2021 February 26 Shift 1

EE24BTECH11001 - ADITYA TRIPATHY

1. The number of seven-digit integers with the sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is :

   a) 77 b) 42 c) 35 d) 82

2. The maximum value of the term independent of $'t'$ in the expression of $\left[\left(tx^{\frac{1}{5}}+\{\frac{1-x}{t}\}\right)^{\frac{1}{10}}\right]^{10}$ where $x \in (0,1)$ is :

   a) $\frac{10!}{\sqrt{3}(5!)^2}$ b) $\frac{2.10!}{3(5!)^2}$ c) $\frac{10!}{3(5!)^2}$ d) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

3. The value of

$$\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}\,dx \tag{1}$$

   where $[x]$ is the greatest integer $\le x$

   a) $100\,(e-1)$ b) $100e$

   c) $100\,(1-e)$ d) $100\,(1+e)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria cout is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

   a) 1 b) 2 c) 16 d) 8

5. If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, then

$$\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times\mathbf{b})))$$

   is equal to

   a) $\frac{1}{2}\|\mathbf{a}\|^4\,\mathbf{b}$
   b) $\mathbf{a}\times\mathbf{b}$
   c) $\|\mathbf{a}\|^4\,\mathbf{b}$
   d) $\mathbf{0}$

6. In an increasing geometric series, the sum of the second and the sixth terms is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} terms is equal to :

   a) 35 b) 30 c) 26 d) 32

7. Consider the three planes

P1 : $3x + 15y + 21z = 9$,
P2 : $x - 3y - z = 5$,and
P3 : $2x + 10y + 14z = 5$

   Then, which of the following is true ?

a) P1 and P3 are parallel.
b) P2 and P3 are parallel.
c) P1 and P2 are parallel.
d) P1, P2 and P3 are parallel.

8. The sum of the infinite series

$$1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \dots \tag{2}$$

is equal to :

a) $\frac{9}{4}$ b) $\frac{15}{4}$ c) $\frac{13}{4}$ d) $\frac{11}{4}$

9. The value of

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} \tag{3}$$

is :

a) -2
b) $(a+1)(a+2)(a+3)$
c) 0
d) $(a+2)(a+3)(a+4)$

10. If

$$\frac{\sin^{-1}}{a} = \frac{\cos^{-1}}{b} = \frac{\tan^{-1}}{c}; 0 < x < 1 \tag{4}$$

, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

a) $\frac{1-y^2}{2y}$

b) $\frac{1-y^2}{1+y^2}$

c) $1 - y^2$

d) $\frac{1-y^2}{y\sqrt{y}}$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

a) 6 b) 1 c) 4 d) 12

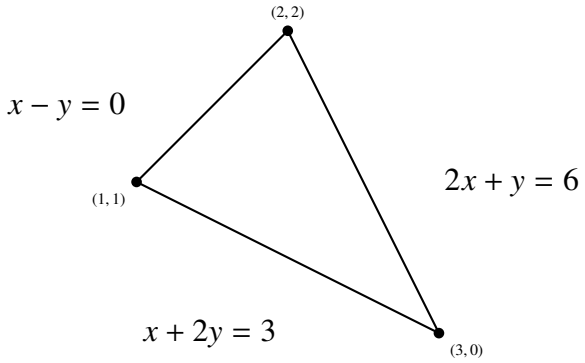
12. The intersection of the three lines x-y = 0, x + 2y = 3 and 2x + y = 6 is a :
a) Equilateral triangle
b) Right angled triangle
c) Isosceles triangle
d) None of the above

13. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^2 + 18x^2 - 19x$ occurs at the point:

a) $(2, 9)$ b) $(2, 2)$ c) $\left(3, \frac{21}{2}\right)$ d) $(0, 0)$

14. Let f be any function defined on $\mathbf{R}$ and let it satisfy the condition:

$$|f(x) - f(y)| \leq \left|\left(x - y^2\right)\right|, \forall x, y \in \mathbf{R} \tag{5}$$



If $f(0) = 1$, then :

a) $f(x) < 0, \forall x \in \mathbf{R}$

b) $f(x)$ can take any vaule in $\mathbf{R}$

c) $f(x) = 0$

d) $f(x) > 0, \forall x \in \mathbf{R}$

15. The value of

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + 3^x}\, dx$$

is :

a) 2π b) 4π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$