Push Down Automata (PDA)

A pushdown Automata is formally defined by 7 tuples as shown below:

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where

Q → A finite set of states.

Σ → A finite set of input symbols.

Γ → A finite stack alphabet

δ → The transition function.

q_0 → The starting state.

Z_0 → The start stack symbol

F → The set of Final / Accepting states.

A Pushdown Automata is a way of implement a context free grammar in a similar in a similar way, we design finite Automata for Regular grammar.

* it is more powerful than FSM.
* FSM has a very limited memory but PDA has more memory.

— PDA — Finite State Machine — stack.

Naan Chomsky gave a mathematical model of grammar which is effective for writing Computer languages.

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The four types of grammar according to Naan
are:

Grammar Type	Grammar Accepted	Language Accepted	Automata
Type 0	Unrestricted Grammar	Recursively Enumerable language	Turing Machine
Type 1	Content Sensitive Grammar	Content Sensitive Language	Linear Bounded Automaton
Type 2	Content free grammar	Content Free language	pushdown Automata
Type 3	Regular grammar	Regular Language	Finite State Automata.

A Stack is a way we arrange elements one on top of another,
A stack does basic two operations:

PUSH: A new element is added at the top of the stack.

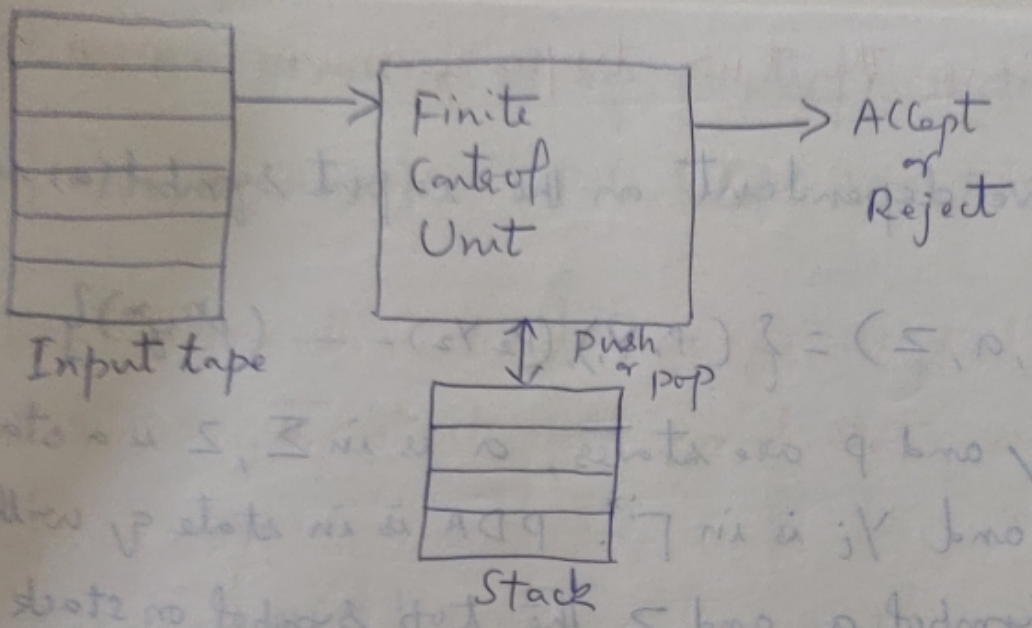
POP: The top element of the stack is read & removed.

A pushdown Automata has 3 components:

i) Input tape.

ii) A finite Control unit.

iii) A Stack with infinite Size.



δ takes as arguments a triple $\delta(q, a, X)$ where:

i) q is a state in Q

ii) a is either an input symbol in Σ or $a = \epsilon$

iii) X is a stack symbol, that is a member of Γ

The output of δ is finite set of pairs (p, γ) where:

p is a new state

γ is a string of stack symbols that replaces X at the top of the stack.

eg:- If $\gamma = \epsilon$ then the stack is popped.

If $\gamma = X$ then the stack is unchanged.

If $\gamma = YZ$ then X is replaced by Z and Y is pushed onto the stack.

1) 08 Specify the two types of moves in PDA

The move dependent on the input symbol (a) sca... is:

$$\delta(q, a, Z) = \{(P_1, Y_1), (P_2 Y_2) \cdots (P^m, Y^m)\}$$

where q and p are states, a is in Σ, Z is a stack symbol and Y_i is in Γ^{i*}. PDA is in state q with input symbol a and Z the top symbol on stack, enter state P_i and replace symbol Z by string Y^i.

The move independent on input symbol is (ϵ-moves):

$$\delta(q, \epsilon, Z) = \{P_1, Y_1), (P_2 Y_2), \cdots (P^m, Y^m)\}.$$

The PDA is in state q, independent of input symbol being scanned and with Z the top symbol on the stack enter a state P_i and replace Z by Y^i.

2) What are the different types of language acceptances by a PDA and define them.

For a PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ we define:

i) Language accepted by final state L(M) as:

$\{w \mid (q_0, w, Z_0) \vdash (P, \epsilon, Y)$ for some P in F and Y in $\Gamma^*\}$

Language accepted by empty / Null stack $N(M)$:

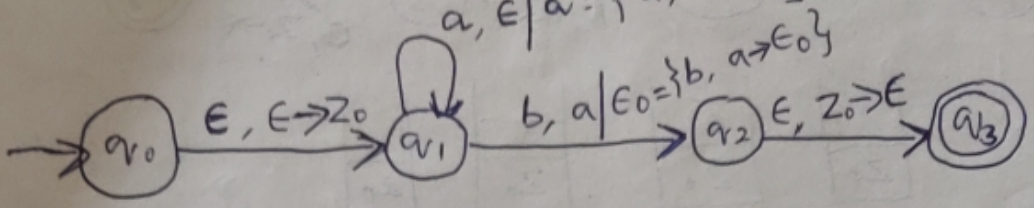
$\{w \mid (q_0, w, Z_0) \vdash (p, \epsilon, \epsilon)$ for some p in $Q\}$

3) Is it true that the language accepted by a PDA and by empty stack and final states are different languages.?

No, because the languages accepted by PDA's by final state are exactly the languages accepted by PDA's by empty stack.

Design a PDA for the language

$L = \{ a^n b^n \mid n \geq 1 \} = L_1 = \{ aabb \}$



$\delta(q_0, a, Z_0) = (q_1, aZ_0)$

$\delta(q_1, a, a) = (q_1, aa)$

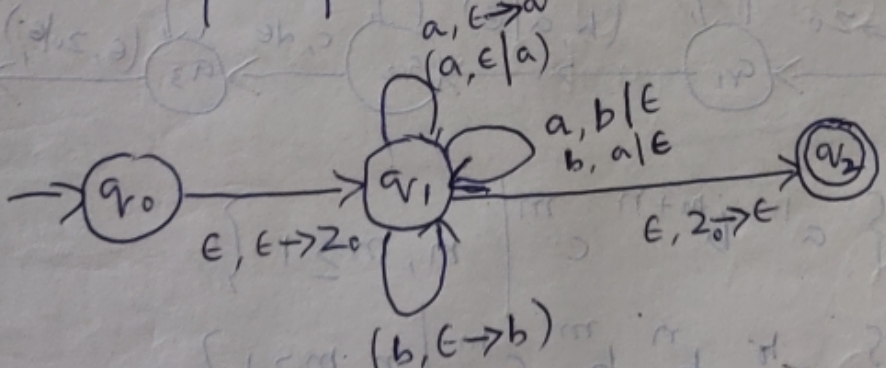
$\delta(q_1, b, a) = (q_2, \epsilon)$

$\delta(q_2, b, a) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, Z_0) = (q_3, Z_0)$

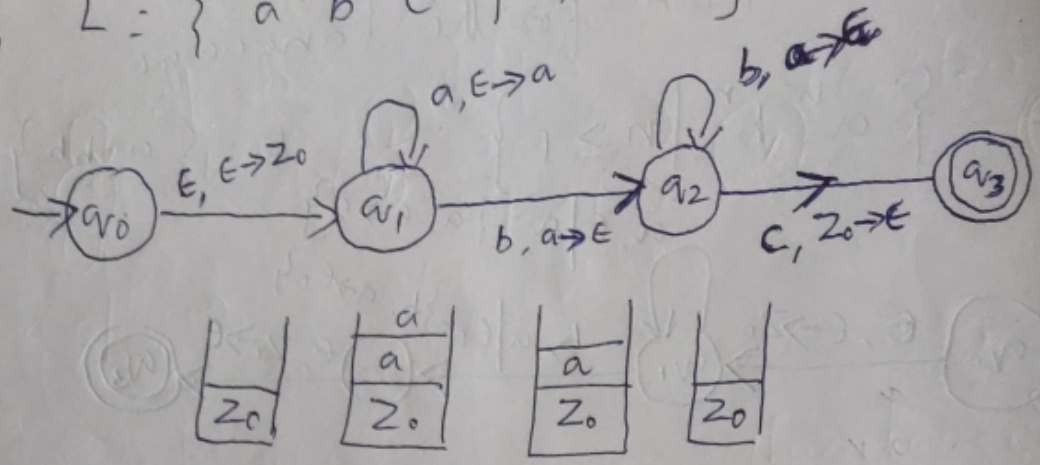
PDA - [Acceptance by Final state / Empty stack acceptance]

2) $L = \{ w \mid n_a(w) = n_b(w) \mid a^n b^n \; n \geq 1 \}$

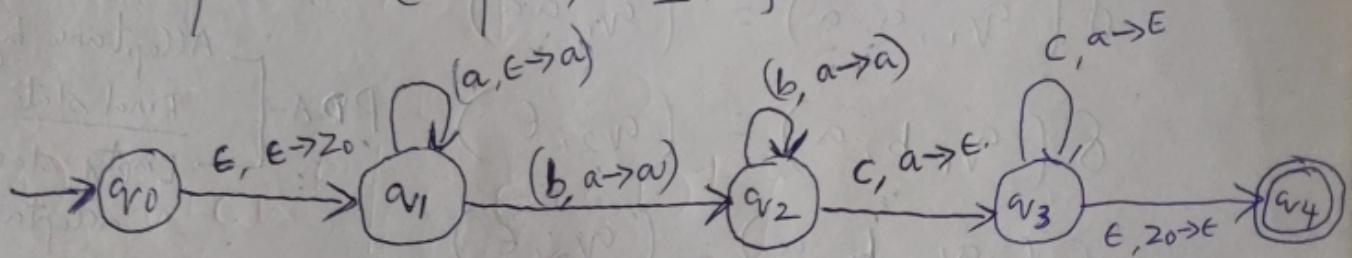


$L = \{ aabb, \; baba, \; bbaa \}$

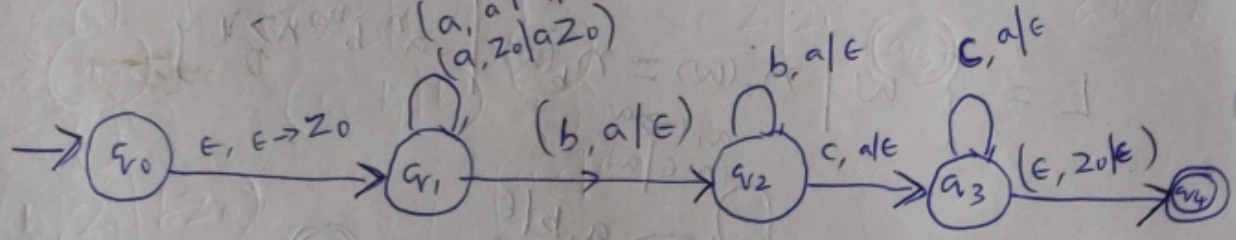
3) $L = \{ a^n b^n c^m \mid n, m \geq 1 \}$



4) $L = \{ a^n b^m c^n \mid n, m \geq 1 \}$



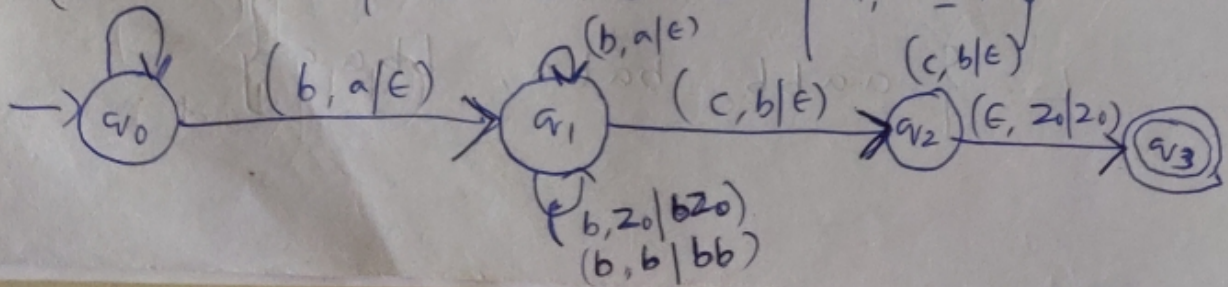
5) $L = \{ a^{m+n}, b^m, b^n \mid m, n \geq 1 \}$



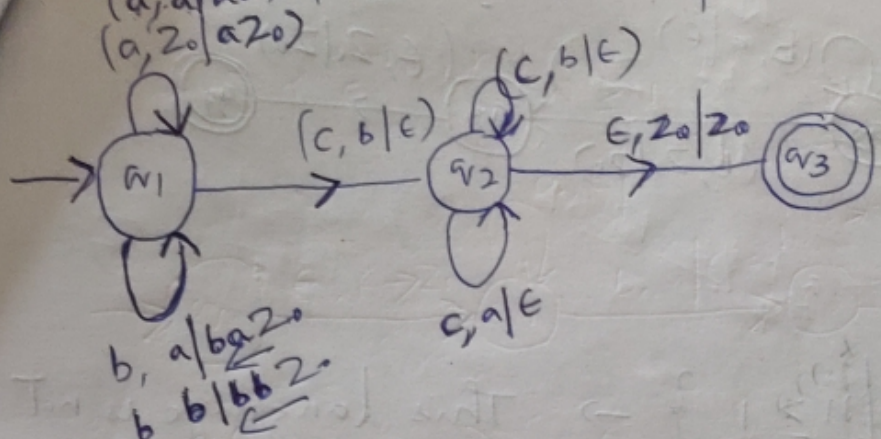
6) $L = \{ a^n b^{m+n} c^m \mid n, m \geq 1 \}$

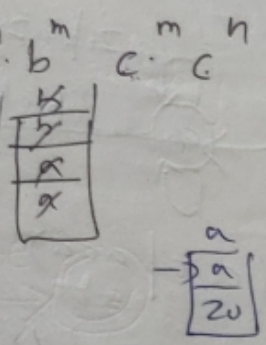
$= \{ a^n . b^n . b^m . c^m \mid n, m \geq 1 \}$

(a, a|aa)
(a, Z0|aZ0)



$L = \{ a^n b^m c^{n+m} \mid n, m \geq 1 \}$





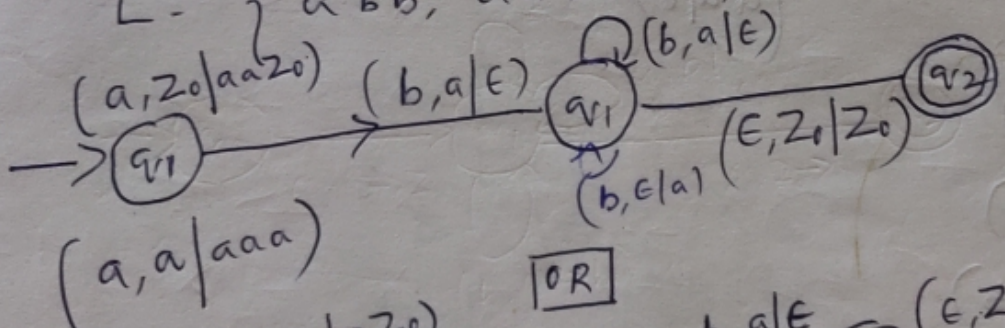
8) $a^n b^n c^m d^m \mid n, m \geq 1$

9) $a^n b^m c^m d^n \mid n, m \geq 1$

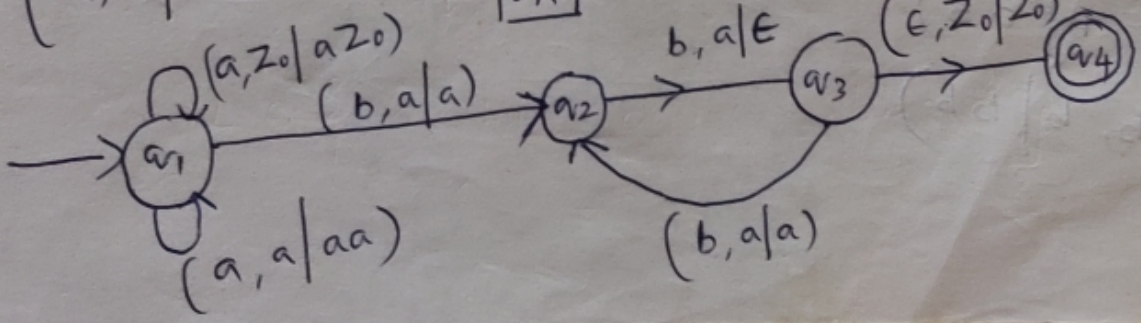
10) $a^n b^m c^n d^m \mid n, m \geq 1$ → This language is not Context free & hence cannot be design using PDA.

11) $L = \{ a^n b^{2n} \mid n \geq 1 \}$

$L: \{ abb, aabbbb, aaabbbbbb \ldots \}$



OR



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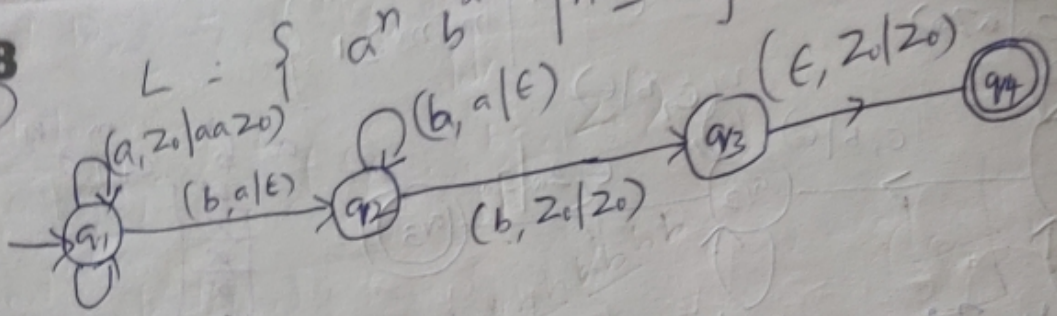
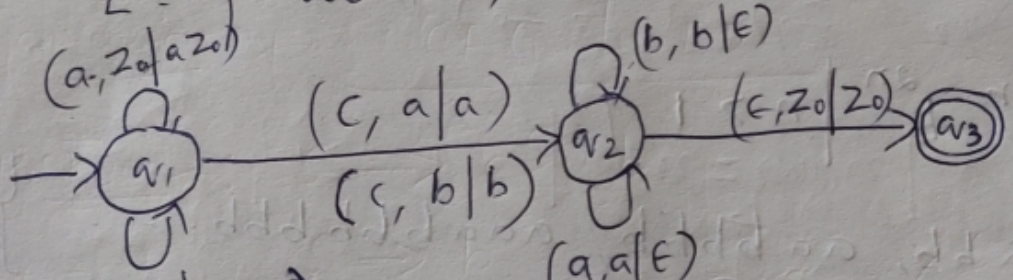
12) $L = \{ a^n b^{2n+1} \mid n \geq 1 \}$



13) $L = \{ a^n b^n c^n \mid n \geq 1 \}$ → This language is not Context free & hence cannot design PDA.

14) $L = \{ wcw^R \mid w \in (a,b)^+ \}$

$L = \{$ abcba, abbcbba ---- $\}$

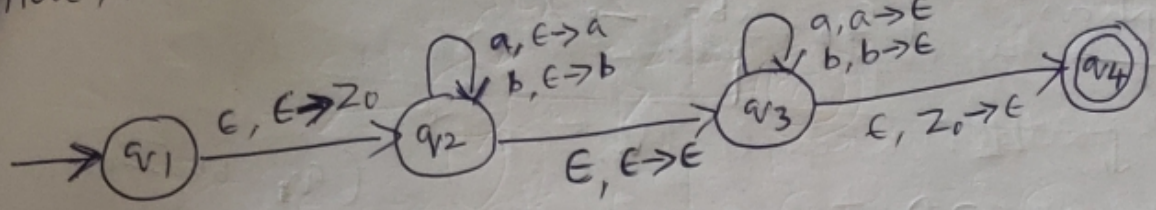


a b ¢ b

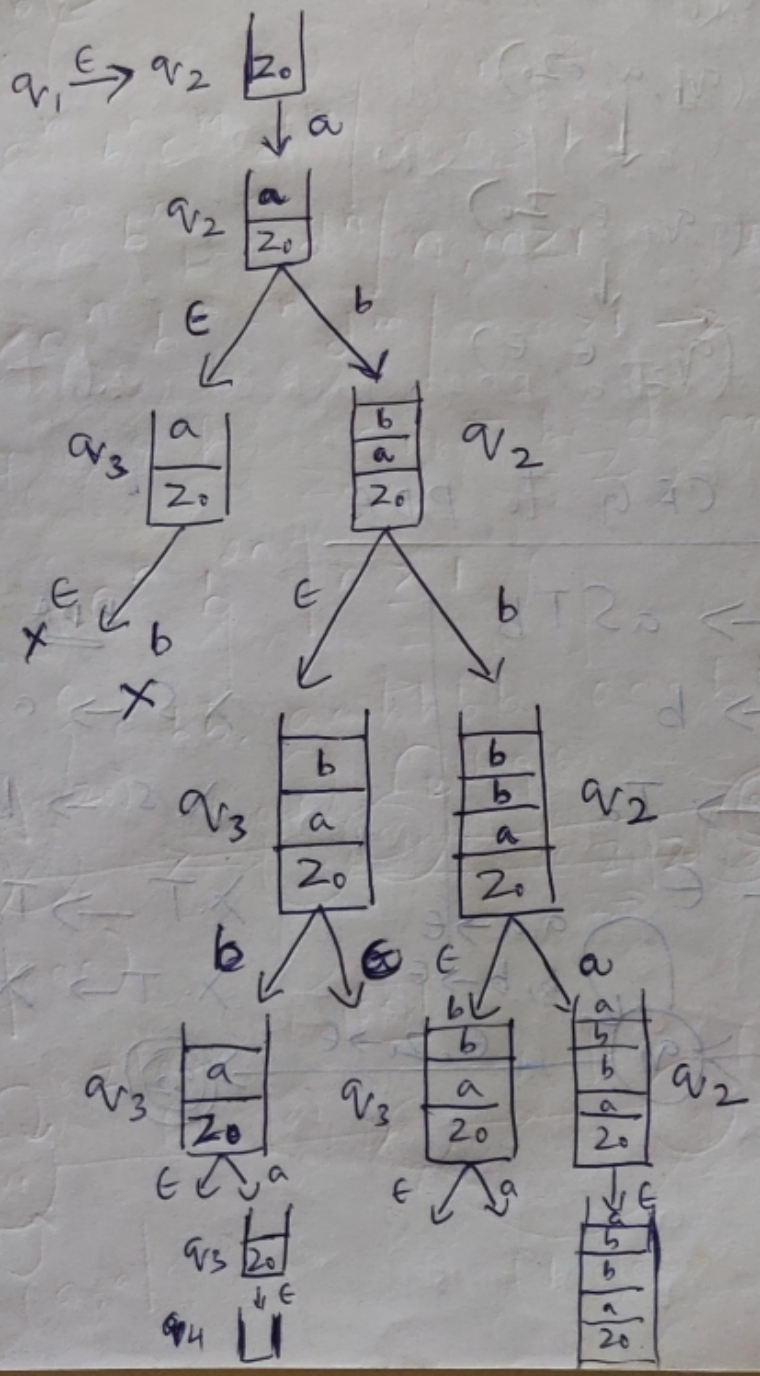
(a, a | aa)
(b, a | ba)
(a, b | ab)
(b, b | bb)

$L = \{ ww^R / w \in (a+b)^+ \}$ (even palindrome)

Here, we will use non deterministic PDA.



Example :- ε a ε b ε b ε a ε



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ID :- (q_0, aba, ϵ)

Instantaneous description

(q_0, aba, Z_0)

$(q_1, ba\ Z_0)$

(q_0, ba, aZ_0)

(q_0, a, baZ_0) (q_1, a, aZ_0)

(q_1, ϵ, Z_0)

$(q_2, \epsilon, \epsilon)$

Conversion from CFG to PDA

1) Grammar : $S \rightarrow aSTb$

$S \rightarrow b$

$T \rightarrow Ta$

$T \rightarrow \epsilon$

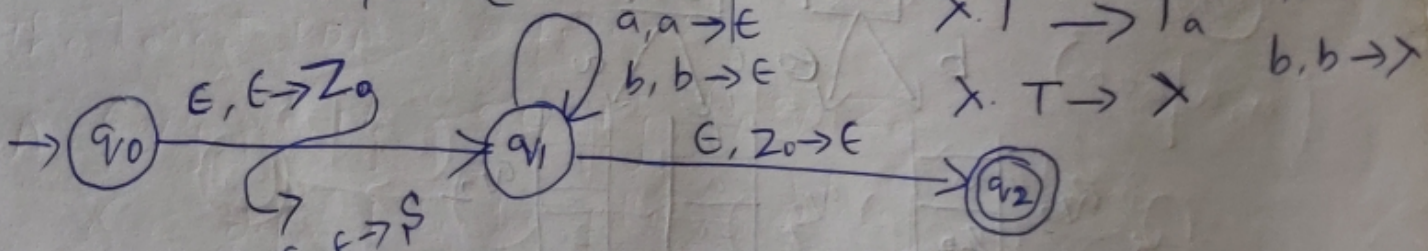
PDA

$\chi . S \rightarrow aSTb$

$\chi . S \rightarrow b$

$\chi . T \rightarrow Ta$

$\chi . T \rightarrow \chi$

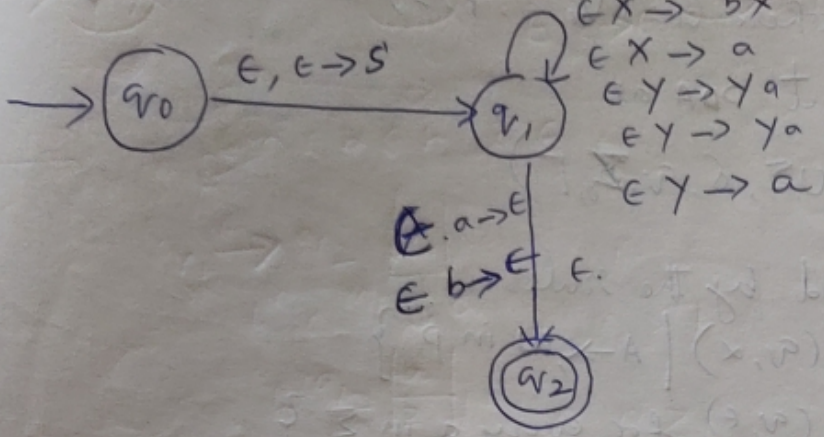


$S \to XY$

$X \to aX \mid bX \mid a$

$Y \to Ya \mid Yb \mid a$

$\in . S \to XY$
$\in X \to aX$
$\in X \to bX$
$\in X \to a$
$\in Y \to Ya$
$\in Y \to Ya$
$\in Y \to a$



3) $S \to aS \mid aA$
$A \to bA \mid b$

$R_1 = (q_0, \in, A) \to (q_0, \alpha)$

$R_2 (q_0, a, a) \to (q_0, \in)$

$S \to aS$
$S \to aA$
$A \to bA$
$A \to b$

$(q_0, \in, S) \to (q_0, aS)$
$(q_0, \in, S) \to (q_0, aA$
$(q_0, \in, A) \to (q_0, bA)$
$(q_0, \in, A) \to (q_0, b)$
$(q_0, a, a) \to (q_0, \in)$
$(q_0, b, b) \to (q_0, \in)$

Push Down Automata to Context free grammar

Theorem :- If L is a CFL, then we can construct a PDA accepting L by empty stack $L = N(A)$

Method :- Let G where $G = (V_N, \Sigma, P, S)$ is a Context free grammar, we construct PDA as

$A = (\{q\}, \Sigma, V_N \cup \Sigma, \delta, q, Z_0, \Gamma)$

where δ is defined by the rules.

$R_1 : \delta(q, \epsilon, A) = \{(q, \alpha) \mid A \to \alpha \text{ in } P\}$

$R_2 : \delta(q, a, a) = \{(q, \epsilon) \text{ for every } a \text{ in } \Sigma\}$

Example :- 1 : Construct a PDA which is Equivalent to the following given CFG.

$S \to OCC$

$C \to OS$

$C \to 1S$

$C \to 0$

Test whether 010^4 is accepted by $N(A)$.

The target PDA is as follows:

$A = \{q\} \{0, 1\} \{S, C, 0, 1\}, \delta, q, Z_0$

δ is defined by the following rules:

$R_1 : \delta(q, \epsilon, S) = \{q, OCC\}$

$\delta(q, \epsilon, C) = \{(q, OS), (q, 1S), (q, 0)\}$

R_2 : $\delta(q, 0, 0) = \{q, \epsilon\}$

R_2 : $\delta(q, 1, 1) = \{q, \wedge\}$

$(q, 010^4, S) \vdash (q, 010^4, 0cc) \vdash (q, 10^4, cc)$

$\vdash (q, 10^4, 1sc) \vdash (q, 0^4, sc)$

$\vdash (q, 0^4\ 0ccc) \vdash (q, 0^3, ccc)$

$\vdash (q, 0^3, 0cc) \vdash (q, 0^2, cc)$

$\vdash (q, 0^2, 0c) \vdash (q, 0, c)$

$\vdash (q, 0, c) \vdash (q, \epsilon, \epsilon)$

Thus $010^4 \in N(A)$

Conversion from PDA to CFG :- If $A=(Q,\Sigma,\Gamma,\delta,q_0,F)$ is a PDA, then CFG is defined as $G=(V,T,P,S)$

Step1 :- Construction of set of non terminals.

$$V=\{S\}\cup\{[q,Z,q'] \mid q,q'\in Q, Z\in\Gamma\}$$

Step2 :- i) S production $S\to[q_0,Z_0,q]$, $q\in Q$

ii) For pop operation

$$\delta(q,a,Z)\to(q',\epsilon)$$

$$[q,Z,q']\to a$$

iii) For push and no operation.

$$\delta(q,a,Z)\to(q_1,Z_1Z_2\cdots Z_m)$$

$$[q,Z,q']\to a[q_1Z_1q_2][q_2Z_2q_3]\cdots\cdots[q_mZ_mq']$$

where $q',q_1,q_2\cdots q_m\in Q$

1) Construct CFG from PDA $A=(\{q_0,q_1\},\{a,b\},\{Z_0,Z\},\delta,q_0,Z_0,\phi\}$ and δ is given by

$\delta(q_0,b,Z_0)=(q_0,ZZ_0)$

$\delta(q_0,b,Z)=(q_0,ZZ)$

$\delta(q_1,b,Z)=(q_1,\epsilon)$

$\delta(q_0,\epsilon,Z_0)=(q_0,\epsilon)$

$\delta(q_0,a,Z)=(q_1,Z)$

$\delta(q_1,a,Z_0)=(q_0,Z_0)$

Step1 :- Construction of set of non terminals.

$$V=\{S\}\cup\{[q,Z,q]\} \mid q,q'\in Q, Z\in\Gamma$$

$V=S,\ [q_0,Z_0,q_0]\quad [q_0,Z_0q_1]$

$[q_0Z_1,q_0]\quad [q_0,Z_1q_1]$

10 $[q_1 Z_0 q_0]$ $[q_1 Z_0 q_1]$

$[q_1 Z q_0]$ $[q_1 Z q_1]$

(2) $S \to [q_0 Z_0 q_0]$

$S \to [q_0 Z_0 q_1]$

(3) $\delta(q_0, b, Z_0) = (q_0, Z Z_0)$

$[q_0 Z_0 q_0] \to b\,[q_0 Z q_0][q_0 Z_0 q_0]$

$[q_0 Z_0 q_0] \to b\,[q_0 Z q_1][q_1 Z_0 q_0]$

$[q_0 Z_0 q_1] \to b\,[q_0 Z q_0][q_0 Z_0 q_1]$

$b\,[q_0 Z q_1][q_1 Z_0 q_1]$

(4) $\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$(q_0, Z_0, q_0) \to \epsilon$

(5) $\delta(q_0, a, Z_0) = (q_1, Z)$

$(q_0, Z, q_0) \to a\,(q_1, Z$