⇒ Context Free Grammar :- (CFG)

It is represented by 4 Tuples,

$CFG = (V, T, P, S)$

$V \rightarrow$ Set of variables (Capital Letters)

$T \rightarrow$ Finite set of Terminals

$P \rightarrow$ Production $(\underset{\text{variable}}{\alpha} \rightarrow \beta)$, $\beta \rightarrow (V \cup T)^*$

$\delta \rightarrow$

$S \rightarrow$ Start Symbol (A)

A language can be derived based on 2 methods

- Left Most Derivation (LMD)
- Right Most Derivation (RMD)

Parse Tree :- Representation of LMD & RMD

Ambigous Grammar :- Different Parse Trees for same grammar

① Derive LMD and RMD for the ~~grammar~~ grammar,

$$E \to E+T \mid T$$

$$T \to T*F \mid F$$

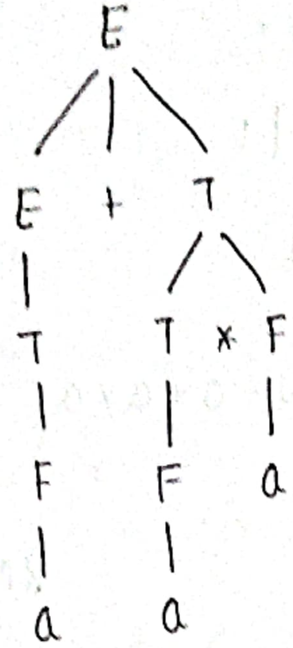
$$F \to (E) \mid a$$

consider the input, $w = a+a*a$

LMD $\to$ $E+T$
$\to$ $T+T$
$\to$ $F+T$
$\to$ $a+T$
$\to$ $a+T*F$
$\to$ $a+F*F$
$\to$ $a+a*F$
$\to$ $a+a*a$

RMD $\to$ $E+T$
$\to$ $E+T*F$
$\to$ $E+T*a$
$\to$ $E+a*a$
$\to$ $T+a*a$
$\to$ $F+a*a$
$\to$ $a+a*a$

Parse Tree :-



② Construct CFG for the language that generates equal number of a's and b's in the form $a^n b^n$.

$G = \{ (S, A), (a, b), S \rightarrow aAb, A \rightarrow aAb \mid \varepsilon \}$

$S \rightarrow aAb$

$A \rightarrow aAb \mid \varepsilon$

LMD $\rightarrow aAb$

$\rightarrow aaAbb$

$\rightarrow aaaAbbb$

$\rightarrow aaa\varepsilon bbb$

$\rightarrow aaabbb$

③ Construct a CFG for the language having any number of a's over the alphabet $\Sigma = \{a\}$ and the production rule is,

$$S \rightarrow aS$$
$$S \rightarrow \varepsilon$$

LMD → aS
↓
aaS
↓
aaaS
↓
aaaaS
↓
aaaaε

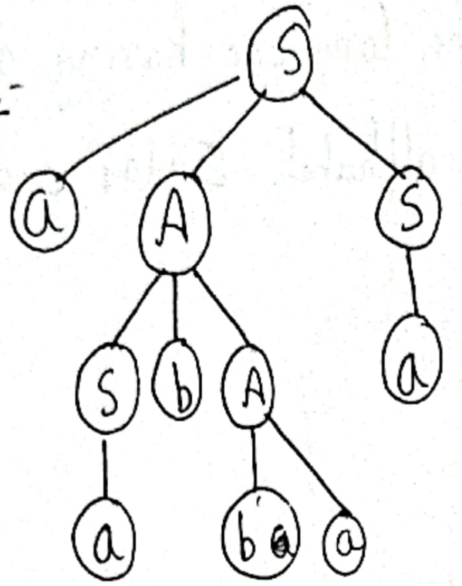
aaaa

④ Consider the following production $S \rightarrow aAS$, $S \rightarrow a$, $A \rightarrow SbA$, $A \rightarrow SS$, $A \rightarrow ba$. Find LMD, RMD and Parse Tree to generate String "aabbaa"

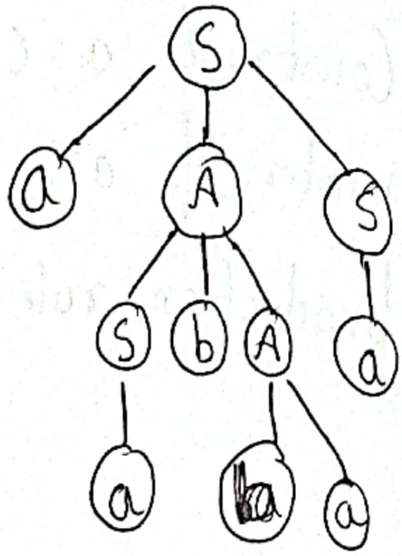
LMD → aAS
↓
aSbAS
↓
aabAS
↓
aabbaS
↓
aabbaa

RMD → aAS
↓
aAa
↓
aSbAa
↓
aSbbaa
↓
aabbaa

LMD:-



RMD:-



⑤ Derive the String "abb" using LMD & RMD for the production given by.

$S \to AB \mid \epsilon$

$A \to aB$

$B \to Sb$

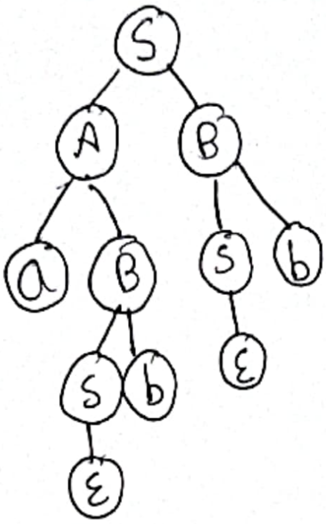
LMD:-

AB
$\downarrow$
aBB
$\downarrow$
$aSbB$
$\downarrow$
$a\varepsilon bB$
$\downarrow$
$a\varepsilon bSb$
$\downarrow$
$a\varepsilon b\varepsilon b$
$\downarrow$
abb

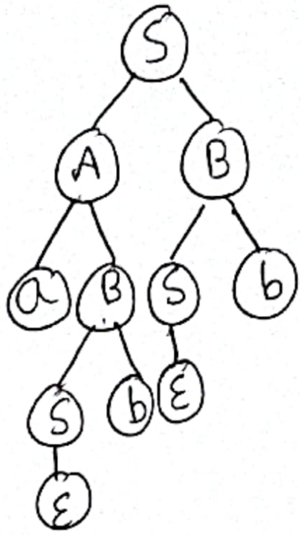
RMD:-

AB
$\downarrow$
ASb
$\downarrow$
$A\varepsilon b$
$\downarrow$
$aB\varepsilon b$
$\downarrow$
$aSbb$
$\downarrow$
$a\varepsilon bb$
$\downarrow$
abb

LMD:-



RMD:-

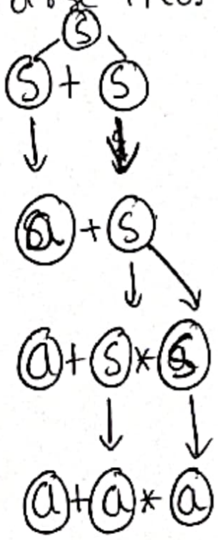


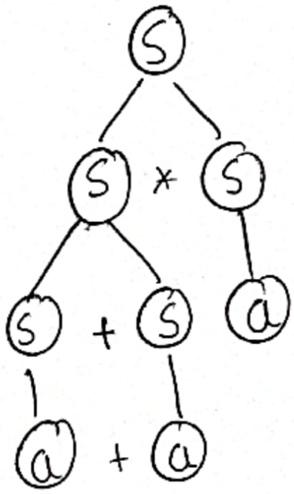
⑥ Generate ambigous Parse Tree for the grammar,

$S \rightarrow S+S \mid S*S \mid a$ for input $a+a*a$

Parse Trees

LMD:-





1) Construct a CFG for the language having any number of a's over the set $\Sigma = \{a\}$

$L = \{a^n, n \geq 0\}$

$L = \{\varepsilon, a, aa, \ldots\}$

Variable $\rightarrow \{A\}$

Terminal $\rightarrow \{a\}$

Production:-

$A \longrightarrow Aa \mid \varepsilon$

Starting Symbol $\rightarrow$ A

$G = \{A, a, A \rightarrow Aa \mid \varepsilon, A\}$

2) Construct a CFG for the language having any number of a's and any number of b's.

$L = \{\varepsilon, a, b, ab, abb \ldots\}$

$R = (a+b)^*$

Variable $\rightarrow \{S\}$ Terminal $\rightarrow \{a, b\}$

Production:-

$S \rightarrow Sa \mid Sb \mid \varepsilon$

③ Construct a CFG for language

$$L = \{a^n b^n \mid n \geq 1\}$$

Variable $\rightarrow \{S\}$

Terminal $\rightarrow \{a, b\}$

~~S → / S / S | a | b~~

$S \rightarrow aSb$

$S \rightarrow ab$

④ Construct a CFG for

(i) $L = \{a^n b^{2n} , n \geq 1\}$

$S \rightarrow aSbb$

$S \rightarrow abb$

(ii) $L = \{ w c w^T \mid w \in (a,b)^{n*}, n \geq 0 \}$

$S \rightarrow STS$

$T \rightarrow c \mid \varepsilon$

$S \rightarrow$ ~~a|b|ba~~

$S \rightarrow a \mid b \mid \varepsilon$

$S \rightarrow aSa$

$S \rightarrow bSa$

$S \rightarrow c$

(iii) $L(G) = \{a^n b^n c^m d^m,\ n, m \geq 0\}$

$L(G) = \{\varepsilon, ab, cd, abcdcd\}$

$S \to aSbS \mid cSd$

$S \to ab \mid cd \mid \varepsilon$

① Check the grammar is ambigious (or) not

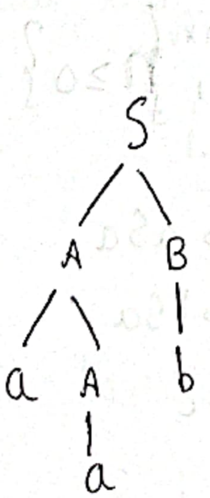
(i) $S \to AB \mid aaB$

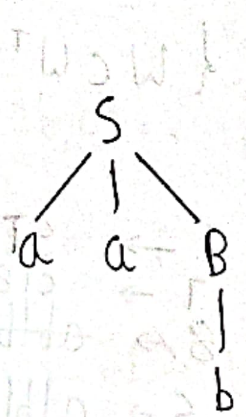
$A \to a \mid Aa$

$B \to b$

LMD $\longrightarrow$ $S \to AB$
$\downarrow\downarrow$
aB
$\downarrow$
ab

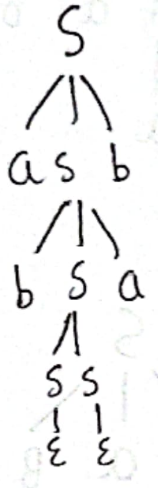
RMD $\longrightarrow$ $S \to aaB$
$\downarrow\downarrow$
aab

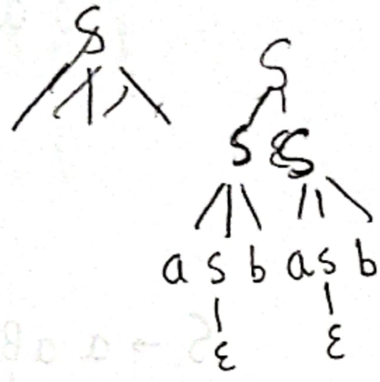




(ii) S.T. $G = (\{S\}, \{a,b\}, S \to aSb \mid bSa \mid SS \mid \varepsilon)$

is ambigious





→ Problems with ~~Un~~ambigious grammar:-

- Associativity
- Precedence

(2) Consider the grammar

$S \to AB \mid aaB$

$A \to Sa \mid Aa$

$B \to b$

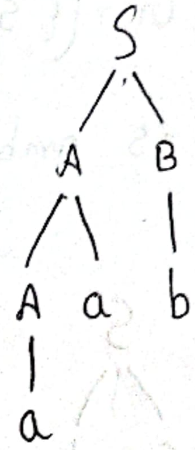
Generate the string aab for the grammar, check the gramar is ambigous. Construct an unabigous grammar equivalent to CFG.

LMD:- $S \rightarrow AB$

Aab

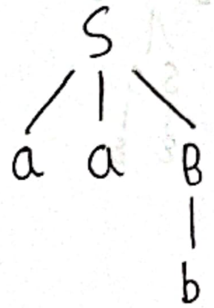
aab

~~RMD~~ $\Rightarrow$



$S \rightarrow aaB$

b



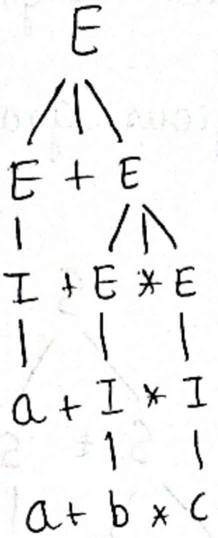
Unambigious grammoris:-

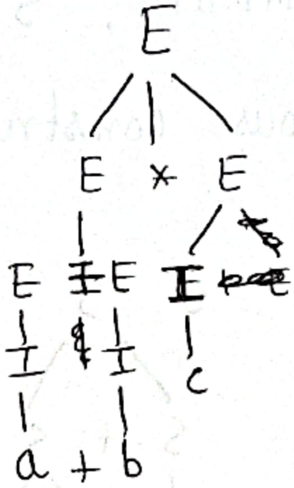
$S \rightarrow AB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

③ Consider the grammor $G = (V, T, P, S)$ with $V = \{E, I\}$, $T = \{a, b, c, d, +, *, (,)\}$, $P \Rightarrow E \rightarrow I$, $E \rightarrow E+E$, $E \rightarrow E*E$, $E \rightarrow (E)$, $I \rightarrow a|b|c$, Construct derivation tree for the string $a+b*c$, check the grammor is ambigious. Construct an unambigous grammor equivalent to G.



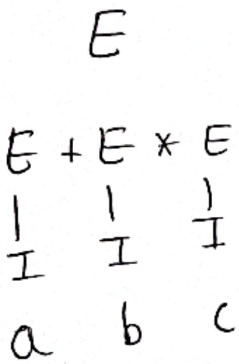


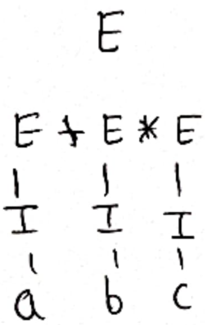
$$E \rightarrow I$$

$$E \rightarrow E + E * E$$

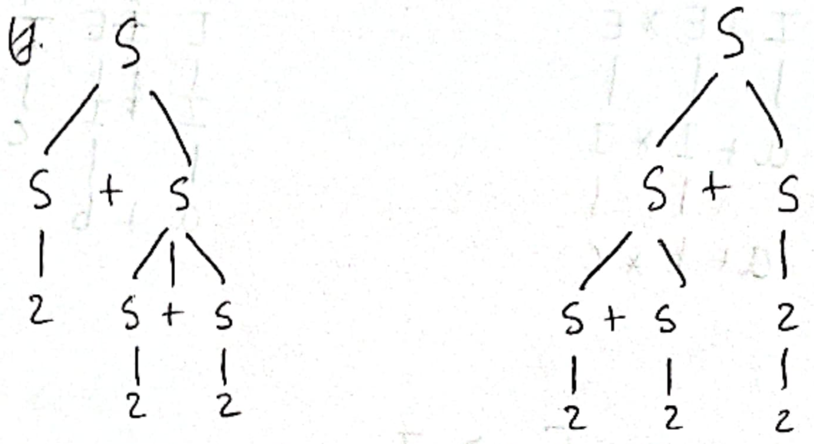
$$E \rightarrow (E)$$

$$I \rightarrow a|b|c$$



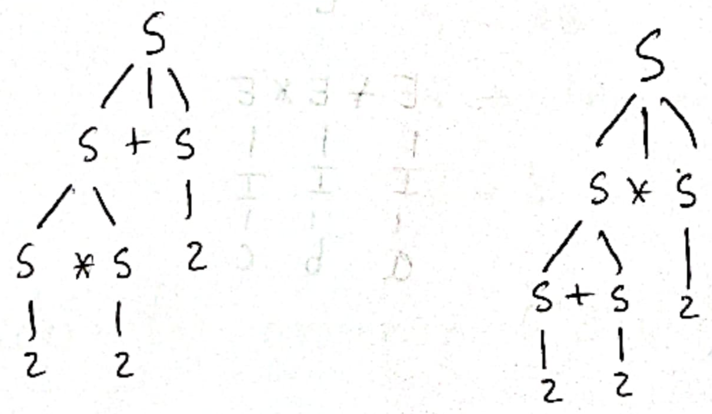


(4) A grammar, $S \rightarrow S+S \mid 2$, Check the grammar is ambigious. Construct an unambigious grammar equivalent to



Unambigous :- $S \rightarrow S+2 \mid 2$

(5) $S \rightarrow S+S \mid S*S \mid 2$



Unambigious :- $S \rightarrow S+S*S \mid 2$

⇒ Chomsky's Normal Form (CNF) :-

① Convert the following CFG to CNF

$$S \to ASA \mid aB$$
$$A \to B \mid S$$
$$B \to b \mid \varepsilon$$

Step ① :- If start symbol (S) appears on right side, create a new start symbol S'

i.e $S' \to S$

$$S \to ASA \mid aB$$
$$A \to B \mid S$$
$$B \to b \mid \varepsilon$$

Step ② :- Removing Null production

$$S' \to S$$
$$S \to ASA \mid aB \mid a$$
$$A \to B \mid S \mid \varepsilon$$
$$B \to b$$

$$S' \to S$$
$$S \to ASA\cancel{B} \mid aB \mid a \mid SA \mid AS \mid S$$
$$A \to B \mid S$$
$$B \to b$$

Step ③ :- Removing Unit production

Unit production :- A variable producing another variable

Unit Productions :- $S' \to S$, $S \to S$, $A \to B$, $A \to S$

$\Rightarrow$ $S \to ASA \mid aB \mid a \mid SA \mid AS$

$S' \to ASA \mid aB \mid a \mid SA \mid AS$

$A \to b \mid ASA \mid aB \mid a \mid SA \mid AS$

$B \to b$

Step ④ :- Maximum variables allowed in CNF is 2, so three variables must be replaced by some other variable

$\therefore$ $S \to ASA$

$S' \to ASA$

$A \to ASA$

Let, $\cancel{SA}$ $X \to SA$

$\therefore$ $S \to AX \mid aB \mid a \mid SA \mid AS$

$S' \to AX \mid aB \mid a \mid SA \mid AS$

$A \to b \mid AX \mid aB \mid a \mid SA \mid AS$

$B \to b$

Step ⑤ :- In CNF only productions having variables ⓞ terminals are allowed but production having one variable and one terminal is not allowed.

$$\therefore \quad Y \to a$$

$$S \to AX \mid \cancel{X}B \mid a \mid SA \mid AS$$

$$S' \to AX \mid \cancel{a}XB \mid a \mid SA \mid AS$$

$$A \to b \mid AX \mid YB \mid a \mid SA \mid AS$$

$$B \to b$$

② Convert CFG to CNF

$$S \to aA \mid aBB$$

$$A \to aaA \mid \varepsilon$$

$$B \to bB \mid bbC$$

$$C \to B$$

①

$$S \to aA \mid aBB$$

$$A \to aaA \mid \varepsilon$$

$$B \to bB \mid bbC$$

$$C \to B$$

(ii) Remove Null Production

A → S

A → BC

$S \to aA \mid aBB \mid a$

$A \to aaA \mid aa$

$B \to bB \mid bbC$

$C \to B$

(iii) Removing unit Production

$S \to aA \mid aBB \mid a$

$A \to aaA \mid aa$

$B \to bB \mid bbC$

$C \to bB \mid bbC$

(iv)

$X \to BB \qquad Z \to a$

$Y \to b$

$S \to \cancel{aA \mid aX \mid} a$

$A \to \cancel{ZA \mid Z}$

$B \to \cancel{bB \mid Y}C$

$C \to \cancel{bB \mid Y}C$

$S \to ZA \mid ZX \mid a$

$A \to ZZA \mid ZZ$

$B \to YB \mid YYC$

$C \to YB \mid YYC$

(v) $P \rightarrow a, \quad Q \rightarrow b \qquad P \rightarrow YY \rightarrow Q \rightarrow$

$X \rightarrow BB \quad Z \rightarrow aa \quad Y \rightarrow bb$

$\therefore \quad S \rightarrow PA \mid PX \mid P$

$A \rightarrow ZA \mid Z$

$B \rightarrow QB \mid YC$

$C \rightarrow QB \mid YC$

Note :- A Chomsky normal form consists of Production