

Soft Inverted Pendulum Robot - Modelling, Simulation and Control

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Model used and Parameters

Modelling scheme: Cosserat Rod Theory

Numerical Solver / Software Used: PyElastica (Python)

Initial Length of rod (L) = 0.13 mts.

Initial radius of the rod (R) = 0.01416 mts.

Density, ρ = 1180 kg/m³

Poisson Ratio, γ = 0.5

Young's Modulus, E = 3.79 MPa

Energy Dissipation, ν = 10

Simulation Time, t_f = 10 secs

Shear Modulus, G = $E / (2 \cdot (1 + \gamma))$

Hinged Planar Case: Stiffness (k) and Damping (β) Experimental Computation

$$\begin{bmatrix} q_0(t_0) & \dot{q}_0(t_0) \\ \vdots & \vdots \\ q_0(t_f) & \dot{q}_0(t_f) \end{bmatrix} \begin{bmatrix} k \\ \beta \end{bmatrix} = \begin{bmatrix} RHS(t_0) \\ \vdots \\ RHS(t_f) \end{bmatrix}$$

$$q = \begin{bmatrix} q_0 \\ \theta \end{bmatrix}$$

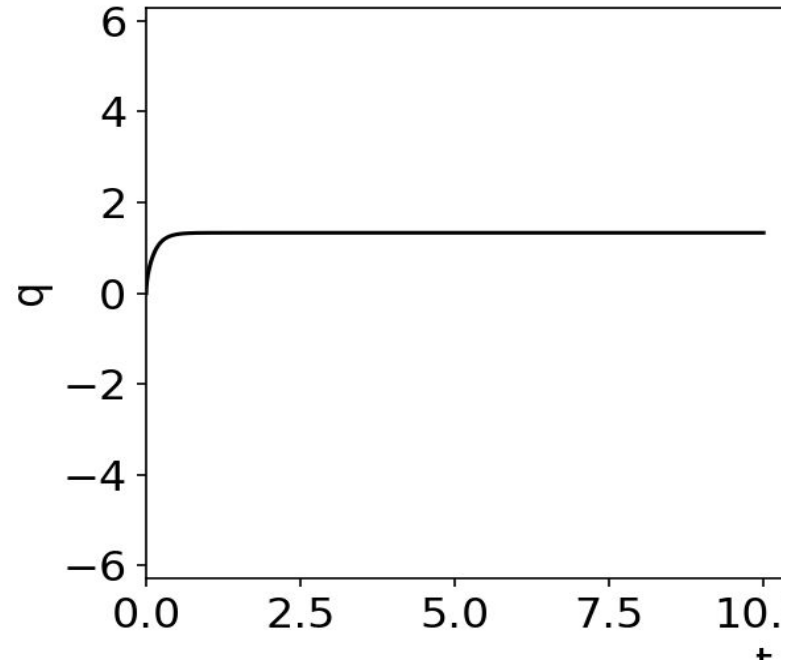
$$RHS(t_i) = \tau - (M(q)q(t_i) + C(q, \dot{q})\dot{q}(t_i))[0]$$

$$\begin{bmatrix} k \\ \beta \end{bmatrix} = \begin{bmatrix} q_0(t_0) & \dot{q}_0(t_0) \\ \vdots & \vdots \\ q_0(t_f) & \dot{q}_0(t_f) \end{bmatrix}^{\dagger} \begin{bmatrix} RHS(t_0) \\ \vdots \\ RHS(t_f) \end{bmatrix}$$

, where, A^{\dagger} means pseudo-inverse of A

Why $\nu = 10$?

As we can see from the figure, the damping seems to be enough to have a smooth convergence towards steady-state



Hinged Planar Case: Stiffness (k) and Damping (β) Experimental Computation

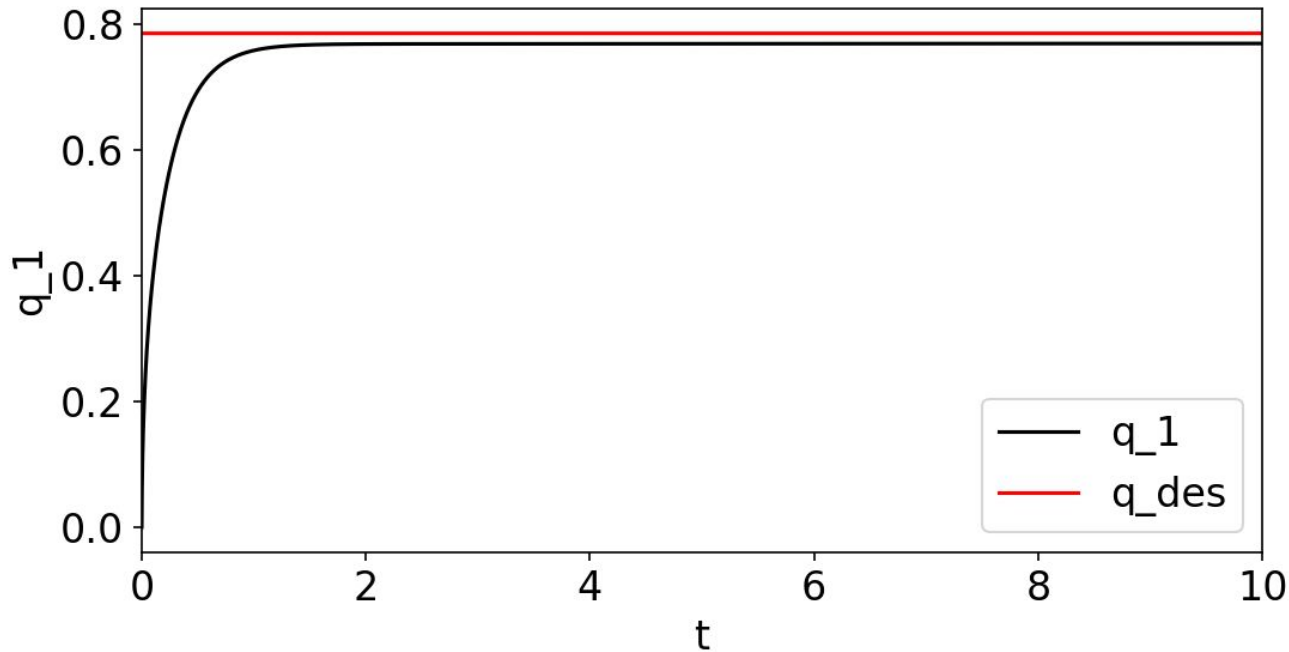
Input Torque (τ) (Nm)	Stiffness (k) (Nm/rad)	Damping (β) (Nsm/rad)	Final Bending Angle (q_0) (rad)
0.3125	0.942	0.07	0.33
0.625	0.942	0.07	0.67
0.9205	0.942	0.07	0.98
1.25	0.942	0.07	1.34
1.5	0.942	0.07	1.60
1.75	0.942	0.07	1.86

Thus, the obtained values for $\nu = 10$ and $E = 3.79$ MPa conditions are:

$k = 0.942$ Nm/rad and $\beta = 0.07$ Nsm/rad

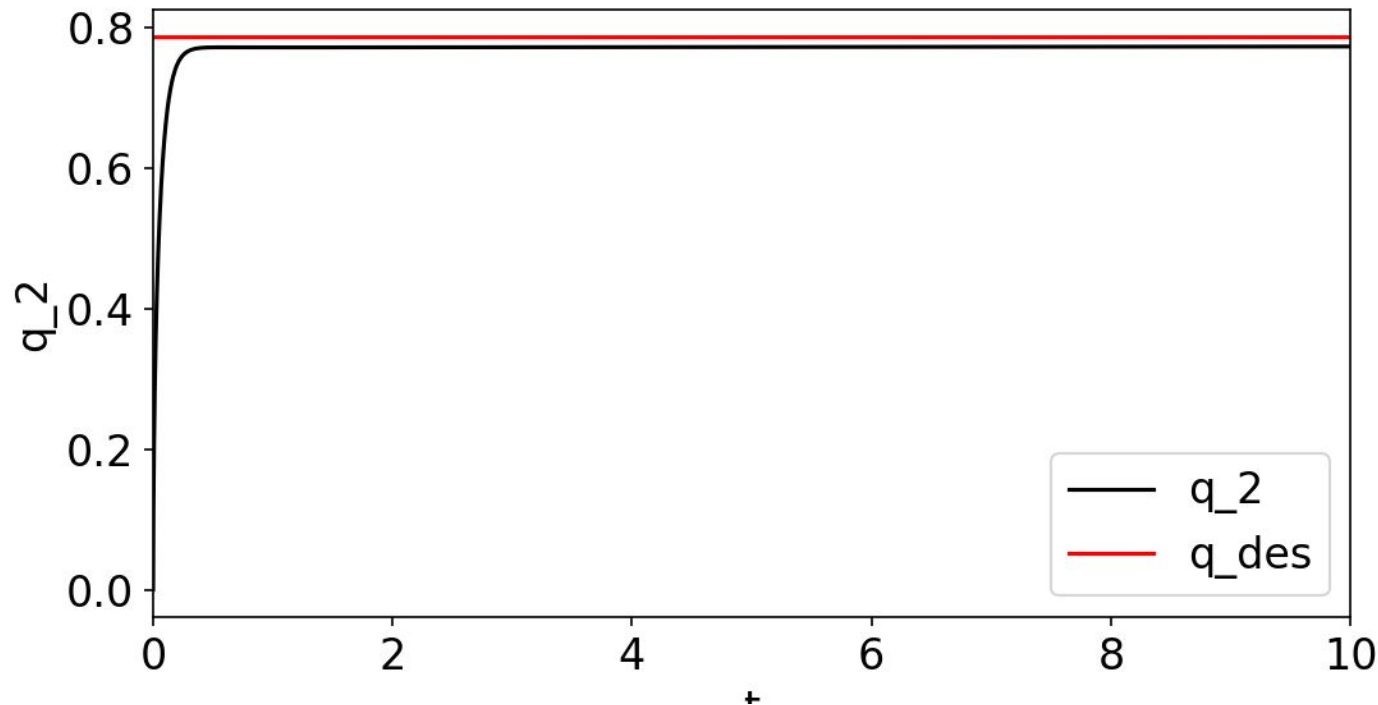
Fixed End 3-segment case PD Joint-Space Kinematic Control:

$$q_{\text{des}1} = q_{\text{des}2} = q_{\text{des}3} = \pi/4, K_p = 1.7275, K_d = 0.0075$$



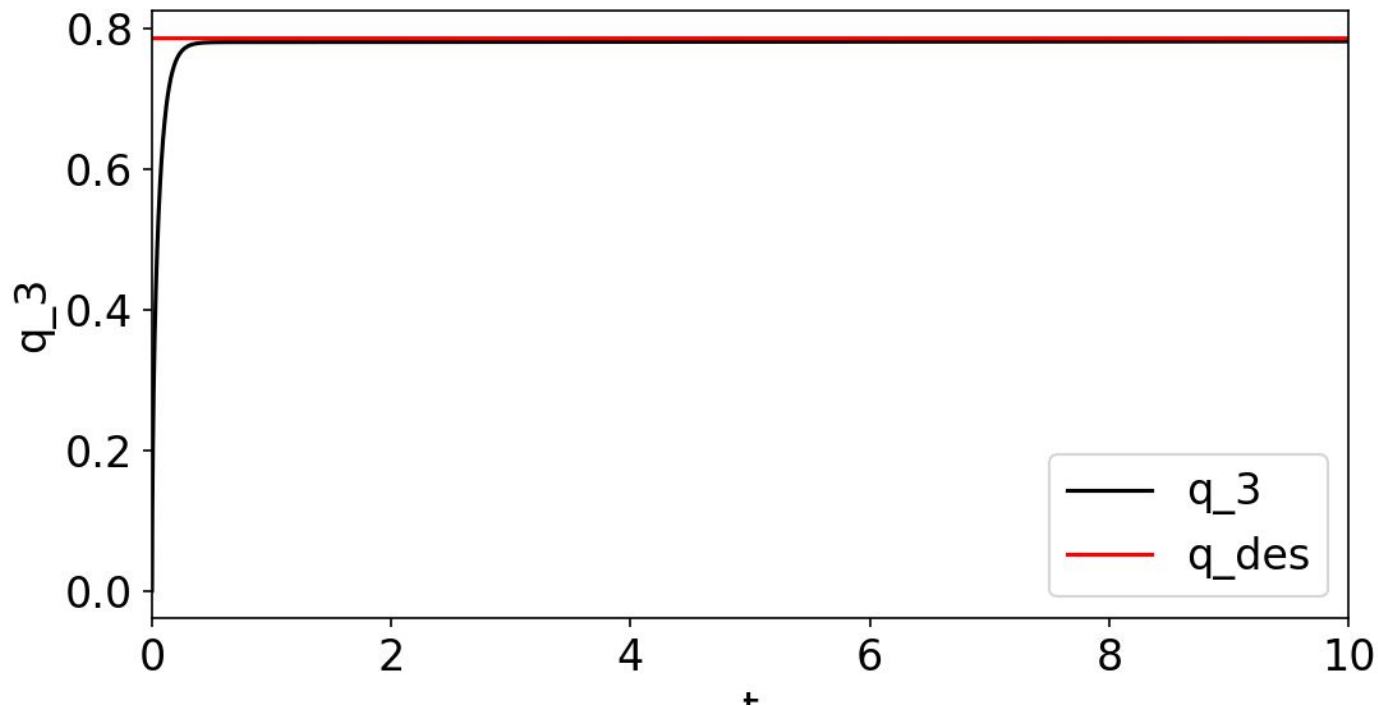
Fixed End 3-segment case PD Joint-Space Control:

$q_{des1} = q_{des2} = q_{des3} = \pi/4$, $K_p = 1.7275$, $K_d = 0.0075$
(contd.)



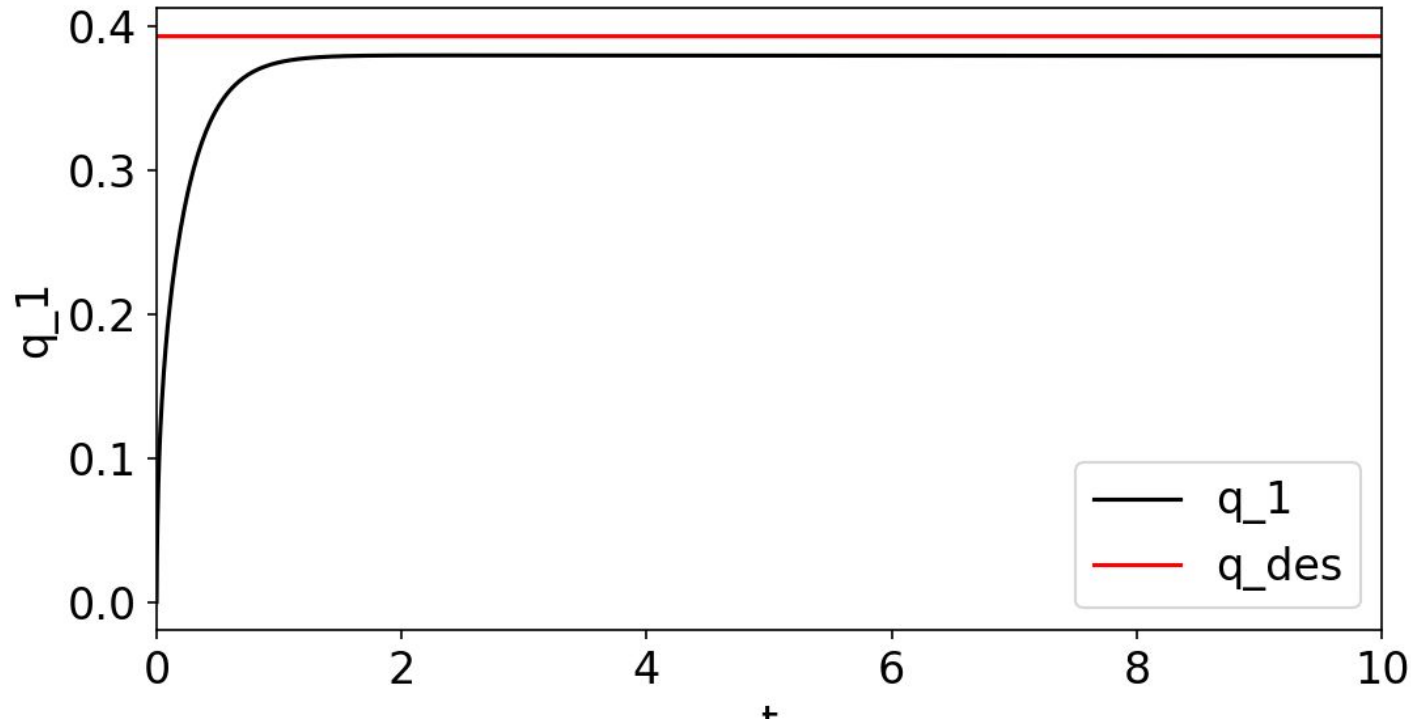
Fixed End 3-segment case PD Joint-Space Control:

$q_{\text{des}1} = q_{\text{des}2} = q_{\text{des}3} = \pi/4$, $K_p = 1.7275$, $K_d = 0.0075$
(contd.)

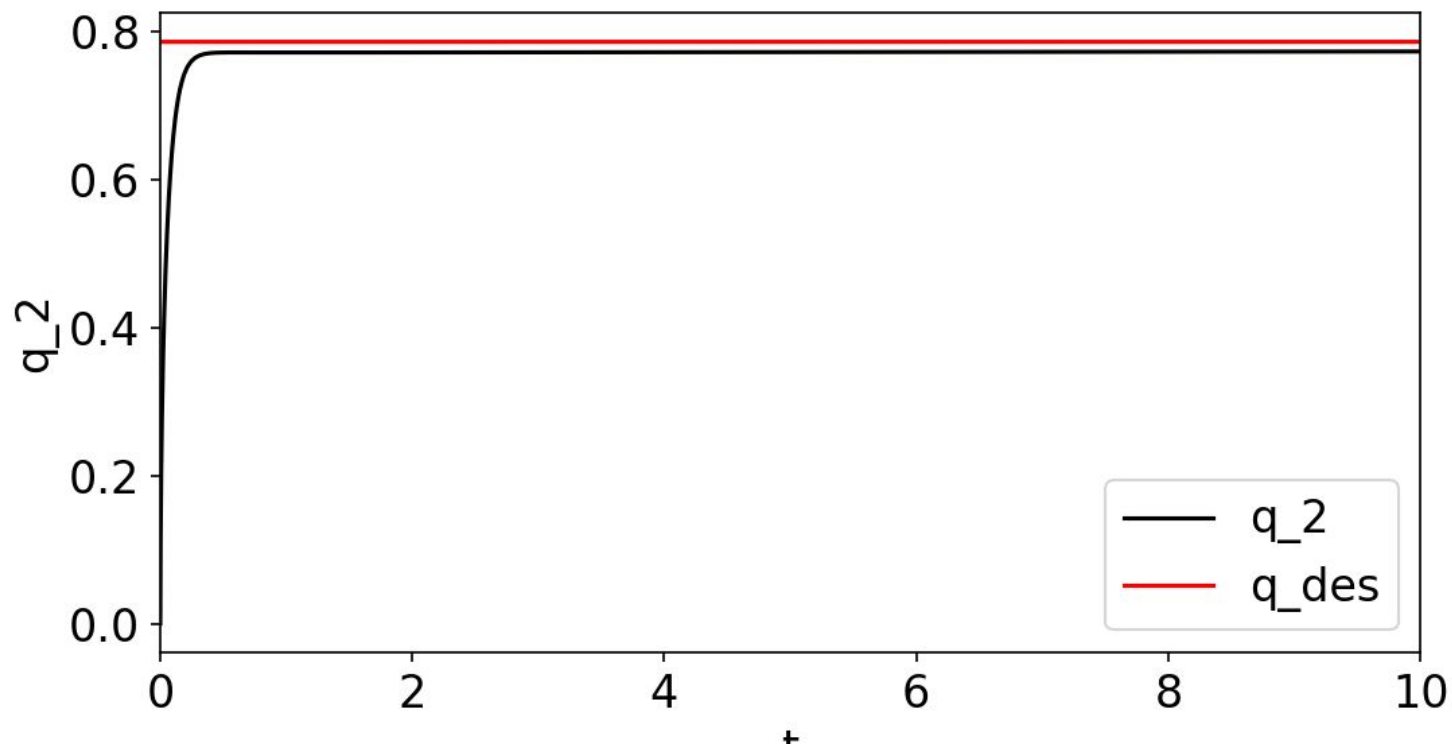


Fixed End 3-segment case PD Joint-Space Control:

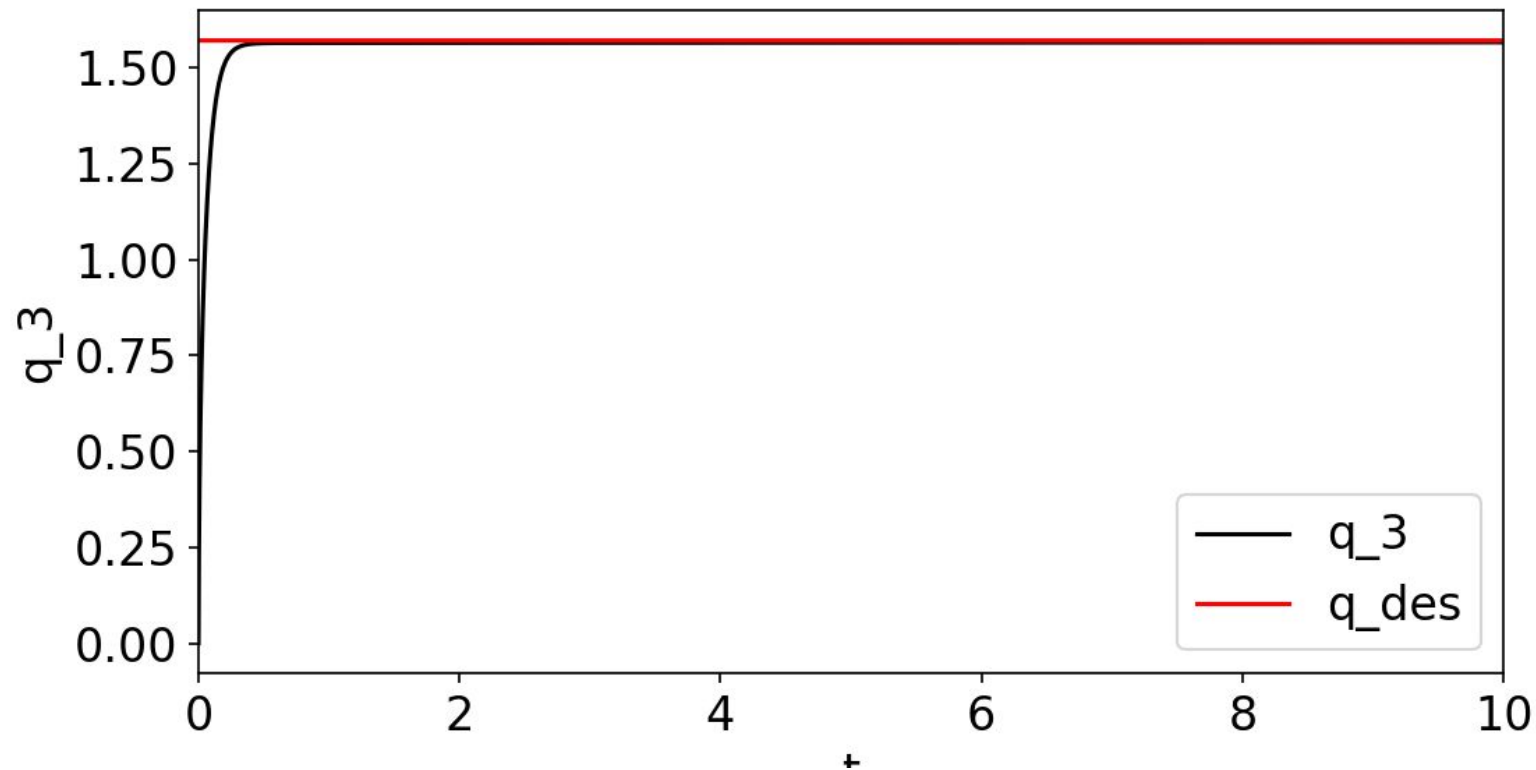
$q_{des1} = \pi/8$, $q_{des2} = \pi/4$, $q_{des3} = \pi/2$, $K_p = 1.7275$, $K_d = 0.0075$



(contd.)



(contd.)



Fixed End 3-segment case PD Joint-Space Control:
Time-varying q_{des} , $K_p = 3.3275$, $K_d = 0.0075$

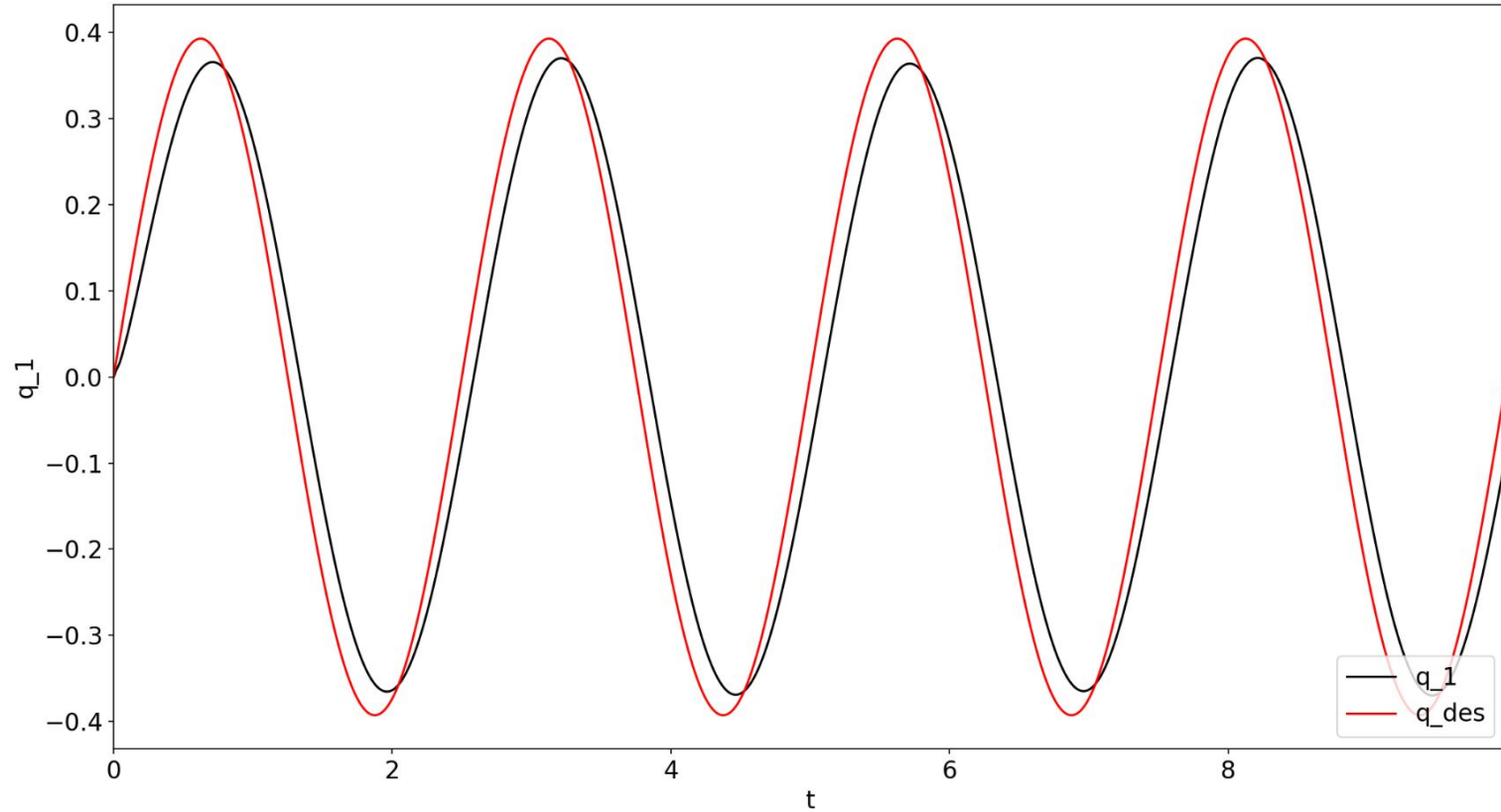
$$q_{des1} = \frac{\pi}{8} \sin\left(\frac{2\pi t}{2.5}\right)$$

$$q_{des2} = \frac{\pi}{4} \cos\left(\frac{2\pi t}{5}\right)$$

$$q_{des3} = \frac{\pi}{2} \sin\left(\frac{2\pi t}{4} + \frac{\pi}{9}\right)$$

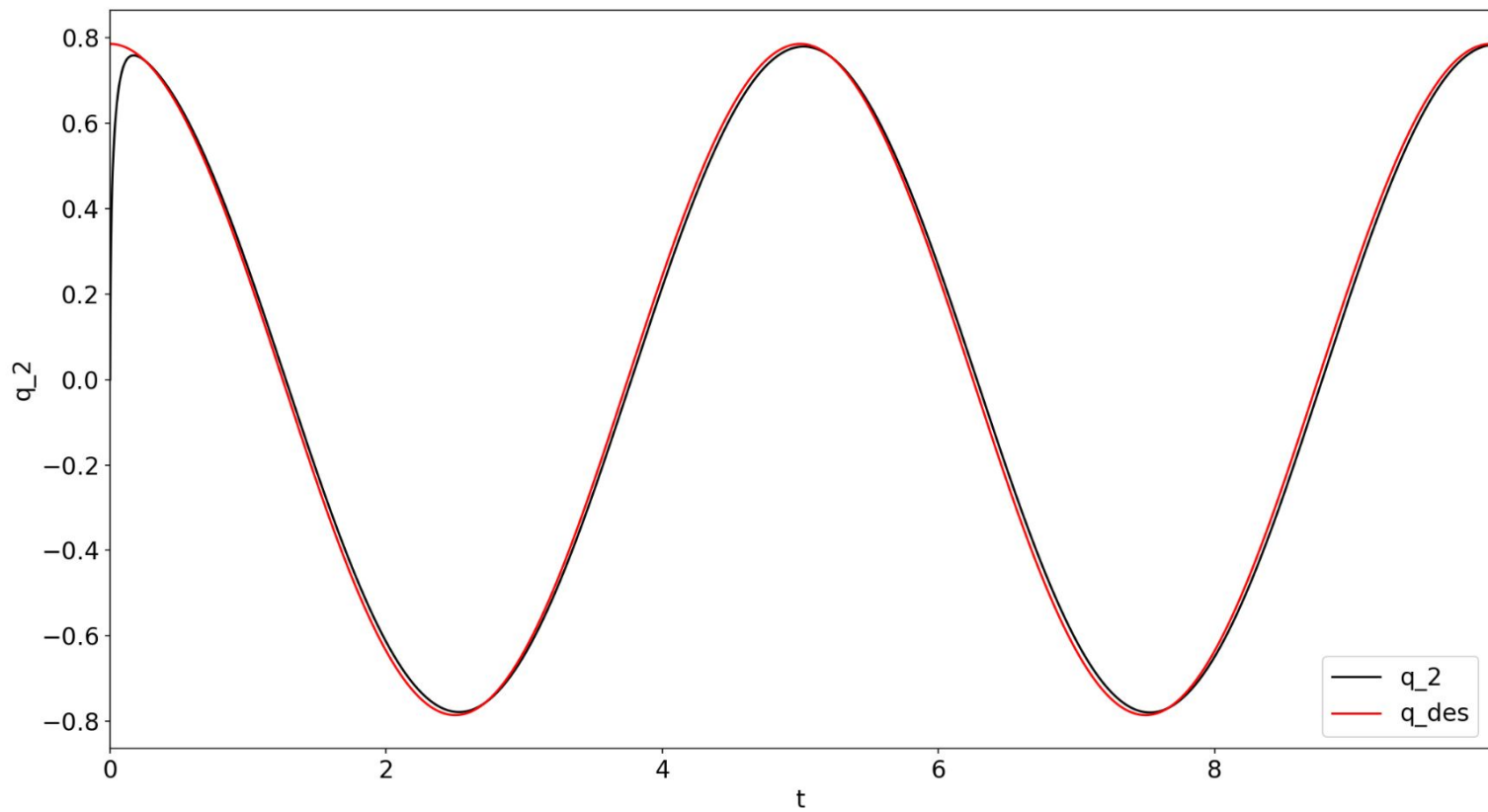
$$\text{Torque} = K_p(q_{des} - q) - K_d\dot{q} + k\ddot{q}$$

(contd.)

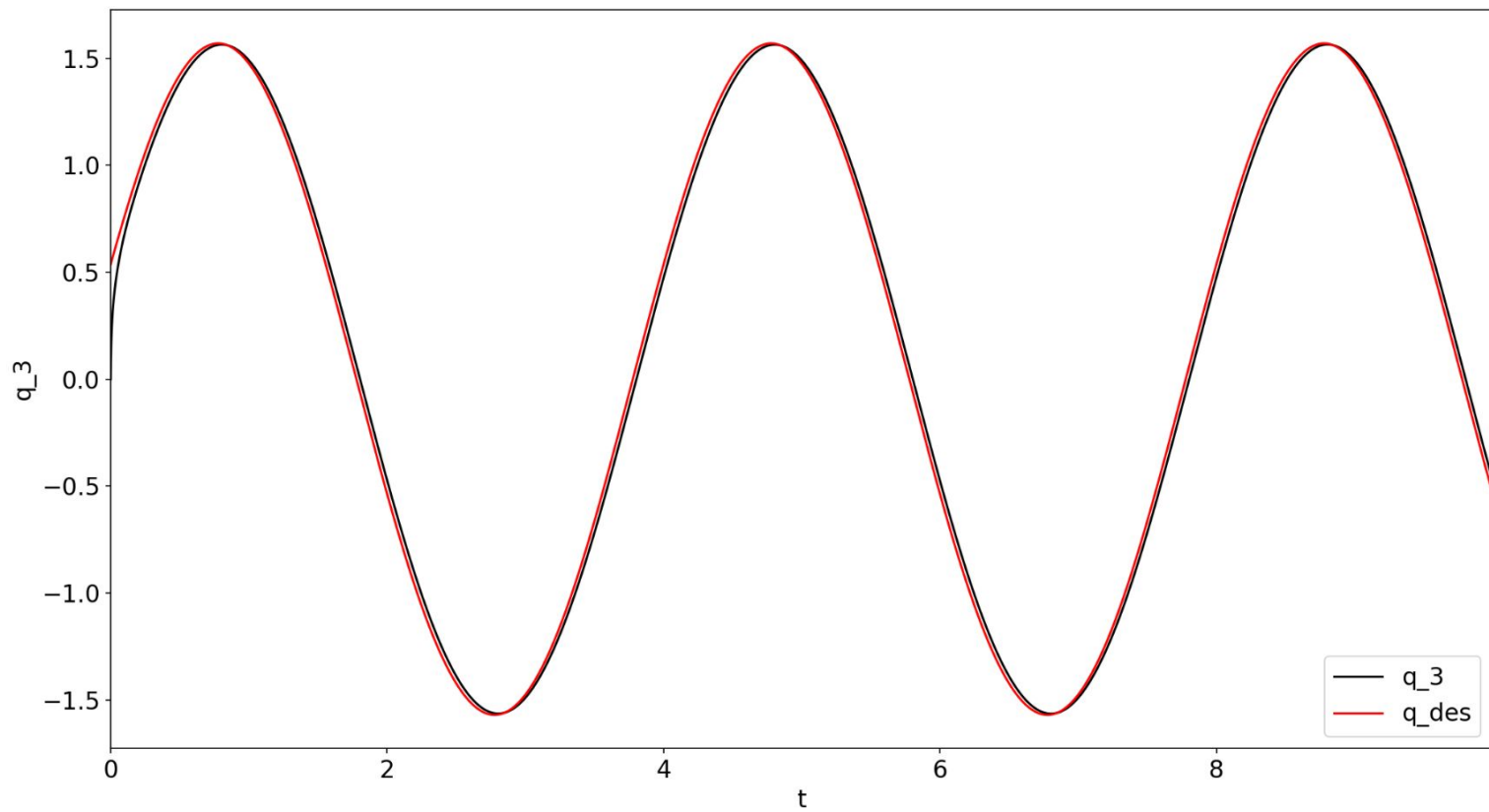


We can see that the max. error is around .0265 radians, i.e., around 1.5° which is acceptable.

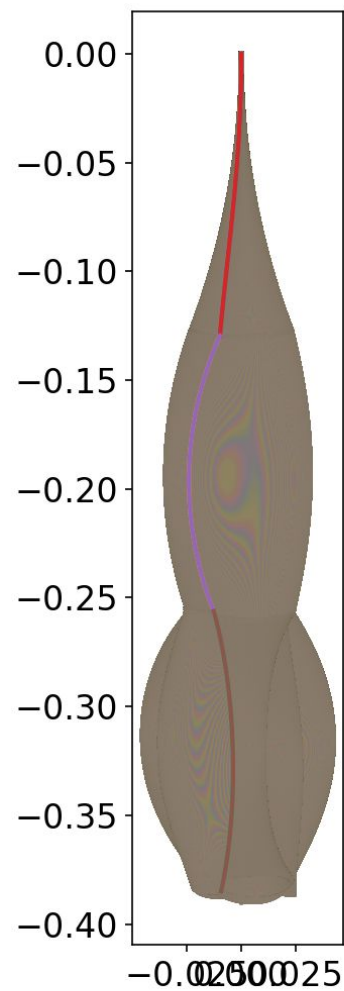
(contd.)



(contd.)



(contd.) Positions

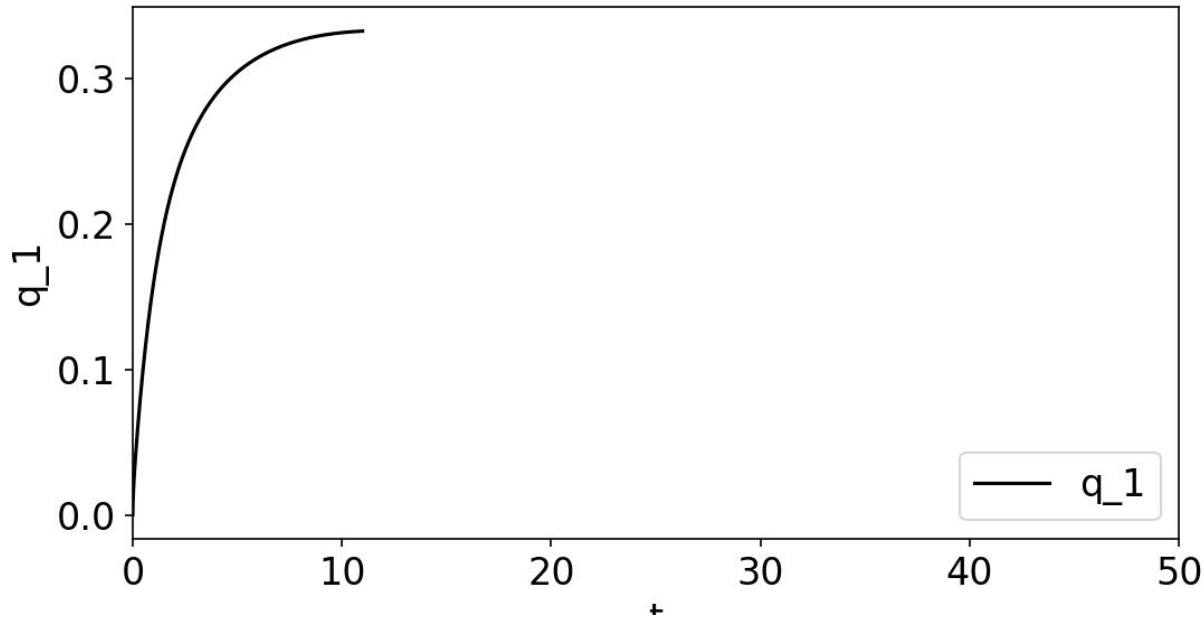


Task Space Control

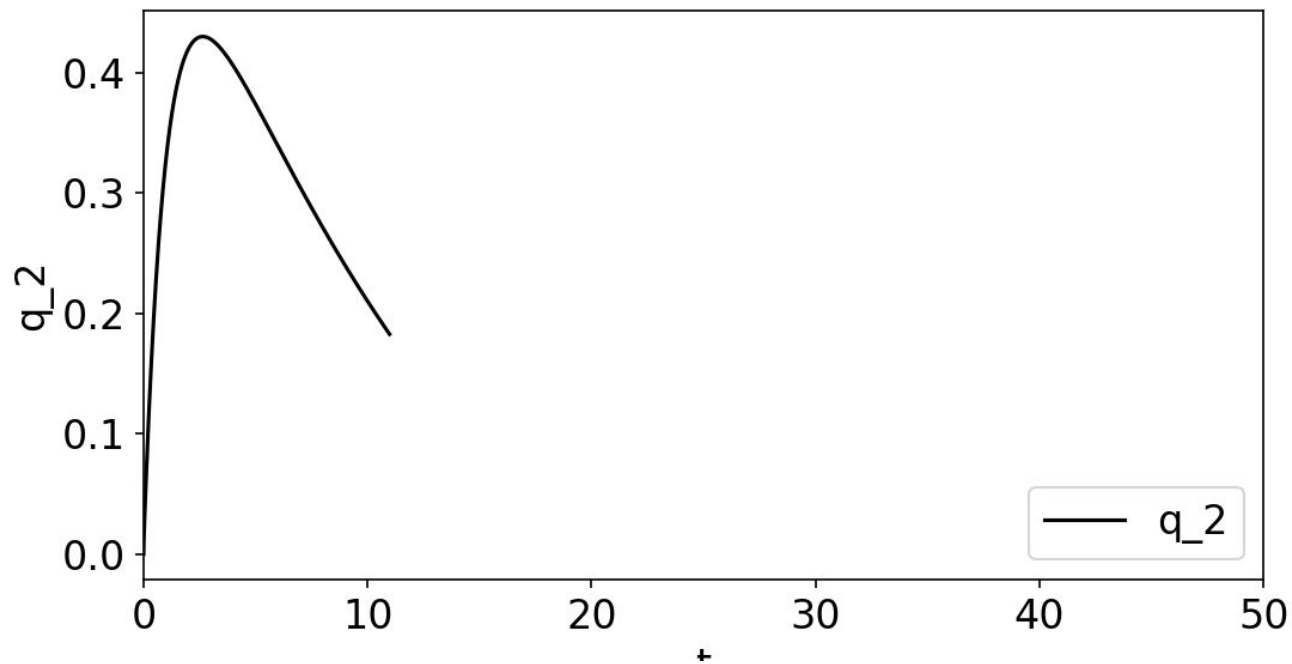
$$F = K_p^*(X_{\text{des}} - X) + K_d^*(\dot{X}_{\text{des}} - \dot{X})$$

$$\text{Torque} = J^T F$$

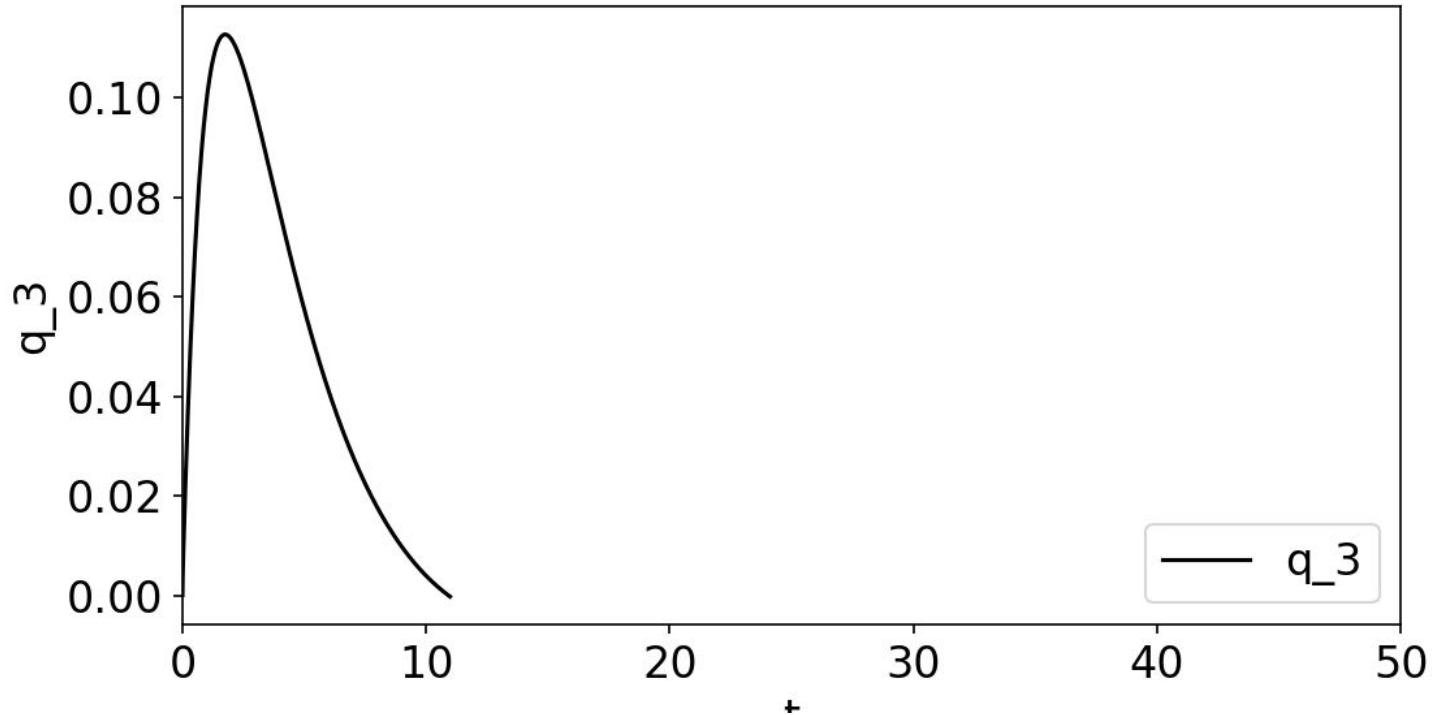
Task Space Control ($K_p = 3.3275$ and $K_d = 0.075$, $n_u=7$):
Case 1 (Constant torque when q_1 , q_2 or $q_3 = 0$)



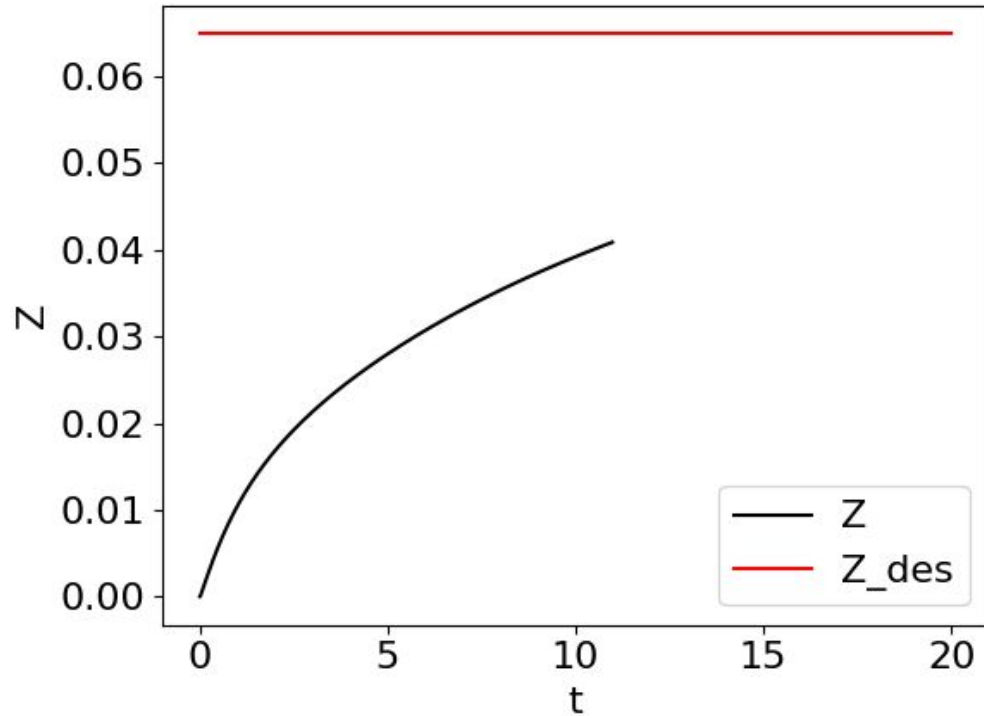
Task Space Control: Case 1 (contd.)



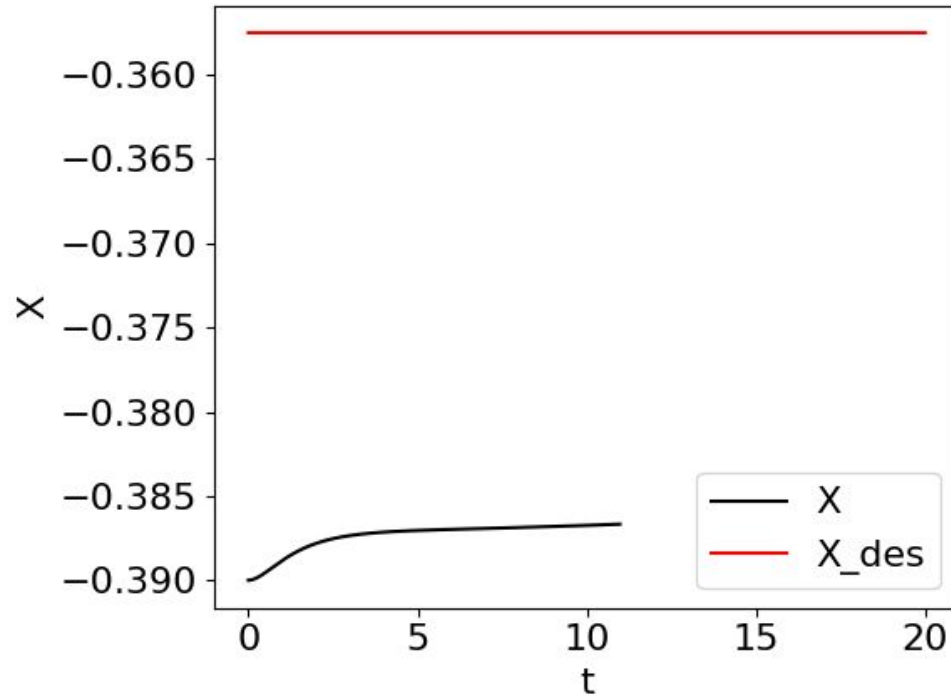
Task Space Control: Case 1 (contd.)



Task Space Control: Case 1 (contd.)



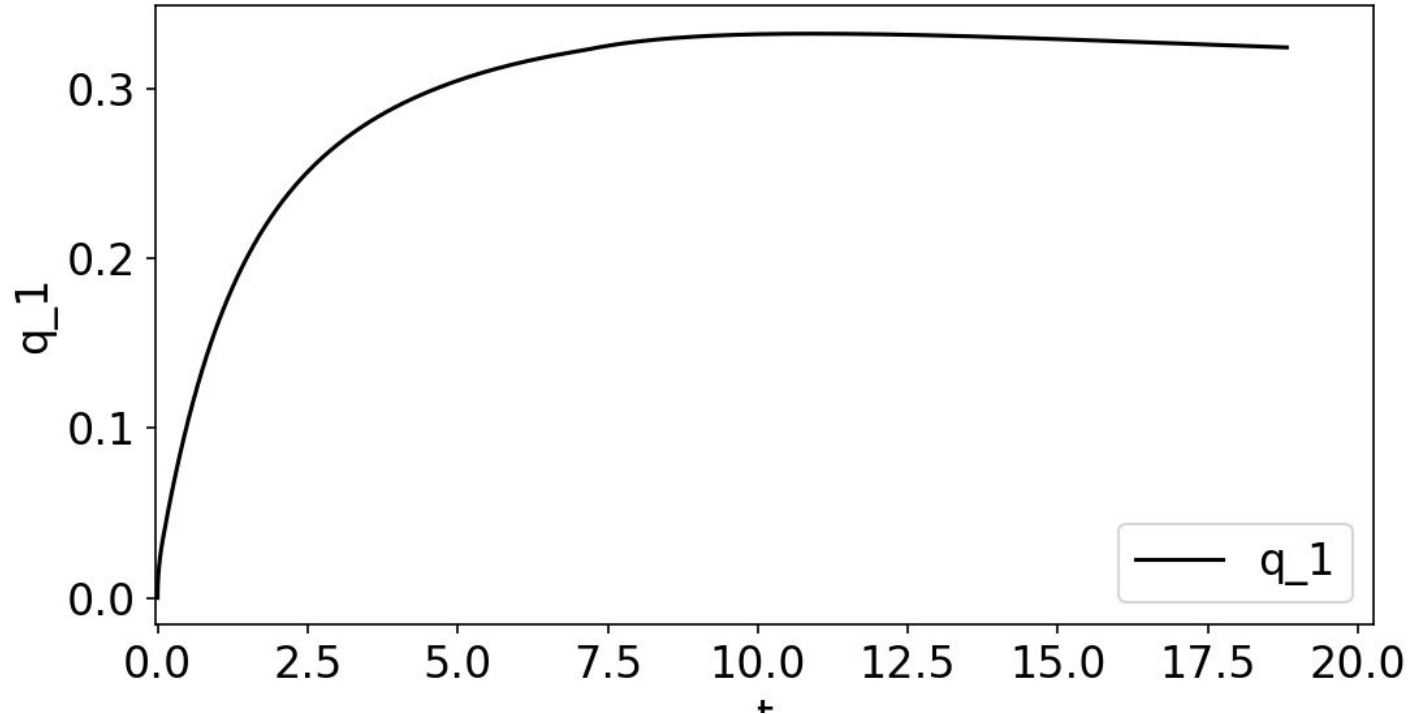
Task Space Control : Case 1 (contd.)



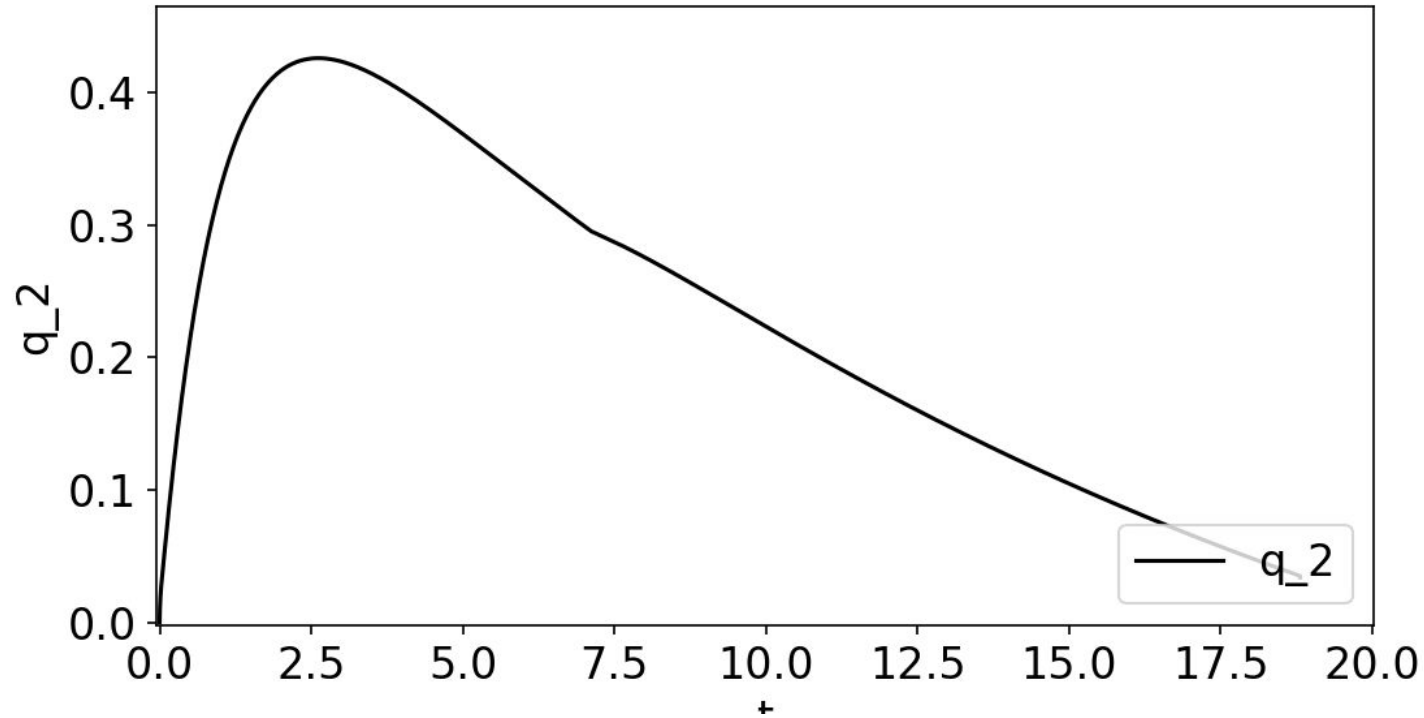
Case 2: Using Margin of 2 degrees

When q is inside the margin, skip to torque at one of the boundaries/margins as per direction of \dot{q} .

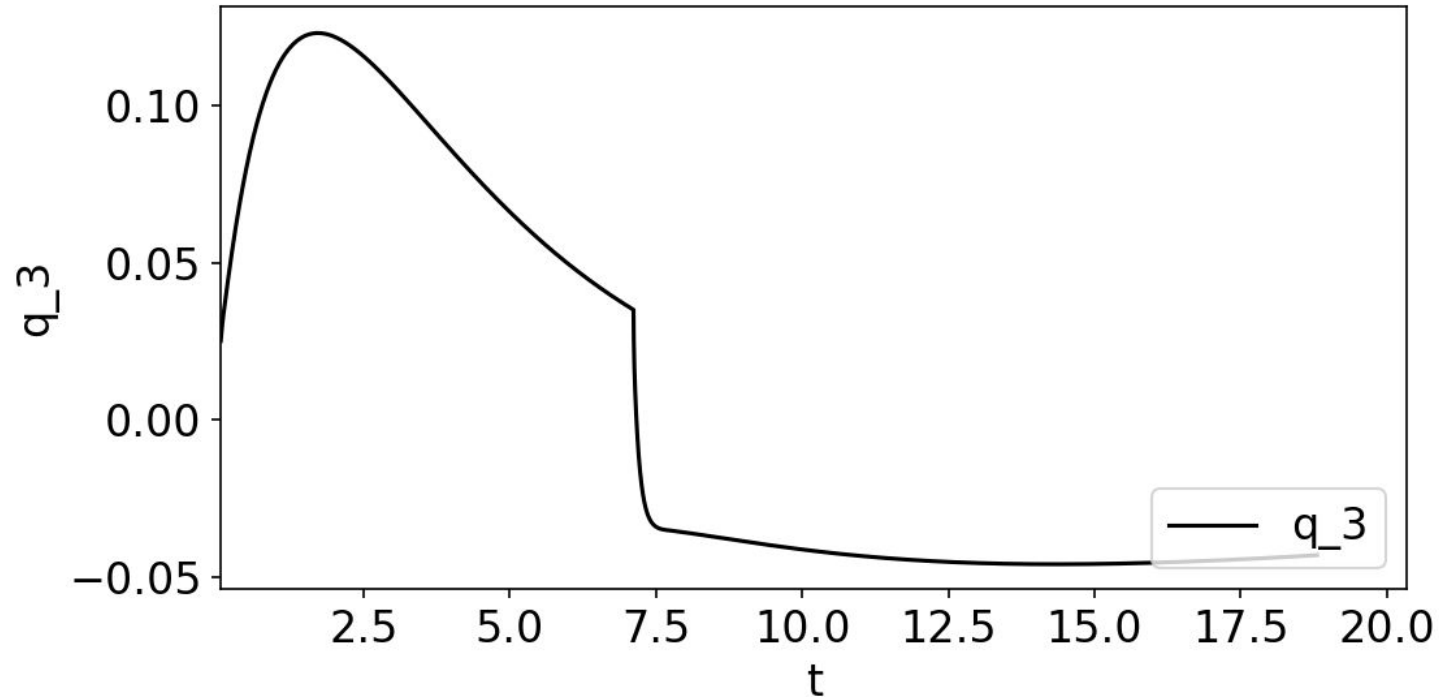
Task Space Control : Case 2 (Using Margin of 2 degrees)



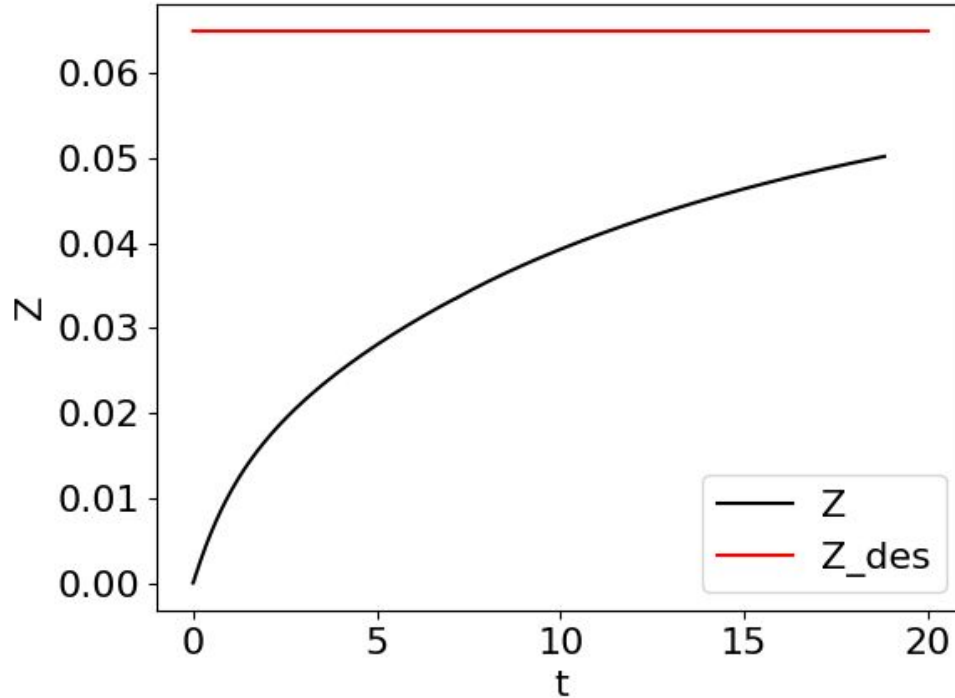
Task Space Control : Case 2 (contd.)



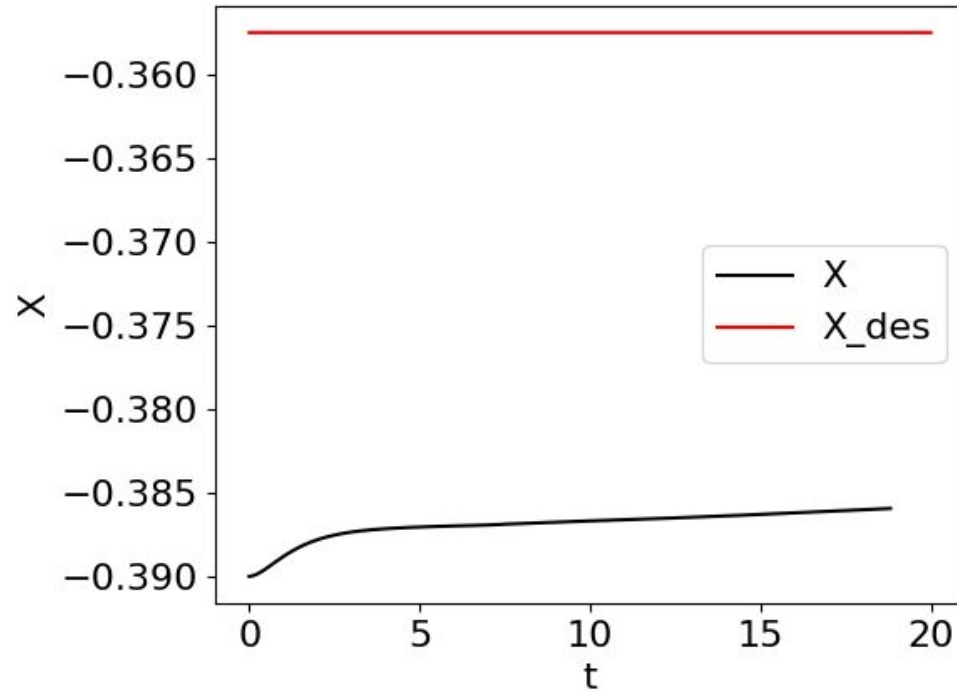
Task Space Control: Case 2 (contd.)



Task Space Control: Case 2 (contd.)



Task Space Control: Case 2 (contd.)



Conclusion

- Joint Space PD Control for Fixed planar case works well with reasonable accuracy.
- Task Space PD Control is not working too well. Giving “Nan”, i.e., “not defined” angle values after a specific amount of time.
- As we can see, around 18 secs, the system goes haywire. Not able to solve this problem yet.
- From q graphs, we can clearly see the skip when in margin of 2 degree around 0. Thus, that logic is working.