## EE24BTECH11001 - Aditya Tripathy

## **Question:**

A coin is tossed twice, what is the probability that atleast one tail occurs?

## **Solution:**

Let Y be the random variable representing the number of tails. Y can be represented as the sum of two bernoulli random variables,  $X_1, X_2$ ,

$$Y = X_1 + X_2 \tag{0.1}$$

The Bernoulli R.V is defined as,

$$X_i = \begin{cases} 0 & \text{Outcome is Heads} \\ 1 & \text{Outcome is Tails} \end{cases}$$
 (0.2)

The PMF of Bernoulli R.V is given by,

$$p_X(n) = \begin{cases} p & n = 0\\ 1 - p & n = 1 \end{cases}$$
 (0.3)

Using properties of Z transform of PMF on eq. (0.1),

$$M_Y(z) = M_{X_1}(z) M_{X_2}(z)$$
(0.4)

$$M_{X_1}(z) = \sum_{k=-\infty}^{\infty} p_{X_1}(k) z^{-k} = p + (1-p) z^{-1}$$
(0.5)

$$M_{X_2}(z) = \sum_{k=-\infty}^{\infty} p_{X_2}(k) z^{-k} = p + (1-p) z^{-1}$$
(0.6)

$$M_Y(z) = \left(p + (1-p)z^{-1}\right)^2 \tag{0.7}$$

$$= \sum_{k=-\infty}^{\infty} {}^{2}C_{k}p^{2-k}(1-p)^{k}z^{-k}$$
 (0.8)

$$p_Y(n) = {}^{2}C_n p^{2-n} (1-p)^n$$
(0.9)

Substituting  $p = \frac{1}{2}$ ,

$$p_Y(n) = {}^{2}C_n \left(\frac{1}{2}\right)^2 \tag{0.10}$$

Using eq. (0.9) the CDF (Cumulative Distribution Function) is given by:

$$F_X(n) = \sum_{k=-\infty}^{n} {}^{2}C_k \left(\frac{1}{2}\right)^2 = \begin{cases} 0 & x < 0 \\ {}^{2}C_0 \left(\frac{1}{2}\right)^2 = \frac{1}{4} & 0 \le x < 1 \\ {}^{2}C_1 \left(\frac{1}{2}\right)^2 + {}^{2}C_0 \left(\frac{1}{2}\right)^2 = \frac{3}{4} & 1 \le x < 2 \\ {}^{2}C_2 \left(\frac{1}{2}\right)^2 + {}^{2}C_1 \left(\frac{1}{2}\right)^2 + {}^{2}C_0 \left(\frac{1}{2}\right)^2 = 1 & x > = 2 \end{cases}$$
 (0.11)

$$p(X \ge 1) = 1 - p(X < 1) \tag{0.12}$$

$$=1-\frac{1}{4}=\frac{3}{4}\tag{0.13}$$

## Simulation:

To run a simulation we need to generate random numbers with uniform probability, which is done as shown below(Algorithm taken from OpenSSL's random uniform.c):

- 1) Generate 1byte(8 bits) of entropy using OpenSSL/rand.h.
- 2) Scale down this number in the range [0, 255] to [0, 1]by dividing by 255.
- 3) Return 0 if the scaled down number is less than p and return 1 otherwise.

The following shows how the relative frequency reaches true probability with increasing number of trials of the event. The blue plot is the n = 2 case and red plot is the n = 10 case.

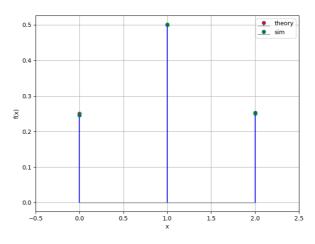


Fig. 3.1: Probability Mass Function

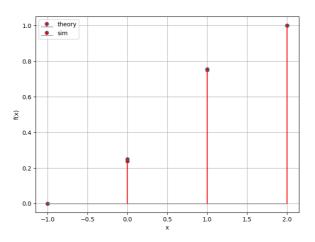


Fig. 3.2: Cumulative Distribution Function

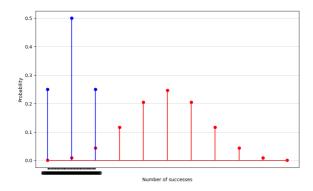


Fig. 3.3: Generating binomial distribution from bernoulli