which is only possible if the amplitude of the fluctuations are high enough at small scales. [**Nakama_2017**]

Related with PBH and stochastic GW is the concept of cosmic horizon reentry. As the universe expands, represented by the scale fact a, comoving length scales[1] between two objects grow along with it. During inflation, a grows exponentially, $a(t) \sim e^{Ht}$, where H is the Hubble constant. But, the horizon stays nearly constant during inflation. Now, the quantity $aH = \dot{a}$ tells us, through its variations in time, whether the comoving length scales grow at a greater or lesser rate than the horizon. During inflation, $a\dot{H} = \ddot{a} > 0$, so the comoving scales grow larger than the Hubble horizon, and after inflation, $a\dot{H} = \ddot{a} < 0$, so the horizon overtakes the comoving lengths, thus an object which was moved outside the horizon during inflation is back inside the horizon. This phenomenon is called Cosmic Horizon Reentry.

The survival of oscillation modes of the inflaton field depend on this phenomenon. The modes which leave the horizon and reenter it undergo a 'classical-to-quantum' transition, which transform them into curvature perturbations. These are the perturbations which have the chance to get highly energetic and yield PBH and the stochastic GW. The modes which never leave the horizon don't undergo the transition, so they don't have a major impact on the inflaton field. [**CHE**]

4.3.2 PBH and Gravitational Wave Bursts

If in a small region, numerous curvature perturbations collapse, then a cluster of PBH could form. They dynamics of such PBH is completely different from that of binary PBH systems. Instead of ending up in traditional bound systems and spiral in, majority of PBH's in a cluster would produce a single scattering event via a hyperbolic encounter, only if their relative velocity or relative distance is high enough to escape getting captured into bound systems. Such events would produce bursts of GW's, which can be detected up to several Gpc.

In hyperbolic encounters, majority of the energy is released near the closest approach. This has a characteristic peak frequency, which mainly depends on the impact parameter b, the eccentricity e and the total mass of the system M. These encounters has a duration of the order of a few milliseconds to several hours.

GW's from such encounters have very different properties and signature when compared to those from traditional binaries, so detecting them would strengthen the possibility of the existence of PBH. [**Garc_a_Bellido_2017**]

[1] Comoving length scales/distances are the measure of distances between fundamental observers, i.e, observers that are moving with the expansion of the universe (Hubble Flow), and doesn't change with time.

5 Types of Gravitational Waves

6 Why study Gravitational Waves

7 Indirect Evidences of Gravitational Waves

8 Direct search for Gravitational waves

9 Detection of Gravitational waves using LIGO

10 Conclusion