Matrices and Determinants

EE24BTECH11001 - ADITYA TRIPATHY

6. If $f(x) =$
$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$
then $f(100)$ is equal to $(1999 - 2Marks)$
   a) 0 b) 1 c) 100 d) -100

7. If the system of equations
$$x - ky - z = 0,$$
$$kx - y - z = 0, x + y - z = 0$$
has a non-zero solution, then the possible values of k are $(2000S)$
   a) -1,2 b) 1,2 c) 0,1 d) -1,1

8. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $(2002S)$
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$$
   a) 3ω b) $3\omega(\omega - 1)$
   c) $3\omega^2$ d) $3\omega(1 - \omega)$

9. The number of values of k for which the system of equations
$$(k+1)x + 8y = 4k;$$
$$kx + (k+3)y = 3k - 1$$
has infinitely many solutions is $(2002S)$
   a) 0 b) 1 c) 2 d) infinte

10. If $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$, then value of α for which $A^2 = B$,is $(2003S)$
   a) 1 b) 4
   c) 2 d) infinite

11. If the system of equations $x + ay = 0, az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is $(2003S)$
   a) -1 b) 1
   c) 0 d) no real values

12. Given
$$2x - y + 2z = 2,$$
$$x - 2y + z = -4,$$
$$x + y + \lambda z = 4$$
then the value of λ such that the given system of equation has NO solution, is $(2004S)$
   a) 3 b) 1 c) 0 d) -3

13. Is $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $\left|A^3\right| = 125$ then the value α is $(2004S)$
   a) ±1 b) ±2 c) ±3 d) ±5

14. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 2005S
and $A^{-1} = \left(\frac{1}{6}\left(A^2 + cA + dI\right)\right)$, then the value of c and d are $(2005S)$
   a) $(-6, -11)$ b) $(6, 11)$
   c) $(-6, 11)$ d) $(6, -11)$

15. If $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$and $A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$ then x is equal to
   a) $\begin{vmatrix} 1 & 2005 \\ 0 & 1 \end{vmatrix}$

b) $\begin{vmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{vmatrix}$

c) $\frac{1}{4}\begin{vmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{vmatrix}$

d) $\frac{1}{4}\begin{vmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{vmatrix}$

16. Consider 3 points

$P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$

and

$$R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$$

where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$.Then, (2008)

a) P lies on the same segment RQ
b) Q lies on the line segment PR
c) R lies on the line segment QP
d) P,Q,R are non-collinear

17. The number of 3x3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ has exactly two distinct solutions is (2008)

a) 0 b) $2^9 - 1$ c) 168 d) 2

18. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is (2008)

a) 2 b) 6 c) 4 d) 8

19. Let $P = \left(a_{ij}\right)$ be 3x3 matrix and let $Q = \left(b_{ij}\right)$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is (2012)

a) 2^{10} b) 2^{11} c) 2^{12} d) 2^{13}

20. If P is a 3x3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3x3 identity matrix, then there exists a column matrix $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (2012)

a) $PX = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ b) $PX = X$

c) $PX = 2X$ d) $PX = -X$

21. Let $P = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$ and I be the identity matrix of order 3. If $Q = \left(q_{ij}\right)$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31}+q_{32}}{q_{21}}$ equals ($JEEAdv$.2016)

a) 52 b) 103 c) 201 d) 205