2008-MA-69-85

EE24BTECH11001 - ADITYA TRIPATHY

69. The initial value problem $u_x + u_y = 1$, $u(s,s) = \sin s, 0 \le s \le 1$, has (2008 − MA)
    a) two solutions
    b) a unique solution
    c) no solution
    d) infinitely many solutions

70. Let $u(x,t) = 0$, be the solution of $u_t t - u_x x = 1, x \in \mathbb{R}, t > 0$ with $u(x,0) = 0, u_t(x,0) = 0, x \in \mathbb{R}$. Then $u\left(\frac{1}{2}, \frac{1}{2}\right)$ is equal to
(2008 − MA)
    a) $\frac{1}{8}$
    b) $-\frac{1}{8}$
    c) $\frac{1}{4}$
    d) $-\frac{1}{4}$

I. Common Data Questions

Common Data for 71, 72, 73
Let $X = C([0,1])$ with sup norm $\|\|_\infty$

71. Let $S = \{x \in X : \|x\|_\infty \le 1\}$. Then (2008 − MA)
    a) S is convex and compact
    b) S is not convex but compact
    c) S is convex but not compact
    d) S is neither convex nor compact
72. Which one of the following is true?
(2008 − MA)
    a) $C^\infty([0,1])$ is dense in X
    b) X is dense $L^\infty([0,1])$
    c) X has a countable basis
    d) There is a sequence in X which is uniformly Cauchy on on $[0,1]$ but does not converge uniformly on $[0,1]$
73. Let $I = \{x \in X : x(0) = 0\}$. Then
(2008 − MA)
    a) I is not an ideal of X
    b) I is an ideal, nut not a prime ideal of X
    c) I is a prime ideal, but not a maximal ideal of X
    d) I is a maximal ideal of X

Common Data for 74, 75
Let $X = C^1([0,1])$ and $Y = C([0,1])$, both with sup norm. Define $F : X \to Y$ by $F(x) = x + x'$ and $f(x) = x(1) + x'(1)$ for $x \in X$.

74. Then (2008 − MA)
    a) F and f are continuos
    b) F is continuous and f is discontinuous
    c) F is discontinuous f is contiuous
    d) F and f are discontinuous
75. Then (2008 − MA)
    a) F and f are closed maps

b) F is a closed map and f is not a closed map
c) F is not a closed map f is a closed map
d) neither F nor f is a closed map

II. Linked Answer Questions: Q76. to Q85 carry two marks each

Statement for Linked Answer Questions 76 , 77

Let $N = \begin{vmatrix} \frac{3}{5} & -\frac{4}{5} & 0 \\ \frac{4}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{vmatrix}$

76. Then N is (2008 – MA)
   a) non-invertible
   b) skew-symmetric
   c) symmetric
   d) orthogonal
77. If M is any 3×3 real matrix, then trace($NMN^{\top}$) is equal to (2008 – MA)
   a) $[i\text{trace}\,(N)]^2\,\text{trace}\,(M)$
   b) $2\text{trace}\,(N) + \text{trace}\,(M)$
   c) $\text{trace}\,(M)$
   d) $[\text{trace}\,(N)]^2 + \text{trace}\,(M)$

Statement for Linked Answer Questions 78 , 79

Let $f(z) = \cos z - \frac{\sin z}{z}$ for non-zero $z \in \mathbb{C}$ and $f(0) = 0$. Also, let $g(z) = \sinh z$ for $z \in \mathbb{C}$.

78. Then $f(z)$ has a zero $z = 0$ or order (2008 – MA)
   a) 0
   b) 1
   c) 2
   d) greater than 2
79. Then $\frac{g(z)}{z(z)}$ has a pole at $z = 0$ or order (2008 – MA)
   a) 1
   b) 2
   c) 3
   d) greater than 3

Statement for Linked Answer Questions 80 , 81

Let $n \geq 3$ be an integer. Let y be the polynomial solution of $\left(1 - x^2\right) y'' - 2xy' + n(n-1)\,y = 0$ satisfying $y(1) = 1$

80. Then the degree of y is (2008 – MA)

   a) n b) $n - 1$ c) less than $n - 1$ d) greater than $n + 1$

81. If $I = \int_{-1}^{1} y(x)\, x^{n-3}\, dx$ and $J = \int_{-1}^{1} y(x)\, x^{n}\, dx$, then (2008 – MA)

   a) $I \neq 0, J \neq 0$ b) $I \neq 0, J = 0$ c) $I = 0, J \neq 0$ d) $I = 0, J = 0$

Statement for Linked Answer Questions 82 , 83

Consider the boundary value problem

$$u_x x + u_y y = 0, x \in (0, \pi), y \in (0, \pi), \tag{1}$$
$$u(x, 0) = u(x, \pi) = u(0, y) = 0. \tag{2}$$

82. Any solution of this boundary value problem is of the form (2008 – MA)
   a) $\sum_{n=1}^{\infty} a_n \sinh nx \sin ny$

b) $\sum_{n=1}^{\infty} a_n \cosh nx \sin ny$
c) $\sum_{n=1}^{\infty} a_n \sinh nx \cos ny$
d) $\sum_{n=1}^{\infty} a_n \cosh nx \cos ny$

83. If an additional boundary condition $u_x(\pi, y) = \sin y$ is satisfied, then $u\left(x, \frac{\pi}{2}\right)$ is equal to $(2008 - MA)$
a) $\frac{\pi}{2}(e^x - e^{-x})(e^x + e^{-x})$
b) $\frac{\pi\left(e^x - e^{-x}\right)}{(e^x + e^{-x})}$
c) $\frac{\pi\left(e^x + e^{-x}\right)}{(e^x - e^{-x})}$
d) $\frac{\pi}{2}(e^{\pi} + e^{-\pi})(e^x - e^{-x})$

Statement for Linked Answer Questions 84 , 85

Let the random variable X follow the exponential distribution with mean 2. Define $Y = [X - 2|X > 2]$.

84. The value of $\Pr(Y \geq t)$ $(2008 - MA)$
a) $e^{\frac{-t}{2}}$
b) e^2
c) $\frac{1}{2}e^{\frac{-t}{2}}$
d) $\frac{1}{2}e^{-t}$

85. The value of $E(Y)$ is equal to $(2008 - MA)$
a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) 2