

Introduction of theory of computation 07:-

Automata theory (also known as Theory of Computation) is a theoretical branch of Computer science & mathematics, which mainly deals with the logic of Computation with respect to simple machines, referred to as automata.

Automata enables the scientists to understand how machines Compute the functions and solve problems. The main motivation behind developing automata theory was to develop methods to describe and analyse the dynamic behavior of discrete systems.

Automata is originated from the word " Automaton" which is closely related to " Automation".

The basic terminologies which is closely related to Automation or automata theory are as follows:

Symbol :- Symbol is the smallest building block, which can be any alphabet, letter or any picture.

e.g:- a, b, c, 0, 1, -- ☺ ☻ ☻

08 Alphabets (Σ) :- Alphabets are set of symbols which are always finite.

e.g. - $\Sigma = \{0, 1\}$ is an alphabet of binary digits.

$\Sigma = \{0, 1, \dots, 9\}$ is an alphabet of decimal digits.

$\Sigma = \{a, b, c\}$

$\Sigma = \{A, B, C, \dots, Z\}$

String :- String is a finite sequence of symbols from some alphabet. String is generally denoted as w and length of a string is denoted as $|w|$.

Empty string is the string with zero occurrence of symbols represented as ϵ (Epsilon)

e.g. - The number of strings that can be generated over the alphabet $\{a, b\}$ of length 2 is $\{aa, ab, ba, bb\}$.

Note! - If the number of Σ 's is represented by $|\Sigma|$, then number of strings of length ' n ', possible over Σ is $|\Sigma|^n$

Language! - A language is a set of strings, chosen from some Σ^* . A language is a subset of Σ^* . A language which can be formed over Σ can be finite or infinite.

eg:- $L_1 = \{ \text{Set of strings of length } 2 \}$
= { aa, ab, ba, bb } 09 \rightarrow Finite language.

$L_2 = \{ \text{Set of strings which starts with 'a'} \}$
= { a, aa, ab, aba, aaa, abb, ... } \rightarrow which
is infinite language.

Theory of Computation is normally divided in 3 parts:

- i) Automata Theory and languages.
- ii) Computability theory
- iii) Complexity theory.

Automata theory :- It deals with the definitions & properties of mathematical models of computation. The finite automata used in text processing, compilers & hardware design. The context free grammars are used in programming language & artificial intelligence.

Computability theory :- In the first half of the 20th century, mathematicians such as Kurt Gödel, Alan Turing, and Alonzo Church discovered that certain basic problems

Cannot be solved by computers

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mathematical statement is true or false.

Computability theory classify problems are solvable, not solvable. Complexity theory classify problems as easy ones & hard ones.

Complexity theory :- it deals with easy problems (sort a million items in a few seconds). Hard problems (schedule a thousand classes in a hundred years). What makes some problems hard and others easy computationally?

Complexity theory addresses this question.

Why to study the theory of computation :-

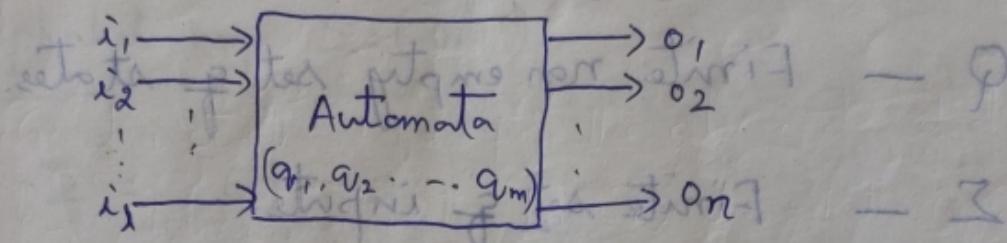
- i) determine what can and cannot be computed.
- ii) How quickly & with how much memory.
- iii) on what type of computational model.

Theory of Computation

Chapter - 1: Introduction.

Automata : - it is a system, used to perform computation without the participation of human. It is generally a mathematical model.

$$(A, \Sigma, \delta, Q, P) = M$$



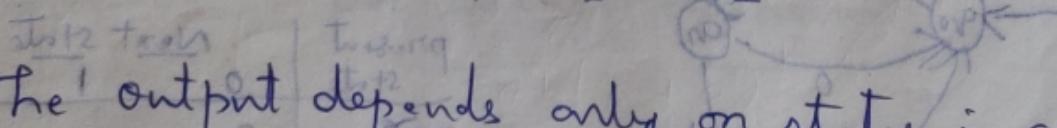
$$\text{Inputs} = \Sigma = \{i_1, i_2, \dots, i_n\} \text{ where } i_j \in \Omega$$

$$\text{states} = Q = \{q_1, q_2, \dots, q_m\} \quad \delta: Q \times \Sigma \rightarrow Q$$

$$\text{outputs} = \Delta = \{o_1, o_2, \dots, o_n\}$$

The output depends only on one input, is called as Automata without memory. (Independent of states)

$$\text{ex: } o_1 \rightarrow i_1, \quad o_2 \rightarrow i_2, \quad \dots$$



The output depends only on states, is called as Moore Machine. (Independent of inputs)

$$\text{ex: } o_1 \rightarrow q_1, \quad o_2 \rightarrow q_2, \quad \dots$$

08 The output depends upon both states & inputs
called as Mealy machine.

$$e.g.: \quad o_1(\delta, i_1), \quad o_2(\delta, i_2)$$

Finite Automata - it is a 5 tuple system.

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q - Finite non empty set of states.

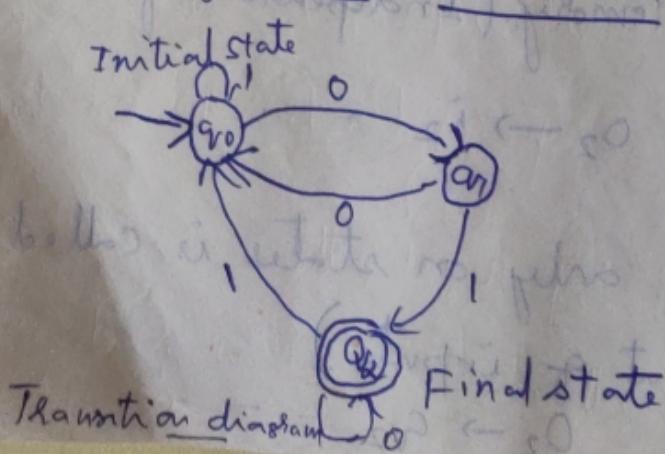
Σ - Finite set of inputs.

δ - transition function (change of state during transaction) $\delta: Q_1 \times \Sigma \rightarrow Q_2$

q_0 - initial state / starting state.

F - set of final states. $F \subseteq Q$

Representation of finite automata by transition diagram.



Transition Table - 8.9

present state	0	1
q_0	q_1, q_2	q_0
q_1	q_0, q_3	q_2
q_2	q_2	q_0

09

representation by 5 tuples - works b/w AFD

$$AFM = \left(\overline{Q}, \Sigma, \delta, q_0, \{q_f\} \right)$$

$$\delta(q_0, 0) = q_1, \quad \delta(q_0, 1) = q_0$$

$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_2$$

$$\delta(q_2, 0) = q_2, \quad \delta(q_2, 1) = q_0$$

Input alphabet :- $\Sigma = \{0, 1\}$

$$\Sigma^0 = |\emptyset| = 0$$

$$\Sigma^1 = \{0, 1\}$$

$$\Sigma^2 = \{00, 01, 10, 11\}$$

$$\Sigma^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$$

$$\Sigma^k = \{\text{Set of all strings over } k\}$$

between ends of AFD all priwords set A - : upto

Constructing DFA addressed type all based, criteria

Type-01 :- For strings ending with a particular sub

string

strings accepted by AFD all words - : I make req

Step 1 :- Decide the minimum number of states required

in the DFA and draw them.

Rule :- All strings ending with 'n' length substring always require minimum $(n+1)$ states in this DFA.

Example :- Consider the regular expression -

$$(a+b)^*z$$

The string of above language will always end on 'z', so if $|z|=n$, then the DFA requires minimum $(n+1)$ states.

Step 2 :- Decide the strings for which you will construct the DFA.

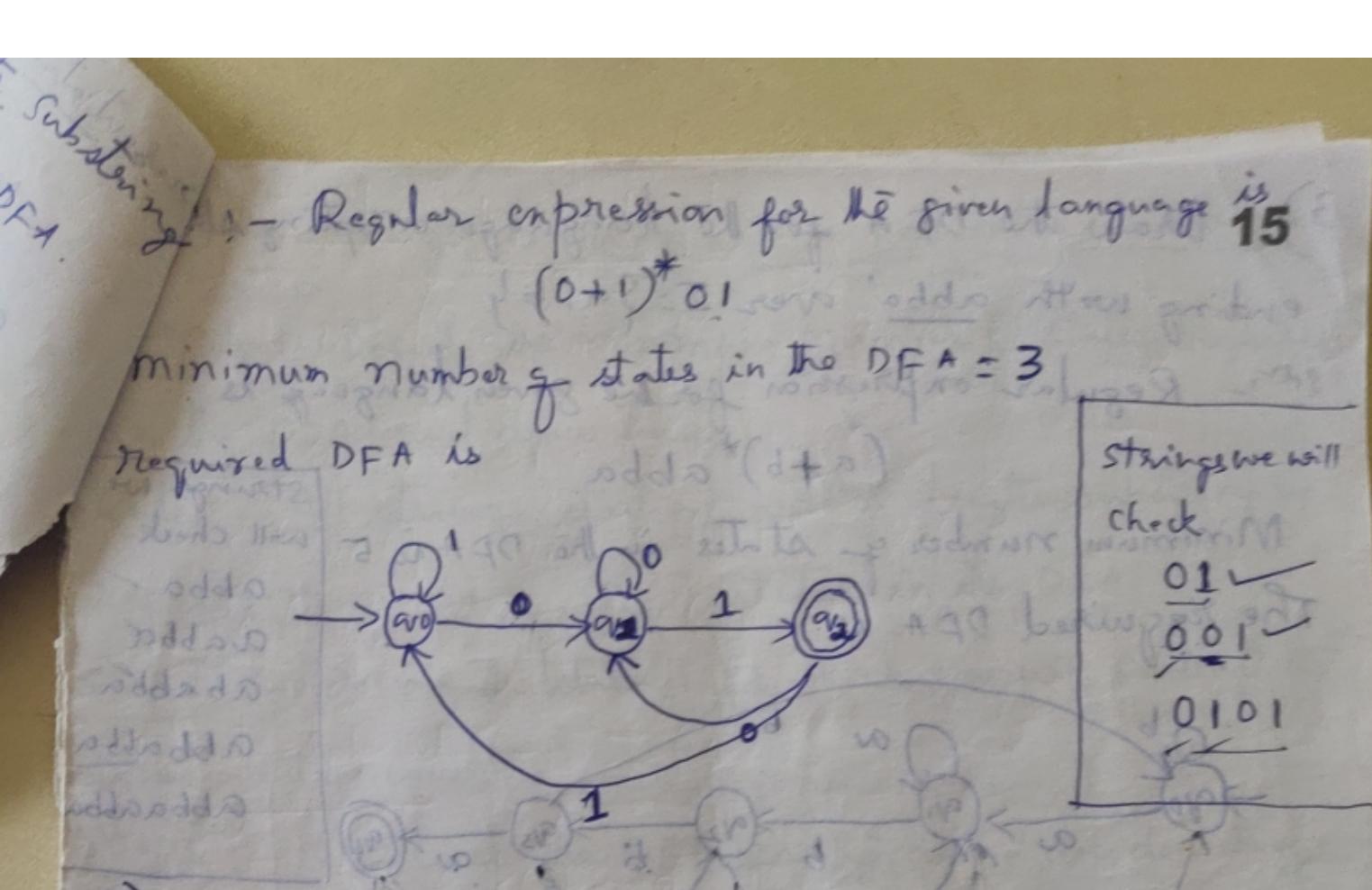
Step 3 :- Construct the DFA for the above decided string.

Remember :- always prefer to go with the existing path.

Create a new path only when you can't find a path to go with.

Step 4 :- After drawing the DFA for the above decided strings, send the left possible combinations to the starting state not over the dead configurations.

Problem 1 :- Draw the DFA for the language accepting strings ending with '01' over the input alphabet $\Sigma = \{0, 1\}$.

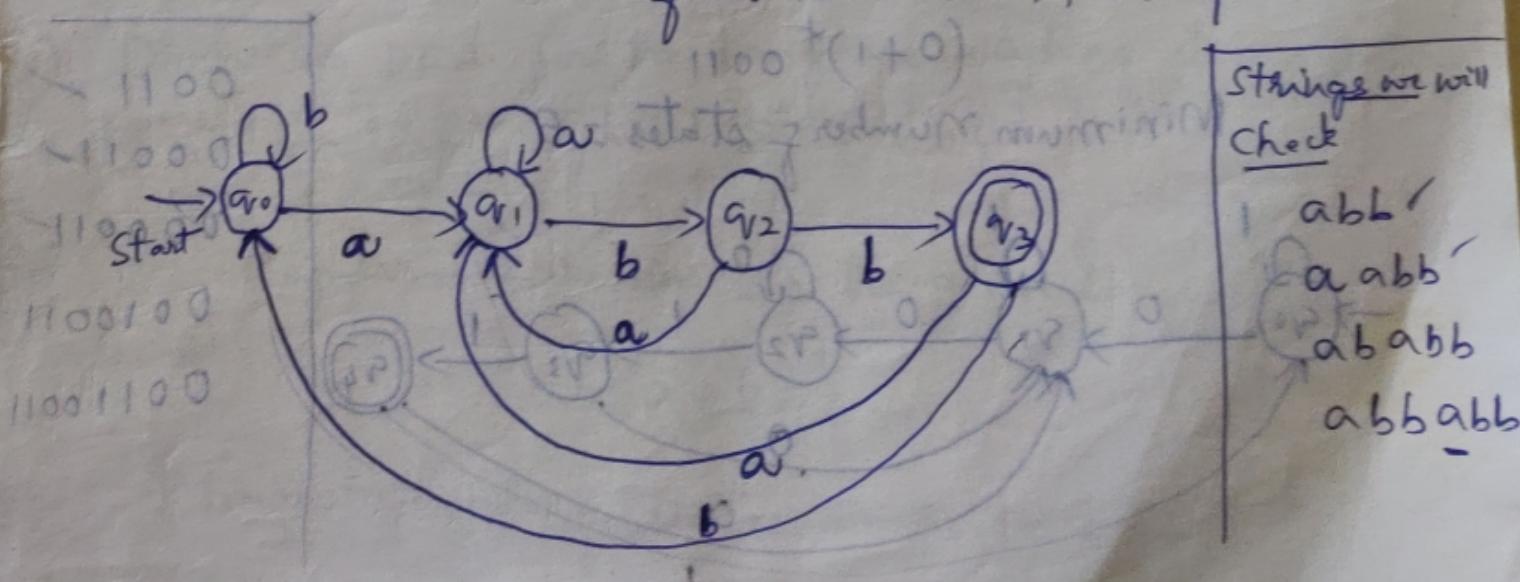


2) Draw the DFA for the language accepting strings ending with abb over the input alphabet $\Sigma = \{a, b\}$

Sol' :- Regular expression for the given language is

$(a+b)^* abb$

The minimum number of states in the DFA = 4

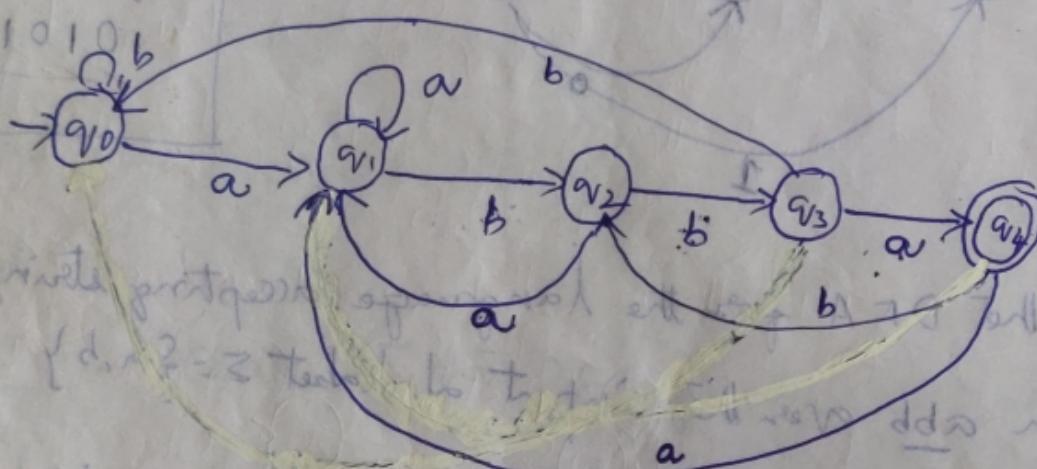


3) 16 Draw the DFA for the language accepting strings ending with abba over $\Sigma = \{a, b\}$

Sol:- Regular expression for the given language is

$$(a+b)^* abba$$

Minimum number of states in the DFA is 5
The required DFA is

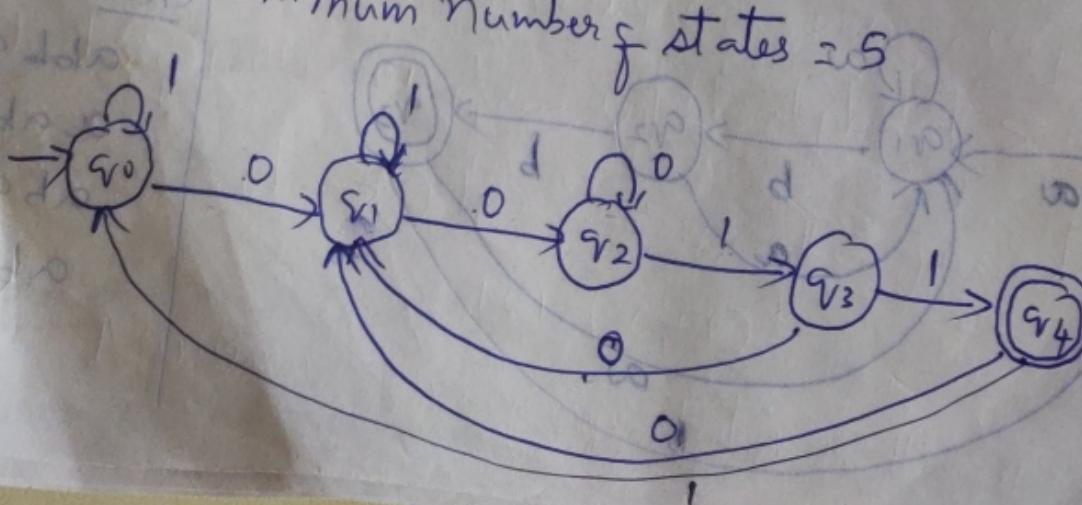


strings we will check
abba
aabba
ababba
abbabba
Abbaabba

4) Draw the DFA for the language accepting strings ending with '0011' over $\Sigma = \{0, 1\}$

$$(0+1)^* 0011$$

Minimum Number of states = 5



0011
00011
000011
0010011
00110011

For strings starting with a particular substring 17

Step 1 :- Decide the minimum number of states required in the DFA and draw them.

Rule :- All strings starting with n length substring will require minimum of $(n+2)$ states in its DFA.

Step 2 :- Decide the strings for which you will construct the DFA.

Step 3 :- Construct the DFA for the above decided strings.

Remember :- Always go with the existing path. Create a new path only when you can't find a path to go with.

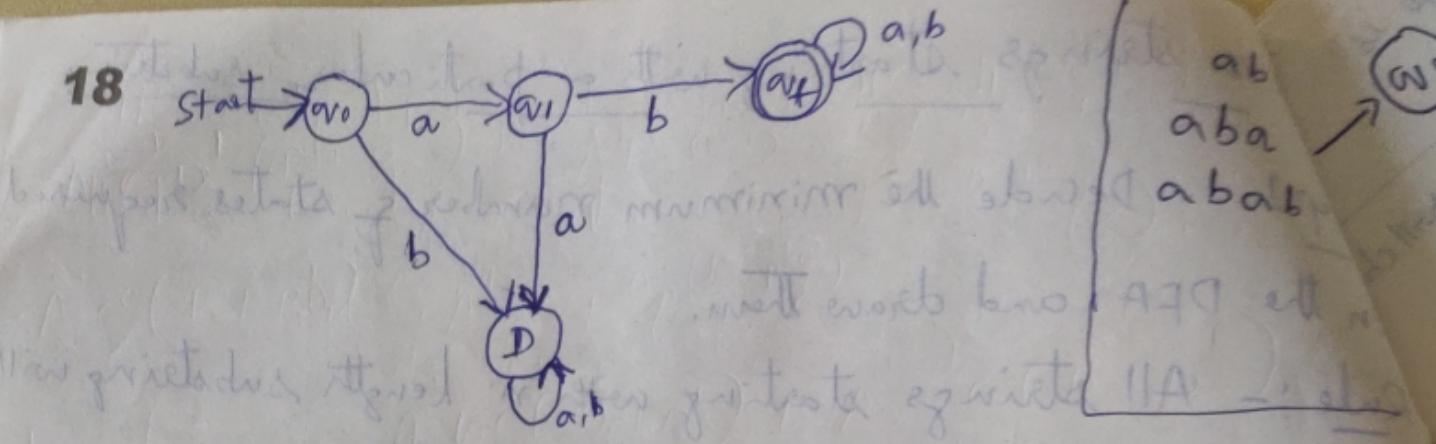
Step 4 :- After drawing the DFA for the above decided strings, send the left possible combinations to the dead state not over the starting state.

i) Draw the DFA for the language accepting strings starting with ab over input alphabet $\Sigma = \{a, b\}$

Sol :- Regular expression for the given language is
 $ab(a+b)^*$

Minimum number of states in the DFA = 4

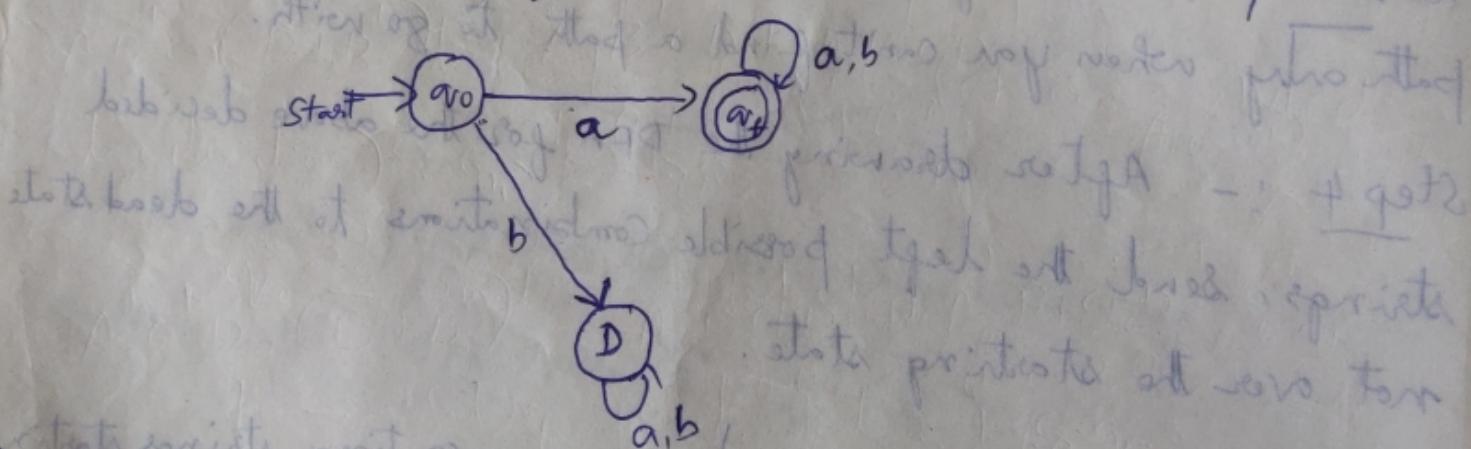
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- 2) Draw the DFA for the language accepting strings starting with 'a' over input alphabet $\Sigma = \{a, b\}$.

Sol :- Regular expression for the given language is $a(a+b)^*$

Minimum number of states in the DFA = 3

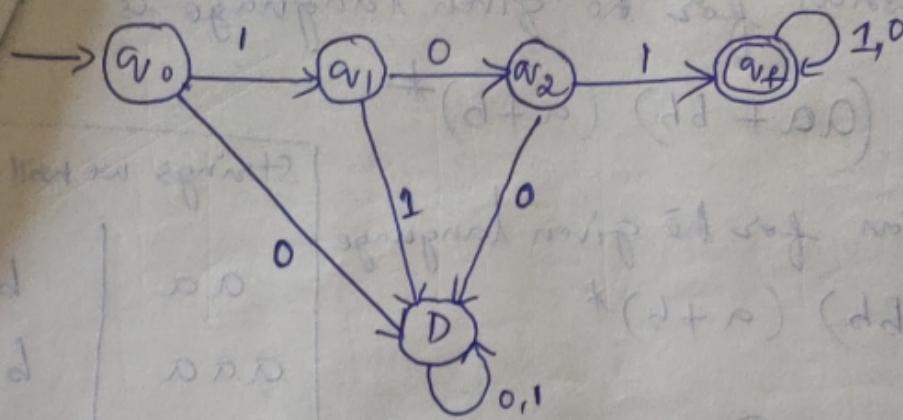


- 3) Draw the DFA for the language accepting strings starting with '101' over input alphabet $\Sigma = \{0, 1\}$.

Sol :- Regular expression for the given language is $101(0+1)^*$

Minimum number of states in the DFA = 5

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101

1011

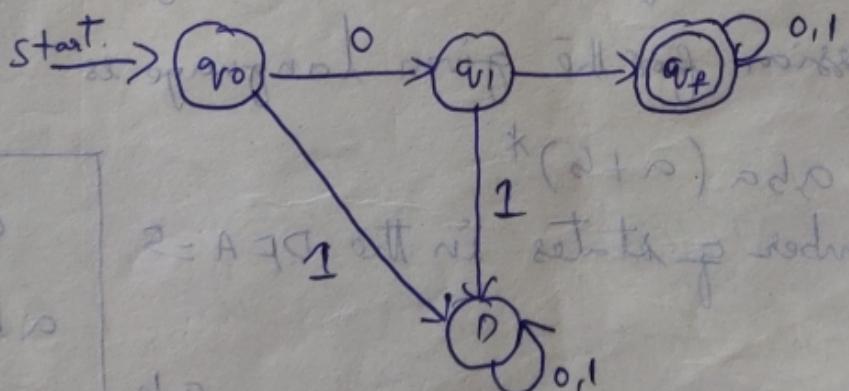
10110

101101

4) Construct a DFA that accepts a language L over $\Sigma = \{0, 1\}$ such that L is the set of all strings starting with '00'.

Regular expression for the given language is

Minimum number of states in the DFA = 4



Strings we will check
00

0000
0000

5) Construct a DFA that accepts a language L over $\Sigma = \{a, b\}$ such that L is the set of all strings starting with 'aa' or 'bb'.

Regular expression for the given language is
08

$$(aa + bb)(a+b)^*$$

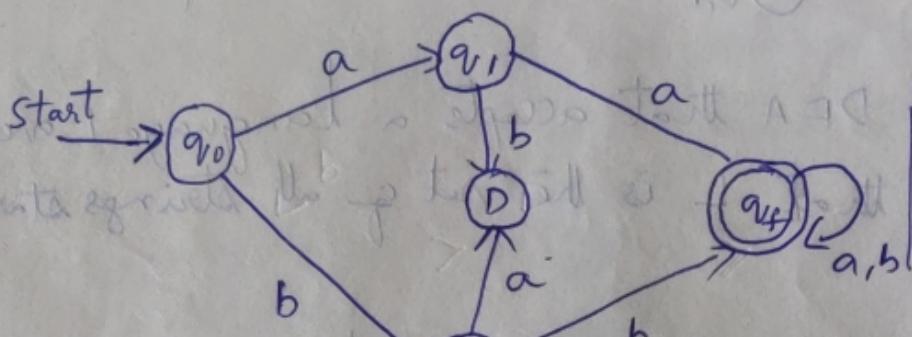
Regular expression for the given language
is $(aa + bb)(a+b)^*$

Strings we will ch

aa bb

aaa bbb

aaaa bbbb

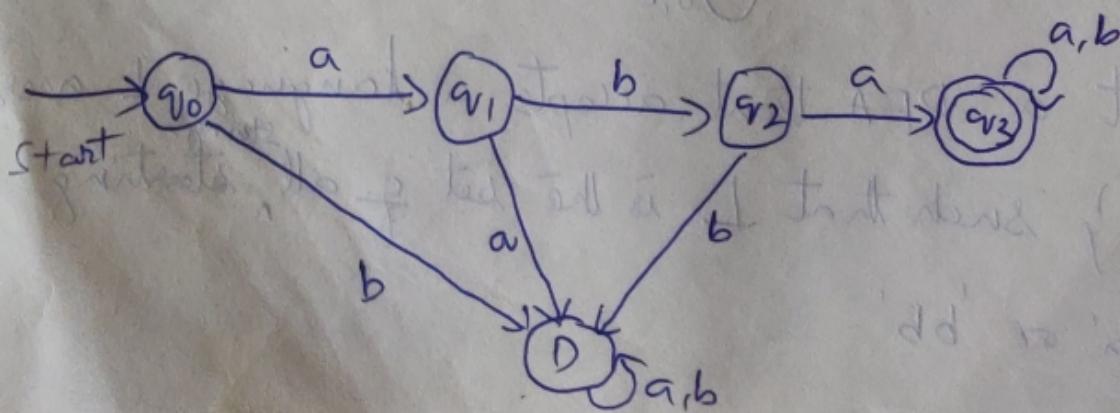


6) Draw the DFA that accepts a language L over $\Sigma = \{a, b\}$ such that L is the set of all strings starting with 'aba'

Regular expression for the given language is

$$aba(a+b)^*$$

Minimum number of states in the DFA = 5



aba

a ba a

a ba ab

a ba aba

Design the DFA that accepts a language L over

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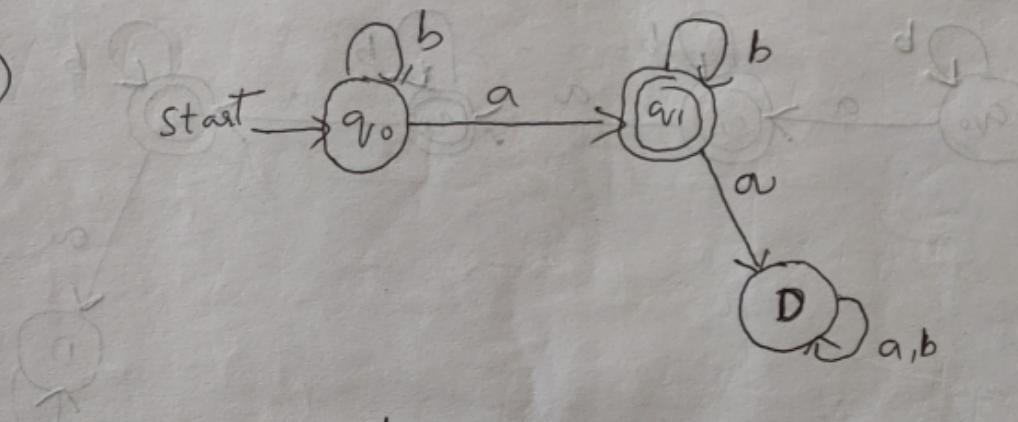
= {a, b} having

i) exactly one 'a'

ii) At least one 'a'

iii) At most one 'a'

i)



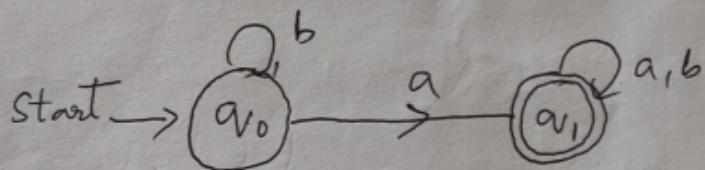
strings

bab ✓

babb ✓

baax

ii)



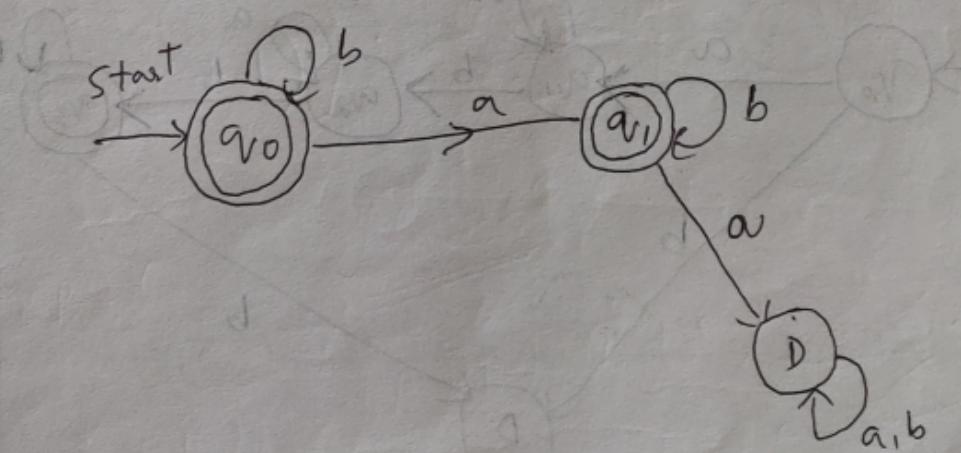
strings

bba ✓

baba ✓

bbb x

iii)



strings

b ✓

bab ✓

babab x

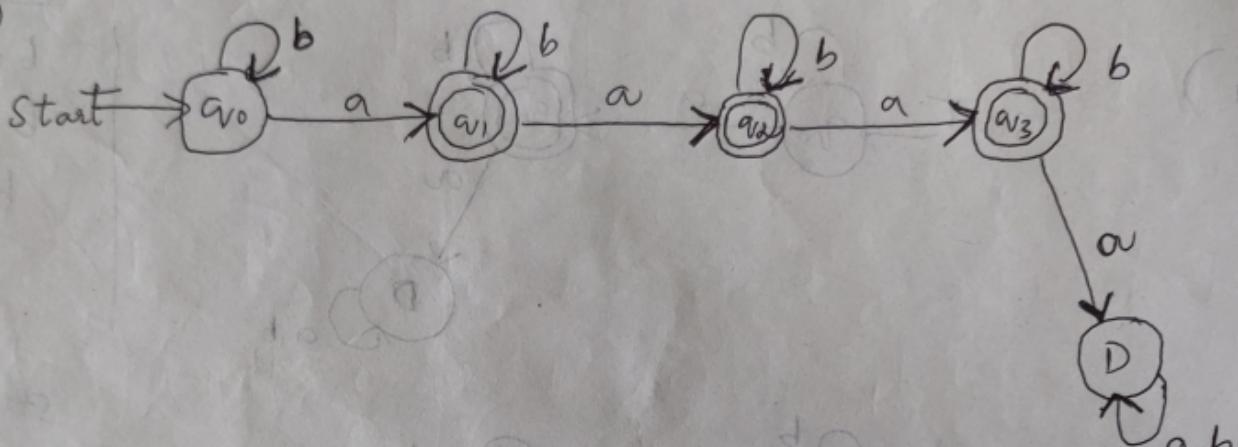
8) 14 Design the DFA that accepts a language L Construct

$\Sigma = \{a, b\}$ such that L is the set of strings

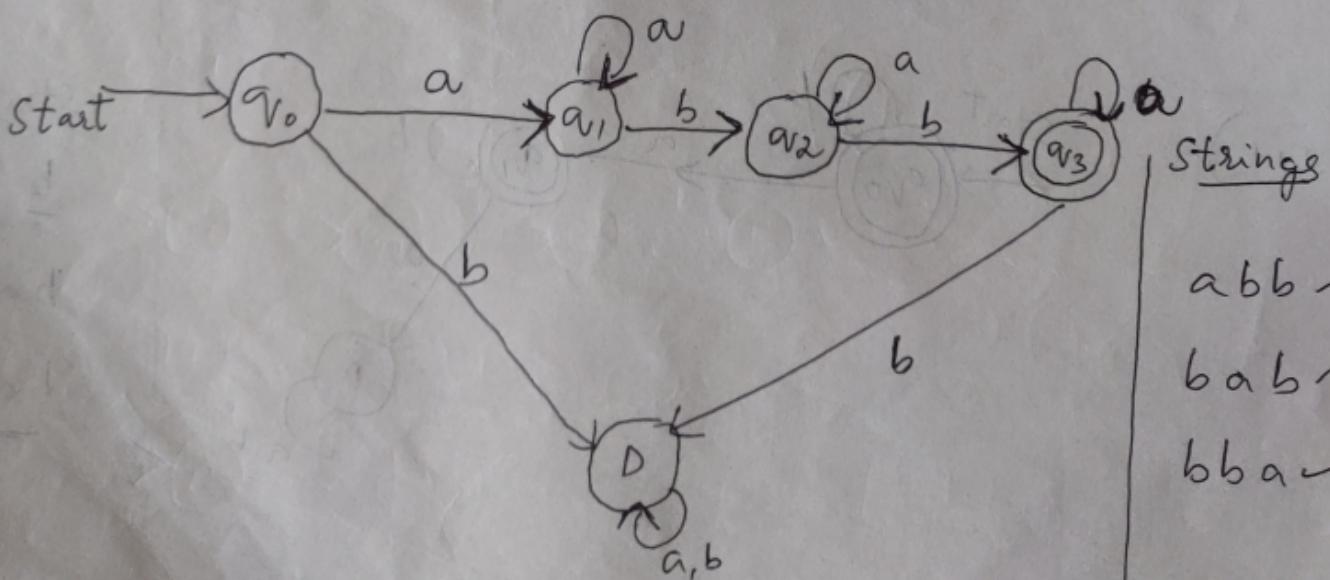
i) No more than '3a' or at most '3a'

ii) with at least 'a' & exactly two b's

i)



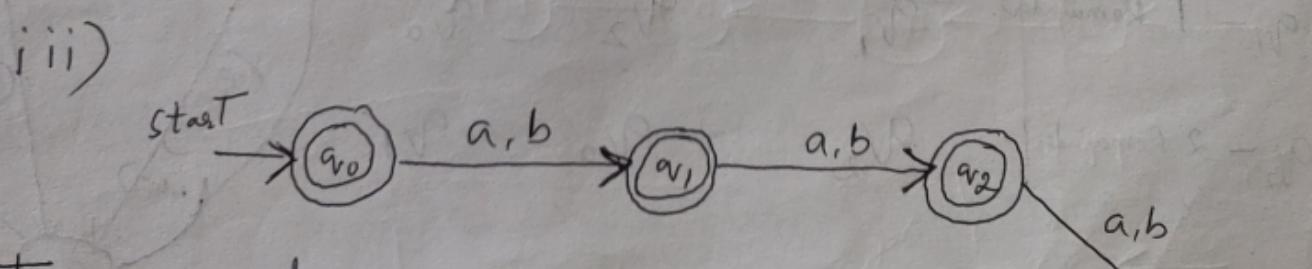
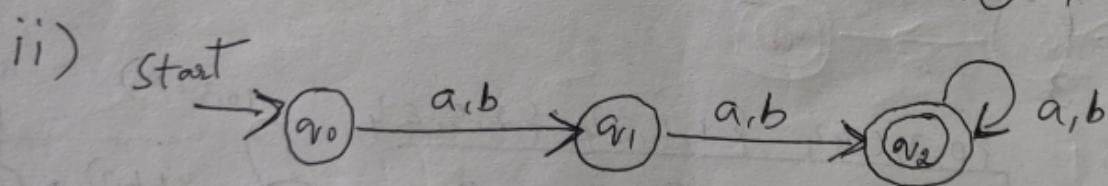
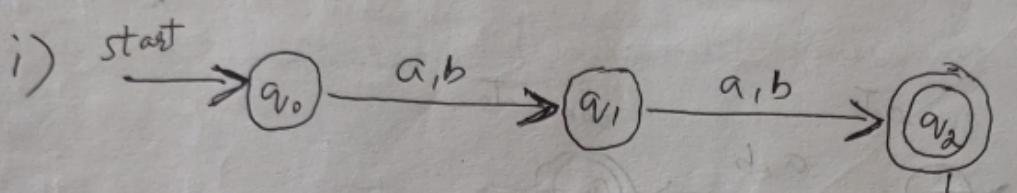
ii)



Construct a DFA that accepts a language L over $\{a, b\}$ such that L is the set of strings

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- The length of the string is exactly 'two'.
- length of the string is at least 'two'
- length of the string at most 2



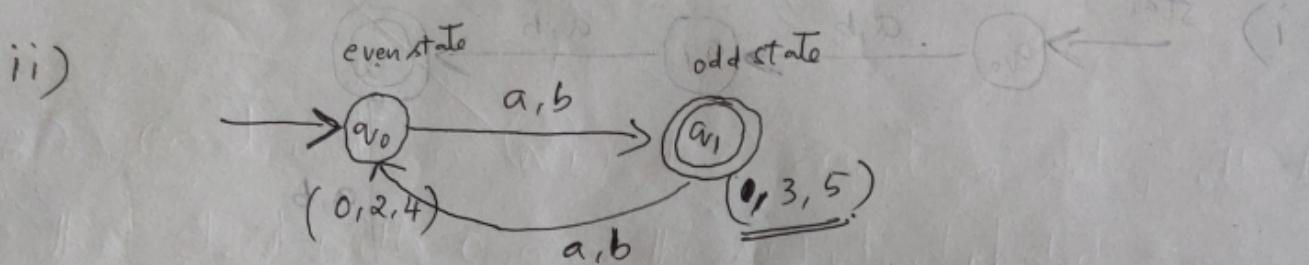
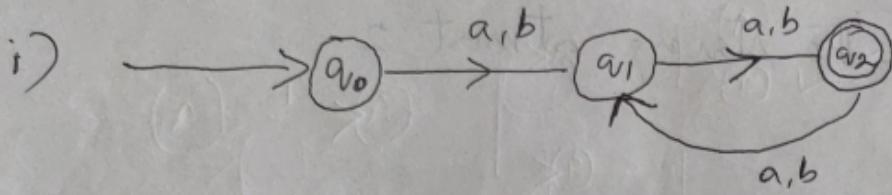
States required: exactly two $\rightarrow n+2$

: at least two $\rightarrow n+1$

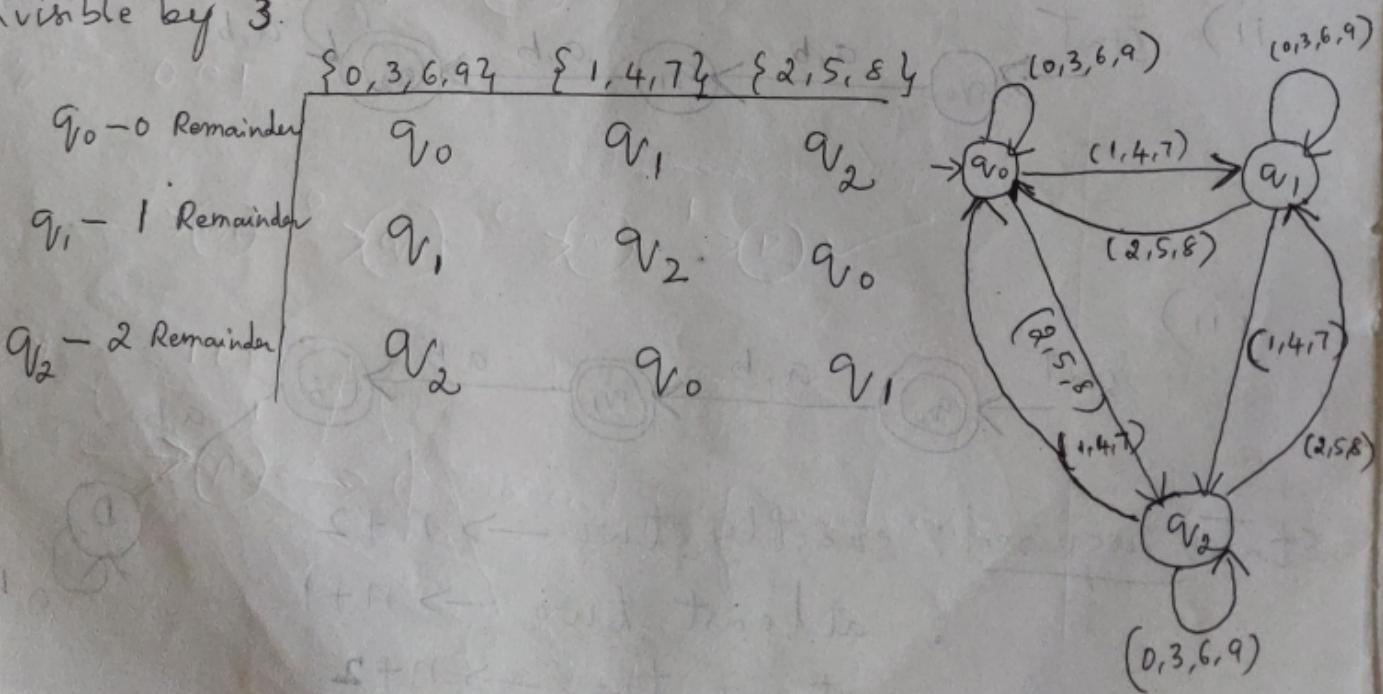
at most two $\rightarrow n+2$

- 10) Construct the DFA that accepts strings over input alphabet $\Sigma = \{a, b\}$ such that i) $|w| \bmod 2 = 0$
16 ii) $|w| \bmod 3 = 1$

$L = \{\epsilon, aa, ab, ba, bb, aaaa, bbbb, \dots\}$



- ii) Construct the DFA that accepts string over the input alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that L is divisible by 3.



Deterministic Finite Automata :- NFA is similar 17

- DFA except following additional features:
- i) Null (or ϵ) move is allowed i.e., it can move forward without reading symbols.
- ii) Ability to transmit to any number of states for a particular input.

NFA has a different transition function, rest is same as

DFA

$$\delta : Q \times (\Sigma \cup \epsilon) \rightarrow 2^V$$

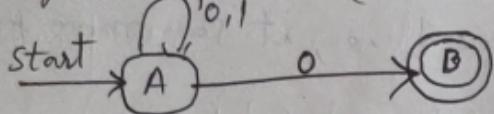
One important thing to note is, In NFA, if any path for an input string leads to a final state, then the input string is accepted.

Some important points :

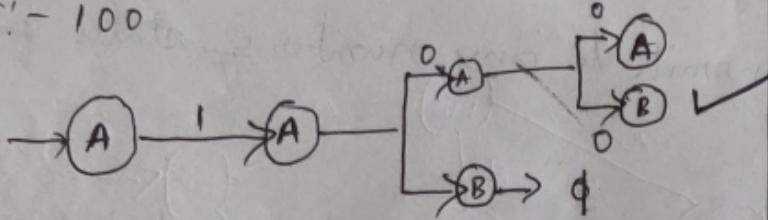
1. Every DFA is NFA, but not vice versa
2. Both NFA and DFA have same power and each NFA can be translated into a DFA.
3. There are multiple final states in both DFA & NFA
4. NFA is more of a theoretical concept.
5. DFA is used in lexical analysis in compiler.

1) Construct a NFA that accepts a set of all strings that end with 0 over the input alphabet $\Sigma = \{0, 1\}$, or

Ans: Regular exp: - $(0+1)^* 0$



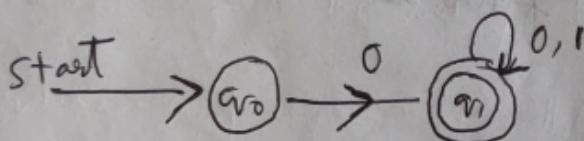
e.g.: - 100



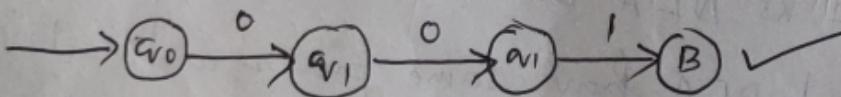
If there is any way to run the machine that ends in any set of states out of which at least one state is a final state, then the NFA accepts.

2) Construct a NFA that accepts a set of all strings that start with 0 over the input alphabet $\Sigma = \{0, 1\}$

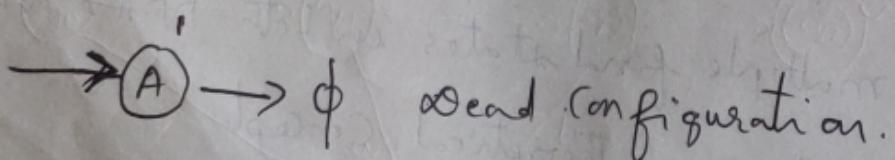
$$L = \{0, 00, 01, 000, 001, 010, \dots\}$$



e.g.: - 001

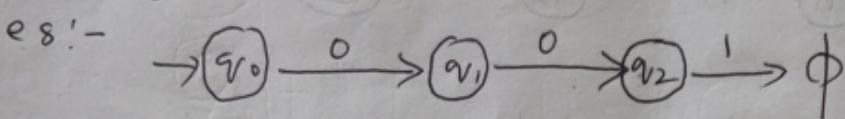
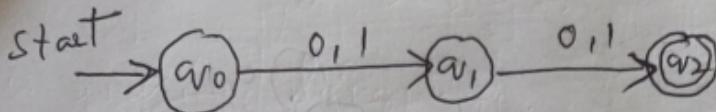


e.g.: - 101



Construct a NFA that accepts sets of all strings over $\{0, 1\}$ of length 2. 11

$$L_1 = \{00, 01, 10, 11\}$$



4) Construct a NFA for the following cases:

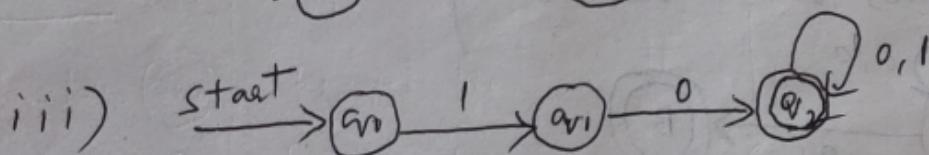
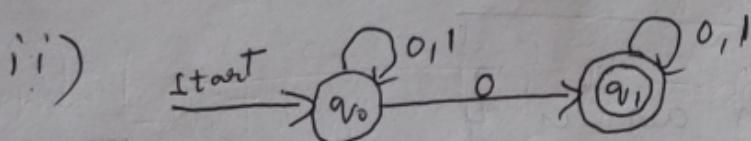
i) $L_1 = \{ \text{set of all strings that ends with '1'} \}$

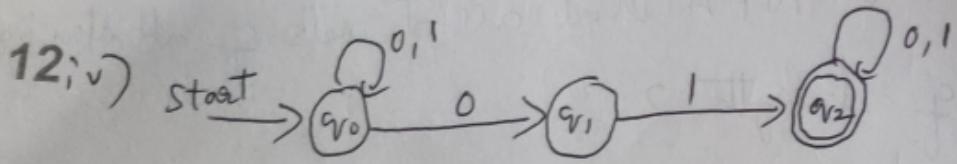
ii) $L_2 = \{ \text{set of all strings that contain '0'} \}$

iii) $L_3 = \{ \text{set of all strings that starts with '10'} \}$

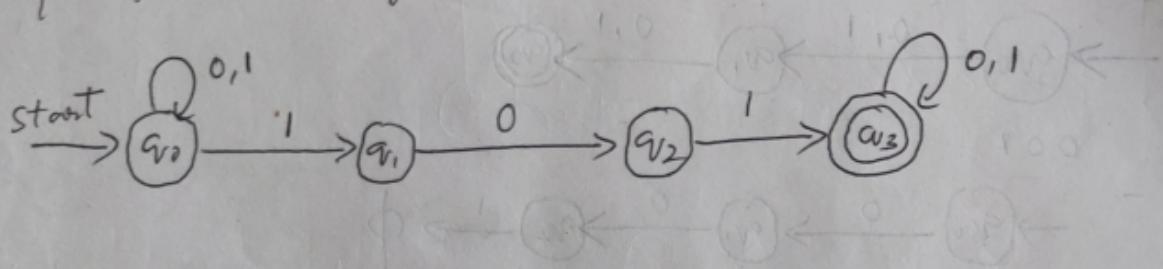
iv) $L_4 = \{ \text{set of all strings that contain '01'} \}$

v) $L_5 = \{ 1, 01, 001, 101, 111, 0101, \dots \}$

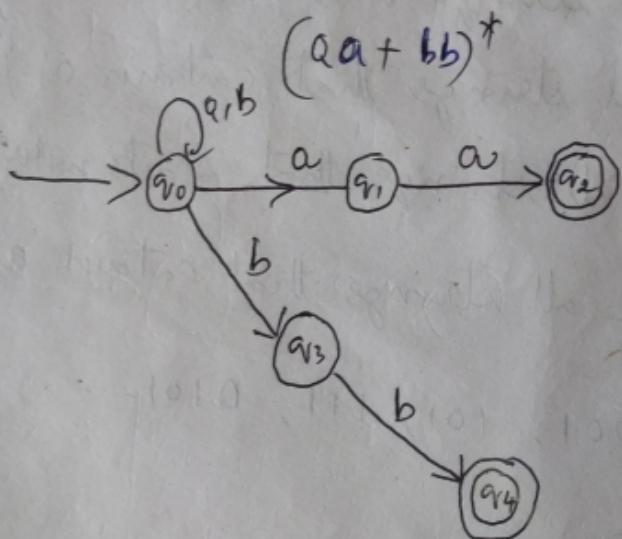




5) Construct a NFA that accepts a set of all strings over $\{0, 1\}$ containing '101' as a substring.

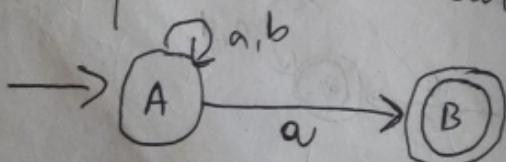


6) Construct a NFA that accepts set of all strings over $\{a, b\}$ ending with aa or bb



Conversion from NFA to DFA

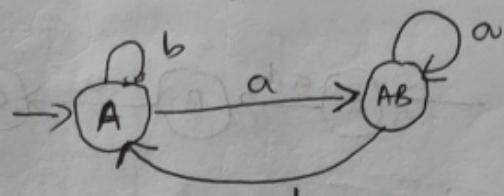
i) $L_1 = \{ \text{ends with an 'a'} \}$



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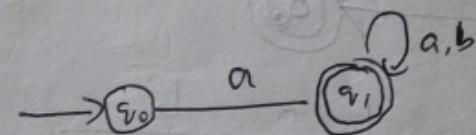
$\rightarrow A$	a	b	Representation of state transition table of NFA.
(B)	$\{\{A, B\}\}$	$\{\{A\}\}$	

$\rightarrow A$	a	b	Representation of state transition table of DFA.
(AB)	$[AB]$	$[A]$	



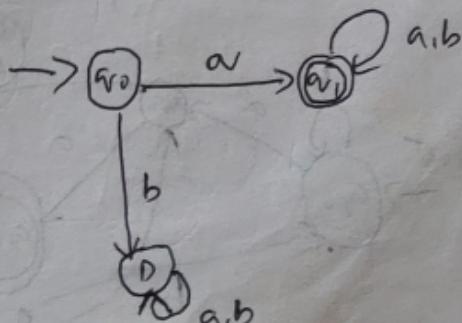
Equivalent of DFA

2) $L_1 = \{ \text{ starts with 'a'} \} \text{ over } \Sigma = \{a, b\}$

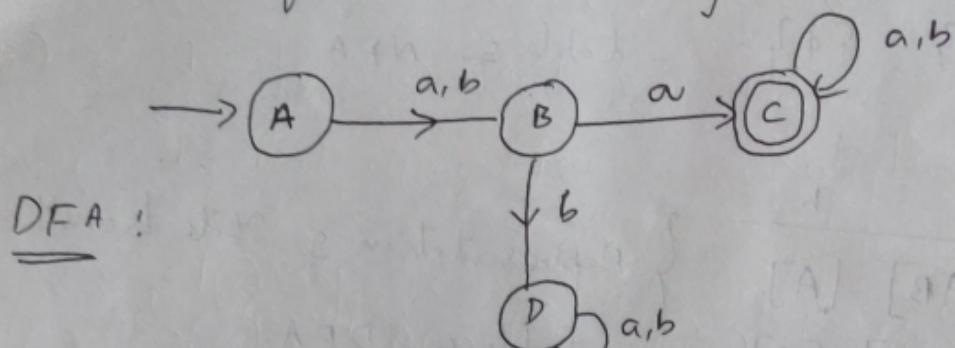


$\rightarrow q_0$	a	b
q_1	q_1	\emptyset
q_1	B	B

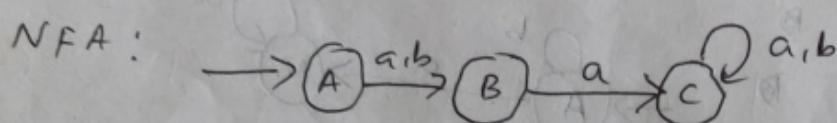
$\rightarrow q_0$	a	b
q_1	q_1	D
D	D	D



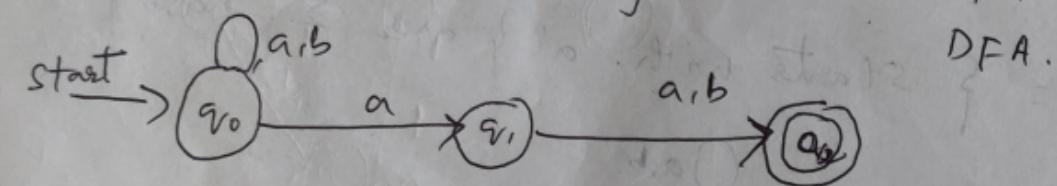
3) 14) $L_1 = \{ \text{set of all strings such that second symbol from LHS is 'a'} \}$. Represent DFA & NFA.



DFA :

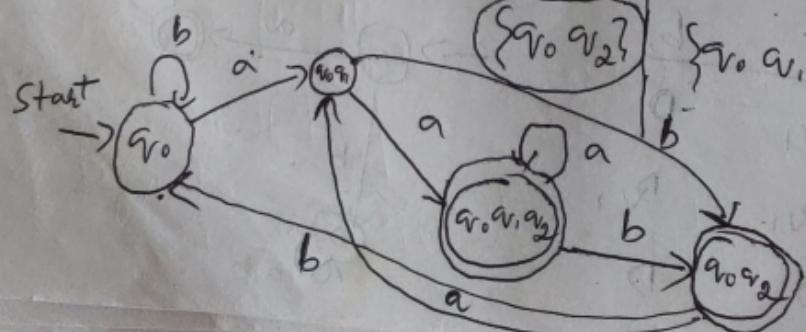


4) $L = \{ \text{set of strings such that second symbol from RHS is 'a'} \}$. Convert the NFA to equivalent DFA.



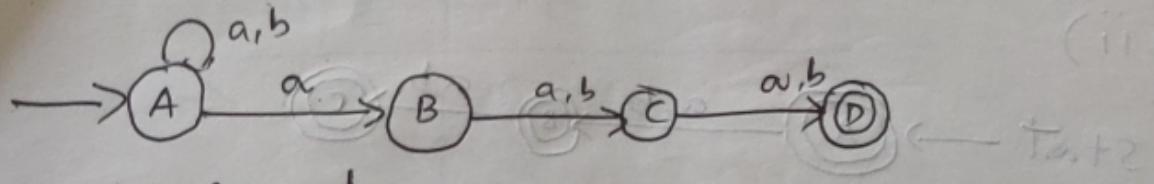
	a	b
q_0	$\{q_0, q_1\}$	q_0
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{\}$	$\{\}$

NFA $\rightarrow q_0 \{q_0, q_1\} \{q_0, q_2\} \{q_0, q_1, q_2\} \{q_0, q_2\} \{q_0, q_1\}$



$L = \{ \text{all strings in which third symbol from RHS is } a \}$

$L_1 = \{ \text{aaa, aab, aba, abb} \dots \}$

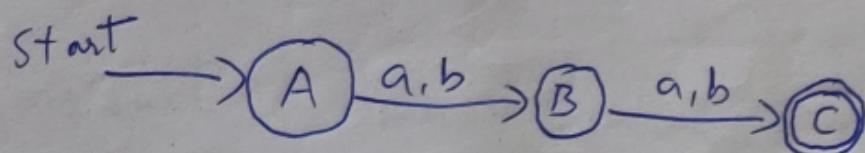


	a	b
A	{A, B}	{A}
B	{C}	{C}
C	{D}	{D}
D	{∅}	{∅}

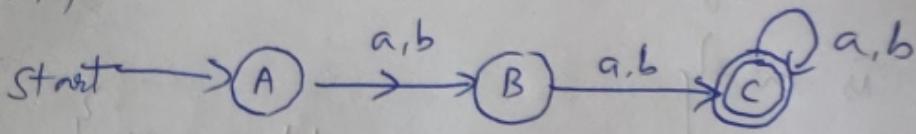
	a	b
[A]	[AB]	[A]
[AB]	[ABC]	[AC]
[Ac]	[ABD]	[AD]
([AD])	[AB]	[A]
[ABC]	[ABCD]	[ACD]
([ABD])	[ABC]	[AC]
([ACD])	[ABD]	[AD]
([ABCD])	[ABCD]	[ACD]

- 6) NFA for strings of length i) exactly 2
 ii) at least 2 iii) at most 2 over the language $\Sigma = \{a, b\}$

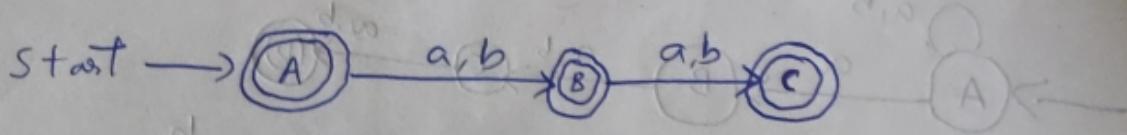
i) $L_1 = \{ ab, aa, ba, bb \}$



12 11)



i ii)



construct a DFA equivalent to $M = (\{q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta, q_1, \{q_4\})$ where δ is given below:

states / Σ	0	1
$\rightarrow q_1$	q_1, q_2	q_1
q_2	q_3	q_2
q_3	q_4	q_4
q_4	q_1	q_3

Ans:-

states / Σ	0	1
$\rightarrow [q_1]$	$[q_1, q_2]$	$[q_1]$
$[q_1, q_2]$	$[q_1, q_2, q_3]$	$[q_1, q_2]$
$[q_1, q_2, q_3]$	$[q_1, q_2, q_3, q_4]$	$[q_1, q_2, q_4]$
$[q_1, q_2, q_3, q_4]$	$[q_1, q_2, q_3, q_4]$	$[q_1, q_2, q_3, q_4]$
$[q_1, q_2, q_4]$	$[q_1, q_2, q_3]$	$[q_1, q_2, q_3]$

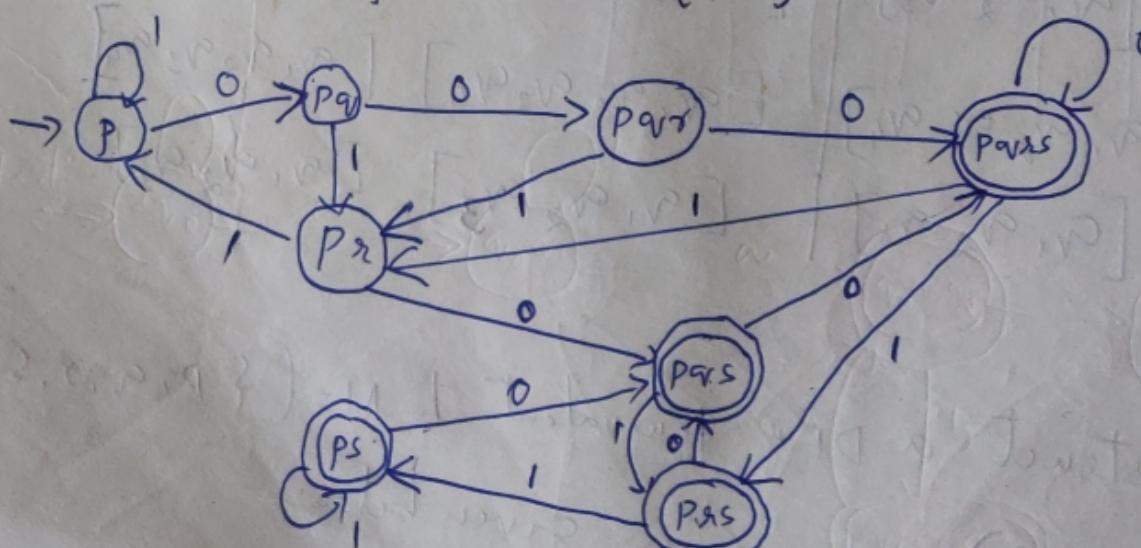
2) Construct a DFA equivalent to $M = (\{p, q, r, s\}, \{0, 1\}, \delta, p, \{s\})$ where δ is given below.

10

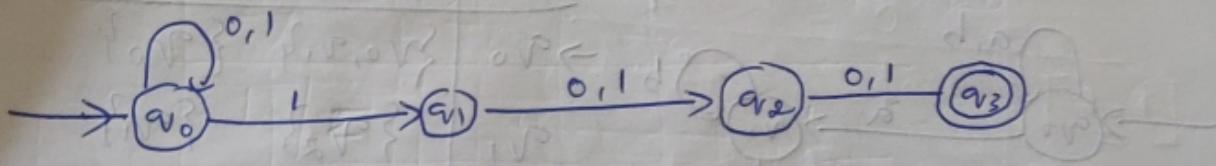
states	0	1
$\rightarrow P$	$\{P, q\}$	$\{P\}$
q	$\{q\}$	$\{q\}$
r	$\{s\}$	q
(S)	$\{s\}$	$\{s\}$

Ans:-

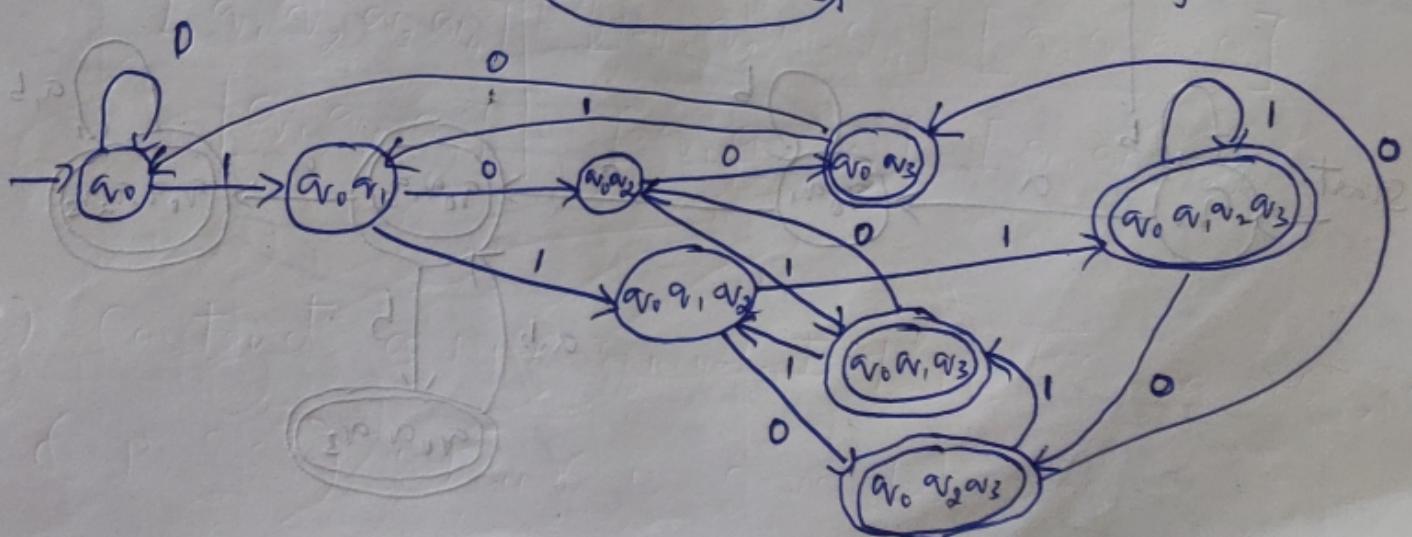
states	0	1
$[P]$	$[Pq]$	$[P]$
$[Pq]$	$[Pqr]$	$[Pr]$
$[Pqr]$	$[Pqrs]$	
$[Pr]$	$[Prs]$	$[P]$
$(Pqrs)$	$[Pqrs]$	
(Prs)	$[Prs]$	
(Pqr)	$[Pqrs]$	$[Pqs]$
(Ps)	$[Pqrs]$	$[Prs]$

Equivalent DFA

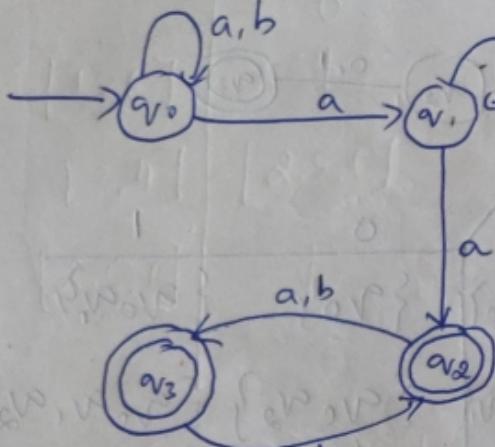
Convert NFA to equivalent DFA for the transition diagram, using subset construction method.



	0	1		0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\rightarrow \{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$	$\{q_0, q_2\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$
q_3^*	$\{\emptyset\}$	$\{\emptyset\}$	$\{q_0, q_3\}$	$\{q_0\}$	$\{q_0, q_1\}$
			$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
			$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
			$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$
			$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$
			$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	



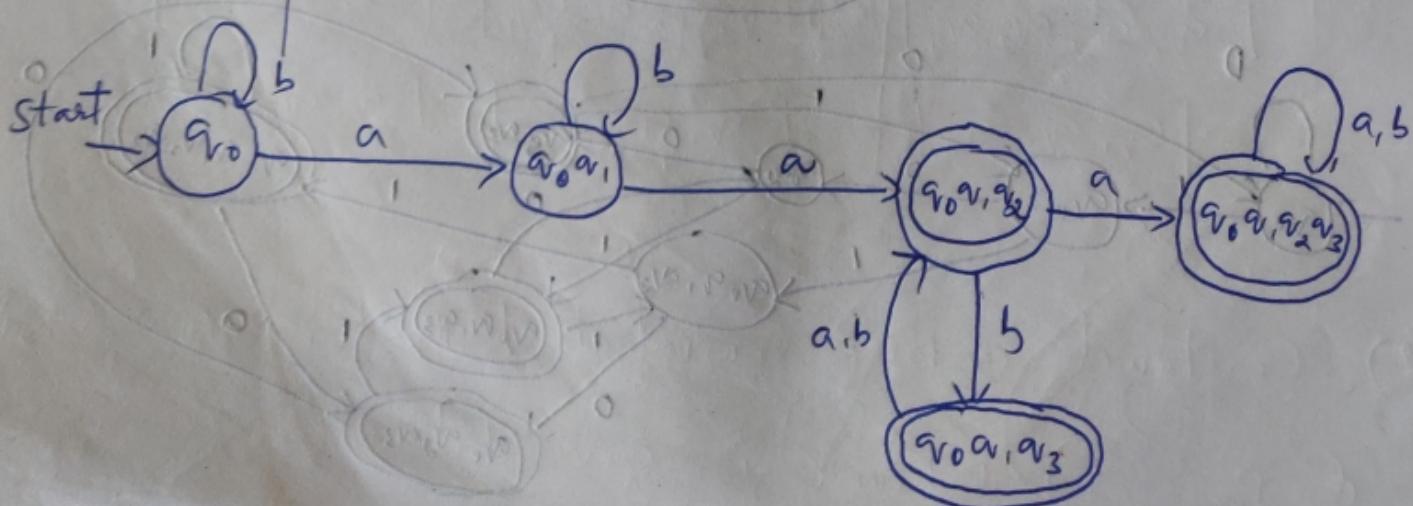
4) 12 Construct an equivalent DFA for the NFA shown below:



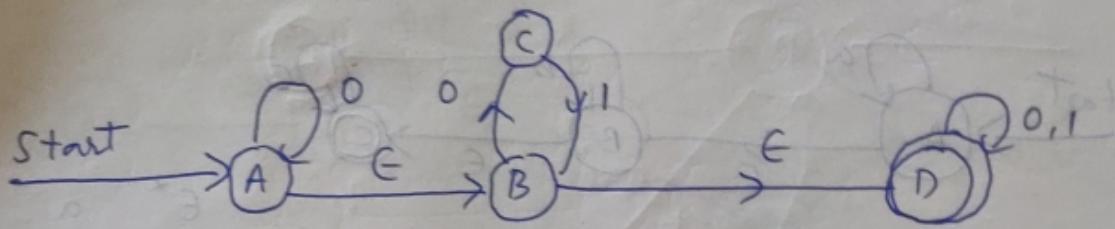
	a	b
$\{q_0, q_1, q_3\}$	$\{q_0\}$	
$\{q_1\}$	$\{q_2\}$	$\{q_1\}$
$\{q_2\}^*$	$\{q_3\}$	$\{q_3\}$
$\{q_3\}$	\emptyset	$\{q_2\}$

Ans. - P.S

	a	b
$\rightarrow q_0$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$



Inversion from ϵ -NFA to NFA



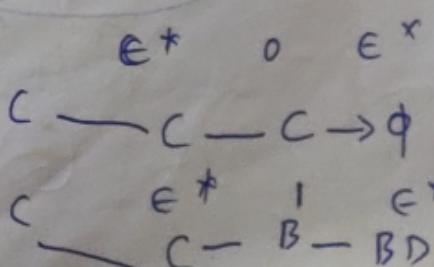
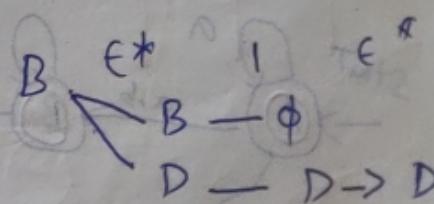
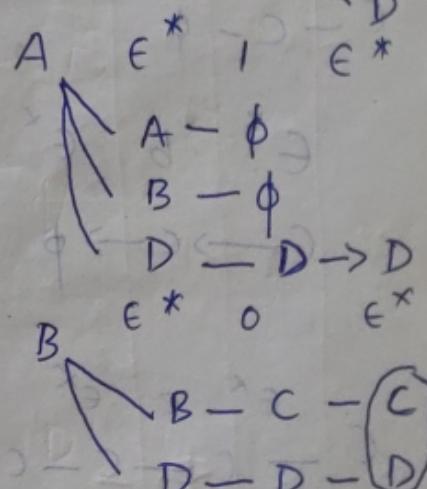
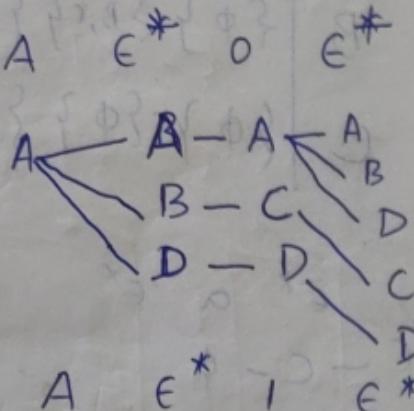
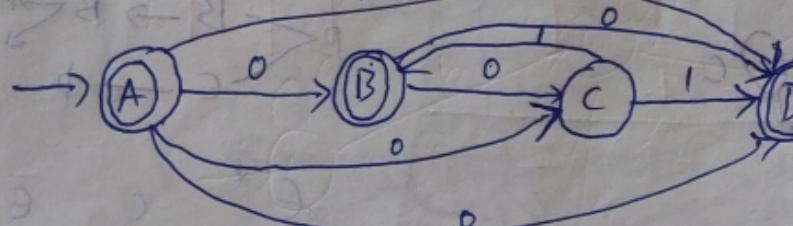
$$\epsilon\text{-closure}(A) = \{A, B, D\}$$

	0	1
A	{ABC D}	{D}
B	{CD}	{D}
C	{φ}	{BD}
D	{D}	{D}

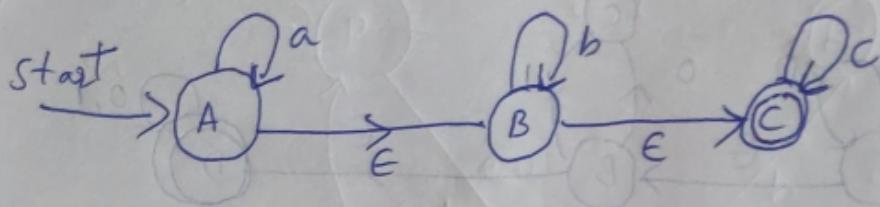
ϵ^* 0 ϵ^*
 $D \rightarrow D - D \rightarrow D$

ϵ^* 1 ϵ^*

$D \rightarrow D - D \rightarrow D$



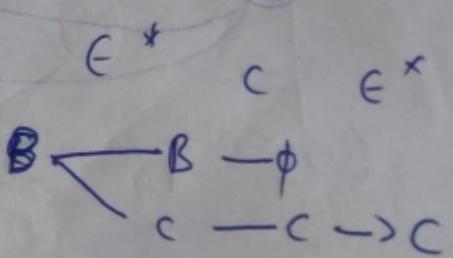
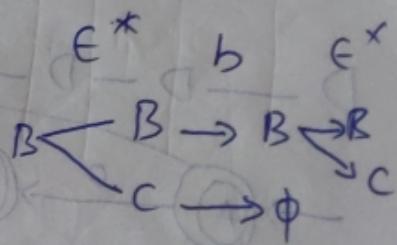
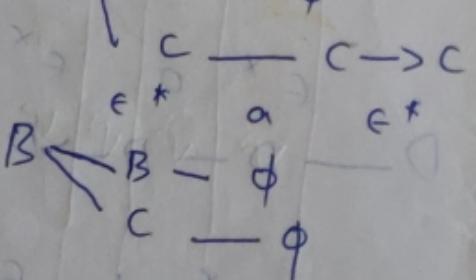
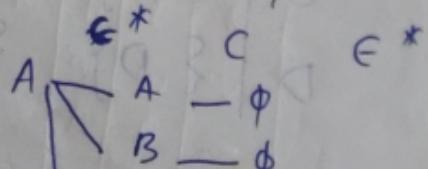
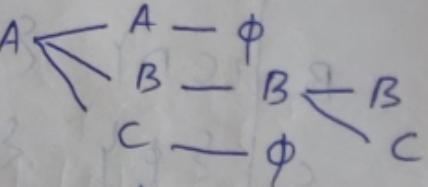
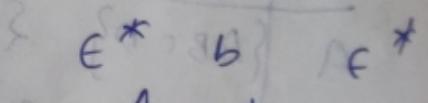
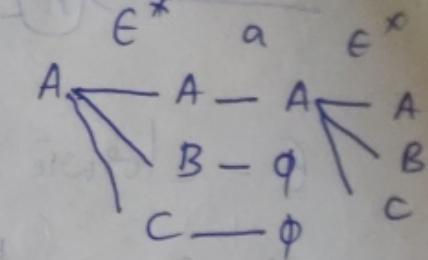
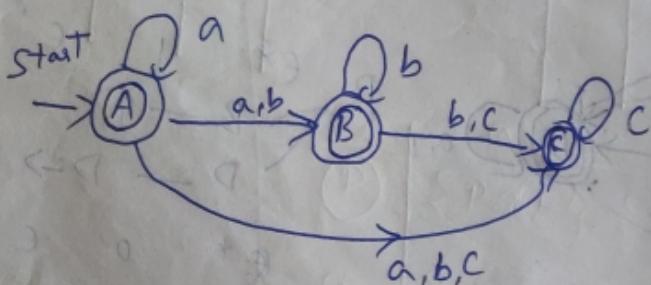
2) 20 ϵ -NFA to NFA

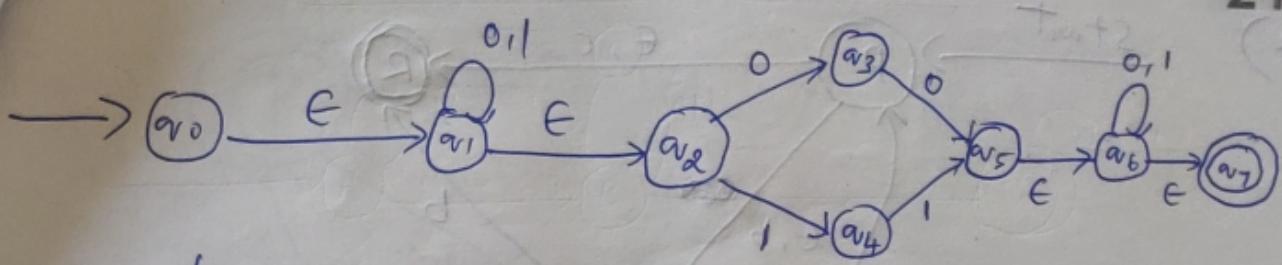


	a	b	c
A	$\{A, B, C\}$	$\{B, C\}$	$\{C\}$
B	$\{\phi\}$	$\{B, C\}$	$\{C\}$
C	$\{\phi\}$	$\{\phi\}$	$\{C\}$

ϵ^* — a — ϵ^*
 ϵ^* — b — ϵ^*
 $C \rightarrow C \rightarrow \phi$

$C \xrightarrow{\epsilon^*} C \xrightarrow{\epsilon^*} C \xrightarrow{\epsilon^*} C$

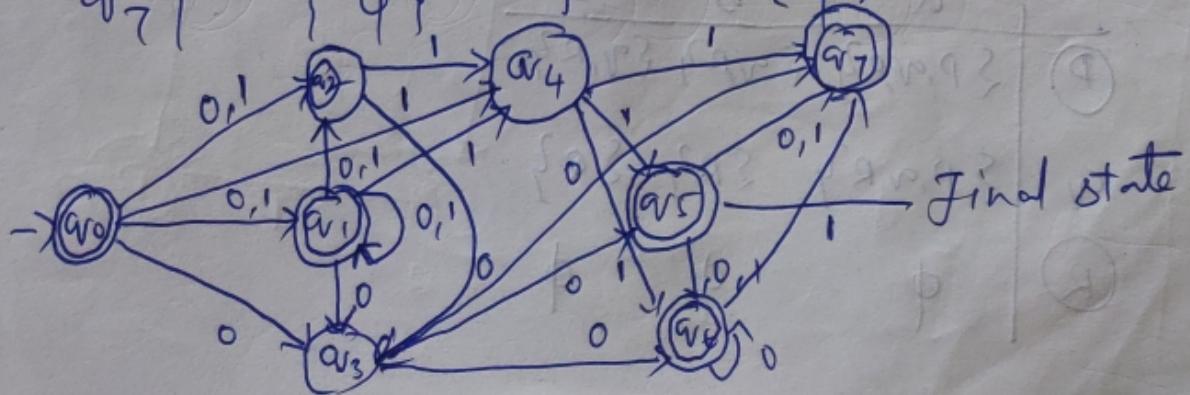




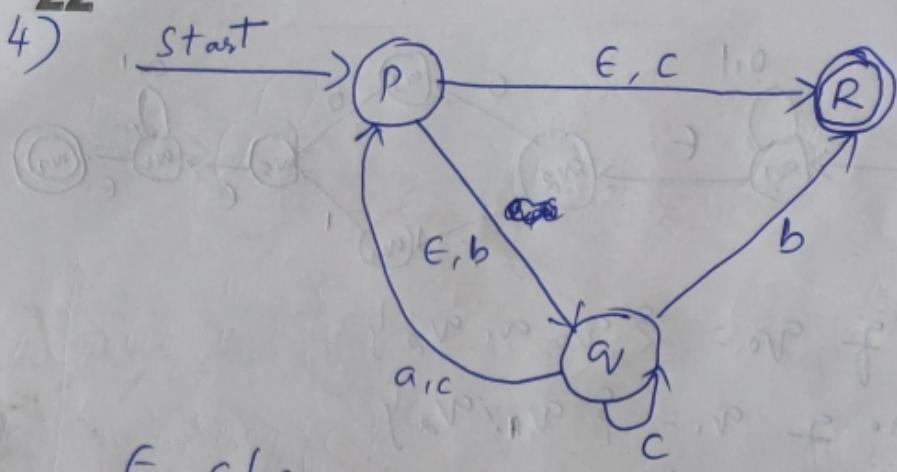
ϵ -closure of $v_0 = \{v_0, v_1, v_2\}$

ϵ -closure of $v_1 = \{v_1, v_2\}$

	0	1
v_0	$\{v_1, v_2, v_3\}$	$\{v_1, v_2, v_4\}$
v_1	$\{v_1, v_2, v_3\}$	$\{v_1, v_2, v_4\}$
v_2	$\{v_3\}$	$\{v_4\}$
v_3	$\{v_5, v_6, v_7\}$	$\{\emptyset\}$
v_4	$\{\emptyset\}$	$\{v_5, v_6, v_7\}$
v_5	$\{v_6, v_7\}$	$\{v_6, v_7\}$
v_6	$\{v_6, v_7\}$	$\{v_6, v_7\}$
v_7	$\{\emptyset\}$	$\{\emptyset\}$



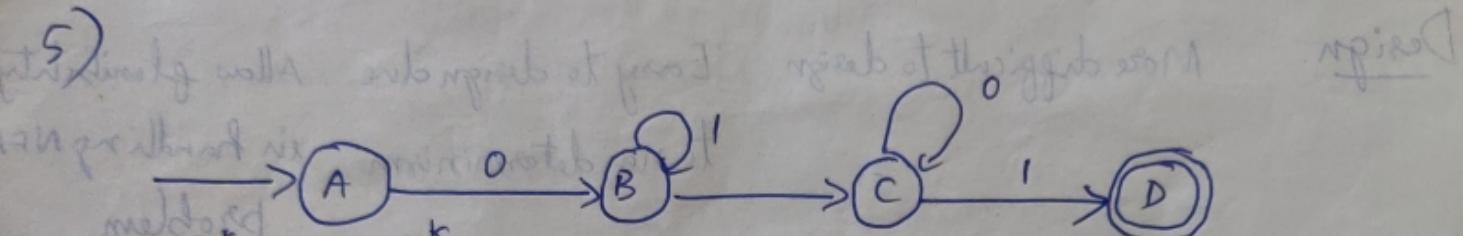
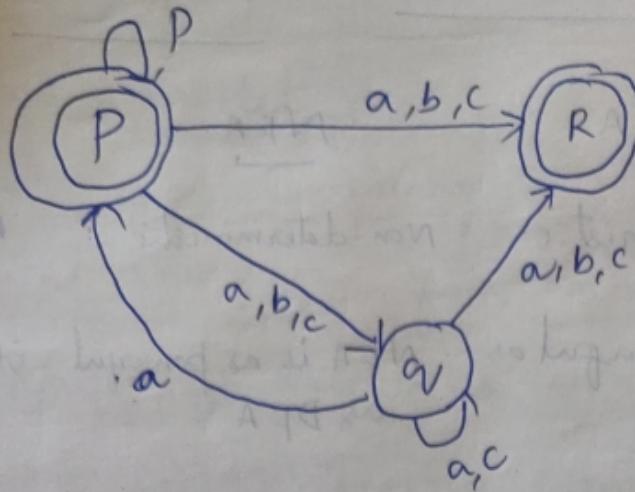
22



$$\text{E-closure}(P) = \{P, Q, R\}$$

$$\text{E-closure}(Q) = \{Q\}$$

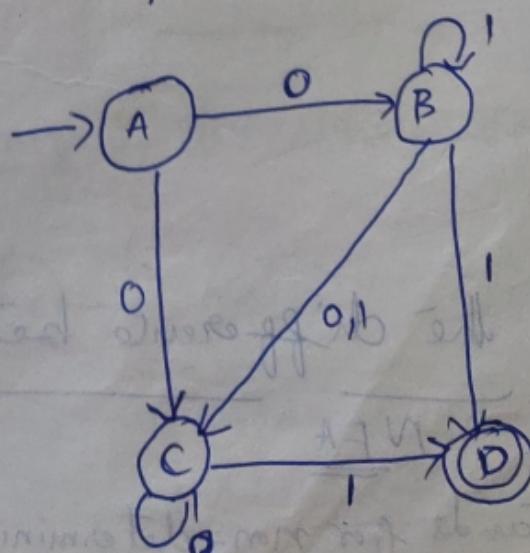
	ϵ^*	a	ϵ^*	ϵ^*	b	ϵ^*	c	ϵ^*
P	P	φ	φ	P	R	φ	R	P
Q	φ	P	P	φ	R	φ	-	Q
R	φ	Q	R	φ	φ	R	R	R
Q	Q	P	P	Q	R	Q	Q	Q
R	R	φ	Y	R	φ	-	R	R
Y	Y	Y	Y	Y	Y	Y	Y	Y
		a		b		c		
(P)			$\{P, Q, R\}$		$\{Q, R\}$	$\{Q, R\}$		
Q			$\{P, Q, R\}$		$\{R\}$	$\{Q\}$		
(R)			φ		φ	φ		



	ϵ^*	0	ϵ^*
A	A	B	B, C
B	B	\emptyset	\emptyset
C	C	C	C
D	D	\emptyset	\emptyset

	ϵ^*	0	ϵ^*
A	A	\emptyset	\emptyset
B	B	B	B, C
C	C	D	D
D	D	\emptyset	\emptyset

	0	1
A	$\{B, C\}$	\emptyset
B	$\{C\}$	$\{B, C, D\}$
C	$\{C\}$	$\{D\}$
D	\emptyset	\emptyset



write the difference between DFA, NFA and G-NFA

20

Parameter

DFA

NFA

G-NFA

Transition

Deterministic

Non-deterministic

Non-Deterministic

Power

FA is powerful as
an NFA

NFA is as powerful
as DFA

It's powerful as
FA

Design

More difficult to design

Easy to design due

to non-determinism

Allow flexibility
in handling N

Acceptance

It is easy to find

it is difficult to

whether $w \in L$ as

find whether $w \in L$

transition are
deterministic

as there are several
paths. Back tracking

is required to explore

several parallel paths.

A transition
is useful in

constructing
a composite

FA with
respect to

Union,

Concatenation

2 stars.

Write the difference between DFA & NFA

NFA

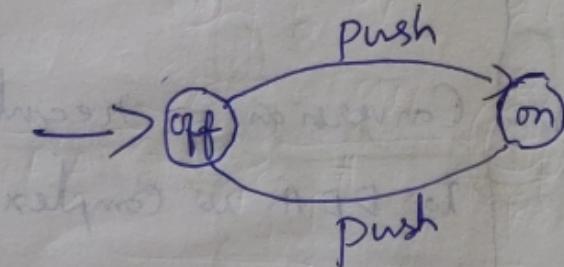
DFA

NFA stands for non-deterministic finite automata | DFA stands for deterministic finite automata.

Applications of Finite Automata :-

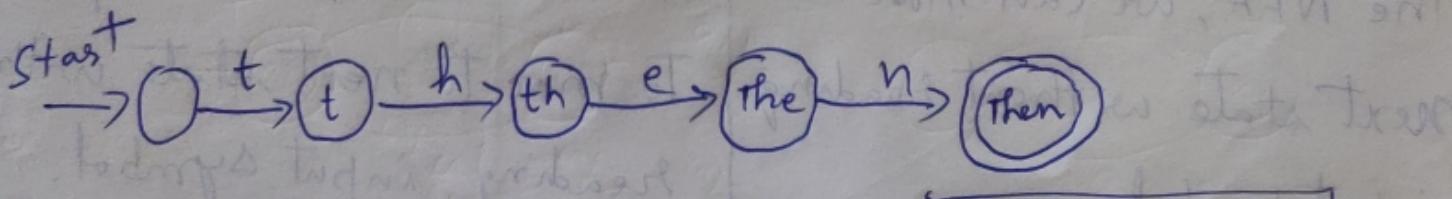
Finite automata are used as a model for

- i) Software for designing digital circuits.
- ii) Lexical analyzer of a compiler.
- iii) Searching for keywords in a file or on the web.
- iv) Software for verifying finite state systems such as communication protocols.
- v) Computer security.
- vi) Computer graphics & fractal compression.
- vii) Virtual currency.
- i) Finite Automata modelling an on/off digital switch.



Model of Computation:
A program.

- ii) Finite Automaton recognizing the string, then



Model of description:
specification

Backtracking is not allowed
in NFA

Easy to design due to non-deterministic transition.

It permits empty string transition.
State string

It requires less memory space.

Every NFA is not a DFA

String is accepted by NFA,
if one of the possible transition
ends in final state

Conversion of regular expression
to NFA is easy

In the NFA, we can move into
next state without reading
input symbol.

Sometimes practical implementation
is not feasible

Backtracking is allowed
in DFA.

More difficult to design
deterministic transition

It cannot permit empty
string transition.

It requires more memory space.

Every DFA is a NFA

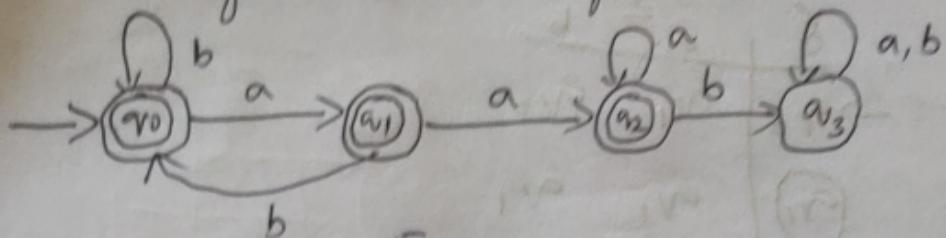
String accepted by DFA, if
it is transition into final
state.

Conversion of regular expression
to DFA is complex.

In the DFA, it is not possible
to move to next state without
reading input symbol.

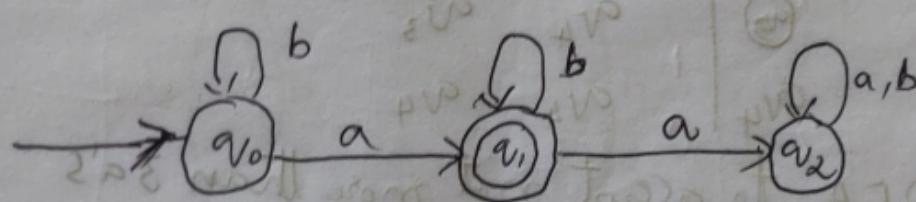
Practical implementation is
feasible.

Obtain a DFA to accept strings of a's and b's except those containing the substring abb. 15



δ	a	b	Σ
Initial $\rightarrow q_0$	q_1	q_0	
q_0	q_2	q_0	
q_1	q_2	q_3	
q_2	q_3		
q_3	q_3	q_3	

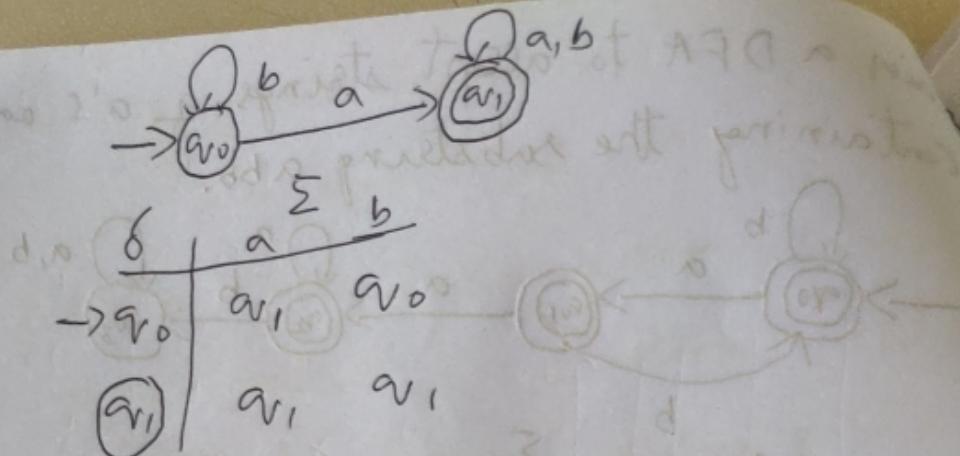
2) obtain a DFA to accept strings of a's & b's having exactly one a, at least one a, not more than three a's



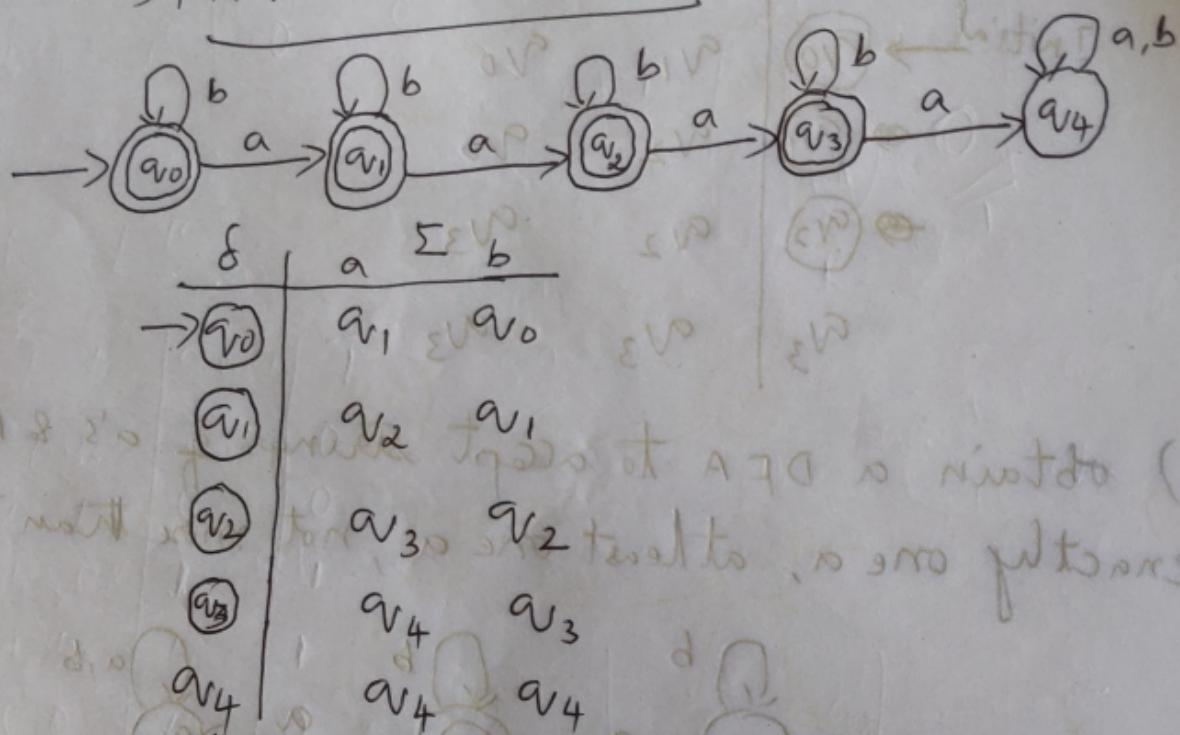
δ	a	b	Σ
$\rightarrow q_0$	q_1	q_0	
q_1	q_2	q_1	
q_2	q_2	q_2	

DFA to accept exactly one a.

16

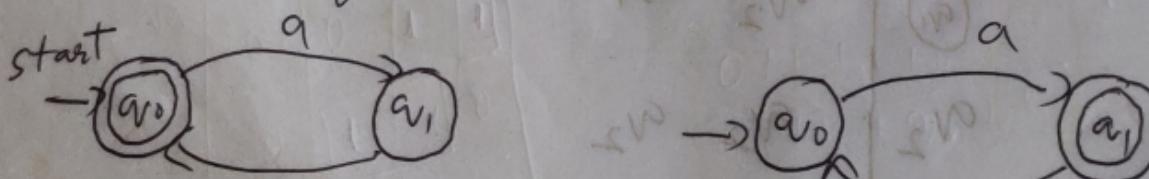


DFA to accept at least one a



DFA to accept not more than 3 a 's

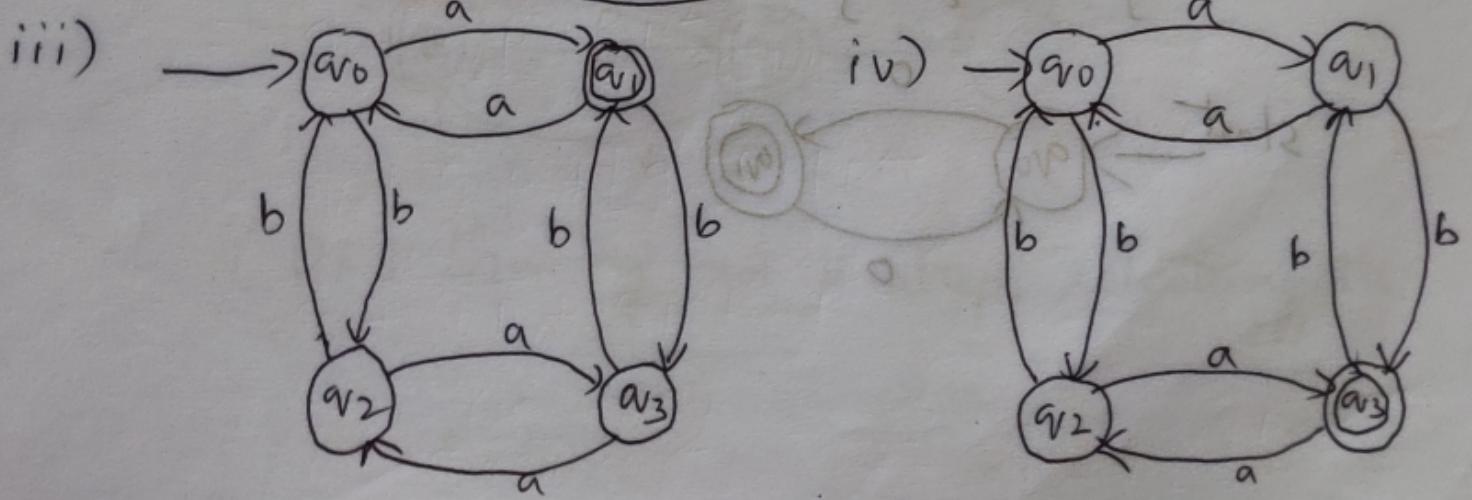
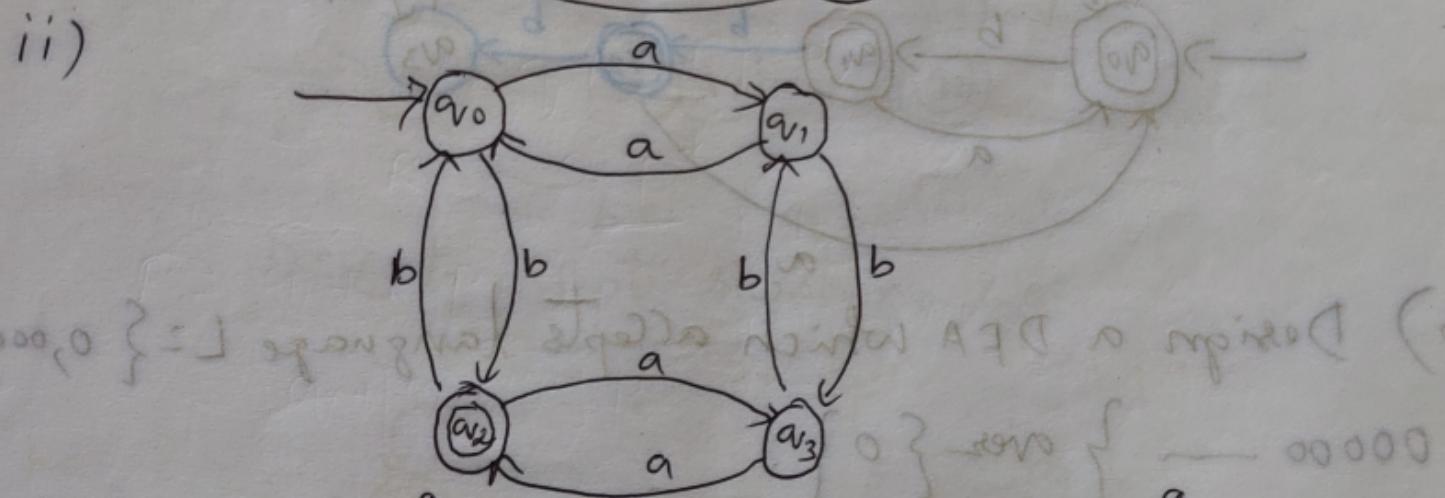
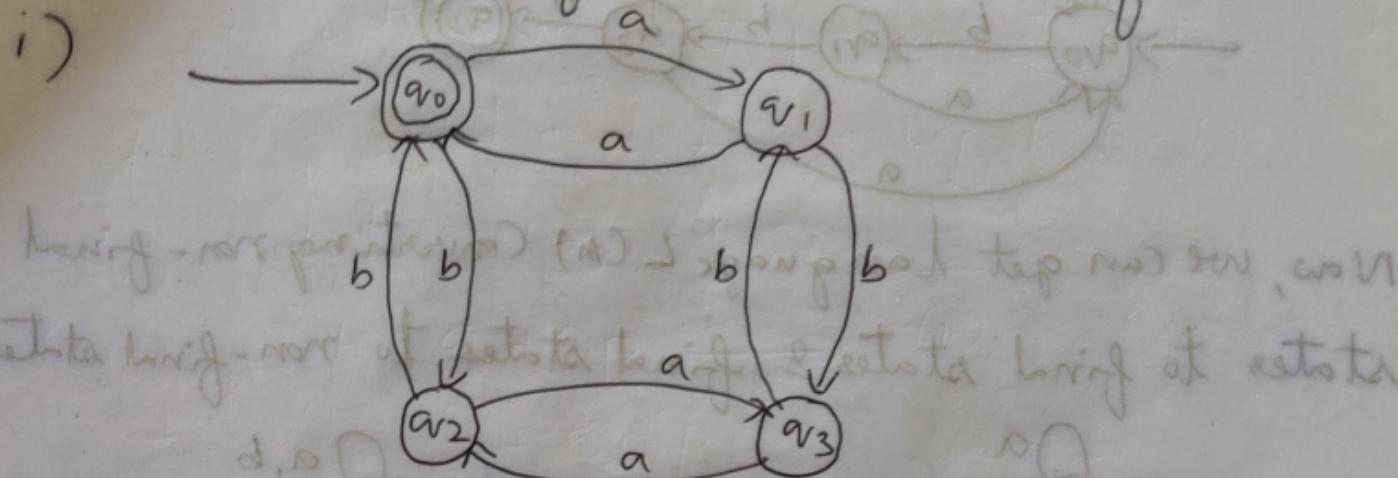
- 3) obtain a DFA to accept even number of a 's and odd number of a 's



even number of a 's odd number of a 's

obtain a DFA to accept strings of α having an even number of a's and b's

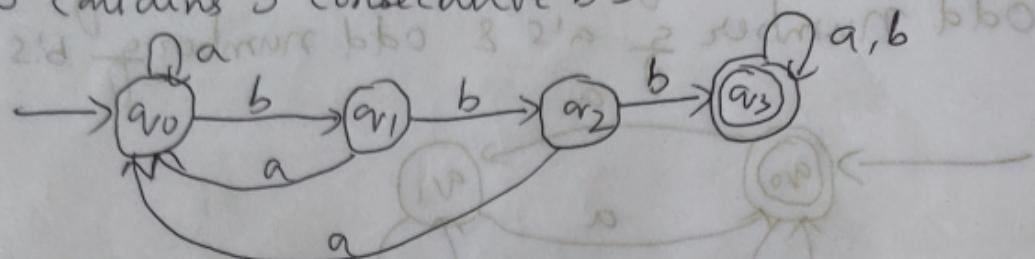
- ii) Even number of a's & odd number of b's
- iii) Odd number of a's & Even number of b's
- iv) Odd number of a's & odd number of b's



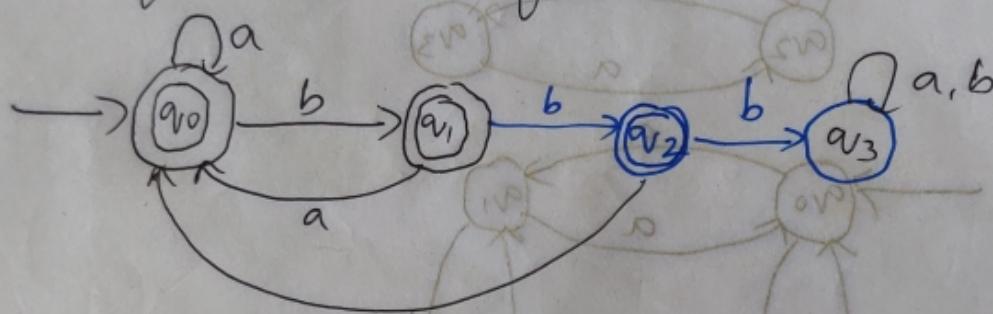
5) obtain a DFA that accepts the language $L = \{ w \mid w \in \{a, b\}^* \text{ and } w \text{ does not contain } 3 \text{ consecutive } b's \}$

we first consider a language $L(M) = \{ w \mid w \in \{a, b\}^* \text{ and } w \text{ contains } 3 \text{ consecutive } b's \}$

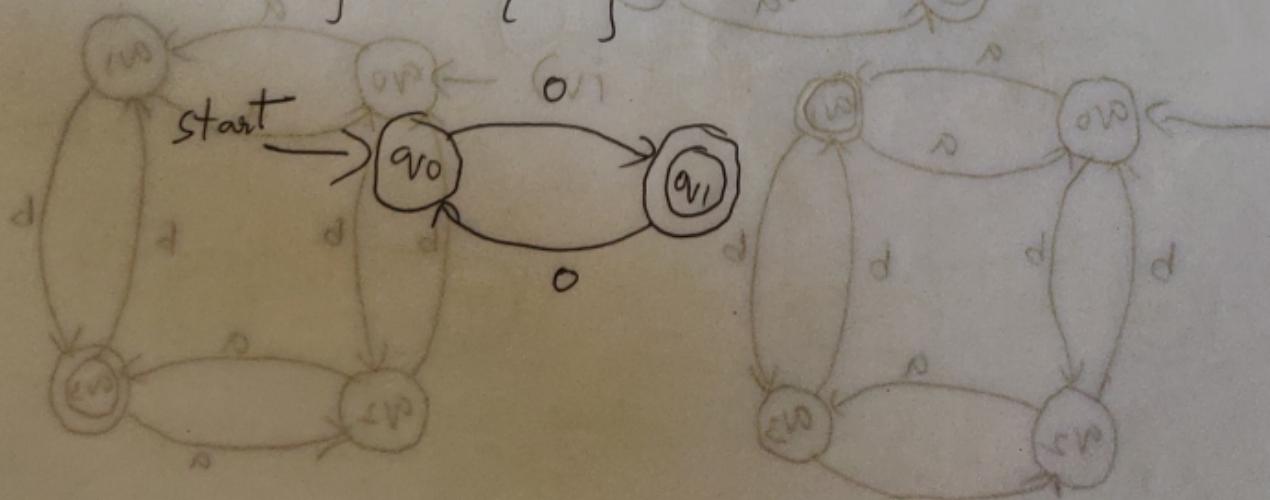
and w contains 3 consecutive b 's



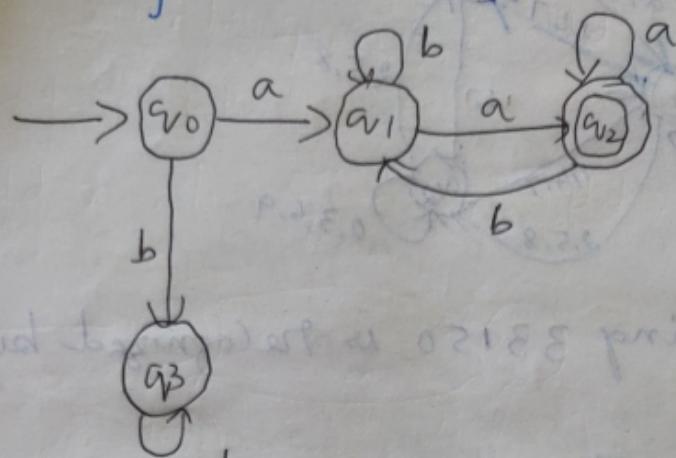
Now, we can get language $L(M)$ Converting non-final states to final states & final states to non-final states.



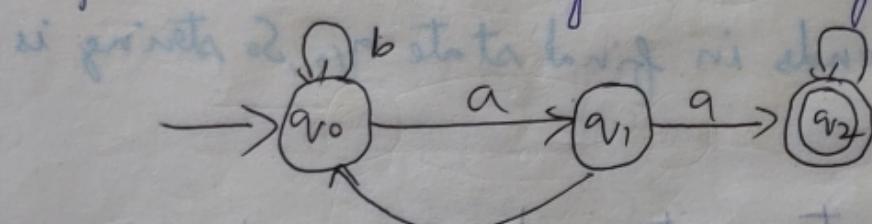
6) Design a DFA which accepts language $L = \{ 0,000, 00000 \} \text{ over } \{ 0 \}$



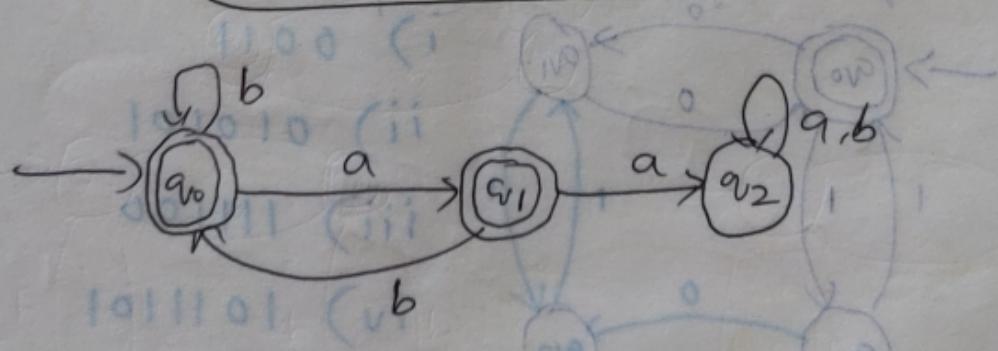
Obtain a DFA to accept the language $L = \{a^n b^n | n \in (a+b)^*\}$. 15



- 8) obtain a DFA to accept strings of a's and b's except those containing substring aa.



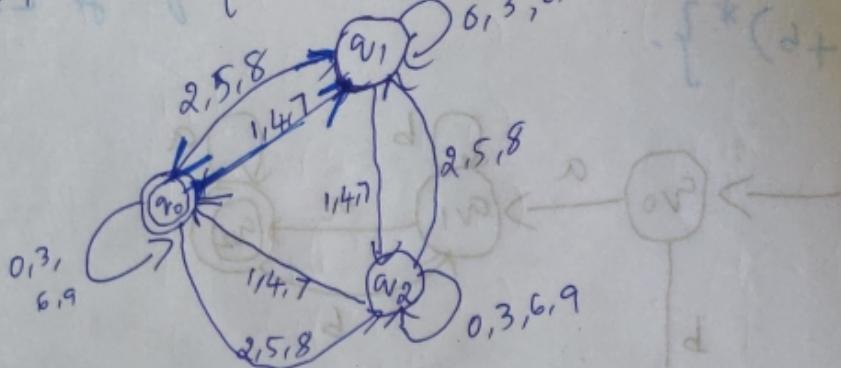
DFA to accept substring aab previously all



DFA showing not to accept substring aa

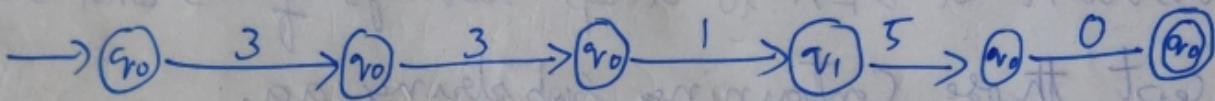
9) Let DFA $M = \{q, \Sigma, \delta, q_0, F\}$ is shown.

16



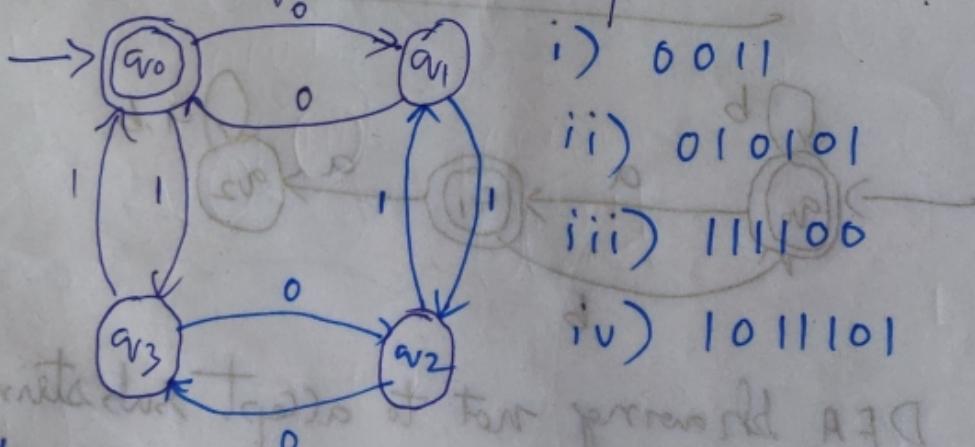
Check that string 33150 is recognized by above DFA or not?

For the string 33150 , the transition sequence is as follows:



Since, transition ends in final state q_0 , so string is recognized.

10) Consider below transition diagram and verify whether the following strings will be accepted or not?



- i) 0011
- ii) 010101
- iii) 111100
- iv) 1011101

i) 0011

$$\delta(q_0, 0011) | \rightarrow \delta(q_1, 011)$$

$$| \rightarrow \delta(q_0, 11)$$

$$| \rightarrow (q_3, 1) | \rightarrow q_0$$

$\therefore 0011$ is accepted.

010101

29

$(q_0, 010101)$	$ - \delta(q_1, 10101)$	iii) 111100
	$ - \delta(q_2, 0101)$	$\delta(q_0, 111100) - \delta(q_3, 11100)$
	$ - \delta(q_3, 101)$	$ - \delta(q_0, 1100)$
	$ - \delta(q_0, 01)$	$ - \delta(q_3, 100)$
	$ - \delta(q_1, 1)$	$ - \delta(q_0, 00)$
	$ - q_2$	$ - \delta(q_1, 0)$
		$ - q_0$

$\therefore 010101$ is not accepted. $\therefore 111100$ is accepted.

iv) 1011101

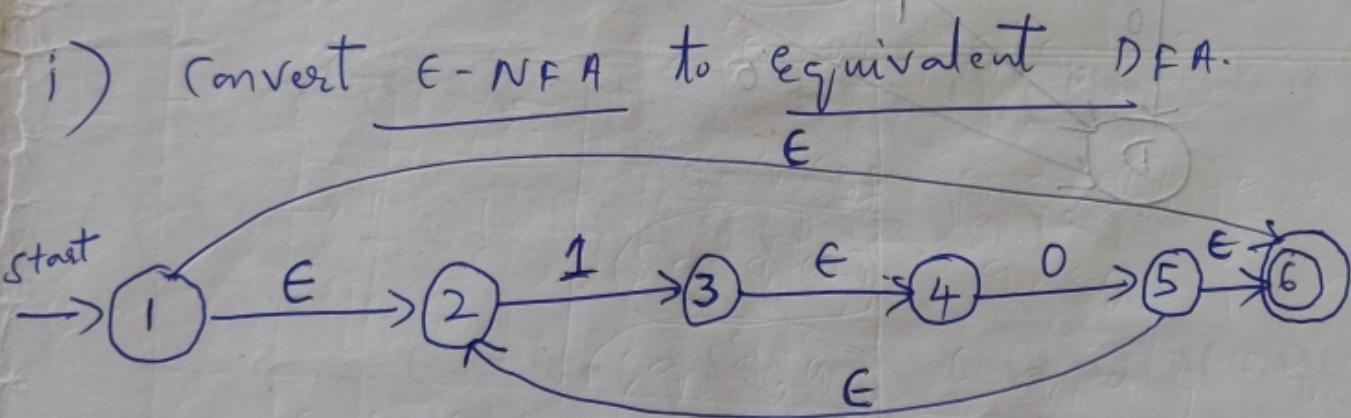
$\delta(q_0, 1011101)$	$ - \delta(q_3, 011101)$
	$ - \delta(q_2, 11101)$
	$ - \delta(q_1, 1101)$
	$ - \delta(q_2, 101)$
	$ - \delta(q_1, 01)$
	$ - \delta(q_0, 1)$
	$ - q_3$

$\therefore 1011101$ is not accepted.

Conversion from ϵ -NFA to DFA :- the steps

for the conversion is as follows:

- i) Find the ϵ -closure of all states from the NFA
- ii) Draw a transition table.
- iii) Start computing the DFA table from the first state and take the resulting states as the next state in each step.



$$\epsilon\text{-closure}(1) = \{1, 2, 6\}$$

$$\epsilon\text{-closure}(2) = \{2\}$$

$$\epsilon\text{-closure}(3) = \{3, 4\}$$

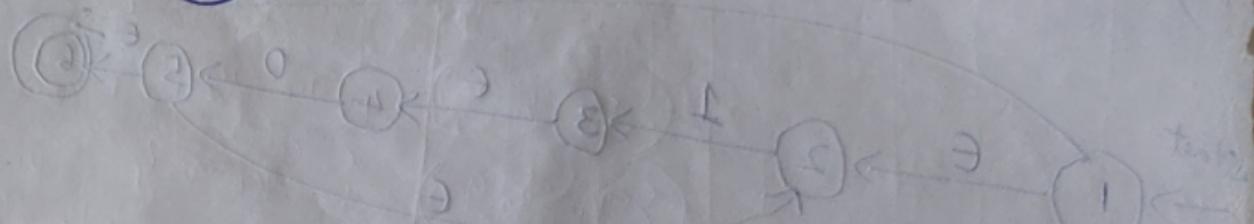
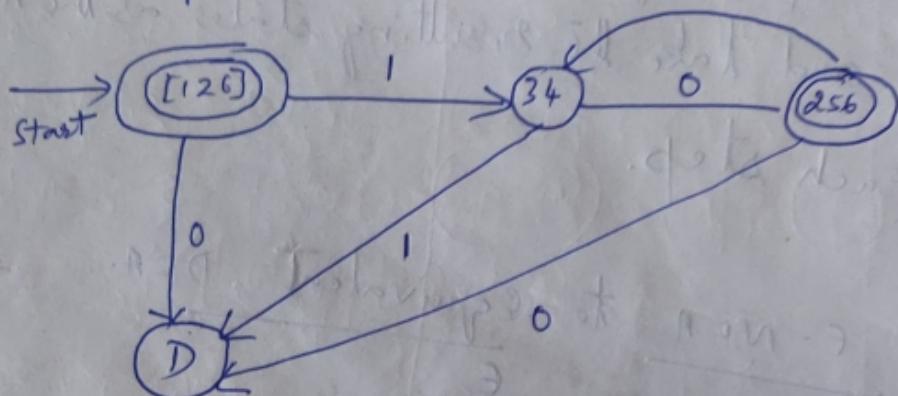
$$\epsilon\text{-closure}(4) = \{4\}$$

$$\epsilon\text{-closure}(5) = \{5, 6\}$$

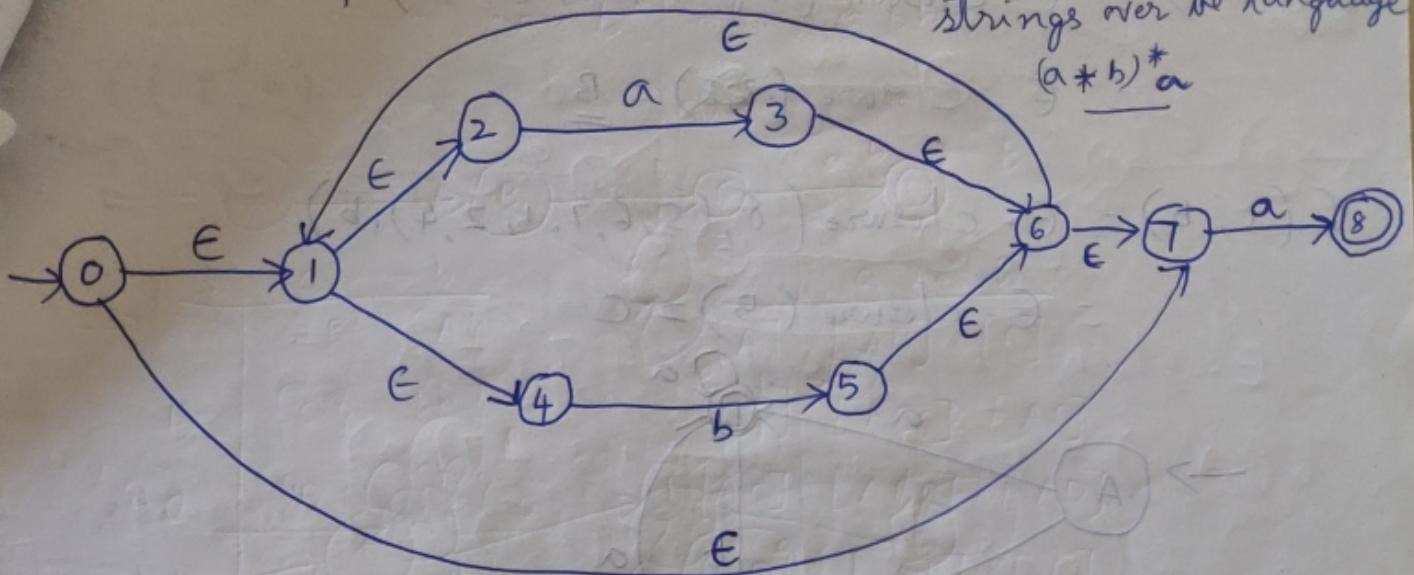
$$\epsilon\text{-closure}(6) = \{6\}$$

2) 14 Draw the transition table

	0	1
[1, 2, 6]	∅	[3, 4]
[3, 4]	[2, 5, 6]	∅
[2, 5, 6]	∅	[3, 4]



Convert ϵ -NFA to DFA for the state diagram 13 over the input alphabet $\Sigma = \{a, b\}$ that should accept the strings over the language



$$\epsilon\text{-closure}(0) = \{0, 1, 2, 4, 7\} = A \text{ (New state in DFA)}$$

$$\begin{aligned}\delta(A, a) &= \epsilon\text{-closure}(\delta(0, 1, 2, 4, 7), a) \\ &= \epsilon\text{-closure}(\delta(0, a) \cup \delta(2, a) \cup \delta(4, a) \cup \delta(7, a)) \\ &= \epsilon\text{-closure}(3, 8) = \{3, 6, 1, 7, 2, 4, 8\} = B\end{aligned}$$

$$\begin{aligned}\delta(A, b) &= \epsilon\text{-closure}(\delta(0, 1, 2, 4, 7), b) \\ &= \epsilon\text{-closure}(5) = \{5, 6, 7, 1, 2, 4\} = C\end{aligned}$$

$$\begin{aligned}\delta(B, a) &= \epsilon\text{-closure}(\delta(3, 6, 1, 7, 2, 4, 8), a) \\ &= \epsilon\text{-closure}(3, 8) = B\end{aligned}$$

$$\delta(B, b) = \epsilon\text{-closure}(\delta(3, 6, 1, 7, 2, 4, 8), b)$$

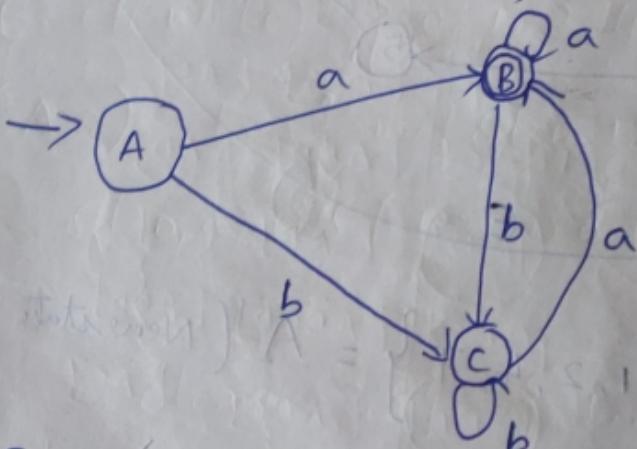
$$14 \quad = \epsilon\text{-closure}(s) = c$$

$$\delta(c, a) = \epsilon\text{-closure}(\delta(s, 6, 7, 12, 4), a)$$

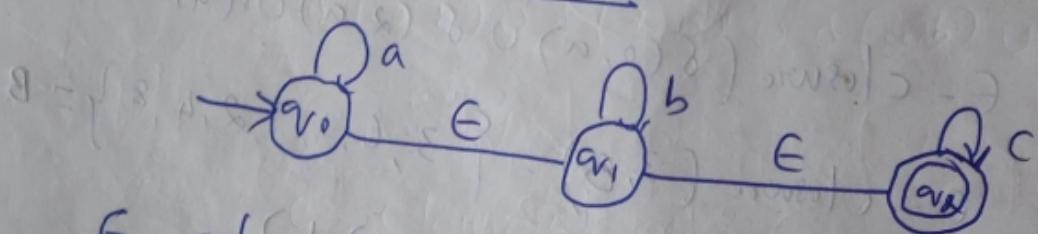
$$= \epsilon\text{-closure}(3, 8) = \underline{B}$$

$$\delta(c, b) = \epsilon\text{-closure}(\delta(s, 6, 7, 1, 2, 4), b)$$

$$= \epsilon\text{-closure}(5) = c$$



2) $\epsilon\text{-NFA} \xrightarrow{\text{to DFA}}$



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\} = A$$

$$\delta(A, a) = \epsilon\text{-closure}(\delta(q_0, q_1, q_2), a) = B$$

$$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), a)$$

$$\delta(A, b) = \epsilon\text{-closure}(\delta(q_0)) = A$$

$$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), b)$$

$$= \epsilon\text{-closure}(q_1) = \{q_1, q_2\} = B$$

$$\delta(A, c) = \text{E-closure}(\delta(a_0, a_1, a_2), c))$$

15

$$= \text{E-closure}(a_2) = \{a_2\} = C$$

$$\delta(B, a) = \text{E-closure}(\delta(a_1, a_2), a)$$

$$= \text{E-closure}(\phi) = D$$

$$\delta(B, b) = \text{E-closure}(\delta(a_1, a_2), b)$$

$$= \text{E-closure}(a_1) = B$$

$$\delta(B, c) = \text{E-closure}(\delta(a_1, a_2), c)$$

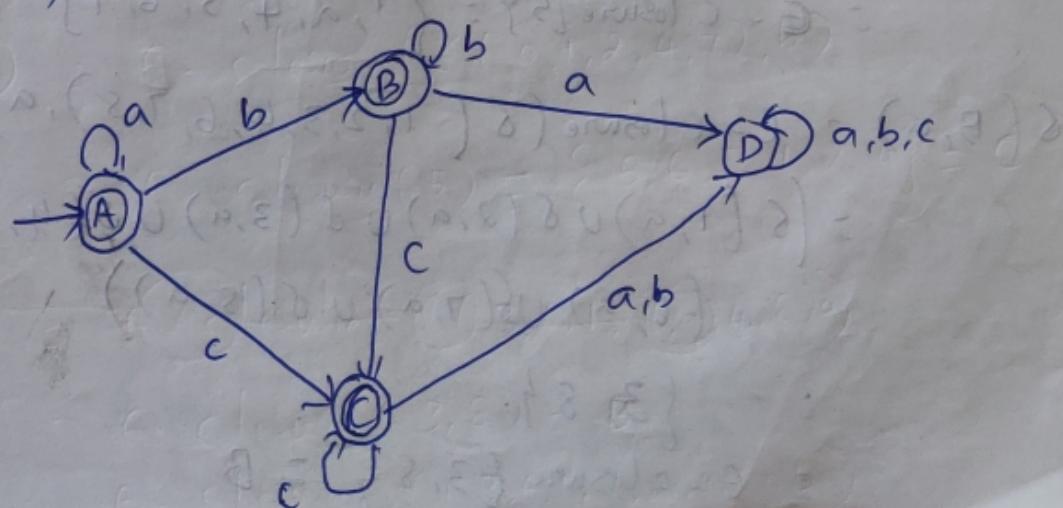
$$= \text{E-closure}(a_2) = C$$

$$\delta(D, a) = \delta(D, b) = \delta(D, c) = D$$

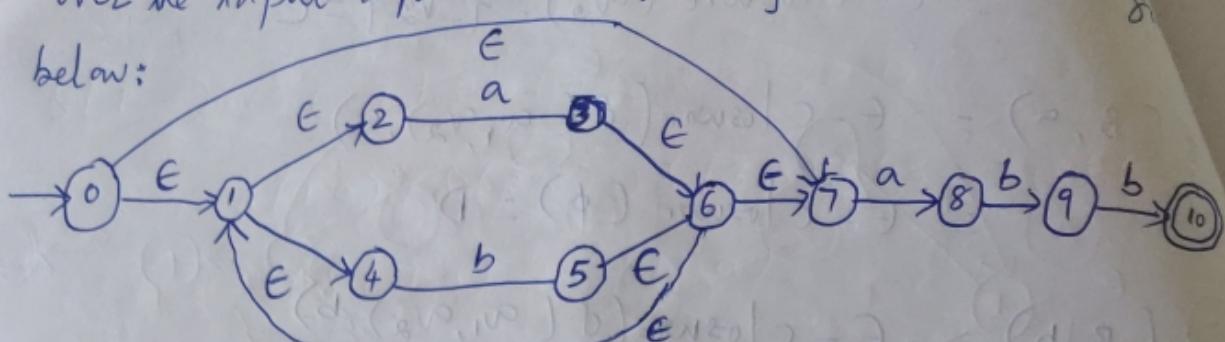
$$\delta(C, a) = \text{E-closure}(\delta(a_2, a)) = \phi = D$$

$$\delta(C, b) = \text{E-closure}(\delta(a_2, b)) = \phi = D$$

$$\delta(C, c) = \text{E-closure}(\delta(a_2, c)) = \phi = D$$



3) **16** Convert ϵ -NFA to DFA for the language over the input alphabet $\Sigma = \{a, b\}$ & ϵ -NFA is below:



$$\text{E-closure}(0) = \{0, 1, 2, 4, 7\} = A$$

$$\delta(A, a) = \text{E-closure}(\delta(0, 1, 2, 4, 7), a)$$

$$= \delta((0, a) \cup \delta(1, a) \cup \delta(2, a) \cup \delta(4, a) \cup \delta(7, a)) \\ = \{3, 8\}$$

$$= \text{E-closure}\{3, 8\} = \{1, 2, 3, 4, 6, 7, 8\} = B$$

$$\delta(A, b) = \text{E-closure}(\delta(0, 1, 2, 4, 7), b)$$

$$= \delta(0, b) \cup \delta(1, b) \cup \delta(2, b) \cup \delta(4, b) \cup \delta(7, b)$$

$$= \{5\}$$

$$= \text{E-closure}\{5\} = \{1, 2, 4, 5, 6, 7\} = C$$

$$\delta(B, a) = \text{E-closure}(\delta(1, 2, 3, 4, 6, 7, 8), a)$$

$$= (\delta(1, a) \cup \delta(2, a) \cup \delta(3, a) \cup \delta(4, a) \cup \delta(6, a) \cup \delta(7, a) \cup \delta(8, a))$$

$$= \{3, 8\}$$

$$= \text{E-closure}\{3, 8\} = B$$

25

$$\delta(B, b) = \text{E-closure}(\delta(1, 2, 3, 4, 6, 7, 8), b)$$

$$\begin{aligned}
 &= \delta(1, b) \cup \delta(2, b) \cup \delta(3, b) \cup \delta(4, b) \cup \delta(6, b) \cup \\
 &\quad \delta(7, b) \cup \delta(8, b) \\
 &= \text{E-closure}(5, 9) \\
 &= \{1, 2, 4, 5, 6, 7, 9\} = D
 \end{aligned}$$

$$\delta(C, a) = \text{E-closure}(\delta(1, 2, 4, 5, 6, 7), a)$$

$$\begin{aligned}
 &= \delta(1, a) \cup \delta(2, a) \cup \delta(4, a) \cup \delta(5, a) \cup \delta(6, a) \\
 &\quad \cup \delta(7, a)
 \end{aligned}$$

$$= \text{E-closure}(3, 8) = \underline{\underline{B}}$$

$$\delta(C, b) = \text{E-closure}(\delta(1, 2, 4, 5, 6, 7), b)$$

$$\begin{aligned}
 &= \delta(1, b) \cup \delta(2, b) \cup \delta(4, b) \cup \delta(5, b) \cup \delta(6, b) \\
 &\quad \cup \delta(7, b)
 \end{aligned}$$

$$= \text{E-closure}(5) = \underline{\underline{C}}$$

$$\delta(D, a) = \text{E-closure}(\delta(1, 2, 4, 5, 6, 7, 9), a)$$

$$\begin{aligned}
 &= \delta(1, a) \cup \delta(2, a) \cup \delta(4, a) \cup \delta(5, a) \cup \\
 &\quad \delta(6, a) \cup \delta(7, a) \cup \delta(9, a)
 \end{aligned}$$

$$= \text{E-closure}(3, 8) = \underline{\underline{B}}$$

$$\delta(D, b) = \text{E-closure}(\delta(1, 2, 4, 5, 6, 7, 9), b)$$

$$\begin{aligned}
 &= \delta(1, b) \cup \delta(2, b) \cup \delta(4, b) \cup \delta(5, b) \cup \\
 &\quad \delta(6, b) \cup \delta(7, b) \cup \delta(9, b)
 \end{aligned}$$

26

$$\epsilon\text{-closure}(5, 10) = E$$

$$= \{1, 2, 3, 5, 6, 7, 10\} = E$$

$$\delta(E, a) = \epsilon\text{-closure}(\delta(1, 2, 3, 5, 6, 7, 10), a)$$

$$= \delta(1, a) \cup \delta(2, a) \cup \delta(3, a) \cup \delta(5, a) \cup \delta(6, a) \\ \cup \delta(7, a) \cup \delta(10, a)$$

$$(n, a) = (3, 8) = \epsilon\text{-closure}(3, 8) = B$$

$$\delta(E, b) = \epsilon\text{-closure}(\delta(1, 2, 3, 5, 6, 7, 10), b)$$

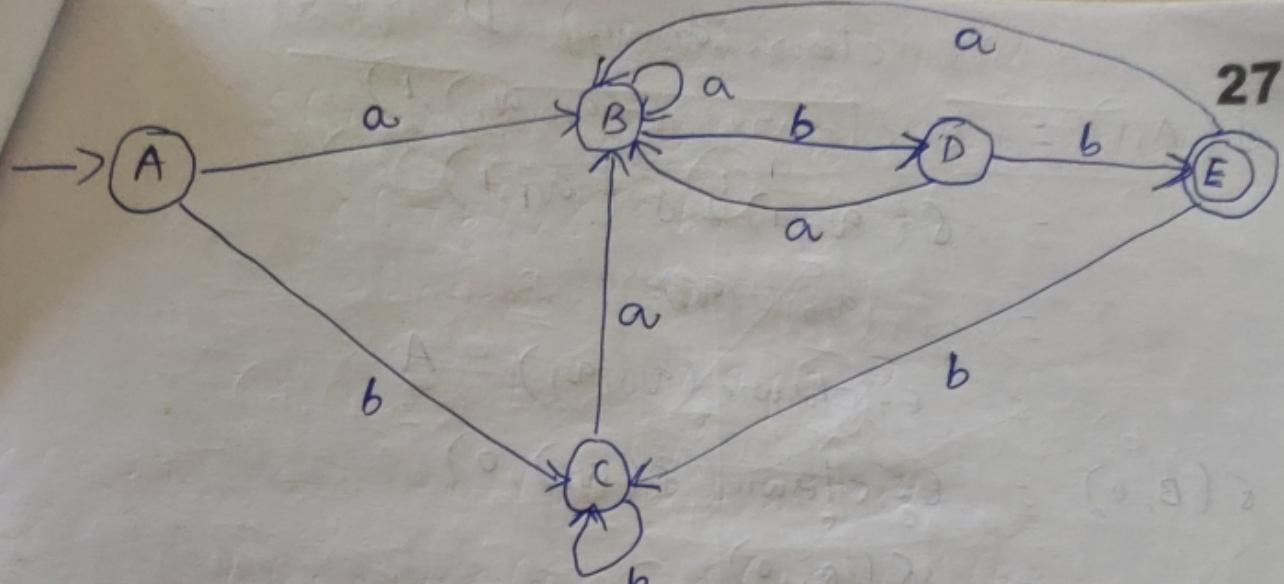
$$= \delta(1, b) \cup \delta(2, b) \cup \delta(3, b) \cup \delta(5, b) \cup \delta(6, b) \\ \cup \delta(7, b) \cup \delta(10, b)$$

$$= \epsilon\text{-closure}(5) = C$$

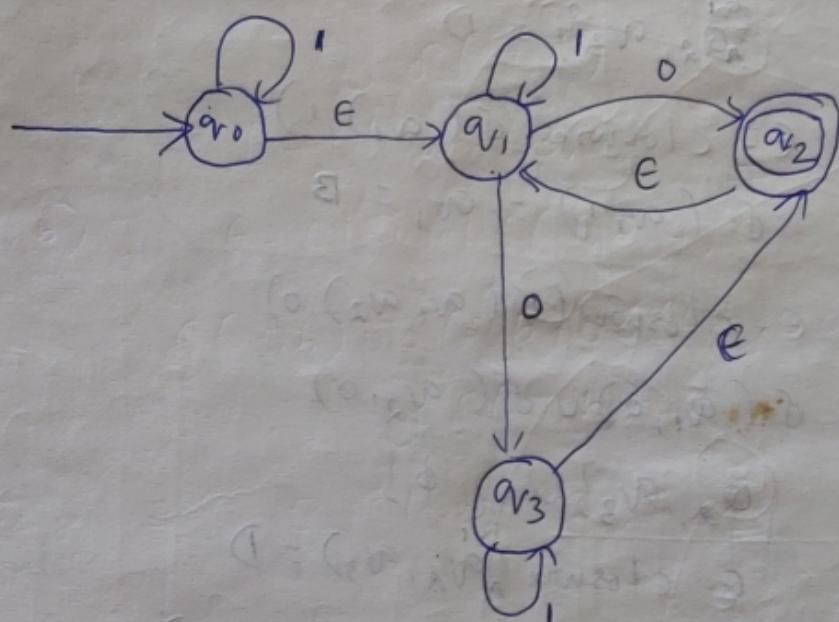
NFA state

NFA state	DFA state	I/P	
		a	b
{0, 1, 2, 4, 7}	A	B	C
{1, 2, 3, 4, 6, 7, 8}	B	B	D
{1, 2, 4, 5, 6, 7}	C	B	C
{1, 2, 4, 5, 6, 7, 9}	D	B	E
{1, 2, 3, 5, 6, 7, 10}	E	B	C





4)



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1\} = A$$

$$\epsilon\text{-closure}(q_1) = \{q_1\} = B$$

$$\epsilon\text{-closure}(q_2) = \{q_1, q_2\} = C$$

$$\epsilon\text{-closure}(q_3) = \{q_2, q_3\} = D$$

$$\delta(A, o) = \epsilon\text{-closure}(\delta(q_0, q_1), o)$$

$$= \delta(q_0, o) \cup \delta(q_1, o)$$

$$= (\emptyset) \cup (q_2, q_3)$$

28

$$\epsilon\text{-closure}(\{q_2, q_3\}) = D$$

$$\begin{aligned}\delta(A, 1) &= \epsilon\text{-closure}(\delta(q_0, q_1), 1) \\&= \delta(q_0, 1) \cup \delta(q_1, 1) \\&= \{q_0\} \cup \{q_1\} \\&= \epsilon\text{-closure}(q_0, q_1) = A\end{aligned}$$

$$\begin{aligned}\delta(B, 0) &= \epsilon\text{-closure}(\delta(q_1), 0) \\&= \delta(q_1, 0) \\&= \{q_1\} = D\end{aligned}$$

$$\begin{aligned}\delta(B, 1) &= \epsilon\text{-closure}(\delta(q_1), 1) \\&= \delta(q_1, 1) = q_1 = B\end{aligned}$$

$$\begin{aligned}\delta(C, 0) &= \epsilon\text{-closure}(\delta(q_1, q_2), 0) \\&= \delta(q_1, 0) \cup \delta(q_2, 0) \\&= \{q_2, q_3\} \cup \{\emptyset\} \\&= \epsilon\text{-closure}(\{q_2, q_3\}) = D\end{aligned}$$

$$\begin{aligned}\delta(C, 1) &= \epsilon\text{-closure}(\delta(q_1, q_2), 1) \\&= \delta(q_1, 1) \cup \{q_2, 1\} \\&= \{q_1\} \cup \{\emptyset\} \\&= \epsilon\text{-closure}(q_1) = q_1 = B\end{aligned}$$

$$\begin{aligned}\delta(D, 0) &= \epsilon\text{-closure}(\delta(D), 0) \\&= \delta(q_2, q_3), 0 \\&= \delta(q_2, 0) \cup \delta(q_3, 0) \\&= \{\emptyset\} \cup \{\emptyset\} = \emptyset = E\end{aligned}$$

$$\delta(\{D\}, 1) = \text{E-closure}(\delta(D), 1)$$

15

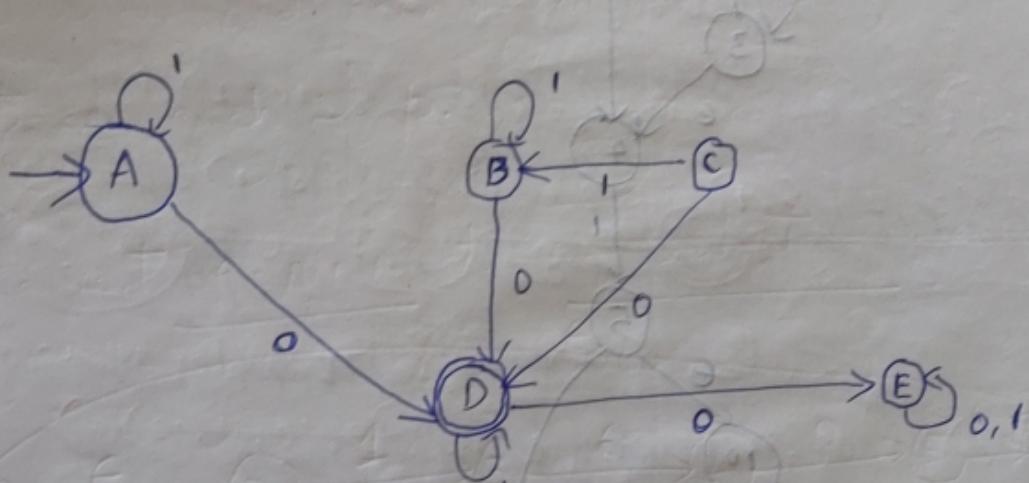
$$= \text{E-closure}(\delta(v_2, v_3), 1)$$

$$= \delta(v_2, 1) \cup \delta(v_3, 1)$$

$$= \{\emptyset\} \cup \{v_3\}$$

$$= \text{E-closure}(v_3) = D$$

$$\delta(E, 0) = \delta(E, 1) = E$$



NFA state

$$A = \{v_0, v_1\}$$

$$B = \{v_1\}$$

$$C = \{v_1, v_2\}$$

$$D = \{v_1, v_2, v_3\}$$

$$E = \{v_4\}$$

DFA state

A

B

C

D

E

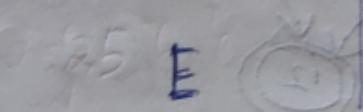
A

B

C

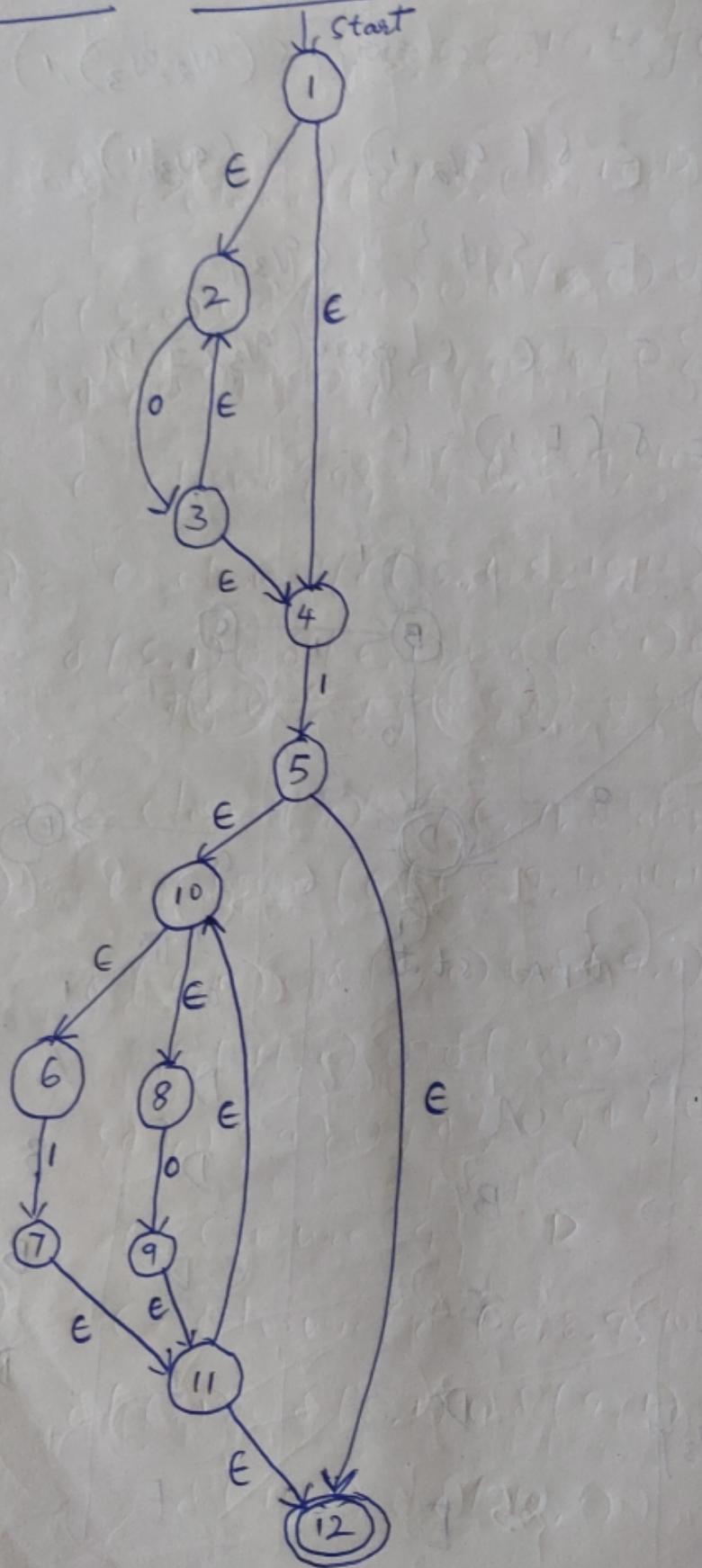
D

E



Convert ϵ -NFA to Equivalent DFA :-

16



$$\text{E-closure}\{1\} = \{1, 2, 4\} = A$$

29

$$\delta(A, 0) = \text{E-closure}(\delta(A, 0))$$

$$= \delta((1, 2, 4), 0)$$

$$= \delta((1, 0) \cup \delta(2, 0) \cup \delta(4, 0))$$

$$= \{\emptyset\} \cup \{3\} \cup \{\emptyset\}$$

$$A = \{1, 2, 4\} \quad \text{E-closure}\{3\} = \{3, 2, 4\} = \{2, 3, 4\} = B$$

$$\delta(A, 1) = \text{E-closure}(\delta(A, 1))$$

$$= \delta((1, 2, 4), 1)$$

$$= \delta((1, 1) \cup \delta(2, 1) \cup \delta(4, 1))$$

$$= \{1, 1\} \cup \{\emptyset\} \cup \{\emptyset\} \cup \{5\}$$

$$\text{E-closure}\{5\} = \{5, 10, 12, 6, 8\}$$

$$= \{5, 6, 8, 10, 12\} = C$$

$$\delta(B, 0) = \text{E-closure}(\delta(2, 3, 4), 0))$$

$$= \delta((2, 0) \cup \delta(3, 0) \cup \delta(4, 0))$$

$$= \{3\} \cup \{\emptyset\} \cup \{\emptyset\}$$

$$\text{E-closure}\{3\} = B$$

$$\delta(B, 1) = \text{E-closure}(\delta(2, 3, 4), 1))$$

$$= \delta((2, 1) \cup \delta(3, 1) \cup \delta(4, 1))$$

$$= \{\emptyset\} \cup \{\emptyset\} \cup \{5\}$$

30

$$\epsilon\text{-closure}\{5\} = \{5, 10, 12, 6, 8\}$$

$$= \{5, 6, 8, 10, 12\} = C$$

$$\delta(C, 0) = \{\epsilon\text{-closure}((\delta(5, 6, 8, 10, 12), 0))\}$$

$$= \delta(5, 0) \cup \delta(6, 0) \cup \delta(8, 0) \cup \delta(10, 0) \cup \delta(12, 0)$$

$$= \{\emptyset\} \cup \{\emptyset\} \cup \{9\} \cup \{\emptyset\} \cup \{\emptyset\}$$

$$\epsilon\text{-closure}\{9\} = \{6, 8, 9, 10, 11, 12\} = D$$

$$\delta(C, 1) = \epsilon\text{-closure}(\delta(5, 6, 8, 10, 12), 1)$$

$$= \delta(5, 1) \cup \delta(6, 1) \cup \delta(8, 1) \cup \delta(10, 1) \cup \delta(12, 1)$$

$$= \{\emptyset\} \cup \{7\} \cup \{\emptyset\} \cup \{\emptyset\} \cup \{\emptyset\}$$

$$= \epsilon\text{-closure}(7) = \{6, 7, 8, 10, 11, 12\} = E$$

$$\delta(D, 0) = \epsilon\text{-closure}(\delta(6, 8, 9, 10, 11, 12), 0)$$

$$= \delta(6, 0) \cup \delta(8, 0) \cup \delta(9, 0) \cup \delta(10, 0) \cup$$

$$\delta(11, 0) \cup \delta(12, 0)$$

$$= \{\emptyset\} \cup \{9\} \cup \{\emptyset\} \cup \{\emptyset\} \cup \{\emptyset\} \cup \{\emptyset\}$$

$$= \epsilon\text{-closure}\{9\} = D$$

$$\delta(D, 1) = \epsilon\text{-closure}(\delta(6, 8, 9, 10, 11, 12), 1)$$

$$= \delta(6, 1) \cup \delta(8, 1) \cup \delta(9, 1) \cup \delta(10, 1) \cup$$

$$\delta(11, 1) \cup \delta(12, 1)$$

$$\{7\} \cup \{\emptyset\} \cup \{1\} \cup \{2\} \cup \{3\}$$

17

$$= E - \text{closure } \{7\} = E$$

$$\delta(E, 0) = E - \text{closure}(\delta(6, 7, 8, 10, 11, 12), 0)$$

$$= \delta(6, 0) \cup \delta(7, 0) \cup \delta(8, 0) \cup \delta(10, 0) \cup \delta(11, 0) \\ \cup \delta(12, 0)$$

$$\{6, 7, 8, 11\} = \{\emptyset\} \cup \{1\} \cup \{2\} \cup \{3\} \cup \{4\} \cup \{5\}$$

$$= E - \text{closure } \{9\} = D$$

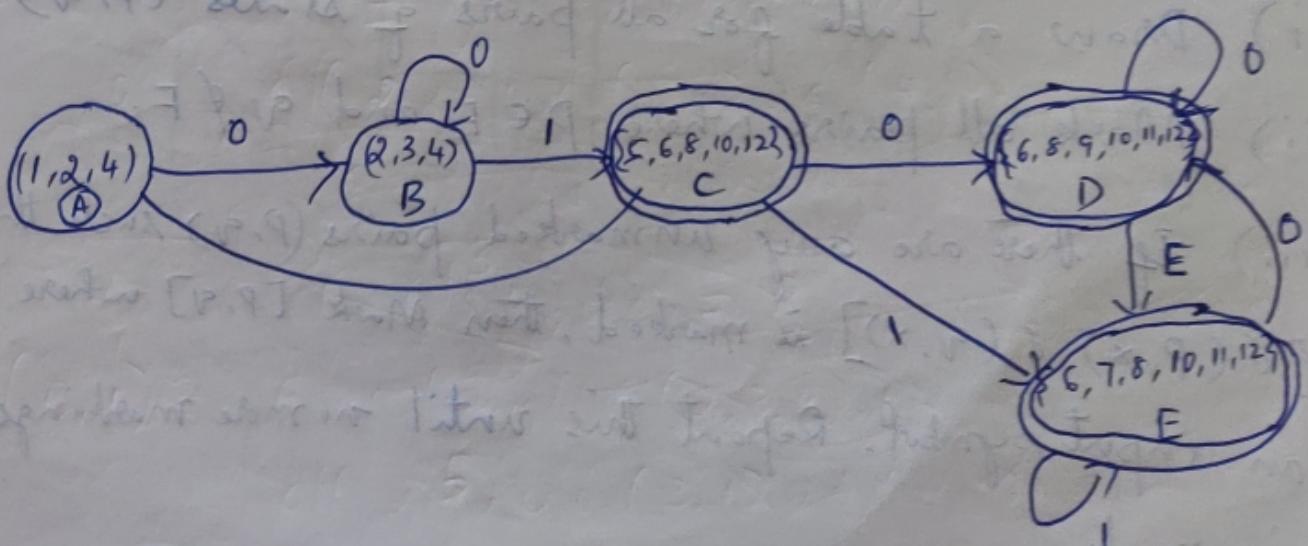
$$\delta(E, 1) = E - \text{closure}(\delta(6, 7, 8, 10, 11, 12), 1)$$

$$= \delta(6, 1) \cup \delta(7, 1) \cup \delta(8, 1) \cup \delta(10, 1) \cup$$

$$\delta(11, 1) \cup \delta(12, 1)$$

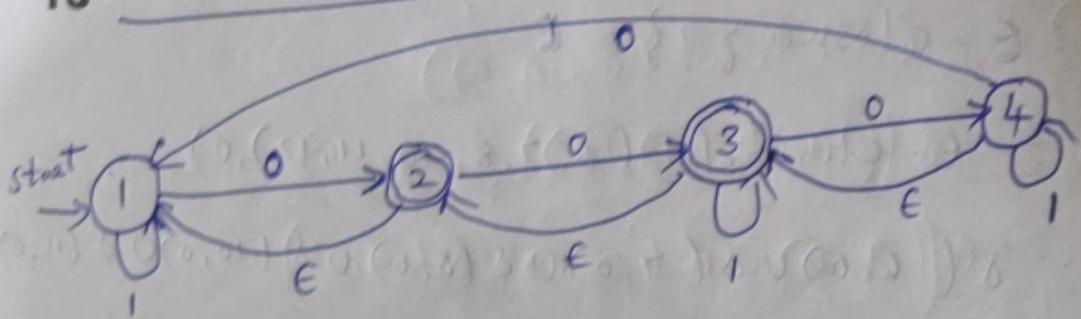
$$= \{7\} \cup \{\emptyset\} \cup \{1\} \cup \{2\} \cup \{3\} \cup \{4\} \cup \{5\}$$

$$= E - \text{closure } \{7\} = E$$



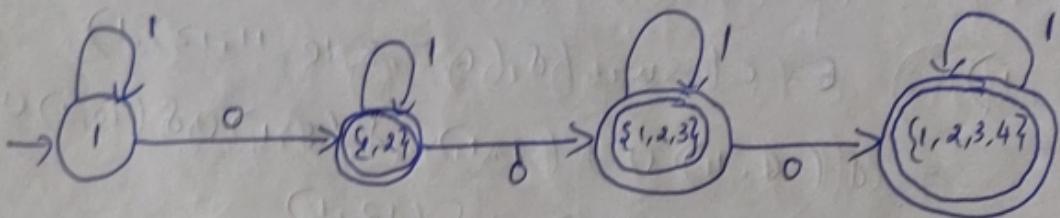
Convert E-NFA to DFA Equivalent

18



$$E\text{-closure}\{1\} = \{1, 2\}, \quad E\text{-closure}\{3\} = \{1, 2, 3\}$$

$$E\text{-closure}\{2\} = \{1, 2\}, \quad E\text{-closure}\{4\} = \{1, 2, 3, 4\}$$



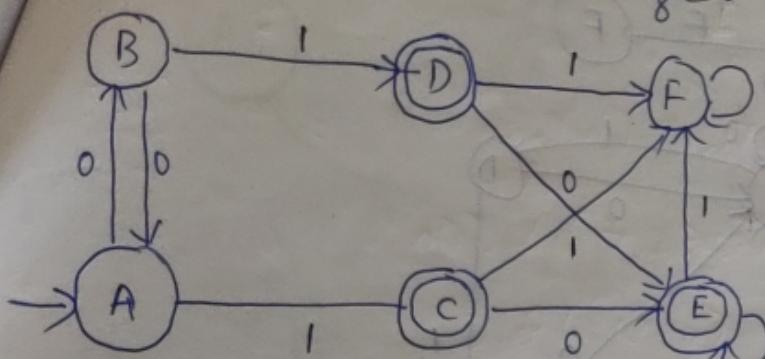
Minimization of DFA by Table Filling Method
(Myhill - Nerode Theorem)

The steps of the algorithm is as follows:

- i) Draw a table for all pairs of states (P, q)
- ii) Mark all pairs where $P \in F$ and $q \notin F$.
- iii) If there are any unmarked pairs (P, q) such that $[\delta(P, x), \delta(q, x)]$ is marked, then Mark $[P, q]$ where 'x' is an input symbol. Repeat this until no more markings can be made.

Combine all the unmarked pairs and make them a single state in the minimized DFA

23



$$\delta(B, 0) = A \quad \delta(B, 1) = D \\ \delta(A, 0) = B \quad \delta(A, 1) = C$$

$$(D, C) \\ \delta(D, 0) = E \quad \delta(D, 1) = F \\ \delta(C, 0) = E \quad \delta(C, 1) = F$$

$$(E, C) \\ \delta(E, 0) = E \quad \delta(E, 1) = F \\ \delta(C, 0) = E \quad \delta(C, 1) = F$$

$$(E, D) \\ \delta(E, 0) = E \quad \delta(E, 1) = F \\ \delta(D, 0) = E \quad \delta(D, 1) = F$$

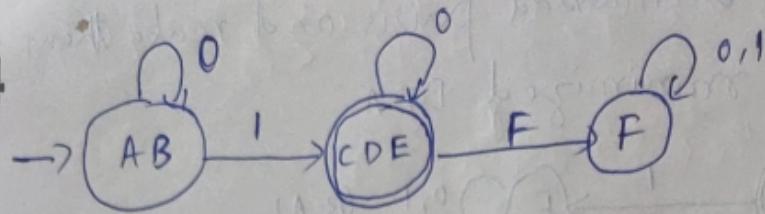
$$(F, A) \\ \delta(F, 0) = F \quad \delta(F, 1) = F \\ \delta(A, 0) = B \quad \delta(A, 1) = C$$

$$(F, B) \\ \delta(F, 0) = F \quad \delta(F, 1) = F \\ \delta(B, 0) = A \quad \delta(B, 1) = C$$

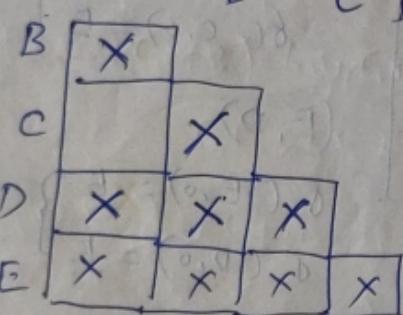
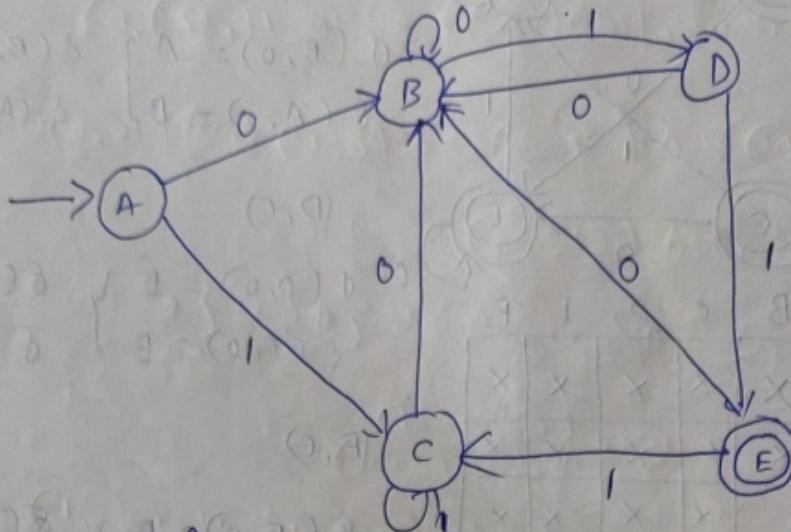
	A	B	C	D	E
A	X	X	X	X	X
B		X	X	X	X
C			X	X	X
D				X	X
E					X
F					

(A, B) (C, D) (E, C) (E, D)

24



2)



$$\delta(D, B)$$

$$\begin{cases} \delta(D, 0) = B \\ \delta(B, 0) = B \end{cases}$$

$$\begin{cases} \delta(D, 1) = E \\ \delta(B, 1) = D \end{cases}$$

$$\delta(D, C)$$

$$\begin{cases} \delta(D, 0) = B \\ \delta(C, 0) = B \end{cases}$$

$$\begin{cases} \delta(D, 1) = E \\ \delta(C, 1) = C \end{cases}$$

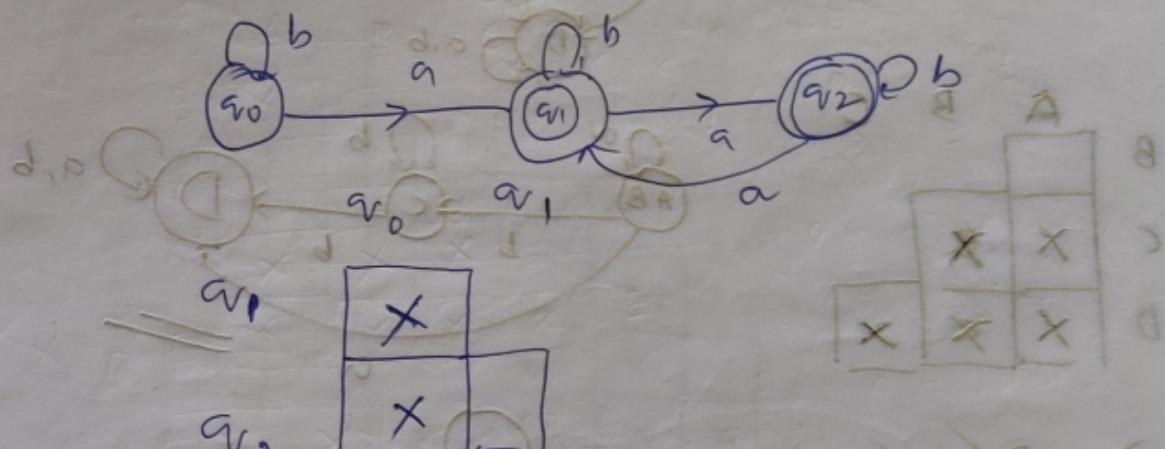
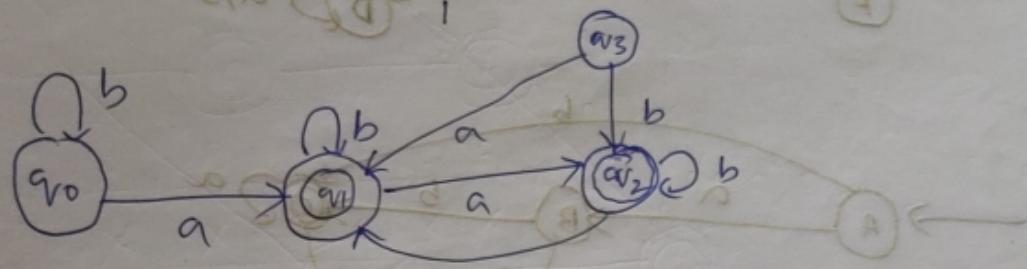
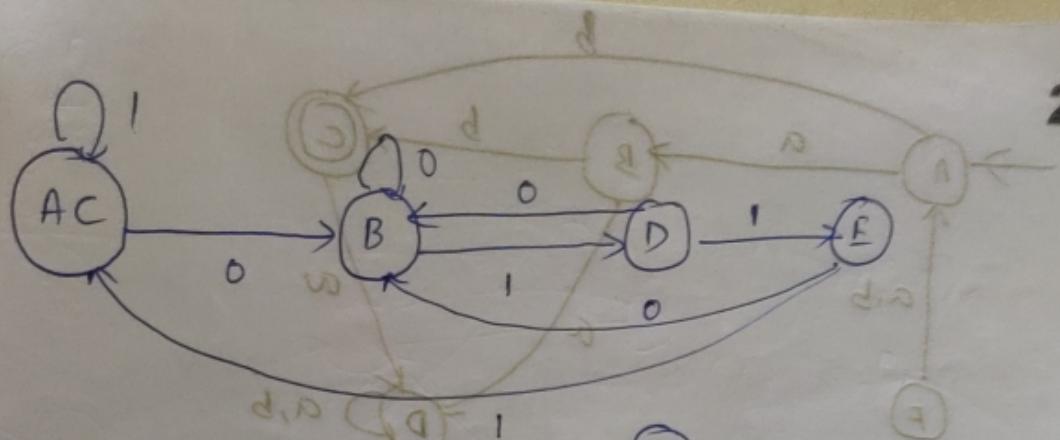
$$\begin{cases} (B, A) = \delta(B, 0) = D \\ \delta(A, 0) = C \end{cases} \quad \begin{cases} \delta(B, 1) = B \\ \delta(A, 1) = B \end{cases}$$

$$\begin{cases} (C, A) = \delta(C, 0) = B \\ \delta(A, 0) = B \end{cases} \quad \begin{cases} \delta(C, 1) = C \\ \delta(A, 1) = C \end{cases}$$

$$\begin{cases} (C, B) = \delta(C, 0) = B \\ \delta(B, 0) = B \end{cases} \quad \begin{cases} \delta(C, 1) = C \\ \delta(B, 1) = D \end{cases}$$

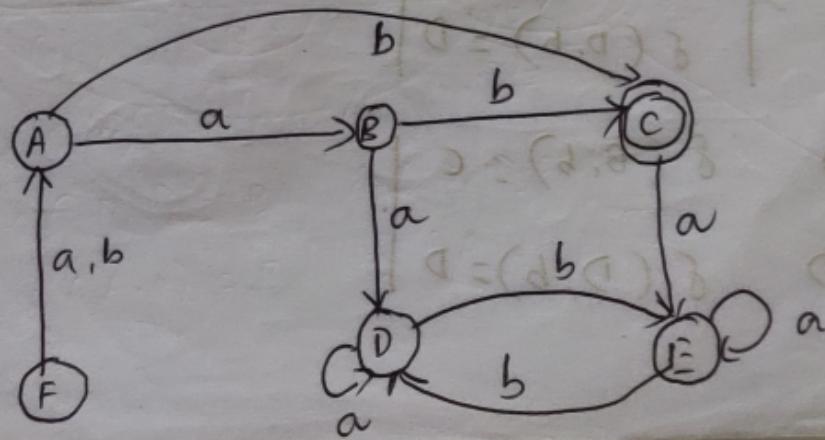
$$\begin{cases} (D, A) = \delta(D, 0) = B \\ \delta(A, 0) = B \end{cases} \quad \begin{cases} \delta(D, 1) = E \\ \delta(A, 1) = C \end{cases}$$

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$$\begin{aligned}
 & S = (d, A) \\
 & Q = (d, A) \\
 & \delta = (d, A) \\
 & \alpha = (d, A) \\
 & \beta = (d, A) \\
 & \gamma = (d, A) \\
 & \delta = (d, A) \\
 & \alpha = (d, A) \\
 & \beta = (d, A) \\
 & \gamma = (d, A)
 \end{aligned}$$

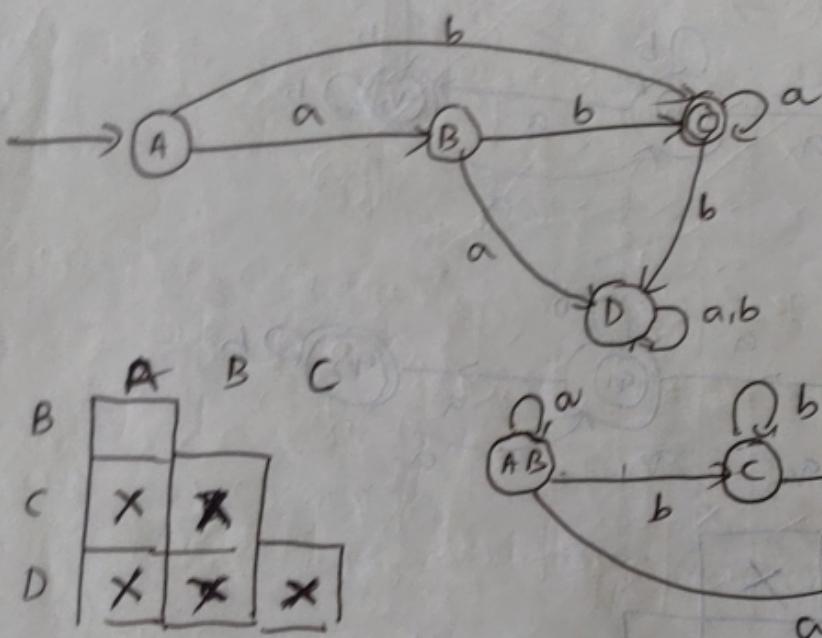
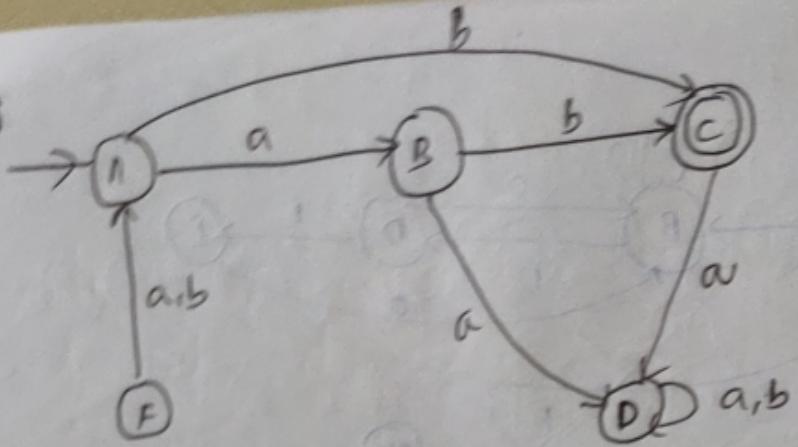
4)



$$Q = (d, A)$$

$$Q = (d, A)$$

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	A	B	C
B	X		
C	X	X	
D	X	X	X

$$\delta(A, a) = B \quad | \quad \delta(A, b) = C \quad | \quad \text{AB}$$

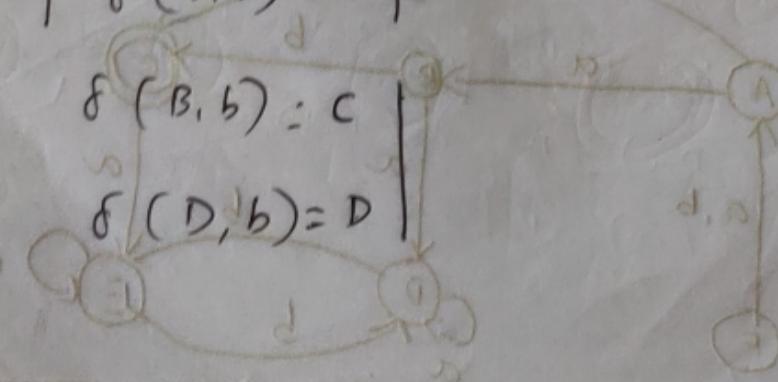
$$\delta(B, a) = D \quad | \quad \delta(B, b) = C \quad | \quad \delta(A, a) = B \quad | \quad \delta(A, b) = C$$

$$\delta(A, a) = B \quad | \quad \delta(A, b) = C \quad | \quad \delta(B, a) = D \quad | \quad \delta(B, b) = C$$

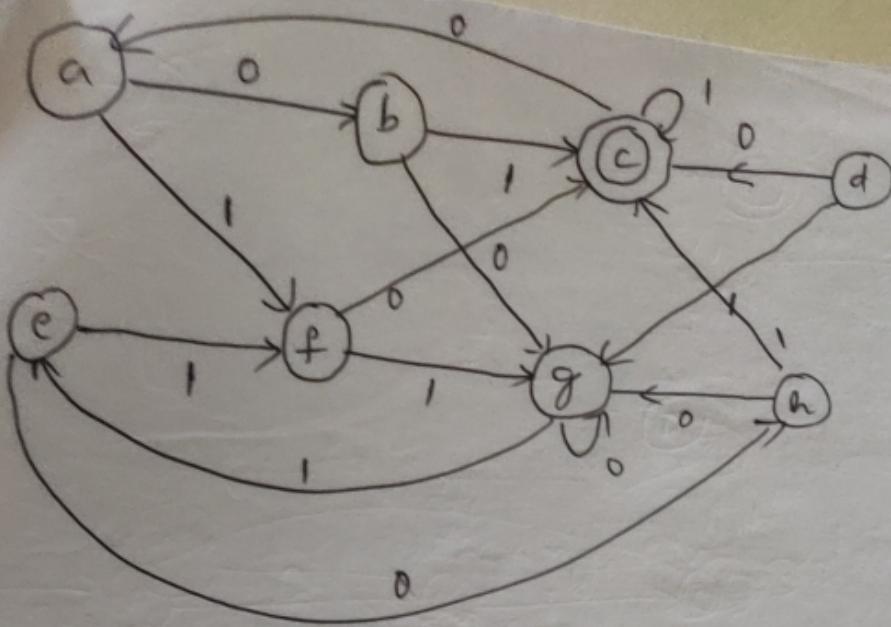
$$\delta(D, a) = D \quad | \quad \delta(D, b) = D \quad |$$

$$\delta(B, a) = D \quad | \quad \delta(B, b) = C \quad |$$

$$\delta(D, a) = D \quad | \quad \delta(D, b) = D \quad |$$



(4)



	a	b	c	d	e	f	g
b	x						
c	x	x					
d	x	x	x				
e		x	x	x			
f	x	x	x			x	
g	x	x	x	x	x	x	
h	x		x	x	x	x	x

