1.8.12

EE24BTECH11001 - Aditya Tripathy

Question:

Find the point on X axis which is equidistant from $\begin{pmatrix}7\\6\end{pmatrix}$ and $\begin{pmatrix}3\\4\end{pmatrix}$.

Solution:

Let the desired point on the X axis be C $\begin{pmatrix}x\\0\end{pmatrix}$. Let A and B be the above points respectively. Let S be any point on the perpendicular bisector of AB

$$\|A - S\| = \|B - S\| \tag{0.1}$$

$$\implies \sqrt{(A-S)^\top (A-S)} = \sqrt{(B-S)^\top (B-S)} \tag{0.2}$$

$$\implies (A-S)^\top (A-S) = (B-S)^\top (B-S) \tag{0.3}$$

$$\|A\|^2 - S^\top A - A^\top S + \|S\|^2 = \|B\|^2 - S^\top B - B^\top S + \|S\|^2 \tag{0.4}$$

$$\implies 2B^\top S - 2A^\top S = \|B\|^2 - \|A\|^2 \tag{0.5}$$

$$\implies 2(B-A)^\top S = \|B\|^2 - \|A\|^2 \tag{0.6}$$

$$\tag{0.7}$$

Now we have the line representing perpendicular bisector of line joining $A$and B. The solution S is found by solving the previous eqution with the eqution for X axis.

$$m^\top S = 0 \tag{0.8}$$

$$\tag{0.9}$$

where m = $\begin{pmatrix}0\\1\end{pmatrix}$.

So, the matrix equation to solve becomes,

$$\begin{pmatrix}B-A & m\end{pmatrix}^\top S = \begin{pmatrix}\frac{\|B\|^2-\|A\|^2}{2}\\0\end{pmatrix} \tag{0.10}$$

On solving,

$$\begin{pmatrix} -4 & -2 \\ 0 & 1 \end{pmatrix} S = \begin{pmatrix} -30 \\ 0 \end{pmatrix} \tag{0.11}$$

$$\begin{pmatrix} -4 & -2 & -30 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 = 2R_2 + R_1} \begin{pmatrix} -4 & 0 & -30 \\ 0 & 1 & 0 \end{pmatrix} \tag{0.12}$$

$$\begin{pmatrix} -4 & 0 & -30 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 = \frac{R_1}{-4}} \begin{pmatrix} 1 & 0 & 7.5 \\ 0 & 1 & 0 \end{pmatrix} \tag{0.13}$$

$$\implies C = \begin{pmatrix} 7.5 \\ 0 \end{pmatrix} \tag{0.14}$$

$$\tag{0.15}$$

Therefore $\begin{pmatrix} 7.5 \\ 0 \end{pmatrix}$ is the required point on X-axis.

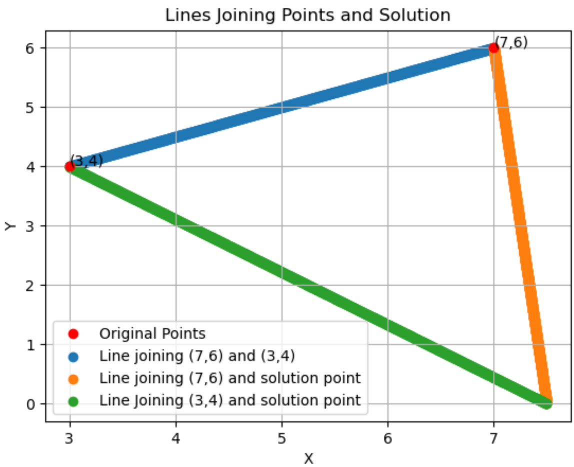

Fig. 0.1: Line joining the three given points