Trignometric Functions and Equations

EE24BTECH11001- ADITYA TRIPATHY

A: Fill In The Blanks

1) Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos x$ is an identity in x, where $C_0, C_1, \cdots, C_n$ are constants and $C_n \neq 0$ then the value of n is $(1981 - 2Marks)$
2) The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real,is $(1987 - 2Marks)$
3) The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is $(1987 - 2Marks)$
4) The sides of a triangle in a given circle subtend angles α, β, γ. The minimum value of arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$, $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to $(1987 - 2Marks)$
5) The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$ $\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$is equal to $(1991 - 2Marks)$
6) If $K = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$ then the numerical value of K is $(1993 - 2Marks)$
7) If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value $\tan A \tan B$ is $(1993 - 2Marks)$
8) General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is $(1996 - 1Mark)$
9) The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are $(1997 - 2Marks)$

B: True / False

1) If $\tan A = \frac{1-\cos B}{\sin B}$, then $\tan 2A = \tan B$ $(1981 - 1Mark)$
2) There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. $(1984 - 1Mark)$

C :MCQs with One Correct Answer

1) If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is (1979)
   a) $\frac{-4}{5}$ but not $\frac{4}{5}$
   b) $\frac{4}{5}$ or $\frac{-4}{5}$
   c) $\frac{4}{5}$ but not $\frac{-4}{5}$
   d) None of These
2) If $\alpha + \beta + \gamma = 2\pi$ (1979)
   a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
   c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   d) None of These
3) Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ (1980)
   a) $1 \leq A \leq 2$
   b) $\frac{3}{4} \leq A \leq 1$
   c) $\frac{13}{16} \leq A \leq 1$
   d) $\frac{3}{4} \leq A \leq \frac{13}{16}$
4) The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$ (1980)
   a) no real solution

b) one real solution
c) more than one real solution
d) None of these

5) The general solution to the trignometric equation $\sin x + \cos x = 1$ is given by $(1981 - 2Marks)$
a) $x = 2n\pi; n = 0, \pm 1, \pm 2 \cdots$
b) $x = 2n\pi + \frac{\pi}{2}, n = 0, \pm 1, \pm 2 \cdots$
c) $x = n\pi + (-1)^n \frac{\pi}{4}, n = 0, \pm 1, \pm 2 \cdots$
d) none of these

6) The value of the expression $\sqrt{3}\,\mathrm{cosec}\,20^\circ - \sec 20^\circ$ is equal to $(1988 - 2Marks)$
a) 2
b) $2\sin 20^\circ / \sin 40^\circ$
c) 4
d) $2\sin 20^\circ / \sin 40^\circ$