Robust Control Of Bevel-Tipped Needle

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Primary Reference:

S. Hans and F. O. M. Joseph, "Robust control of a bevel-tip needle for medical interventional procedures," in IEEE/CAA Journal of Automatica Sinica, vol. 7, no. 1, pp. 244-256, January 2020, doi: 10.1109/JAS.2019.1911660.

References

[1] S. Hans and F. O. M. Joseph, "Robust control of a bevel-tip needle for medical interventional procedures," in IEEE/CAA Journal of Automatica Sinica, vol. 7, no. 1, pp. 244-256, January 2020, doi: 10.1109/JAS.2019.1911660.

[2] S. P. Bhat and D. S. Bernstein, "Geometric homogeneity with applications to finite-time stability", Math. Control Signals Syst., vol. 17, no. 2, pp. 101–127, 2005

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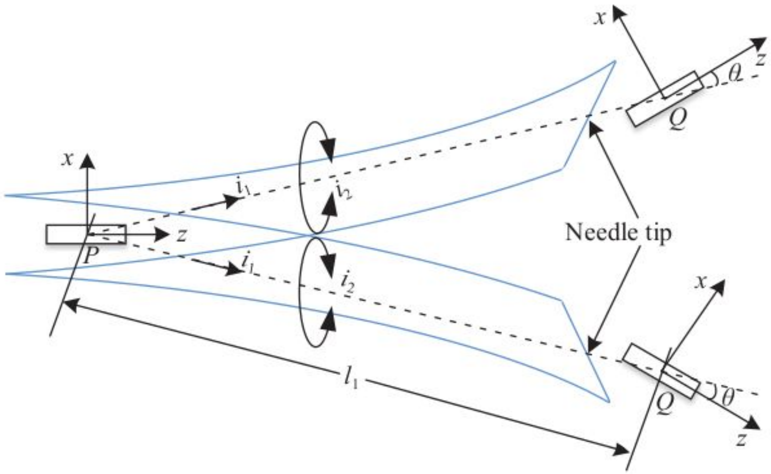
[5] A. Moreno and M. Osorio, "Strict Lyapunov functions for the super-twisting algorithm," IEEE Trans. Automatic Control, vol. 57, no. 4, pp. 1035–1040, April 2012.

[6] A. Levant, "Robust exact differentiation via sliding mode technique", Automatica, vol. 34, pp. 379–384, 1998.

Introduction and Literature Review

- The merits of sliding mode based robust control strategies involve disturbance rejection, parameter uncertainty, and unmodeled dynamics, which are essential for addressing any practical applications
- Many papers have used Sliding-Mode control techniques to drive steerable needle.
- Aims:
    - To implement robust SMC that rejects disturbance
    - Avoid Chattering using ISMC + STA and Continuous Control



Kinematic model and State-Space Equations

$$g_{PQ} = \begin{bmatrix} R_{PQ} & p_{PQ} \\ 0 & 1 \end{bmatrix} \in SE(3)$$

$$Velocity,\ v = \dot{p_1} i_1 + \dot{p_2} i_2$$

$$\dot{p_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ k \\ 0 \end{bmatrix},\ \dot{p_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

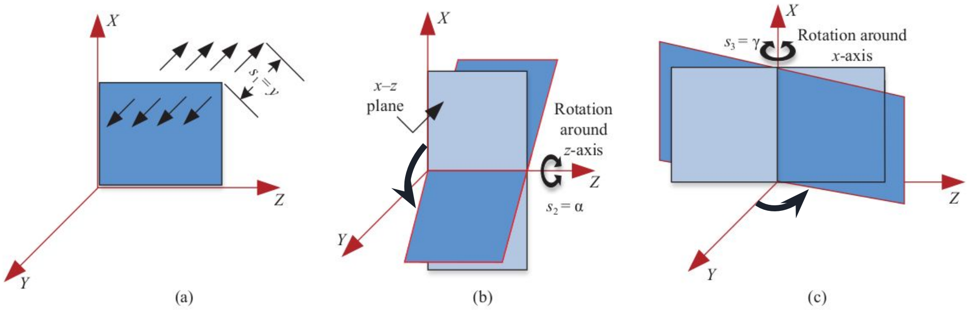
$$v = J\dot{q},\ q = \begin{bmatrix} x \\ y \\ z \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$s = [s_1\ s_2\ s_3]^T = [y\ \alpha\ \gamma]^T$$

$$\dot{s_1} = -\sin s_2$$

$$\dot{s_2} = k \cos s_3 \sec s_2$$

$$\dot{s_3} = -k \cos s_3 \tan s_2 + i$$



Input-Output Feedback Linearization

$$\dot{\psi} = \frac{\delta h(s)}{\delta s}[f_0(s) + g_0(s)i]$$

$$\Rightarrow \; L_{f_0}h(s) = \frac{\delta h(s)}{\delta s} f_0(s) = \dot{\psi} = -\sin s_2$$

$$L_{g_0}h(s) = \frac{\delta h(s)}{\delta s} g_0(s) = 0$$

$$L_{f_0}^2 h(s) = \frac{\delta L_{f_0}h(s)}{\delta s} f_0(s) = \begin{bmatrix} 0 & -\cos s_2 & 0 \end{bmatrix} \begin{bmatrix} -\sin s_2 \\ k\cos s_3 \sec s_2 \\ -k\cos s_3 \tan s_2 \end{bmatrix} = -k\cos s_3$$

$$L_{g_0}L_{f_0}h(s) = \frac{\delta L_{f_0}h(s)}{\delta s} g_0(s) = \begin{bmatrix} 0 & -\cos s_2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$L_{g_0}L_{f_0}^2 h(s) = \frac{\delta L_{f_0}^2 h(s)}{\delta s} g_0(s) = \begin{bmatrix} 0 & 0 & k\sin s_3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = k\sin s_3$$

$$\Rightarrow \; \textit{Relative Degree}, \; \rho = 3 = n$$

$$L_{f_0}^3 h(s) = \frac{\delta L_{f_0}^2 h(s)}{\delta s} f_0(s) = \begin{bmatrix} 0 & 0 & k\sin s_3 \end{bmatrix} \begin{bmatrix} -\sin s_2 \\ k\cos s_3 \sec s_2 \\ -k\cos s_3 \tan s_2 \end{bmatrix}$$

$$= -k^2 \sin s_3 \cos s_3 \tan s_2$$

$$r = T(s) = \xi = \begin{bmatrix} h(s) \\ L_{f_0}h(s) \\ L_{f_0}^2 h(s) \end{bmatrix}$$

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} s_1 \\ -\sin s_2 \\ -k\cos s_3 \end{bmatrix}$$

$$\dot{r_1} = r_2$$

$$\dot{r_2} = r_3$$

$$\dot{r_3} = \nu_1 = f(r) + g(r)i$$

$$, where, \; \nu_1 = \gamma(s)(i - \alpha(s))$$

$$\gamma(s) = L_{g_0}L_{f_0}^2 h(s) = k\sin s_3$$

$$\alpha(s) = \frac{-L_{f_0}^3 h(s)}{L_{g_0}L_{f_0}^2 h(s)} = \frac{k^2 \sin s_3 \cos s_3 \tan s_2}{k\sin s_3} = k\cos s_3 \tan s_2$$

$$\Rightarrow \; \nu_1 = -k^2 \sin s_3 \cos s_3 \tan s_2 + k\sin s_3 i$$

Conventional SMC

$$
\begin{aligned}
\dot{r_1} &= r_2 \\
\dot{r_2} &= r_3 \\
\dot{r_3} &= \nu_1 \;=\; f(r) + g(r)i + d \\
f(r) &= k\sin\left[\cos^{-1}\left(\frac{-r_3}{k}\right)\right] r_3 \tan\left[\sin^{-1}(-r_2)\right] \\
g(r) &= k\sin\left[\cos^{-1}\left(\frac{-r_3}{k}\right)\right] \\
d &= 2\sin t + 3
\end{aligned}
$$

The sliding surface has been considered as follows:

$$
\begin{aligned}
\zeta &= c_1 r_1 + c_2 r_2 + r_3 \\
\dot{\zeta} &= c_1 r_2 + c_2 r_3 + f(r) + g(r)i + d \\
i &= -g^{-1}(r)[c_1 r_2 + c_2 r_3 + f(r) + K sign(\zeta) + \eta\zeta] \\
\Rightarrow\; \dot{\zeta} &= -K sign(\zeta) - \eta\zeta + 2\sin t + 3
\end{aligned}
\tag{8}
$$

The parameters $c_1 = 10,\; c_2 = 5,\; K = 6,\; \eta = 4$ give a desired rate of convergence of states to zero.

Continuous Finite-time Feedback Control

Theorem 1 [6]. *Let* $a_1, ...a_n > 0$ *be such that the polynomial* $\Gamma^n + a_n\Gamma^{n-1} + ... + a_2\Gamma + a_1$ *is* ***Hurwitz****, and there exists* $\epsilon \in (0,1)$ *such that, for every* $b \in (1-\epsilon, 1)$*, the* ***origin*** *is a* ***globally finite-time stable equilibrium*** *for the undisturbed system* (6) *under the feedback control* $i = g^{-1}(r)[u - f(r)]$ *by assuming:*

$$u = -a_1|r_1|^{b_1} sign(r_1) - ... - a_n|r_n|^{b_n} sign(r_n) \tag{9}$$

,where, $b_1, ..., b_n$ *satisfy*

$$b_{w-1} = \frac{b_w b_{w+1}}{2b_{w+1} - b_w}, \quad w = 2,..,n \tag{10}$$

, with, $b_{n+1} = 1$ *and* $b_n = b$*.*

$$i = g^{-1}(r)[u - f(r)]$$

$$u_{nom} = -a_1|r_1|^{b_1} sign(r_1) - a_2|r_2|^{b_2} sign(r_2) - a_3|r_3|^{b_3} sign(r_3) \tag{11}$$

, where, controller gains were chosen as $a_1 = 15,\ a_2 = 23,\ a_3 = 9,\ b_1 = \frac{1}{4},\ b_2 = \frac{1}{3},\ b_3 = \frac{1}{2}$.

ISMC With Discontinuous control

$U_{nom} \rightarrow$ Performance

$U_{discon} \rightarrow$ Disturbance Rejection

$$u = u_{nom} + u_{discon}$$

$$u_{discon} = -a_4 sign(\zeta)$$

$$Sliding\ variable,\ \zeta = r_3 - r_{30} - \int_0^t u_{nom} dt,\ r_{30}\ is\ initial\ condition\ of\ r_3$$

$$\Rightarrow\ \dot{\zeta} = u + d - u_{nom} = 0$$

$$\Rightarrow\ u_{nom} + u_{discon} + d - u_{nom} = 0$$

$$\Rightarrow\ u_{discon} = -d$$

$$\Rightarrow\ [-a_4 sign(\zeta)]_{eq} = -d \qquad (12)$$

$a_4 = 10$

Higher Order Sliding Mode Control

- Higher Order SMC means that the equation for the higher order derivative of sliding variable must contain the discontinuity such that all lower derivatives converge to zero in finite-time.
- The **sliding manifold** in this type of control is not just **$\zeta=0$**, but, a set :
- $\boldsymbol{\Phi = \{\zeta=0, d^e\zeta/dt^e = 0\}}$
- e goes from 1 to m-1, where, m is the order of sliding mode

Super Twisting Algo (STA)

The **assumptions** in **STA** are as follows [7]:

$$\textit{Sliding variable}, \; \dot{\zeta} = h_1(x) + g_1(x)u + d(t)$$

$$u = -\lambda|\zeta|^{1/2} sign(\zeta) + \nu$$

$$\dot{\nu} = \begin{cases} -u, & |u| > U_M \\ -\alpha sign(\zeta), & |u| \leq U_M \end{cases}$$

$$|\dot{h_1}| + |\dot{g_1}|U_M + |\dot{d}(t)| \leq C$$

$$0 < K_m \leq g_1(x) \leq K_M$$

$$|h_1| < q|g_1|U_M, \; q \in (0,1)$$

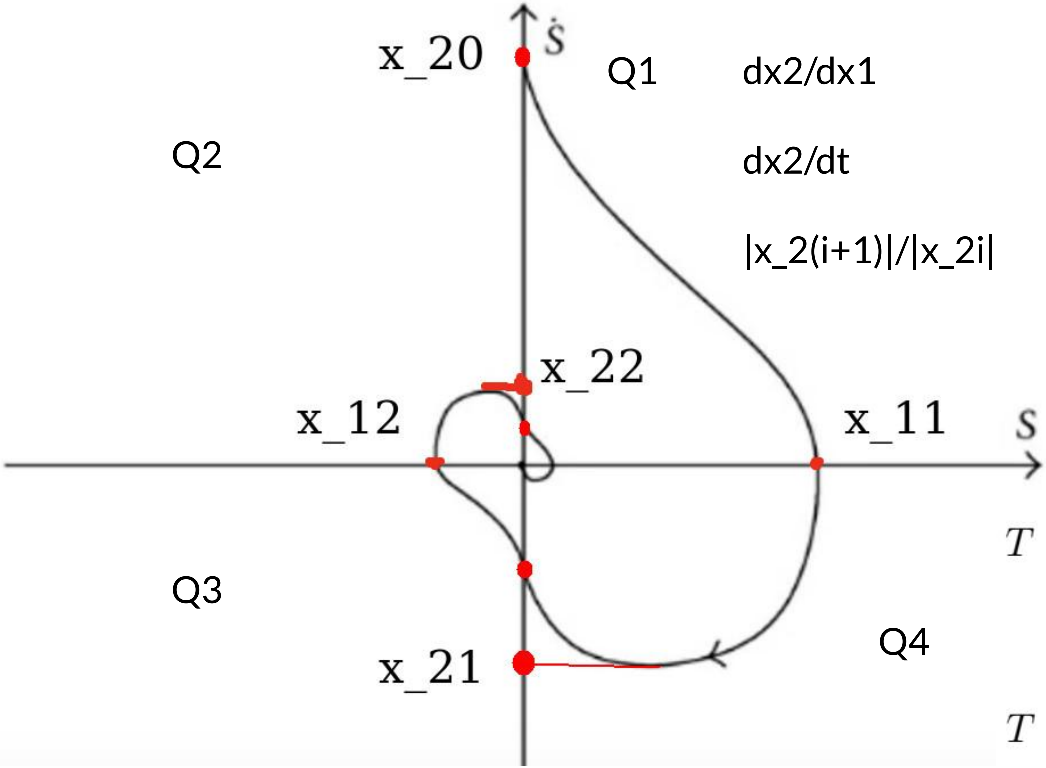
$$|\dot{d}(t)| \leq C_1$$

$$|d(t)| \leq C_2$$

We know that $d(t)$ satisfies the assumptions $|d(t)| \leq C_2$ and $|\dot{d}(t)| \leq C_1$, with, $C_1 = \tau$ and $C_2 = d_m$ as given in the paper [1]. The **conditions on controller gains** are given below:

$$\begin{aligned} & K_m\alpha > C \\ & \lambda > \sqrt{\frac{2}{K_m\alpha - C}} \times \frac{K_m\alpha + C}{K_m} \times \frac{1+q}{1-q} \end{aligned} \tag{16}$$

Finite-time Stability of STA (Majorant Curve Analysis)



dx2/dx1

dx2/dt

|x_2(i+1)|/|x_2i|

$$T = \Sigma_{i=1}^{\infty} T_i$$

$$= p|x_{20}|\Sigma_{i=0}^{\infty} r^i$$

$$T = \frac{p|x_{20}|}{1-r} > 0 \; and \; finite$$

ISMC + STA

$$u = u_{nom} + u_{STC}$$

$$\dot{\zeta} = u + d - u_{nom}$$

$$\dot{\zeta} = u_{STC} + d$$

$$u_{STC} = -a_5|\zeta|^{1/2} sign(\zeta) + \Theta$$

$$\dot{\zeta} = -a_5|\zeta|^{1/2} sign(\zeta) + \Theta + d$$

$$\dot{\Theta} = -a_6 sign(\zeta)$$

$$O = \Theta + d$$

$$\dot{\zeta} = -a_5|\zeta|^{1/2} sign(\zeta) + O$$

$$\dot{O} = -a_6 sign(\zeta) + \dot{d}$$

$$\dot{\zeta} = u + d - u_{nom} = 0$$

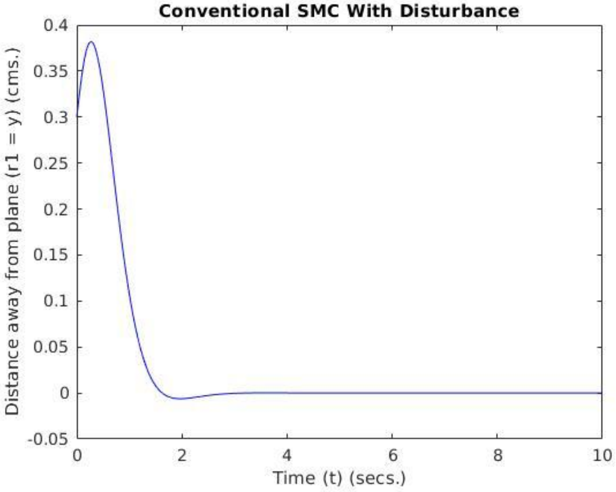
$$\Rightarrow u + d = u_{nom}$$

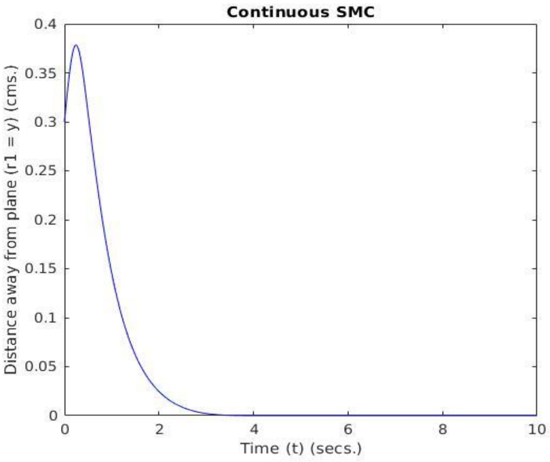
$$u_{STC} = -d$$

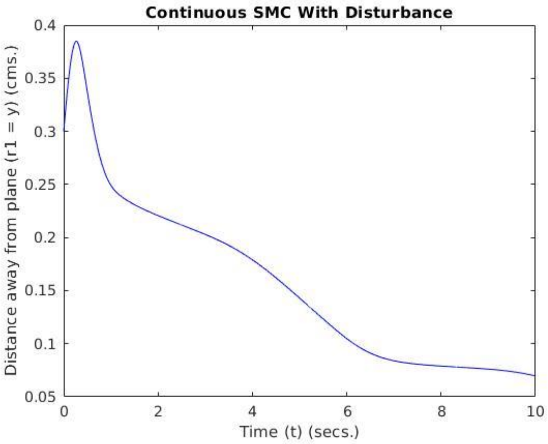
a_5 = 0.13

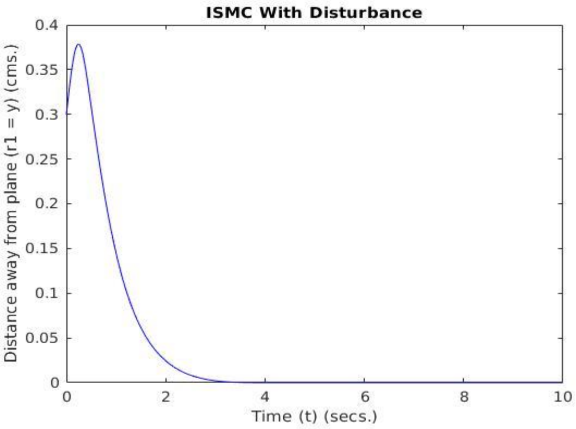
a_6 = 300

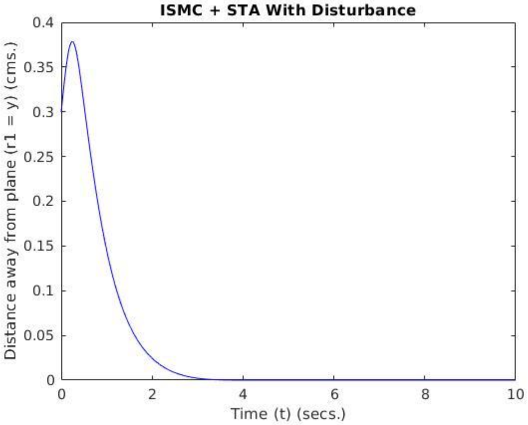
Simulation: r_1 State (Distance from x-z plane)



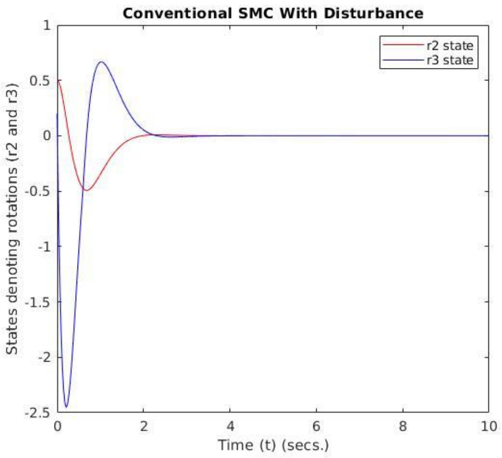


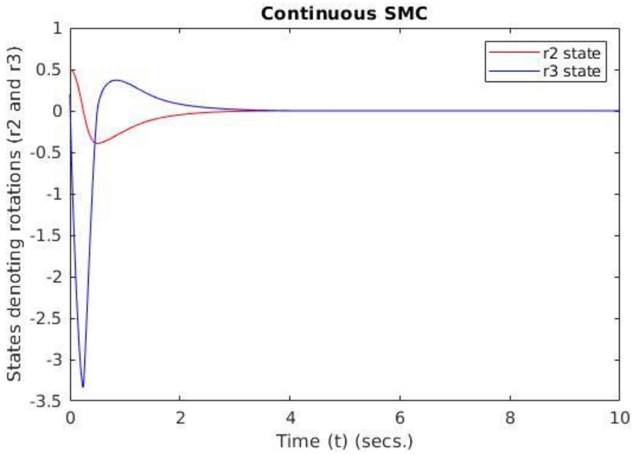


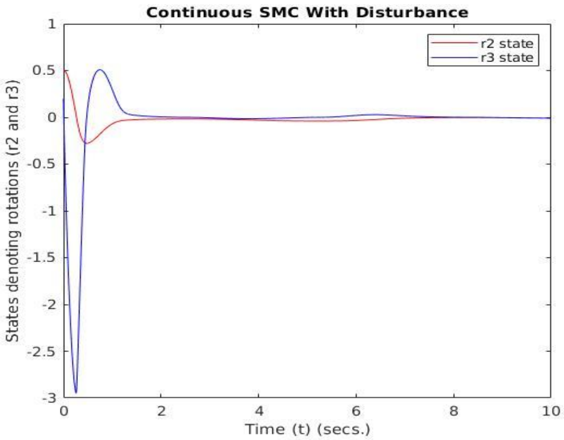


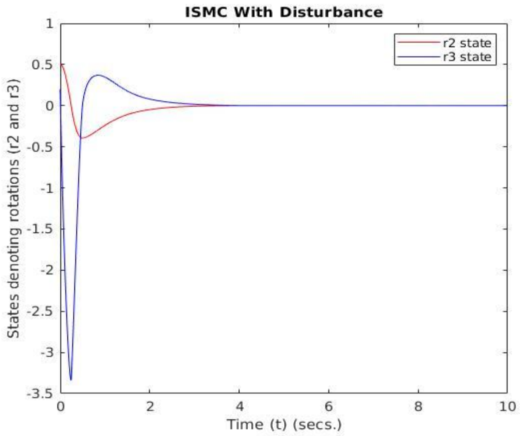


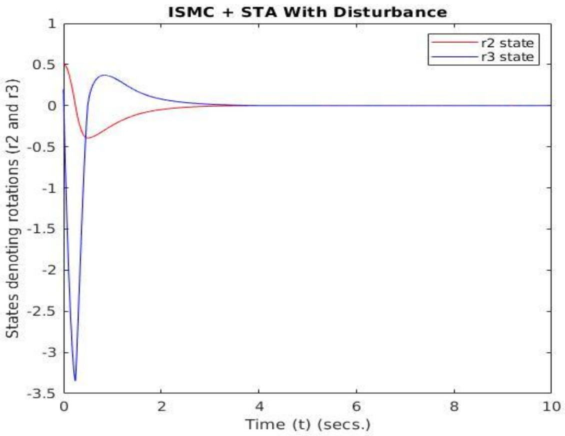
Simulation: r_2 and r_3 States (Representing Rotations)

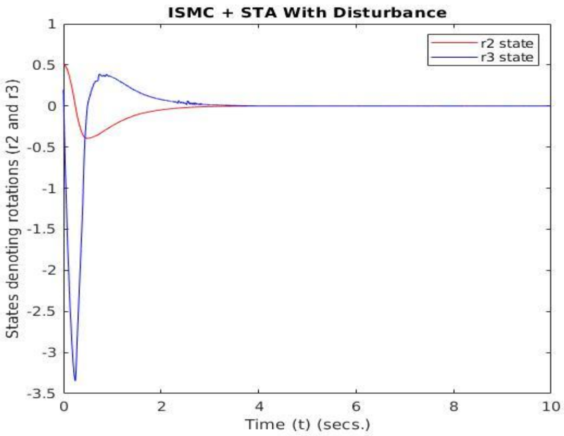




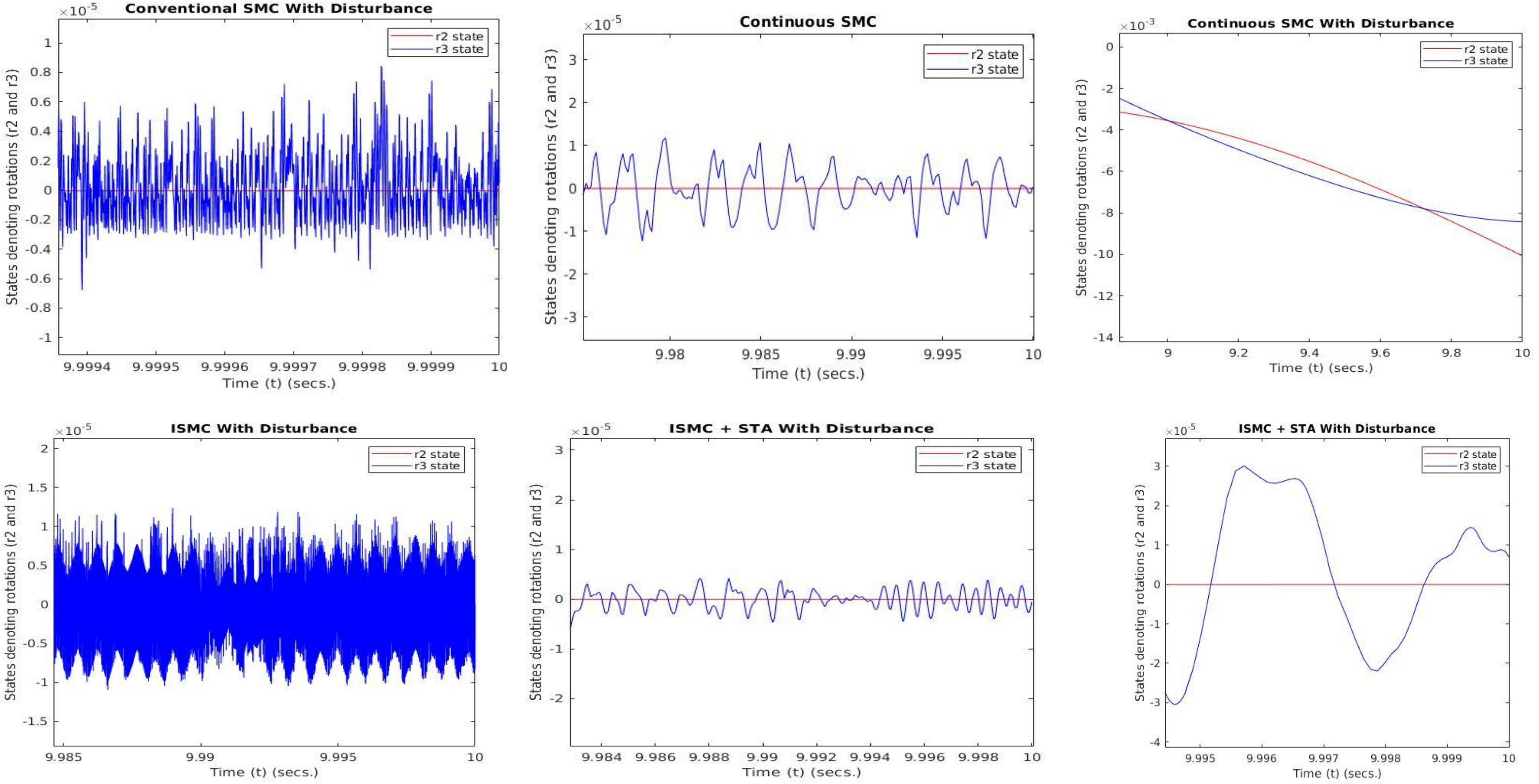




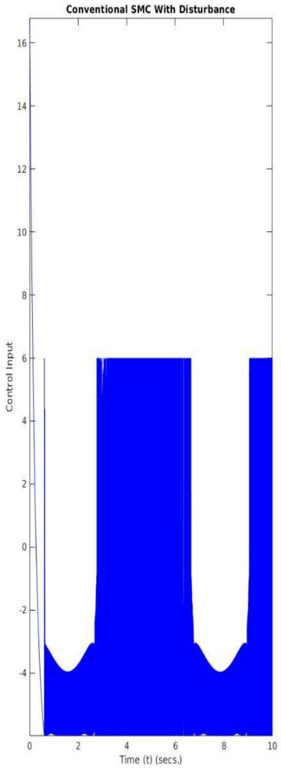


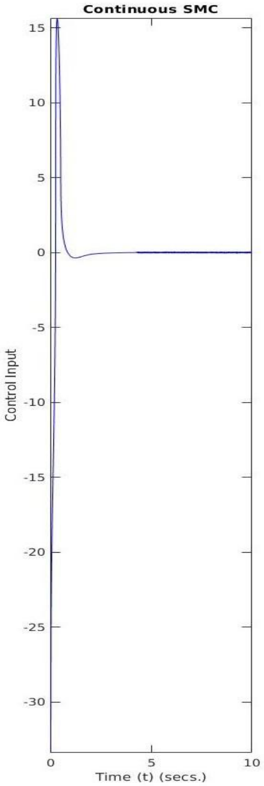


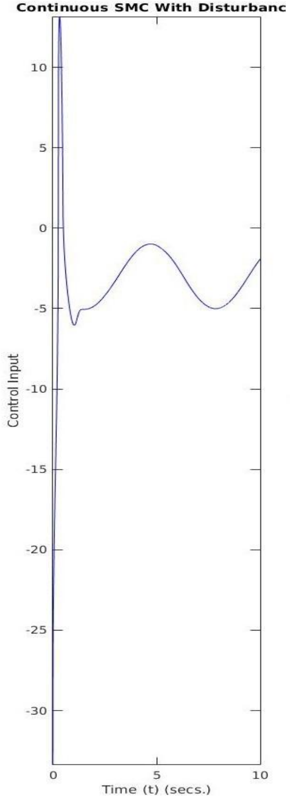
Simulation: r_2 and r_3 States Zoomed

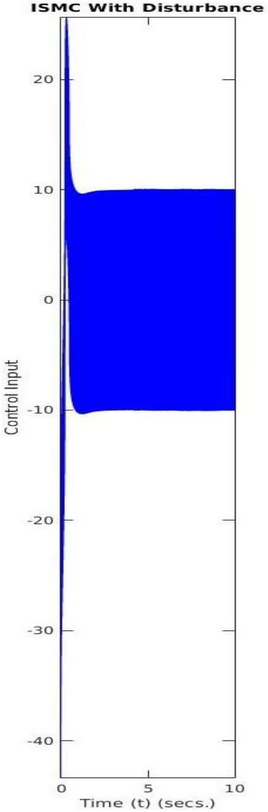


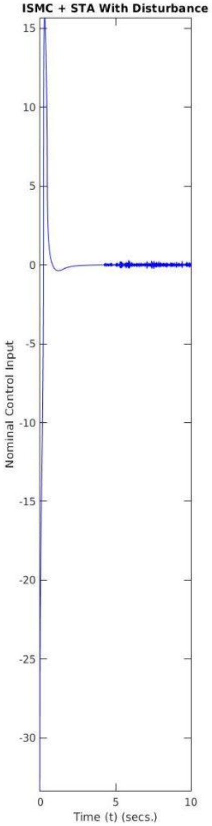
Simulation: Control Input “u”

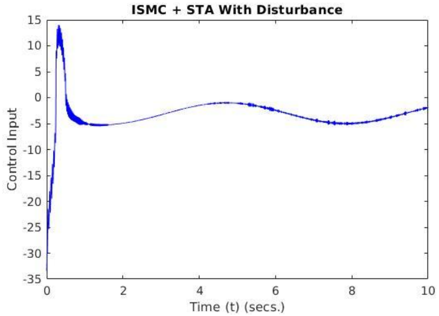




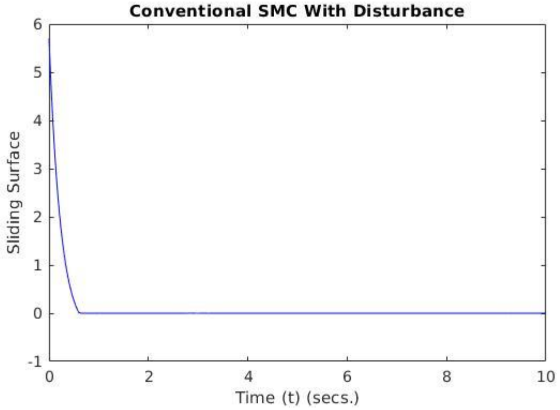


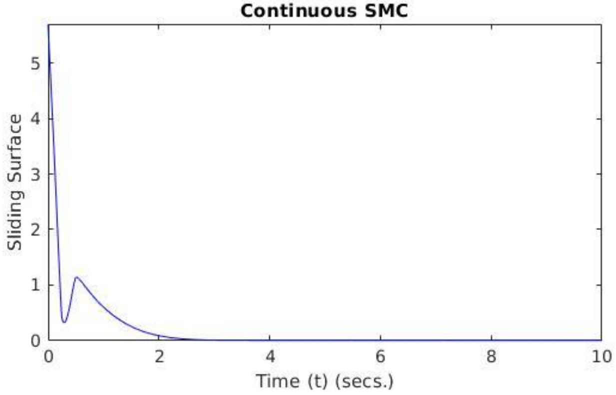


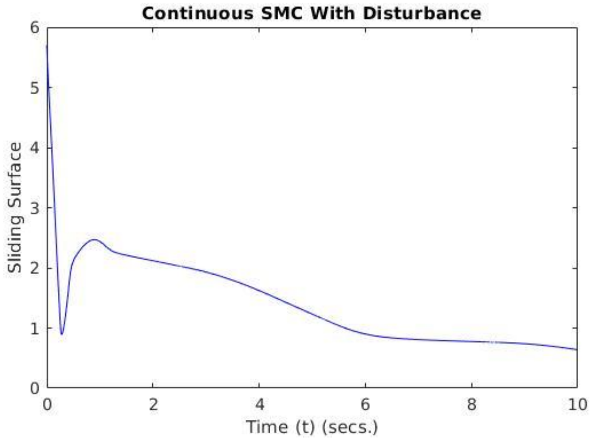


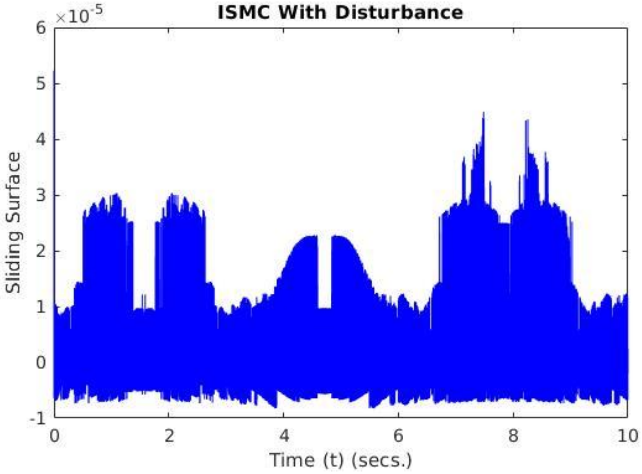


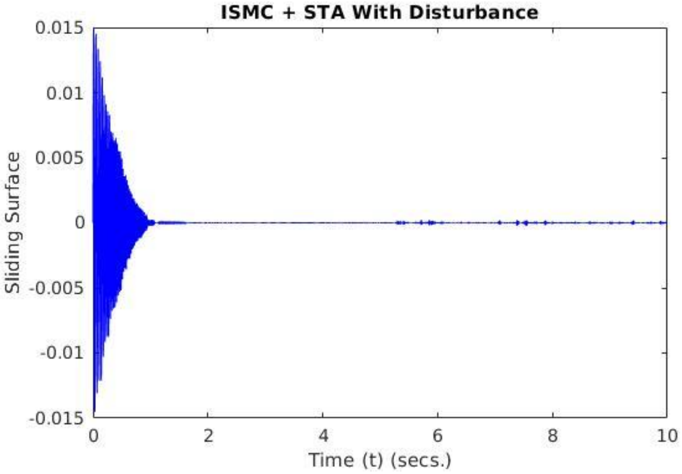
Simulation: Sliding Variable



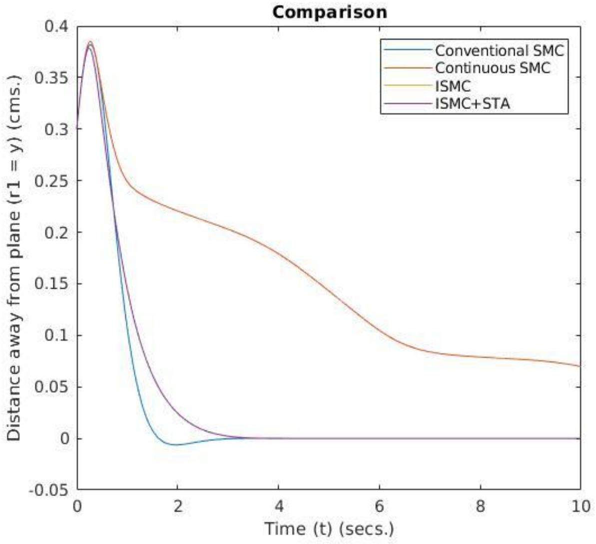




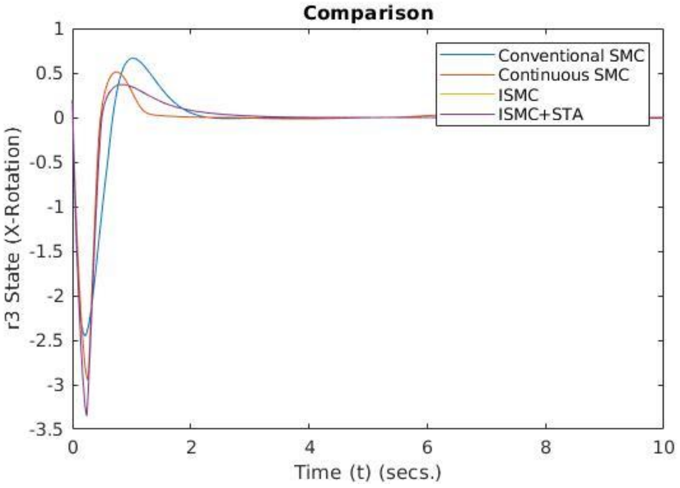




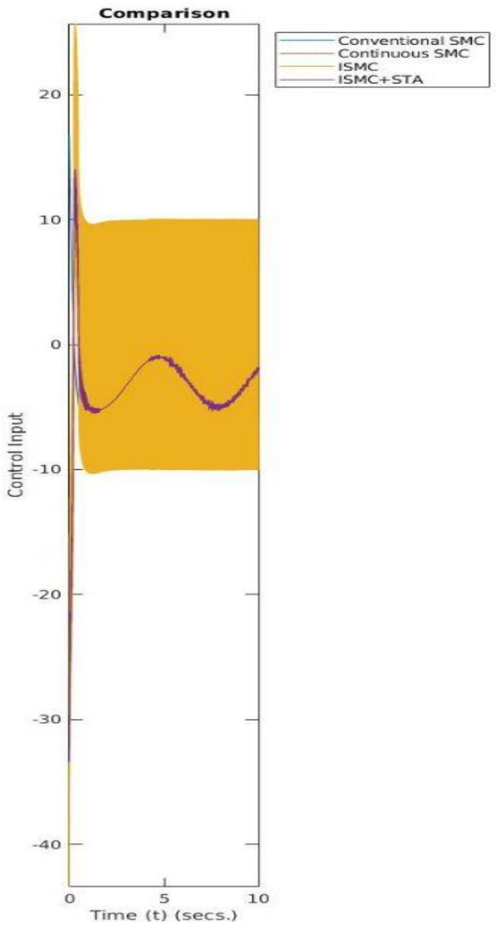
Comparison



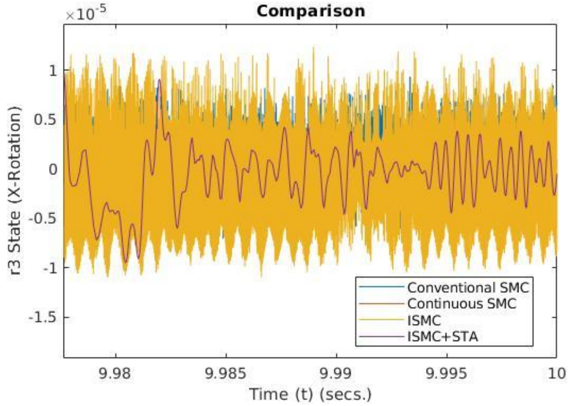
Comparison
Distance away from plane (r1 = y) (cms.)
Time (t) (secs.)
Conventional SMC
Continuous SMC
ISMC
ISMC+STA



Comparison
r3 State (X-Rotation)
Time (t) (secs.)
Conventional SMC
Continuous SMC
ISMC
ISMC+STA



Comparison
Control Input
Time (t) (secs.)
Conventional SMC
Continuous SMC
ISMC
ISMC+STA



Comparison
×10^-5
r3 State (X-Rotation)
Time (t) (secs.)
Conventional SMC
Continuous SMC
ISMC
ISMC+STA

THANKS
Q&A