Assignment 2

Roll Number - Name

6. If $f(x) =$
$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$$
then $f(100)$ is equal to (1999 − 2Marks)
   (a) 0 (b) 1 (c) 100 (d) -100

7. If the system of equations $x - ky - z = 0, kx - y - z = 0, x + y - z = 0$ has a non-zero solution, then the possible values of k are (2000S)
   (a) -1,2 (b) 1,2 (c) 0,1 (d) -1,1

8. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant (2002S)
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$$
   (a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

9. The number of values of k for which the system of equations $(k+1)x + 8y = 4k; kx + (k+3)y = 3k - 1$ has infinitely many solutions is (2002S)
   (a) 0 (b) 1 (c) 2 (d) infinte

10. If $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$, then value of α for which $A^2 = B$,is (2003S)
   (a) 1 (b) 4
   (c) 2 (d) infinite

11. If the system of equations $x + ay = 0, az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is (2003S)
   (a) -1 (b) 1
   (c) 0 (d) no real values

12. Given $2x - y + 2z = 2, x - 2y + z = -4, x + y + \lambda z = 4$ then the value of λ such that the given system of equation has NO solution, is (2004S)
   (a) 3 (b) 1 (c) 0 (d) -3

13. Is $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $|A^3| = 125$ then the value α is (2004S)
   (a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

14. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 2005S
and $A^{-1} = \left(\frac{1}{6}\left(A^2 + cA + dI\right)\right)$, then the value of c and d are (2005S)
   (a) $(-6, -11)$ (b) $(6, 11)$ (c) $(-6, 11)$ (d) $(6, -11)$

15. If $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ and $A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$ then x is equal to
   (a) $\begin{vmatrix} 1 & 2005 \\ 0 & 1 \end{vmatrix}$
   (b) $\begin{vmatrix} 4 + 2005\sqrt{3} & 6015 \\ 2005 & 4 - 2005\sqrt{3} \end{vmatrix}$
   (c) $\frac{1}{4}\begin{vmatrix} 2 + \sqrt{3} & 1 \\ -1 & 2 - \sqrt{3} \end{vmatrix}$
   (d) $\frac{1}{4}\begin{vmatrix} 2005 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2005 \end{vmatrix}$

16. Consider 3 points $P = (-\sin(\beta - \alpha), -\cos\beta), Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$ where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$.Then, (2008)
   (a) P lies on the same segment RQ

(b) Q lies on the line segment PR
(c) R lies on the line segment QP
(d) P,Q,R are non-collinear

17. the number of 3x3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ has exactly two distinct solutions is (2008)

(a) 0 (b) $2^9 - 1$ (c) 168 (d) 2

18. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is (2008)

(a) 2 (b) 6 (c) 4 (d) 8

19. Let $P = \left(a_{ij}\right)$ be 3x3 matrix and let $Q = \left(b_{ij}\right)$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is (2012)

(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

20. If P is a 3x3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3x3 identity matrix, then there exists a column matrix $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (2012)

(a) $PX = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ (b) $PX = X$

(c) $PX = 2X$ (d) $PX = -X$

21. Let $P = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$ and I be the identity matrix of order 3. If $Q = \left(q_{ij}\right)$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31}+q_{32}}{q_{21}}$ equals ($JEEAdv$.2016)

(a) 52 (b) 103 (c) 201 (d) 205