2021 February 26 Shift 1

EE24BTECH11001 - ADITYA TRIPATHY

1. The number of seven-digit integers with the sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is : $(2021-Feb-26-S1)$

   a) 77 b) 42 c) 35 d) 82

2. The maximum value of the term independent of $'t'$ in the expression of $\left[\left(tx^{\frac{1}{5}}+\{\frac{1-x}{t}\}\right)^{\frac{1}{10}}\right]^{10}$ where $x \in (0,1)$ is : $(2021-Feb-26-S1)$

   a) $\frac{10!}{\sqrt{3}(5!)^2}$ b) $\frac{2.10!}{3(5!)^2}$ c) $\frac{10!}{3(5!)^2}$ d) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

3. The value of

$$\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}\,dx \tag{1}$$

   where $[x]$ is the greatest integer $\le x$ $(2021-Feb-26-S1)$

   a) $100\,(e-1)$ b) $100e$

   c) $100\,(1-e)$ d) $100\,(1+e)$

4. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria cout is 1000 at initial time $t=0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e(\frac{6}{5})}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to : $(2021-Feb-26-S1)$

   a) 1 b) 2 c) 16 d) 8

5. If $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, then

$$\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times(\mathbf{a}\times\mathbf{b})))$$

   is equal to $(2021-Feb-26-S1)$

   a) $\frac{1}{2}\|\mathbf{a}\|^4\,\mathbf{b}$
   b) $\mathbf{a}\times\mathbf{b}$
   c) $\|\mathbf{a}\|^4\,\mathbf{b}$
   d) $\mathbf{0}$

6. In an increasing geometric series, the sum of the second and the sixth terms is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} terms is equal to : $(2021-Feb-26-S1)$

   a) 35 b) 30 c) 26 d) 32

7. Consider the three planes
P1 : $3x+15y+21z=9$,
P2 : $x-3y-z=5$,and
P3 : $2x+10y+14z=5$
   Then, which of the following is true ? $(2021-Feb-26-S1)$

a) P1 and P3 are parallel.
b) P2 and P3 are parallel.
c) P1 and P2 are parallel.
d) P1, P2 and P3 are parallel.

8. The sum of the infinite series

$$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots \tag{2}$$

is equal to : $(2021-Feb-26-S1)$

a) $\frac{9}{4}$ b) $\frac{15}{4}$ c) $\frac{13}{4}$ d) $\frac{11}{4}$

9. The value of

$$\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix} \tag{3}$$

is : $(2021-Feb-26-S1)$

a) -2
b) $(a+1)(a+2)(a+3)$
c) 0
d) $(a+2)(a+3)(a+4)$

10. If

$$\frac{\sin^{-1}}{a}=\frac{\cos^{-1}}{b}=\frac{\tan^{-1}}{c};0<x<1 \tag{4}$$

, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is : $(2021-Feb-26-S1)$

a) $\frac{1-y^2}{2y}$

b) $\frac{1-y^2}{1+y^2}$

c) $1-y^2$

d) $\frac{1-y^2}{y\sqrt{y}}$

11. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is: $(2021-Feb-26-S1)$

a) 6 b) 1 c) 4 d) 12

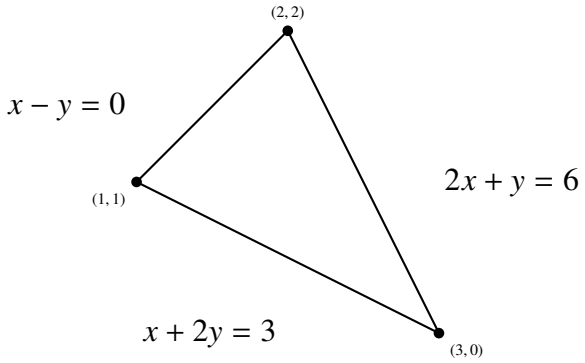
12. The intersection of the three lines x-y = 0, x + 2y = 3 and 2x + y = 6 is a : $(2021-Feb-26-S1)$
a) Equilateral triangle
b) Right angled triangle
c) Isosceles triangle
d) None of the above

13. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^2+18x^2-19x$ occurs at the point: $(2021-Feb-26-S1)$

a) $(2,9)$ b) $(2,2)$ c) $\left(3,\frac{21}{2}\right)$ d) $(0,0)$

14. Let f be any function defined on $\mathbf{R}$ and let it satisfy the condition:

$$|f(x)-f(y)|\leq\left|\left(x-y^2\right)\right|,\forall x,y\in\mathbf{R} \tag{5}$$



If $f(0) = 1$, then : $(2021 - Feb - 26 - S1)$

a) $f(x) < 0, \forall x \in \mathbf{R}$
b) $f(x)$ can take any vaule in $\mathbf{R}$
c) $f(x) = 0$
d) $f(x) > 0, \forall x \in \mathbf{R}$

15. The value of

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1 + 3^x}\, dx$$

is : $(2021 - Feb - 26 - S1)$

a) 2π b) 4π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$