

9.1.2

EE24BTECH11001 - Aditya Tripathy

Question:

Plot a solution to the following differential equation:

$$y' + 5y = 0$$

Solution:

The aim is to find the difference equation using the trapezoidal law using the following initial conditions, $x_0 = 0, y_0 = 1$

$$y' = -5y \quad (0.1)$$

$$\int_{y_n}^{y_{n+1}} dy = \int_{x_n}^{x_{n+1}} -5y dx \quad (0.2)$$

$$(0.3)$$

Using the trapezoidal rule,

$$J = \int_a^b f(x) dx \quad (0.4)$$

$$\frac{y_{n+1} - y_n}{1} \left(\frac{1}{2} + \frac{1}{2} \right) = -5 \frac{x_{n+1} - x_n}{1} \left(\frac{y_n}{2} + \frac{y_{n+1}}{2} \right) \quad (0.5)$$

$$\rightarrow y_{n+1} = \frac{(2 - 5h)y_n}{5h + 2} \quad (0.6)$$

To check how close the approximate graph is to the actual solution, we will solve the original equation using a Laplace Transform method:

Let $\mathcal{L}(y) = Y$

$$(sY - y_0) + 5Y = 0 \quad (0.7)$$

$$(s + 5)Y = y_0 \quad (0.8)$$

$$Y = \frac{1}{s + 5} \quad (0.9)$$

$$(0.10)$$

Substituting $y_0 = 1$,

$$Y = \frac{1}{s + 5} \quad (0.11)$$

$$(0.12)$$

Now, we take the inverse laplace transform to get a solution,

$$\mathcal{L}^{-1}\left(\frac{1}{s+5}\right) = e^{-5x}u(x) \quad (0.13)$$

For the following approximate graph, I chose $h = 0.01$ and $h = 0.1$.

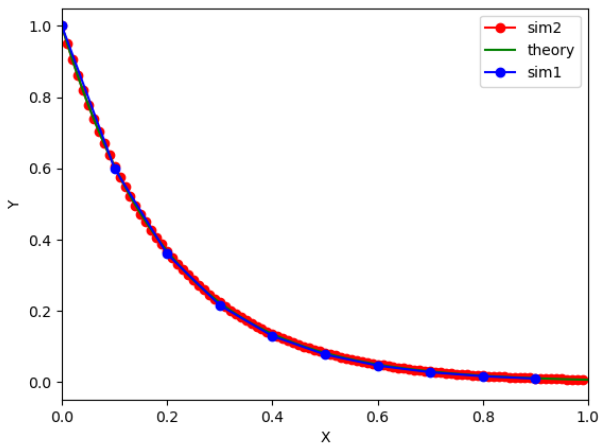


Fig. 0.1: Approximate solution of the DE