

# 10.4.1.2

EE24BTECH11001 - Aditya Tripathy

**Question:**

Find the roots of the equation  $x^2 - 2x = (-2)(3 - x)$

**Solution:**

First, we simplify the given equation,

$$x^2 - 4x + 6 = 0 \quad (0.1)$$

Below are two methods to find the solutions of this quadratic equation,

Fixed Point Iterations: Take an initial guess  $x_0$ . The update difference equation will use the following function:

$$x = g(x) \quad (0.2)$$

For our problem,

$$g(x) = \frac{1}{4}x^2 + \frac{3}{2} \quad (0.3)$$

Now the update equation will be,

$$x_{n+1} = g(x_n) \quad (0.4)$$

When we try to run the iterations however, we realize that whatever be the initial guess, the subsequent updated values grow without bound. This is because of the following theorem

**Theorem:** (0.5)

Let  $x = s$  be a solution of  $x = g(x)$  and suppose that  $g$  has a continuous derivative in some interval  $J$  containing  $s$ . Then if  $|g'| \leq K < 1$  in  $J$ , the iteration process defined above converges for any  $x_0$  in  $J$ . The limit of the sequence  $[x_n]$  is  $s$

Since there is no solution (evident by quadratic formula) there exists no interval  $J$  for which the process converges to a point.

The same behaviour is shown by the Newton-Raphson Method, Start with an initial guess  $x_0$ , and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (0.6)$$

where ,

$$f(x) = x^2 - 4x + 6 \quad (0.7)$$

$$f'(x) = 2x - 4 \quad (0.8)$$

The behaviour shown here is that regardless of which guess we take, it reaches a point of extrema(derivative  $\approx 0$ ) and then the process halts, or the updated point grow with bound. To get the complex solutions, however , we can Just take the initial guess point to be a random complex number.

The ouput of a program written to find roots is shown below:

Running Newton iterations:

Failure

Trying fixed point iterations:

x got too big

Trying complex Newton's iterations:

Solution = 2.000000 + 1.414214 i

And on a second run,

Running Newton iterations:

x got too big

Trying fixed point iterations:

x got too big

Trying complex Newton's iterations:

Solution = 2.000000 + -1.414214 i