2023-Jan-25-S1

EE24BTECH11001 - ADITYA TRIPATHY

16. The statement $(p \wedge (\sim q)) \implies (p \implies (\sim q))$ (2023-Jan)

a) equivalent to $(\sim p) \vee (\sim q)$
b) a tautology
c) equivalent $p \vee q$
d) a contradiction

17. Let $f : (0, 1) \rightarrow \mathbb{R}$ a be a function defined by

$$f(x) = \frac{1}{1 - e^{-x}} \tag{1}$$

and,

$$g(x) = (f(-x) - f(x)) \tag{2}$$

. Consider the two statements

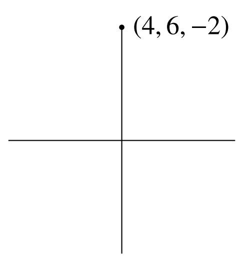
a) g is an increasing function $(0, 1)$
b) g is one-one in $(0, 1)$

Then, (2023-Jan)

a) Only 1 is true
b) Only 2 is true
c) Neither 1 nor 2 is true
d) Both 1 and 2 are true

18. The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios 3, 3, -1 is equal to : (2023-Jan)

a) 3
b) $\sqrt{6}$
c) $2\sqrt{3}$
d) $\sqrt{14}$



19. Let $x, y, z, > 1$ and

$$A = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_x y & 3 \end{vmatrix} \tag{3}$$

Then $\left|\text{adj}\left(\text{adj}A^2\right)\right|$ is equal to : (2023-Jan)

a) 6^4 b) 2^8 c) 4^8 d) 2^4

20. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^1 0$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}^2\right)$ is equal to : (2023-Jan)

a) 4895 b) 1210 c) 5445 d) 3025

21. Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements of multiple of 3 is : (2023-Jan)

22. For some $a, b, c \in \mathbb{N}$, lat $f(x) = ax - 3$ and $g(x) = x^b + c, x \in \mathbb{R}$. If $(fog)^{-1}(x) = \left(\frac{x-7}{2}^{\frac{1}{3}}\right)$, then $(fog)(ac) + (gof)(b)$ is equal to : (2023-Jan)

23. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let $\mathbb{N}$ be the normal to H at a point in the first quadrant and the parallel to the $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to : (2023-Jan)

24. Let

$$S = \{\alpha : \log_2\left(9^{2\alpha-4} + 14\right) - \log_2\left(\frac{5}{2}3^{2\alpha-4} + 1\right) = 2\} \tag{4}$$

Then the maximum value of β for which the equation

$$x^2 - 2\left(\sum_{\alpha\in S}\alpha\right)^2 x + \sum_{\alpha\in S}(\alpha+1)^2\beta - 0 \tag{5}$$

has real roots, is : (2023-Jan)

25. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is : (2023-Jan)

26. Let A_1, A_2, A_3 be the tree A.P. with the same common difference d an having their first terms as $A, A+1, A+2$, respectively. Let a, b, c be the $7^{th}, 9^{th}, 17^{th}$ terms of A_1, A_2, A_3, respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0 \tag{6}$$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c-a-b$ and common difference is $\frac{d}{12}$, is equal to : (2023-Jan)

27. If the sum of all solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3} \tag{7}$$

(2023-Jan)

28. Let the equation of the plane passing through the line

$$x - 2y - z - 5 = 0 = x + y + 3z - 5 \tag{8}$$

and parallel to the line

$$x + y + 2z - 7 = 0 = 2x + 3y + z - 2 \tag{9}$$

be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is : (2023-Jan)

29. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5, is : (2023-Jan)

30. If the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, the α^2 is equal to : (2023-Jan)