7.2.21

EE24BTECH11001 - Aditya Tripathy

Question:

In each of the following exercises, find the equation of the circle with the following parameters.

Centre $\frac{1}{2}, \frac{1}{4}$ and Radius $\frac{1}{12}$.

Solution:

The general equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

Substituting the numerical values, we have:

$$\mathbf{u} = -\mathbf{c} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{4} \end{pmatrix}, \quad r = \frac{1}{12} \tag{2}$$

Calculating f:

$$f = \|\mathbf{u}\|^2 - r^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{12}\right)^2 \tag{3}$$

This simplifies to:

$$f = \frac{1}{4} + \frac{1}{16} - \frac{1}{144} = \frac{36}{144} + \frac{9}{144} - \frac{1}{144} = \frac{44}{144} = \frac{11}{36} \tag{4}$$

Thus, the equation of the circle is:

$$\|\mathbf{x}\|^2 - \begin{pmatrix} 1 & \frac{1}{2} \end{pmatrix} \mathbf{x} + \frac{11}{36} = 0 \tag{5}$$

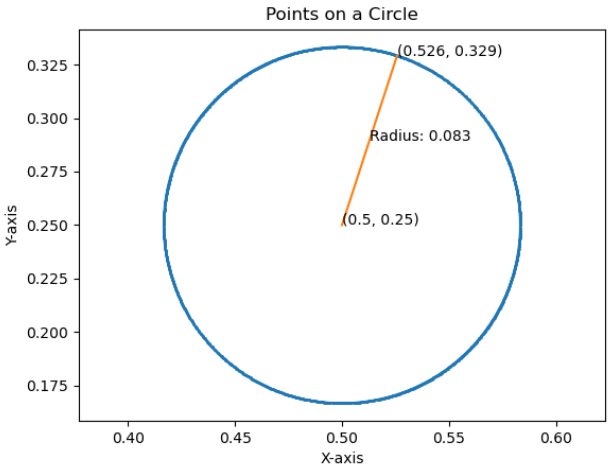

Fig. 0: Equilateral triangle of side 5cm