Assignment 1

Aditya Tripathy
EE24BTECH11001
GROUP 9

February 3, 2025

1 Question

Obtain the 1's and 2's complements of the following binary numbers:

(a) 11100010

(b) 00011000

(c) 10111101

(d) 10100101

(e) 11000011

(f) 01011000

2 Solution

To take the 1's complement of a binary number, we just to swap 0's and 1's.
For 2' complement, we add 1 to 1's complement of the binary number.

1. 11100010

   - 1's complement: 00011101
   - Add 1:

$$\begin{array}{r} {}^{1} \\ 00011101 \\ + \quad 1 \\ \hline 00011110 \end{array}$$

   - 2's complement: 00011110

2. 00011000

   - 1's complement: 11100111

- Add 1:

$$\begin{array}{r} {\scriptstyle 111} \\ 11100111 \\ + \quad 1 \\ \hline 11101000 \end{array}$$

- 2's complement: 11101000

3. 10111101
   - 1's complement: 01000010
   - Add 1:

$$\begin{array}{r} 01000010 \\ + \quad 1 \\ \hline 01000011 \end{array}$$

   - 2's complement: 01000011

4. 10100101
   - 1's complement: 01011010
   - Add 1:

$$\begin{array}{r} 01011010 \\ + \quad 1 \\ \hline 01011011 \end{array}$$

   - 2's complement: 01011011

5. 11000011
   - 1's complement: 00111100
   - Add 1:

$$\begin{array}{r} 00111100 \\ + \quad 1 \\ \hline 00111101 \end{array}$$

   - 2's complement: 00111101

(f) 01011000

- 1's complement: 10100111
- Add 1:

$$\begin{array}{r} {\scriptstyle 111} \\ 10100111 \\ + \quad 1 \\ \hline 10101000 \end{array}$$

- 2's complement: 10101000

Question 2
Determine the base of the numbers in each case for the following operations to be correct:

1. $\frac{67}{5} = 11$

2. $15 \times 3 = 51$

3. $123 + 120 = 303$

Solution:

1. Let the base of the numbers be k. In base 10,

$$\begin{aligned} 67 &= 6 \times k^1 + 7 \times k^0 \\ 11 &= 1 \times k^1 + 1 \times k^0 \\ 5 &= 5 \times k^0 \end{aligned}$$

   The equation in base 10 will be

$$\begin{aligned} \frac{6k+7}{5} &= k+1 \\ 6k+7 &= 5k+5 \\ 6k-5k &= 5-7 \\ k &= -2 \end{aligned}$$

   The base comes out to be -2 which is not possible. Hence there is no base for which the given equation makes sense.

2. Let the base of the numbers be k

$$\begin{aligned} 15 &= 1 \times k^1 + 5 \times k^0 \\ 51 &= 5 \times k^1 + 1 \times k^0 \\ 3 &= 3 \times k^0 \end{aligned}$$

   The equation in base 10 will be

$$\begin{aligned} 3(k+5) &= 5k+1 \\ 3k+15 &= 5k+1 \\ 2k &= 14 \\ k &= 7 \end{aligned}$$

   Hence the equation holds true in base 7.

3. Let the base of the numbers be k. The value of the number in base 10 will be

$$\begin{aligned} 123 &= 1 \times k^2 + 2 \times k^1 + 3 \times k^0 \\ 120 &= 1 \times k^2 + 2 \times k^1 + 0 \times k^0 \\ 303 &= 3 \times k^2 + 0 \times k^1 + 3 \times k^0 \end{aligned}$$

Now,

$$\begin{aligned} (k^2 + 2k + 3) + (k^2 + 2k) &= 3k^2 + 3 \\ 2k^2 + 4k + 3 &= 3k^2 + 3 \\ k^2 - 4k &= 0 \\ k &= 0 \\ k &= 4 \end{aligned}$$

Base 0 is meaningless for the given problem, hence the base in which the given equation holds is base 4.

Question 3

The solutions to the quadratic equation $x^2 - 13x + 22 = 0$ are $x = 7$ and $x = 2$. What is the base of the numbers?

Solution

Let the numbers be in base k then, in base 10 the equation becomes

$$\begin{aligned} 13 &= 1 \times k^1 + 3 \times k^0 \\ 22 &= 2 \times k^1 + 2 \times k^0 \\ x^2 - (k+3)x + (2k+2) &= 0 \end{aligned}$$

Since 2 and 7 are given to be solutions in the problem statement,

$$\begin{aligned} 49 - 7(k+3) + (2k+2) &= 0 \\ k &= 6 \end{aligned}$$

$$\begin{aligned} 4 - 2(k+3) + (2k+2) &= 0 \\ 0 &= 0 \end{aligned}$$

Arbitrary values of k can be used for $x = 2$ to be a solution, however $k = 6$ is the only value for $x = 7$ to be the solution. Hence the value of k is 6.

Question 4

How many printing characters are there in ASCII? How many of them are special characters (not letters or numerals)?

Solution

There are 95 printing characters in ASCII out of which 33 are special characters

Question 5

What bit must be complemented to change an ASCII letter from capital to lowercase and vice versa?

Solution

Capital alphabets start from index 65 while lowercase alphabets start from index 97. Since the difference between the two is $32 = 2^5$ the 6th bit (corresponding to 2^5) must be complemented to change an ASCII letter from capital to lowercase and vice versa.