9.6.1

EE24BTECH11001 - Aditya Tripathy

Question:
Plot a solution to the following differential equation:
$y' + 2y = \sin x$
Solution:
To plot a curve in the solution family, we take the initial condition to be
$x_0 = 0, y_0 = 1$
Using Euler's Method, we represent the the differential equation in the following difference equations:

$$x_{n+1} = x_n + h \tag{0.1}$$
$$y_{n+1} - y_n + 2hy_n = h \sin x_n \tag{0.2}$$
$$\rightarrow y_{n+1} = (1 - 2h) y_n + h \sin x_n \tag{0.3}$$

Now we can iteratively generate points which lie close to the graph.
To check how close the approximate graph is to the actual solution, we will solve the original equation using a Laplace Transform method:
Let $\mathcal{L}(y) = Y$

$$(sY - y_0) + 2Y = \mathcal{L}(\sin x) \tag{0.4}$$
$$\mathcal{L}(\sin x) = \int_0^\infty e^{-sx} \sin x = \frac{1}{s^2 + 1} \tag{0.5}$$
$$(s + 2) Y = y_0 + \frac{1}{s^2 + 1} \tag{0.6}$$
$$Y = \frac{y_0}{s + 2} + \frac{1}{(s^2 + 1)(s + 2)} \tag{0.7}$$
$$\tag{0.8}$$

Using method of partial fractions,

$$\frac{1}{(s^2 + 1) s + 2} = \frac{a}{s + 2} + \frac{bs + c}{s^2 + 1} \tag{0.9}$$
$$\tag{0.10}$$

On solving we get,

$$a = \frac{1}{5} \tag{0.11}$$
$$b = \frac{-1}{5} \tag{0.12}$$
$$c = \frac{2}{5} \tag{0.13}$$
$$\tag{0.14}$$

Substituting $y_0 = 1$,

$$Y = \frac{1}{s+2} + \frac{0.2}{s+2} + \frac{-0.2s}{s^2+1} + \frac{0.4}{s^2+1} \tag{0.15}$$
$$\rightarrow Y = \frac{1.2}{s+2} + \frac{-0.2s}{s^2+1} + \frac{0.4}{s^2+1} \tag{0.16}$$
$$\tag{0.17}$$

Now, we take the inverse laplace transform to get a solution,

$$\mathcal{L}^{-1}\left(\frac{1}{s+2}\right) = e^{-2x}u(x) \tag{0.18}$$
$$\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) = \sin x u(x) \tag{0.19}$$
$$\mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) = \cos x u(x) \tag{0.20}$$

Therefore the final solution to the differential equation is,

$$y(x) = \left(1.2e^{-2x} - 0.2\cos x + 0.4\sin x\right)u(x) \tag{0.21}$$

For the following approximate graph, I chose $h = 0.01$ and $h = 0.1$.

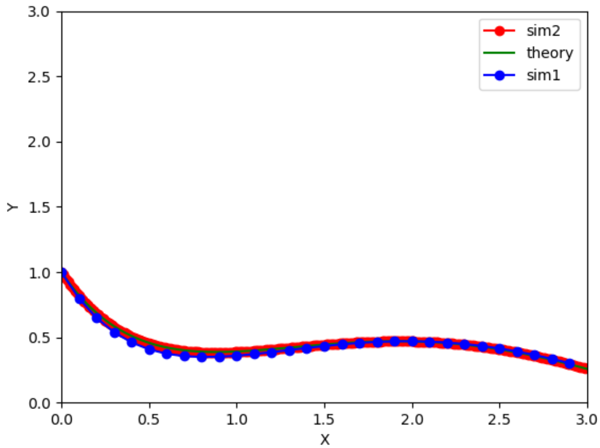

Fig. 0.1: Approximate solution of the DE