9.1.2

EE24BTECH11001 - Aditya Tripathy

Question:
Plot a solution to the following differential equation:
$y' + 5y = 0$
Solution:
The aim is to find the difference equation using the trapezoidal law using the following initial conditions, $x_0 = 0, y_0 = 1$

$$y' = -5y \tag{0.1}$$
$$\int_{y_n}^{y_{n+1}} dy = \int_{x_n}^{x_{n+1}} -5y\,dx \tag{0.2}$$
$$\tag{0.3}$$

Using the trapezoidal rule,

$$J = \int_a^b f(x)\, dx \tag{0.4}$$
$$\approx h\left(\frac{1}{2}f(x) + f(x_1) + f(x_2)\cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{0.5}$$
$$\frac{y_{n+1} - y_n}{1}\left(\frac{1}{2} + \frac{1}{2}\right) = -5\frac{x_{n+1} - x_n}{1}\left(\frac{y_n}{2} + \frac{y_{n+1}}{2}\right) \tag{0.6}$$
$$\to y_{n+1} = \frac{(2-5h)\,y_n}{5h+2} \tag{0.7}$$

To check how close the approximate graph is to the actual solution, we will solve the original equation using a Laplace Transform method:
Let $\mathcal{L}(y) = Y$

$$(sY - y_0) + 5Y = 0 \tag{0.8}$$
$$(s+5)\,Y = y_0 \tag{0.9}$$
$$Y = \frac{1}{s+5} \tag{0.10}$$
$$\tag{0.11}$$

Substituting $y_0 = 1$,

$$Y = \frac{1}{s+5} \tag{0.12}$$

$$\tag{0.13}$$

Now, we take the inverse laplace transform to get a solution,

$$\mathcal{L}^{-1}\left(\frac{1}{s+5}\right) = e^{-5x}u(x) \tag{0.14}$$

For the following approximate graph, I chose $h = 0.01$ and $h = 0.1$.

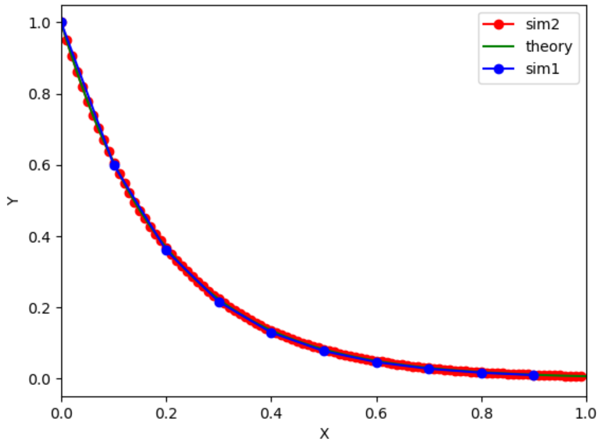

Fig. 0.1: Approximate solution of the DE