10.4.1.2

EE24BTECH11001 - Aditya Tripathy

Question:
Find the roots of the equation $x^2 - 2x = (-2)(3 - x)$
Solution:
On simplification, the equation to solve is,

$$x^2 - 4x + 6 = 0 \tag{0.1}$$

The motive now is to solve the equation at hand using a matrix based approach.
Recall that for a matrix A of order n, the characteristic equation is given by,

$$det(A - \lambda I) = a_n\lambda^n + a_{n-1}\lambda^{n-1} \cdots + a_0 = 0 \tag{0.2}$$

Recall that the solutions to the characteristic polynomial are the eigenvalues of the the matrix A. So if we can somehow construct the corresponding matrix from the characteristic polynomial, our job will be finished, since we can find the eigen values using the QR alogorithm.
Companion matrix: A matrix is said to be the companion of a polynomial $f(x)$ if $det(A - \lambda I) = 0 \implies f(x) = 0$.
For,

$$f(x) = c_0 + c_1x \cdots + x^n \tag{0.3}$$

The companion matrix is,

$$\begin{pmatrix} 0 & 0 & \cdots & 0 & -c_0 \\ 1 & 0 & \cdots & 0 & -c_1 \\ 0 & 1 & \cdots & 0 & -c_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -c_{n-1} \end{pmatrix} \tag{0.4}$$

For the equation at hand, the companion matrix is,

$$\begin{pmatrix} 0 & -6 \\ 1 & 4 \end{pmatrix} \tag{0.5}$$

Using the QR algorithm we can now solve for the eigenvalues and thus the solutions for the given equation.
On running the code to solve for eigenvalues we get:

```
  Upper Hessenberg Matrix
0.000000000 +0.000000000i -6.000000000 +0.000000000i
1.000000000 +0.000000000i 4.000000000 +0.000000000i
eigenvalue 1: 2.000000 + 1.414214i
```

```
eigenvalue 2: 2.000000 + -1.414214i
```