2024 April 5 Shift 1

EE24BTECH11001 - ADITYA TRIPATHY

1. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point $(3, 2)$ and parallel to the coordinate axes touch it. Then the shortest distance of the circle from the point $(5, 5)$ is : $(2024 - Apr)$

   a) 5 b) $4\sqrt{2}$ c) 4 d) $2\sqrt{2}$

2. Let a rectangle $ABCD$ of sides 2 and 4 be inscribed in another rectangle $PQRS$ such that the vertices of the rectangle $ABCD$ lie on the sides og the rectangle $PQRS$. Let a and b be the sides of the rectangle $PQRS$ when its area is maximum. Then $(a + b)^2$ is equal to :

   a) 80 b) 60 c) 72 d) 64

3. If

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \dots \frac{1}{\sqrt{99} + \sqrt{100}} = m \tag{1}$$

and

$$\frac{1}{1.2} + \frac{1}{2.3} + \dots \frac{1}{99.100} = n \tag{2}$$

then the point (m, n) lies on the line $(2024 - Apr)$

   a) $11(x - 1) - 100(y - 2)$
   b) $11(x - 2) - 100(y - 1)$
   c) $11(x - 1) - 100y$
   d) $11x - 100y$

4. Let d be this distance of the point of intersection of the lines

$$\frac{x-6}{3} = \frac{y}{2} = \frac{z+1}{1} \tag{3}$$

and

$$\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2} \tag{4}$$

from the point $(7, 8, 9)$. Then $d^2 + 6$ is equal to : $(2024 - Apr)$

   a) 72 b) 78 c) 69 d) 75

5. Let the line $2x + 3y - k = 0, k > 0$, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then $2m + n$ is equal to : $(2024 - Apr)$

   a) 11 b) 10 c) 13 d) 12

6. The coefficients a, b, c in the quadratic equation $ax^2+bx+c = 0$ are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The probability of this equation having repeated roots is : $(2024 - Apr)$

a) $\frac{3}{128}$ b) $\frac{1}{64}$ c) $\frac{1}{128}$ d) $\frac{3}{256}$

7. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to : $(2024 - Apr)$
   a) $\frac{4}{(3\sqrt{6}-2)}$
   b) $\frac{6-\sqrt{6}}{(3\sqrt{6}-2)}$
   c) $\frac{4}{(3\sqrt{6}+2)}$
   d) $\frac{6+\sqrt{6}}{(3\sqrt{6}+2)}$
8. For the function

$$f(x) = \sin x + 3x - \frac{2}{\pi}\left(x^2 + x\right), \text{where} x \in \left[0, \frac{\pi}{2}\right] \tag{5}$$

   Consider the follwing two statements,
   1. f is increasing in $\left(0, \frac{\pi}{2}\right)$
   2. f' is decreasing in $\left(0, \frac{\pi}{2}\right)$

   $(2024 - Apr)$
   a) Only 2 is true.
   b) neither 1 nor 2 is true.
   c) both 1 and 2 are true.
   d) only 1 is true.
9. Let $f(x) = x^5 + 2x^3 + 3x + 1, x \in \mathbb{R}$, and $g(x)$ be a function such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. Then $\frac{g(7)}{g'(7)}$ is equal to: $(2024 - Apr)$

   a) 7 b) 42 c) 14 d) 1

10. If the system of equations

$$11x + y + \lambda z = -5 \tag{6}$$
$$2x + 3y + 5z = 3 \tag{7}$$
$$8x - 19y - 39z = \mu \tag{8}$$

   , has infinitely many solutions, then $\lambda^4 - \mu$ is equal to : $(2024 - Apr)$

   a) 45 b) 51 c) 47 d) 49

11. The value of

$$\int_{-\pi}^{\pi} \frac{2y(1 + \sin y)}{1 + \cos^2 y}\, dy \tag{9}$$

   is : $(2024 - Apr)$

   a) $\frac{\pi}{2}$ b) $\frac{\pi^2}{2}$ c) π^2 d) $2\pi^2$

12. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda} = 4 - z$ makes right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$ the value of $4\lambda + 9\mu$ is : $(2024 - Apr)$

   a) 13 b) 5 c) 4 d) 6

13. If $A(1, -1, 2), B(5, 7, -6), C(3, 4, -10)$ and $D(-1, -4, -2)$ are the vertices of a quadrilateral $ABCD$, then its area is : $(2024 - Apr)$

a) $12\sqrt{29}$ b) $24\sqrt{29}$ c) $48\sqrt{7}$ d) $24\sqrt{7}$

14. Let A and B be the two square matrices of order 3 such that $|A| = 3$ and $|B| = 8$. Then $\left|A^{\top}A\,(\operatorname{adj}(2A))^{-1}\left(\operatorname{adj}(4b)\left(\operatorname{adj}(AB)^{-1}AA^{\top}\right)\right)\right|$ is equal to : $(2024 - Apr)$

a) 64 b) 81 c) 108 d) 32

15. Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$ is : $(2024 - Apr)$

a) 120 b) 180 c) 480 d) 240