Experiment 04

Aditya Tripathy - ee24btech11001, Durgi Swaraj Sharma - ee24btech11018

Abstract
In Experiment-04, we try to capture LC oscillations.

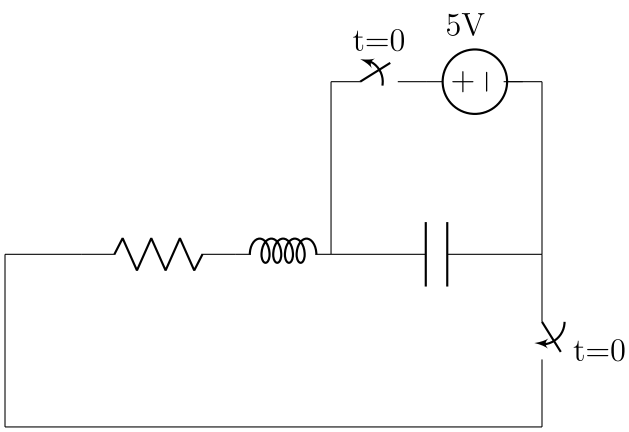
1 Objective

To study the response of a series RLC circuit with a precharged capacitor.

2 Apparatus

- Oscilloscope
- Regulated DC power supply
- Connecting wires and probes
- Unpolarised capacitor ($560pF$)
- Inductor ($2.2mH$)
- Switch (Button switch)

3 Theory



The resposnse to the circuit shown is the solution to the initial value problem:

$$L\frac{di}{dt} + iR + \frac{q}{C} = 0 \tag{1}$$

with $q(0) = CV_0$ and $i(0) = 0$

Since,

$$q = CV_c \tag{2}$$
$$\rightarrow i = C\frac{dV_c}{dt} \tag{3}$$

Now the equation becomes,

$$LC\frac{d^2V_c}{dt^2} + RC\frac{dV_c}{dt} + V_c = 0 \tag{4}$$

with $V_c(0) = V_0$ and $V_c{}'(0) = 0$
The complementary solution to the differential equation is given by

$$y_c = c_1e^{s_1t} + ce^{s_2t} \tag{5}$$

where s_1, s_2 are solutions to the following quadratic equation

$$LCs^2 + RCs + 1 = 0 \tag{6}$$

Therefore,

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{7}$$

Since we wish to study the underdamped case for RLC response,

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0 \tag{8}$$

Now the complementary solution can be written more conveniently as

$$y_c = e^{-\beta}(c_1\cos(w_dt) + c_2\sin(w_dt)) \tag{9}$$

where

$$\beta = \text{Damping Factor} = \frac{R}{2L} \tag{10}$$
$$w_n = \text{Natural Frequency} = \frac{1}{\sqrt{LC}} \tag{11}$$
$$w_d = \text{Damped Resonance Frequency} = \sqrt{w_n{}^2 - \beta^2} \tag{12}$$

Now plugging in the initial conditions we get,

$$c_1 = V_0 \tag{13}$$
$$c_2 = \frac{RV_0}{2Lw_d} \tag{14}$$

Therefore

$$V_c(t) = V_0e^{-\beta t}(\cos(w_dt) + \left(\frac{R}{2Lw_d}\right)\sin(w_dt)) \tag{15}$$

4 Procedure

1. Connections
   - Connect the inductor and capacitor in series.
   - Connect the 5V DC Voltage source in parallel to the capacitor.
   - Complete the circuit by adding a switch.
   - Connect the probe across the inductor to capture the voltage response.
2. Device Setup
   - To capture the response for the first few cycles, set an appropriate trigger level, set "Sweep = Normal" under "Mode Coupling" and press the "Single" button on the oscilloscope.
3. To capture the RLC oscillations, remove the wires connecting DC power supply to capacitor and press the button switch.

5 Readings

Peak No.	Voltage Value	Time Difference(μs)
1.	3.6	0
2.	3.2	2.96
3.	3.0	5.92
4.	2.8	8.88
5.	2.6	11.92
6.	2.2	17.84

It should be noted that the time difference is measured with respect to first peak.

6 Results

Experimental value of $w_d = \frac{2\pi}{2.96\mu s} = 2122697.738912022\ rad/s$

Experimental value of $\beta = \frac{\frac{1}{\Delta T_2}\ln\frac{V_1}{V_2}+\frac{1}{\Delta T_3}\ln\frac{V_1}{V_3}+\frac{1}{\Delta T_4}\ln\frac{V_1}{V_4}+\frac{1}{\Delta T_5}\ln\frac{V_1}{V_5}+\frac{1}{\Delta T_6}\ln\frac{V_1}{V_6}}{5} = 30759.2043\ \Omega/H$

7 Response captured

Figure 1: Response

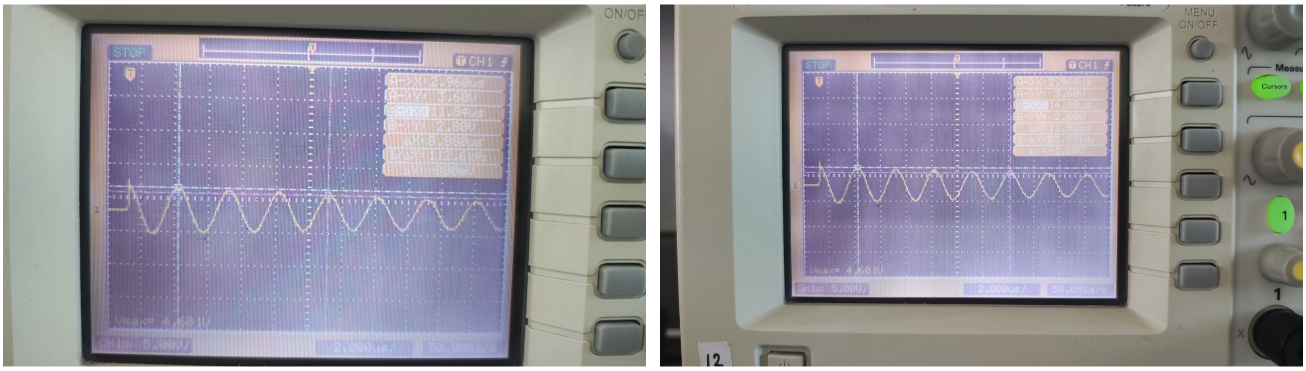


Figure 2: Response

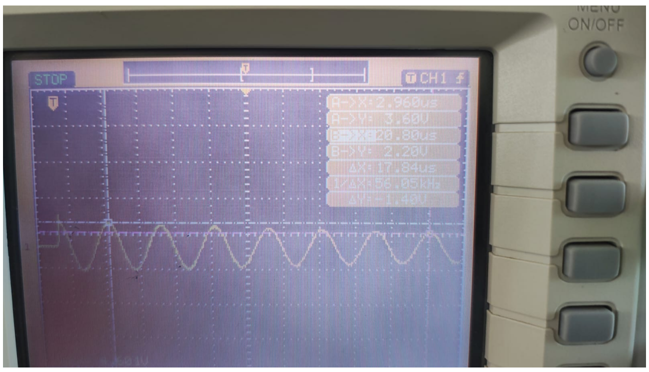


Figure 3: Response