

# Assignment 2

Roll Number - Name

6. If  $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$  then  $f(100)$  is equal to (1999 - 2Marks)
- (a) 0 (b) 1 (c) 100 (d) -100
7. If the system of equations  $x - ky - z = 0, kx - y - z = 0, x + y - z = 0$  has a non-zero solution, then the possible values of  $k$  are (2000S)
- (a) -1,2 (b) 1,2 (c) 0,1 (d) -1,1
8. Let  $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ . Then the value of the determinant (2002S)
- $$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 - \omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$$
- (a)  $3\omega$  (b)  $3\omega$  (c)  $3\omega^2$  (d)  $3\omega(1 - \omega)$
9. The number of values of  $k$  for which the system of equations  $(k+1)x + 8y = 4k; kx + (k+3)y = 3k - 1$  has infinitely many solutions is (2002S)
- (a) 0 (b) 1 (c) 2 (d) infinite
10. If  $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ , then value of  $\alpha$  for which  $A^2 = B$ , is (2003S)
- (a) 1 (b) 4 (c) 2 (d) infinite
11. If the system of equations  $x + ay = 0, az + y = 0$  and  $ax + z = 0$  has infinite solutions, then the value of  $a$  is (2003S)
- (a) -1 (b) 1 (c) 0 (d) no real values
12. Given  $2x - y + 2z = 2, x - 2y + z = -4, x + y + \lambda z = 4$  then the value of  $\lambda$  such that the given system of equation has NO solution, is (2004S)
- (a) 3 (b) 1 (c) 0 (d) -3
13. Is  $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$  and  $|A^3| = 125$  then the value  $\alpha$  is (2004S)
- (a)  $\pm 1$  (b)  $\pm 2$  (c)  $\pm 3$  (d)  $\pm 5$
14.  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{pmatrix}$  and  $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  2005S  
and  $A^{-1} = \left(\frac{1}{6}(A^2 + cA + dI)\right)$ , then the value of  $c$  and  $d$  are (2005S)
- (a)  $(-6, -11)$  (b)  $(6, 11)$  (c)  $(-6, 11)$  (d)  $(6, -11)$
15. If  $P = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$  and  $A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$  and  $Q = PAP^T$  and  $x = P^T Q^{2005} P$  then  $x$  is equal to
- (a)  $\begin{vmatrix} 1 & 2005 \\ 0 & 1 \end{vmatrix}$  (b)  $\begin{vmatrix} 4 + 2005\sqrt{3} & 6015 \\ 2005 & 4 - 2005\sqrt{3} \end{vmatrix}$  (c)  $\frac{1}{4} \begin{vmatrix} 2 + \sqrt{3} & 1 \\ -1 & 2 - \sqrt{3} \end{vmatrix}$  (d)  $\frac{1}{4} \begin{vmatrix} 2005 & 2 - \sqrt{3} \\ 2 + \sqrt{3} & 2005 \end{vmatrix}$
16. Consider 3 points  $P = (-\sin(\beta - \alpha), -\cos\beta)$ ,  $Q = (\cos(\beta - \alpha), \sin\beta)$  and  $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$  where  $0 < \alpha, \beta, \theta < \frac{\pi}{4}$ . Then, (2008)
- (a)  $P$  lies on the same segment  $RQ$

- (b)  $Q$  lies on the line segment  $PR$   
 (c)  $R$  lies on the line segment  $QP$   
 (d)  $P, Q, R$  are non-collinear

(a) 52      (b) 103      (c) 201      (d) 205

17. the number of  $3 \times 3$  matrices  $A$  whose entries are

either 0 or 1 and for which the system  $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} =$

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  has exactly two distinct solutions is (2008)

(a) 0      (b)  $2^9 - 1$       (c) 168      (d) 2

18. Let  $\omega \neq 1$  be a cube root of unity and  $S$  be the set of all non-singular matrices of the form

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$$

where each of  $a, b$  and  $c$  is either  $\omega$  or  $\omega^2$ . Then the number of distinct matrices in the set  $S$  is (2008)

(a) 2      (b) 6      (c) 4      (d) 8

19. Let  $P = (a_{ij})$  be  $3 \times 3$  matrix and let  $Q = (b_{ij})$ , where  $b_{ij} = 2^{i+j}a_{ij}$  for  $1 \leq i, j \leq 3$ . If the determinant of  $P$  is 2, then the determinant of the matrix  $Q$  is (2012)

(a)  $2^{10}$       (b)  $2^{11}$       (c)  $2^{12}$       (d)  $2^{13}$

20. If  $P$  is a  $3 \times 3$  matrix such that  $P^T = 2P + I$ , where  $P^T$  is the transpose of  $P$  and  $I$  is the  $3 \times 3$  identity matrix, then there exists a column

matrix  $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  (2012)

(a)  $PX = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$       (b)  $PX = X$

(c)  $PX = 2X$       (d)  $PX = -X$

21. Let  $P = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{pmatrix}$  and  $I$  be the identity matrix

of order 3. If  $Q = (q_{ij})$  is a matrix such that  $P^{50} - Q = I$ , then  $\frac{q_{31} + q_{32}}{q_{21}}$  equals (JEEAdv.2016)