Global Optimizations

Types of optimizations

Classification of optimization (based on their scope)

- Local (within basic blocks)
- Global Intra-procedural
- Global Inter-procedural

Classification based on their positioning:

- High level optimizations (use the program structure to optimize).
- Low level optimizations (work on medium/lower level IR)

Optimization classification (contd)

Classification with respect to their dependence on the target machine.

Machine independent

- applicable across broad range of machines
- Examples
  - move evaluation to a less frequently executed place
  - remove redundant (unreachable, useless) code.
- create opportunities.

Machine dependent

- capitalize on machine-specific properties
- improve mapping from IR onto machine
- strength reduction.
- replace sequence of instructions with more powerful one
  (use “exotic” instructions)

Optimization

Desirable properties of an optimizing compiler

- code at least as good as an assembler programmer
- stable, robust performance (predictability)
- architectural strengths fully exploited
- architectural weaknesses fully hidden
- broad, efficient support for language features
- instantaneous compilation

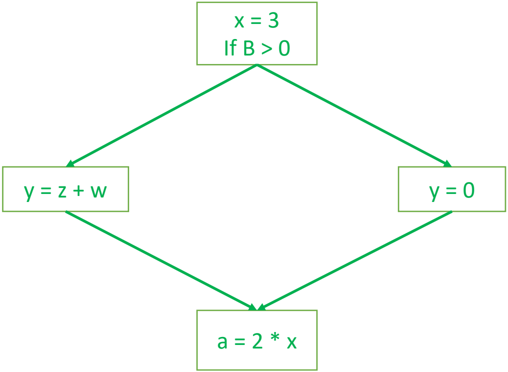
From local to global optimization

- Recall the local basic-block optimizations
  - Constant propagation
  - Dead code elimination

```
x = 3                  x = 3
y = z * w     -->      y = z * w     -->     y = z * w
q = x + y              q = 3 + y             q = 3 + y
```

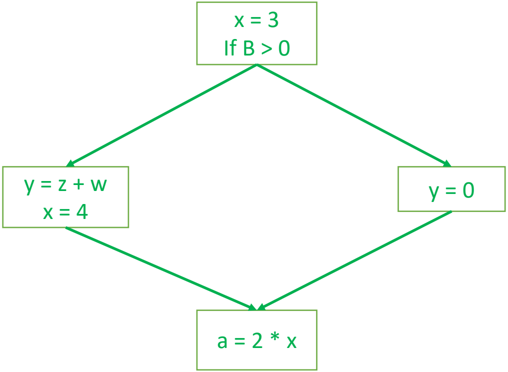
From local to global optimization

Can these optimizations be directly extended to an entire control-flow graph?



From local to global optimization

Can these optimizations be directly extended to an entire control-flow graph? There are situations where it is incorrect to globally propagate constants:



From local to global optimization: Constant Propagation

Correctness Criterion

To replace a use of x by constant k, on every path to the use of x, the last assignment to x is $x = k$.

- This correctness criterion is non-trivial to check.
- ‘Every path’ includes paths around loops and through branches of conditionals.
- This requires a ‘global’ analysis, i.e. an analysis of the entire control-flow graph.

Global Analysis

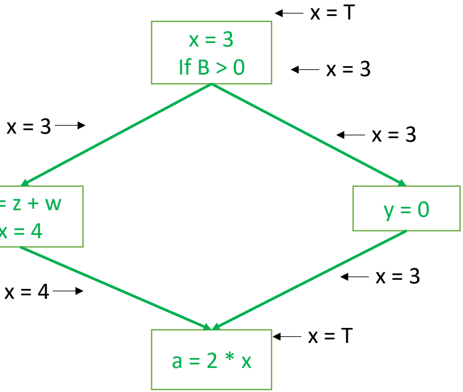
Global optimization tasks share several traits:

- The optimization depends on knowing some property X at a particular point in program execution.
- Proving X at any point requires knowledge of the entire function.
- It is OK to be conservative. If the optimization requires X to be true, then the analysis can output one of two things:
  - X is definitely true
  - Don't know if X is true.
- Global dataflow analysis is a standard technique for performing global optimizations.

Global Constant Propagation

- To perform global constant propagation at a program point, we need to know whether a variable will always have a constant value at the point.
- We associate one of the following values with variable x at every program point:
    - $\perp$: This program point is not reachable.
    - c: x has a constant value c
    - $\top$: x is not a constant

Example



Defining Global Constant Propagation Analysis

- We associate two functions $in, out : Var \rightarrow \mathbb{Z} \cup \{\top, \bot\}$ with each basic block.
- Global Constant Propagation Analysis is a forward analysis: in of a basic block is defined in terms of out of predecessors.
- For a basic block b, In_b is defined as follows:

$$in_b(x) = \begin{cases} \top & \exists p \in Pred(b).\ out_p(x) = \top \\ c & \forall p \in Pred(b).\ out_p(x) = c \vee out_p(x) = \bot \\ \top & \exists p_1, p_2 \in Pred(b).\ out_{p_1}(x) \neq out_{p_2}(x) \\ \bot & \forall p \in Pred(b).\ out_p(x) = \bot \end{cases}$$

The above definition is also called a meet operation.
We use the following notation for the above definition:
$in_b = \bigvee_{p \in Pred(b)} out_p$

Defining Global Constant Propagation Analysis

out_b is defined in terms of in_b and the statements in basic block b

For simplicity, assume that we have a separate basic block for each statement:

$$out_b(x) = \begin{cases} \bot & in_b(x) = \bot \\ c & b: \texttt{x = c} \text{ where } \texttt{c} \in \mathbb{Z} \\ \top & b: \texttt{x = e} \text{ where } \texttt{e} \text{ is an expression} \\ In_b(x) & b: \texttt{y = } \ldots \end{cases}$$

The above definition is also called a transfer function.

We can express the definition as a function f_b such that $f_b(in_b) = out_b$.

Defining Global Constant Propagation Analysis

$$out_b(x) = \begin{cases} \bot & in_b(x) = \bot \\ c & b: \texttt{x = c} \text{ where } c \in \mathbb{Z} \\ e[In_b] & b: \texttt{x = e} \text{ where } e \text{ is an expression} \\ In_b(x) & b: \texttt{y = } \ldots \end{cases}$$

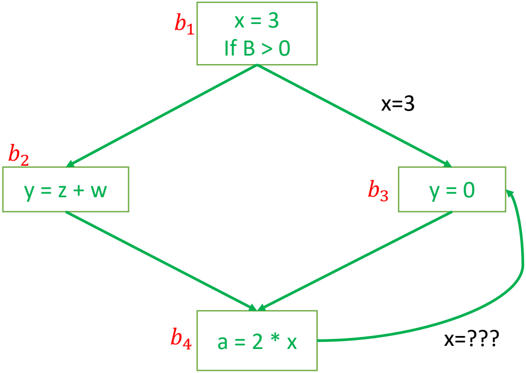
We can also do constant folding while evaluating expressions.

Given a function $f : Var \to \mathbb{Z} \cup \{\top, \bot\}$, $e[f]$ denotes the evaluation of expression e using function f.
While evaluating, $\top + c = \top$ (similar for other arithmetic operators).

Iterative method for computing In, Out

```
$N$ : Set of nodes of CFG;
$Start$ : Entry basic blocks of CFG (i.e. successors of entry);
foreach $n \in Start$ do
    $in_n \leftarrow \lambda v.\top$;
end
foreach $n \in N - Start$ do
    $in_n \leftarrow \lambda v.\bot$;
    $out_n \leftarrow \lambda v.\bot$;
end
repeat
    foreach n ∈ Nodes do
        $in'_n \leftarrow in_n$;
        $out'_n \leftarrow out_n$;
        $in_n \leftarrow \bigvee_{p \in Pred(n)} out_p$;
        $out_n \leftarrow f_n(in_n)$ ;
    end
until $\forall n, in'_n = in_n \wedge out'_n = out_n$ ;
```

Why do we need $\perp$?



- To compute in_{b_3}, we need out_{b_4}, but for that we need out_{b_3}!
- We will encounter similar problems whenever we have loops.
- Hence, we initialize in and out values with $\perp$, which intuitively means that 'so far as we know, control never reaches this point'

Orderings

- We can simplify the presentation of the data-flow analysis by ordering the values.
  - $\forall c \in \mathbb{Z}. \top \leq c \leq \bot$
  - Formally, $\leq$ is a partial order on the set $\mathbb{Z} \cup \{\bot, \top\}$.
- We can define the greatest lower bound of a set of values.
  - Formally, $(\mathbb{Z} \cup \{\bot, \top\}, \leq)$ forms a meet semi-lattice, and hence the *glb* always exists for any set of values.

$$in_b(x) = \begin{cases} \top & \exists p \in Pred(b).\ out_p(x) = \top \\ \top & \exists p_1, p_2 \in Pred(b).\ out_{p_1}(x) \neq out_{p_2}(x) \\ \bot & \forall p \in Pred(b).\ out_p(x) = \bot \\ c & \forall p \in Pred(b).\ out_p(x) = c \vee out_p(x) = \bot \end{cases}$$

- Notice that $in_b(x) = glb(\{out_p(x) \mid p \in Pred(b)\})$.
  - The greatest lower bound is also called **meet**.

Orderings

- Every data-flow analysis can be represented by defining its meet semi-lattice, with the corresponding meet operation being used in the iterative method.
  - Useful for proving the soundness of the analysis, for comparing precision of different analyses, and for proving termination of the iterative method.
  - Dataflow analysis/Abstract Interpretation covered in detail in advanced courses: CS5030, CS6013.
- Termination argument: We start with the highest value ($\perp$) and we only move down.
  - $\perp$ can change to a constant value, which can change to $\top$.
  - Thus, each $in(x)$ or $out(x)$ can change at most twice at any basic block.
  - Maximum number of iterations = 2 * 2 * Number of variables * Number of basic blocks.

Liveness Analysis... revisited

- We can represent liveness analysis in the dataflow analysis framework.
- Let Var be the set of variables. Then, the meet semi-lattice would be $(\mathbb{P}(Var), \supseteq)$.
  - The glb operation is set union.
  - The analysis works in the backward direction. Hence, $out_b = \bigcup_{s \in succ(b)} in_s$.
- The transfer function is $f_b(S) = use_b \cup (S - def_b)$.
  - use_b are variables which are used before they are (possibly) defined in b. Can be determined using the next-use algorithm.
  - def_b are variables which are defined in b.

Types of program analysis

Classification of analysis (based on their view)

```
if (cond) {
   a = ...
   b = ...
} else {
   a = ...
   c = ...
}
// Which of the variables may be assigned? -- {a,b,c}
// Which of the variables must be assigned? -- {a}
```

- May analysis – the analysis holds on at least one data flow path.
- Must analysis – the analysis must hold on all data flow paths.
  - What can we say about constant propagation analysis? May or Must?
  - What can we say about liveness analysis? May or Must?

In the last lecture

- To perform global optimization, we require information which must hold globally, i.e. across all possible executions.
  - Infeasible to compute directly, as there are an infinite number of program paths.
- Global Dataflow Analysis solves this problem by performing local analysis at each basic block, which is then systematically propagated across basic blocks.
  - Information is propagated only from predecessors/successors through meet functions.
  - Local information is obtained using transfer functions.
- Dataflow analysis is sound, i.e. it correctly computes dataflow facts that hold across all paths.
- Dataflow analysis is conservative and efficient.

Classification of analysis (contd)

Classification of analysis (based on precision)

- Flow sensitive / insensitive.
  - Insensitive - the analysis should hold at every program point; does not depend on the control flow.
  - Sensitive - Each program point has its own analysis.

```
if (c) {
  a = 2;
  b = a;
  c = 3;
  print (a, b, c); // constants?
} else {
  a = 3
  b = a;
  c = 3;
  print (a, b, c); // constants?
}
```

Classification of analysis (contd)

- Context sensitive and insensitive

```
a = foo(2);

b = foo (3);

c = bar (2);

d = bar(2);

print (a, b, c, d); // a, b, c, d constants?

int foo(int x) { return x }
int bar(int x) { return x * x }
```

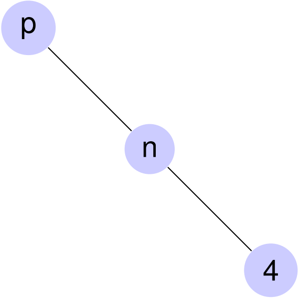
Alias Analysis

Alias analysis: problem of identifying storage locations that can be accessed by more than one way.

Are variable a and b aliases? ⇒ a and b refer to the same location? Modifying the contents of a, modifies the contents of b.

Necessary for performing many optimizations such as constant/copy propagation, common sub-expression elimination, dead code elimination, etc.

```
foo(){
  int *p;
  int n;
  p = &n;
  n = 4;
  print ("%d", *p);
}
```



Alias analysis (contd)

```
extern int *q;
foo( ) {
  int a, k;
  k = a + 5;
  f (a, &k);
  *q = 13;
  k = a + 5;  /* Assignment is redundant? */
              /* Expression is redundant? */
  ...
}
```

Only if a) the assignment to `*q` does not change `k` or `a`,
b) the function call `f`, does not change `*k`.

Loop unrolling

(Example) Matrix-matrix multiply

```
do i ← 1, n, 1
    do j ← 1, n, 1
        c(i,j) ← 0
        do k ← 1, n, 1
            c(i,j) ← c(i,j) + a(i,k) * b(k,j)
```

- All the array elements are floating point values.
- $2n^3$ flops, n^3 loop increments and branches
- each iteration does 2 loads and 2 flops

Example: loop unrolling

Matrix-matrix multiply (assume 4-word cache line)

```
do i ← 1, n, 1
   do j ← 1, n, 1
      c(i,j) ← 0
      do k ← 1, n, 4
         c(i,j) ← c(i,j) + a(i,k) ⋆ b(k,j)
         c(i,j) ← c(i,j) + a(i,k+1) ⋆ b(k+1,j)
         c(i,j) ← c(i,j) + a(i,k+2) ⋆ b(k+2,j)
         c(i,j) ← c(i,j) + a(i,k+3) ⋆ b(k+3,j)
```

- $2n^3$ flops, $\frac{n^3}{4}$ loop increments and branches
- each iteration does 8 loads and 8 flops
- memory traffic is better
  - `c(i,j)` is reused (put it in a register)
  - `a(i,k)` reference are from cache
  - `b(k,j)` is problematic

Example: loop unrolling

Matrix-matrix multiply (to improve traffic on `b`)

```
do j ← 1, n, 1
   do i ← 1, n, 4
      c(i,j) ← 0
      do k ← 1, n, 4
         c(i,j) ← c(i,j) + a(i,k) * b(k,j)
            + a(i,k+1) * b(k+1,j) + a(i,k+2) * b(k+2,j)
            + a(i,k+3) * b(k+3,j)
         c(i+1,j) ← c(i+1,j) + a(i+1,k) * b(k,j)
            + a(i+1,k+1) * b(k+1,j)
            + a(i+1,k+2) * b(k+2,j)
            + a(i+1,k+3) * b(k+3,j)
         c(i+2,j) ← c(i+2,j) + a(i+2,k) * b(k,j)
            + a(i+2,k+1) * b(k+1,j)
            + a(i+2,k+2) * b(k+2,j)
            + a(i+2,k+3) * b(k+3,j)
         c(i+3,j) ← c(i+3,j) + a(i+3,k) * b(k,j)
            + a(i+3,k+1) * b(k+1,j)
            + a(i+3,k+2) * b(k+2,j)
            + a(i+3,k+3) * b(k+3,j)
```

Example: loop unrolling

What happened?

- interchanged `i` and `j` loops
- unrolled `i` loop
- fused inner loops
- $2n^3$ flops, $\frac{n^3}{16}$ loop increments and branches
- first assignment does 8 loads and 8 flops
- 2nd through 4th do 4 loads and 8 flops
- memory traffic is better
  - `c(i,j)` is reused (register)
  - `a(i,k)` references are from cache
  - `b(k,j)` is reused (register)

Loop optimizations: factoring loop-invariants

Loop invariants: expressions constant within loop body

Goal: move the loop invariant computation to outside the loop.

The loop independent code executes only once, instead of many times the loop might.

Example: loop invariants

```
foreach i=1 .. 100 do
    foreach j=1 .. 100 do
        foreach k=1 .. 100 do
            A[i,j,k] = i * j * k;
        end
    end
end
```

- 3 million index operations
- 2 million multiplications

Example: loop invariants (cont.)

Factoring the inner loop:

```
foreach i=1 .. 100 do
    foreach j=1 .. 100 do
        t1 = &A[i][j];
        t2 = i * j ;
        foreach k=1 .. 100 do
            t1[k] = t2 * k;
        end
    end
end
```

And the second loop:

```
foreach i=1 .. 100 do
    t3 = &A[i];
    foreach j=1 .. 100 do
        t1 = &t3[j];
        t2 = i * j ;
        foreach k=1 .. 100 do
            t1[k] = t2 * k;
        end
    end
end
```

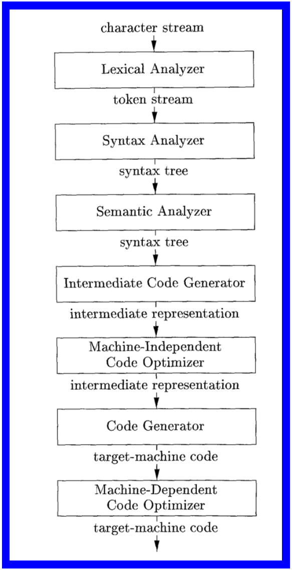
Optimization - overview

Compilers are engineered objects

- minimize running time of compiled code
- minimize compile time
- use reasonable compile-time space
- find a reasonable trade-off

Thus, results are sometimes unexpected

Back to first lecture



Front end responsibilities:

- Recognize syntactically legal code; report errors.
- Recognize semantically legal code; report errors.
- Produce IR.

Back end responsibilities:

- Optimizations, code generation.

Our target

- five out of seven phases.
- glance over optimizations.