

1.8.12

EE24BTECH11001 - Aditya Tripathy

Question:

Find the point on X axis which is equidistant from $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Solution:

Let A and B be the above points respectively. Let S be any point on the perpendicular bisector of AB .

$$\|A - S\| = \|B - S\| \quad (0.1)$$

$$\Rightarrow \sqrt{(A - S)^\top (A - S)} = \sqrt{(B - S)^\top (B - S)} \quad (0.2)$$

$$\Rightarrow (A - S)^\top (A - S) = (B - S)^\top (B - S) \quad (0.3)$$

$$\|A\|^2 - S^\top A - A^\top S + \|S\|^2 = \|B\|^2 - S^\top B - B^\top S + \|S\|^2 \quad (0.4)$$

$$\Rightarrow 2B^\top S - 2A^\top S = \|B\|^2 - \|A\|^2 \quad (0.5)$$

$$\Rightarrow 2(B - A)^\top S = \|B\|^2 - \|A\|^2 \quad (0.6)$$

$$(0.7)$$

Equation of X -axis

$$m^\top S = 0 \quad (0.8)$$

$$m = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.9)$$

So, the matrix equation to solve becomes,

$$(B - A \quad m)^\top S = \begin{pmatrix} \frac{\|B\|^2 - \|A\|^2}{2} \\ 0 \end{pmatrix} \quad (0.10)$$

$$\begin{pmatrix} -4 & -2 \\ 0 & 1 \end{pmatrix} S = \begin{pmatrix} -30 \\ 0 \end{pmatrix} \quad (0.11)$$

$$\begin{pmatrix} -4 & -2 & -30 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 = 2R_2 + R_1} \begin{pmatrix} -4 & 0 & -30 \\ 0 & 1 & 0 \end{pmatrix} \quad (0.12)$$

$$\begin{pmatrix} -4 & 0 & -30 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 = \frac{R_1}{-4}} \begin{pmatrix} 1 & 0 & 7.5 \\ 0 & 1 & 0 \end{pmatrix} \quad (0.13)$$

$$\Rightarrow C = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix} \quad (0.14)$$

$$(0.15)$$

Therefore $\begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$ is the required point on X -axis.

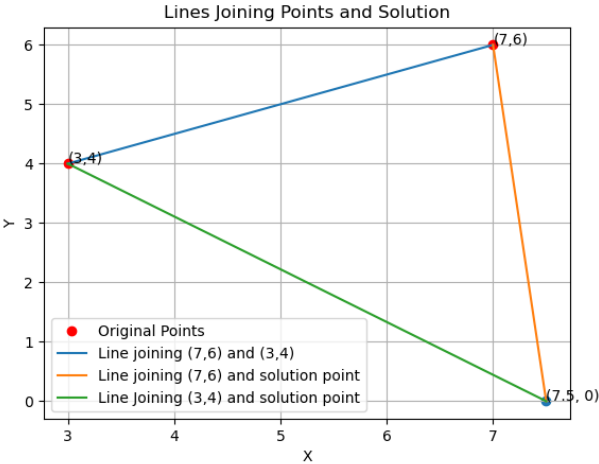


Fig. 0.1: Line joining the three given points