Trignometric Functions and Equations

EE24BTECH11001- ADITYA TRIPATHY

A: Fill In The Blanks

1) Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos x$ is an identity in x, where $C_0, C_1, \cdots, C_n$ are constants and $C_n \neq 0$ then the value of n is (*1981 - 2 Marks*)
2) The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real,is (*1987 - 2 Mark*)
3) The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is (*1987 - 2 mark*)
4) The sides of a triangle in a given circle subtend angles α, β, γ. The minimum value of arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$, $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to (*1987 - 2 Marks*)
5) The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$ $\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$is equal to (*1991 - 2 Marks*)
6) If $K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18})$ then the numerical value of K is (*1993 - 2 Marks*)
7) If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value $\tan A \tan B$ is (*1993 - 2 Marks*)
8) General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is (*1996 - 1 Mark*)
9) The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are (*1997 - 2 Marks*)

B: True / False

1) If $\tan A = \frac{1-\cos B}{\sin B}$, then $\tan 2A = \tan B$ (*1981 - 1 Marks*)
2) There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. (*1984 - 1 Marks*)

C :MCQs with One Correct Answer

1) If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is (*1979*)
   (a) $\frac{-4}{5}$ but not $\frac{4}{5}$
   (b) $\frac{4}{5}$ or $\frac{-4}{5}$
   (c) $\frac{4}{5}$ but not $\frac{-4}{5}$
   (d) None of These
2) If $\alpha + \beta + \gamma = 2\pi$ (*1979*)
   (a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
   (c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (d) None of These
3) Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ (*1980*)
   (a) $1 \leq A \leq 2$
   (b) $\frac{3}{4} \leq A \leq 1$
   (c) $\frac{13}{16} \leq A \leq 1$
   (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$
4) The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$ (*1980*)
   (a) no real solution
   (b) one real solution
   (c) more than one real solution
   (d) None of these
5) The general solution to the trignometric equation $\sin x + \cos x = 1$ is given by (*1981 - 2 Marks*)
   (a) $x = 2n\pi; n = 0, \pm1, \pm2 \cdots$
   (b) $x = 2n\pi + \frac{\pi}{2}, n = 0, \pm1, \pm2 \cdots$
   (c) $x = n\pi + (-1)^n \frac{\pi}{4}, n = 0, \pm1, \pm2 \cdots$
   (d) none of these
6) The value of the expression $\sqrt{3}\,\mathrm{cosec}\, 20^\circ - \sec 20^\circ$ is equal to (*1988 - 2 Marks*)
   (a) 2
   (b) $2\sin 20^\circ/\sin 40^\circ$
   (c) 4
   (d) $2\sin 20^\circ/\sin 40^\circ$