Numerical Solutions to Equations

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Outline

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Problem

Problem Statement

Solve the equation $x^2 - 2x = -2(3 - x)$ or $x^2 - 4x + 6 = 0$

Fixed Point Iteration

Towards the Update Eqn

Take an initial guess x_0. The update difference equation will use the following function:

$$x = g(x) \tag{3.1}$$

For our problem,

$$g(x) = \frac{1}{4}x^2 + \frac{3}{2} \tag{3.2}$$

Update Equation

Now the update equation will be,

$$x_{n+1} = g\left(x_n\right) \tag{3.3}$$

When we try to run the iterations however, we realize that whatever be the initial guess, the subsequent updated values grow without bound. This is becaue of the following theorem

Theorem

Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative in some interval J containing s. Then if $|g'| \leq K < 1$ in J, the iteration process defined above converges for any x_0 in J. The limit of the sequence $[x_n]$ is s

Conclusion 1

Since there is no solution (evident by quadratic formula) there exists no interval J for which the process converges to a point.

Newton Raphson Method

The Method

tart with an initial guess x_0, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)} \tag{4.1}$$

where ,

$$f\left(x\right) = x^2 - 4x + 6 \tag{4.2}$$

$$f'\left(x\right) = 2x - 4 \tag{4.3}$$

Behaviour of the method

The behaviour shown here is that regardless of which guess we take, it reaches a point of extrema(derivative ≈ 0) and then the process halts, or the updated point grow with bound.

The Solution

Complex initital guesses!

To get the complex solutions, however , we can just take the initial guess point to be a random complex number.

Code Output

$$\text{Running Newton iterations:} \tag{5.1}$$

$$\text{x got too big} \tag{5.2}$$

$$\text{Trying fixed point iterations:} \tag{5.3}$$

$$\text{x got too big} \tag{5.4}$$

$$\text{Trying complex Newton's iterations:} \tag{5.5}$$

$$\text{Solution} = 2.000000 + 1.414214 \text{ i} \tag{5.6}$$

$$\tag{5.7}$$

Code Output

And on a second run,

$$\text{Running Newton iterations:} \tag{5.8}$$

$$\text{Failure} \tag{5.9}$$

$$\text{Trying fixed point iterations:} \tag{5.10}$$

$$\text{x got too big} \tag{5.11}$$

$$\text{Trying complex Newton's iterations:} \tag{5.12}$$

$$\text{Solution} = 2.000000 + \text{-}1.414214 \text{ i} \tag{5.13}$$

$$\tag{5.14}$$