Trignometric Functions and Equations

EE24BTECH11001- ADITYA TRIPATHY

A: Fill In The Blanks

1) Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos x$ is an identity in x, where $C_0, C_1, \cdots, C_n$ are constants and $C_n \neq 0$ then the value of n is

(*1981 - 2 Marks*)

2) The solution set of the system of equations $x+y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real,is

(*1987 - 2 Mark*)

3) The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is

(*1987 - 2 mark*)

4) The sides of a triangle in a given circle subtend angles α, β, γ. The minimum value of arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$, $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to

(*1987 - 2 Marks*)

5) The value of

$$\sin\tfrac{\pi}{14}\sin\tfrac{3\pi}{14}\sin\tfrac{5\pi}{14}\sin\tfrac{7\pi}{14}\sin\tfrac{9\pi}{14}\sin\tfrac{11\pi}{14}\sin\tfrac{13\pi}{14} is equal to \quad (1)$$

(*1991 - 2 Marks*)

6) If $K = \sin(\frac{\pi}{18})\sin(\frac{5\pi}{18})\sin(\frac{7\pi}{18})$ then the numerical value of K is

(*1993 - 2 Marks*)

7) If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value $\tan A \tan B$ is

(*1993 - 2 Marks*)

8) General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is

(*1996 - 1 Mark*)

9) The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are

(*1997 - 2 Marks*)

B: True / False

1) If $\tan A = \frac{1-\cos B}{\sin B}$, then $\tan 2A = \tan B$

(*1981 - 1 Marks*)

2) There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$.

(*1984 - 1 Marks*)

C :MCQs w ith One Correct Answer

1) If $\tan\theta = -\frac{4}{3}$ then $\sin\theta$ is

(*1979*)

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$
(b) $\frac{4}{5}$ or $\frac{-4}{5}$
(c) $\frac{4}{5}$ but not $\frac{-4}{5}$
(d) None of These

2) If $\alpha + \beta + \gamma = 2\pi$

(*1979*)

(a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
(b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
(c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
(d) None of These

3) Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ

(*1980*)

(a) $1 \leq A \leq 2$
(b) $\frac{3}{4} \leq A \leq 1$
(c) $\frac{13}{16} \leq A \leq 1$
(d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

4) The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$

(*1980*)

(a) no real solution
(b) one real solution
(c) more than one real solution
(d) None of these

5) The general solution to the trignometric equation $sinx + cosx = 1$ is given by

(*1981 - 2 Marks*)

(a) $x = 2n\pi; n = 0, \pm1, \pm2\cdots$
(b) $x = 2n\pi + \frac{\pi}{2}, n = 0, \pm1, \pm2\cdots$
(c) $x = n\pi + (-1)^n\frac{\pi}{4}, n = 0, \pm1, \pm2\cdots$
(d) none of these

6) The value of the expression $\sqrt{3}\,\mathrm{cosec}\,20^\circ - \sec 20^\circ$ is equal to

(*1988 - 2 Marks*)

(a) 2
(b) $2 \sin 20° / \sin 40°$
(c) 4
(d) $2 \sin 20° / \sin 40°$