

9.6.1

EE24BTECH11001 - Aditya Tripathy

Question:

Find the area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Solution:

Integral to calculate,

$$J = \int_{-a}^a 2\frac{b}{a} \sqrt{a^2 - x^2} dx \quad (0.1)$$

$$(0.2)$$

Using the trapezoidal rule,

$$J = \int_a^b f(x) dx \approx h \left(\frac{1}{2} f(x) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (0.3)$$

$$h = \frac{b-a}{n} \quad (0.4)$$

$$J = j_n, \text{ where, } j_{i+1} = j_i + k \frac{f(x_{n+1}) + f(x_n)}{2} \quad (0.5)$$

$$\rightarrow j_{i+1} = j_i + \frac{bk}{a} \left(\sqrt{a^2 - x_{n+1}^2} + \sqrt{a^2 - x_n^2} \right) \quad (0.6)$$

$$x_{n+1} = x_n + k \quad (0.7)$$

Theoretical Solution:

$$J = \int_{-2}^2 2\frac{b}{a} \sqrt{a^2 - x^2} dx \quad (0.8)$$

$$(0.9)$$

Using the following result,

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \quad (0.10)$$

$$J = 2\frac{b}{a} \left(\frac{\pi a^2}{2} \right) = \pi ab \quad (0.11)$$

Substituting $a = 2, b = 3$

$$J = 6\pi \approx 18.84955592153876 \quad (0.12)$$

Computational solution : 18.849477036874855