CS3388B, Winter 2023

Problem Set 5

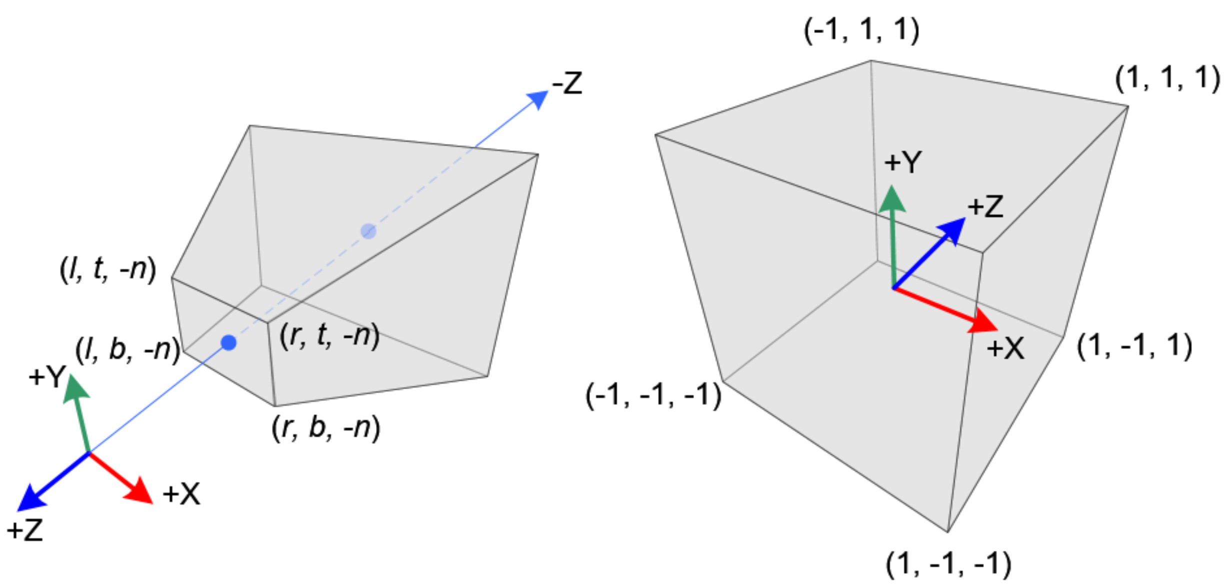
Due: February 17, 2023

Exercise 1.

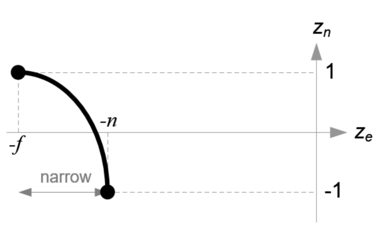
Our goal is to create a perspective projection matrix. Recall its definition given in Lecture 9.

(a) Give the projection matrix that results from a vertical field of view of 50°, an aspect ratio of 2.0, a near clipping plane of 1 and far clipping plane of 10.

Hint: consider that the field-of-view and the near clipping plane together create a right triangle. You can then use SOH-CAH-TOA to find the value of t. Once you know t, aspect ratio gives you r.



(b) To make sure you did this correctly, take the point $(0,0,-5.5)$. If you multiply this point by your projection matrix, and then do perspective divide, you should get $(0,0,0)$, right? Indeed, $z = -5.5$ is halfway between your near clipping plane and far clipping plane and therefore should be mapped to $z = 0$? No, if you notice your entry in the projection matrix gives a non-linear mapping. You'll get something close to $\frac{9}{11}$.



```
[1.072253      0.          0.          0.     ]
[                                            ]
[   0.      2.144507       0.          0.     ]
[                                            ]
[   0.         0.      -1.222222   -2.222222]
[                                            ]
[   0.         0.      -1.000000       0.     ]
```

Exercise 2.

Consider the elementary rotation matrices:

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos(\gamma) & -sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Convince yourself that the order of rotations matter.

Compute the matrix product $R_1 = R_x R_y R_z$. Compute the matrix product $R_2 = R_z R_y R_x$. You should find that they are not equal.

You can check your answers against the table of combinations found at `https://en.wikipedia.org/wiki/Euler_angles#Rotation_matrix`

Exercise 3.

Find the View matrix that results from a camera being placed at $(5,3,5)$ and looking at $(2,0,-1)$. Both these points are given in world coordinates.

```
[0.894427       0.       -0.447214    -2.236068]
[                                              ]
[-0.182574   0.912871    -0.365148       -0.   ]
[                                              ]
[0.408248    0.408248    0.816497     -7.348470]
[                                              ]
[   0.          0.          0.        1.000000 ]
```