1.8.12

EE24BTECH11001 - Aditya Tripathy

Question:

Find the point on X axis which is equidistant from $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Solution:

Let the desired point on the X axis be S $\begin{pmatrix} x \\ 0 \end{pmatrix}$. Let A and B be the above points respectively.

$$\|A - S\| = \|B - S\| \tag{0.1}$$

$$\implies \sqrt{(A - S)^\top (A - S)} = \sqrt{(B - S)^\top (B - S)} \tag{0.2}$$

$$\implies (A - S)^\top (A - S) = (B - S)^\top (B - S) \tag{0.3}$$

$$\|A\|^2 - S^\top A - A^\top S + \|S\|^2 = \|B\|^2 - S^\top B - B^\top S + \|S\|^2 \tag{0.4}$$

$$\implies 2B^\top S - 2A^\top S = \|B\|^2 - \|A\|^2 \tag{0.5}$$

$$\implies 2(B - A)^\top S = \|B\|^2 - \|A\|^2 \tag{0.6}$$

$$\implies 2S\|B - A\|^2 = (B - A)(\|B\|^2 - \|A\|^2) \tag{0.7}$$

$$\implies S = \frac{(B - A)(\|B\|^2 - \|A\|^2)}{2(\|B - A\|)^2} \tag{0.8}$$

$$\tag{0.9}$$

Plugging in the values we get

$$\frac{60\begin{pmatrix} -4 \\ -2 \end{pmatrix}}{2()} \tag{0.10}$$

Since there is only one non-zero row, rank = 1. Hence the points are collinear.

Fig. 0.1: Line joining the three given points