Markov Chains to Model Population Movement

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1 Objective

- Provide a way to model populations at different locations (states) and how the population changes over time after certain events occur.
- Use the predicted movement of populations to estimate profits or rewards earned based on the scenario, hence providing strategies to maximize the rewards.
- These techniques may be extended to other utilities such as website traffic, or any scenario that involves making estimations on the behavior of a population and optimizing certain metrics based on this movement.

2 Problem Statement

Consider the following problem statement, to illustrate the utility of the provided techniques:

- As the organizer of a festival, you must choose the optimal time to announce a surprise event.
- The festival has 3 locations: a food stall, a merch stall, and the main stage where organized events take place. The surprise event will occur at the main stage.
- Data is available on the movement of the population after announcing surprise events at different times in the past.
- There are random variables that give the average profit earned at a location, given the number of people at that location: (fig. 1)
  - The profits at the food stall and the merch stall follow a logistic curve with population at the location. This is due to a limited number of staff being unable to manage large crowds beyond a certain point.
  - The profits at the main stage (after announcing the event) follow a quadratic curve with population. This is because large crowds generate buzz for PR and make the event more attractive for the future.
- **Data Description:**
  - The dataset contains the initial and final populations at the different states (locations) before and after announcing an event at the main stage.
  - The data was created by generating random numbers of people at each state for the initial populations, then performing transition simulations for each person using synthetic probability transition matrices.
  - There are 10 instances of initial and final population pairs for each transition matrix used in the simulations.

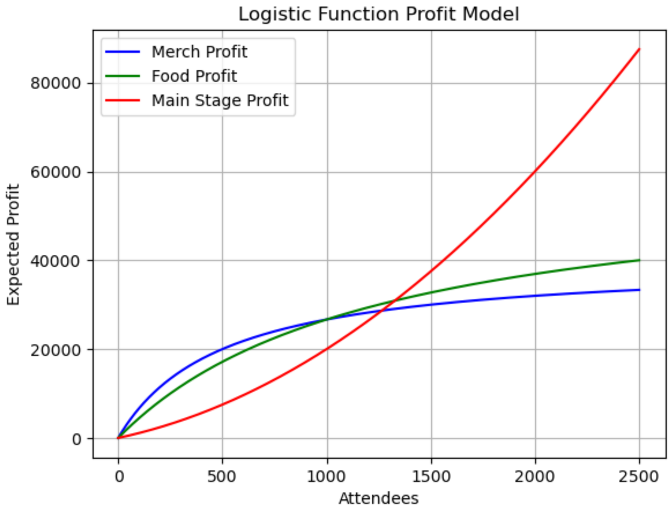

Figure 1: Profit vs. Population at Different Locations

3 Solution

This project models the movement of people between the different locations using a Markov model, where the movement at each time step is governed by a transition matrix.

- Based on the data provided, we can learn the transition matrix parameters by maximizing the likelihood of the given data, i.e., by finding the Maximum Likelihood Estimate (MLE).
- This is done using the `cvxpy` library:
  - We define the cost function to optimize based on MLE and include constraints on the probabilities.
  - We evaluate the prediction using the Weighted Absolute Percentage Error (WAPE)[1]: "How much on average does the predicted value vary from the true value, relative to the true value?".

4 Testing and Results

We show the results obtained and compare them with the true transition probabilities of the synthetic data.

Table I: Comparison of True and Predicted Transition Matrices

True Matrix 1	**Predicted Matrix 1**
$\begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.1 & 0.3 & 0.6 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.2997 & 0.0985 & 0.6018 \\ 0.0959 & 0.2906 & 0.6135 \\ 0.0919 & 0.1029 & 0.8052 \end{bmatrix}$
True Matrix 2	**Predicted Matrix 2**
$\begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.1 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.1079 & 0.4068 & 0.4853 \\ 0.0945 & 0.4960 & 0.4094 \\ 0.1001 & 0.3008 & 0.5991 \end{bmatrix}$
True Matrix 3	**Predicted Matrix 3**
$\begin{bmatrix} 0.15 & 0.05 & 0.8 \\ 0.05 & 0.15 & 0.8 \\ 0.05 & 0.05 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.1473 & 0.0571 & 0.7955 \\ 0.0432 & 0.1391 & 0.8178 \\ 0.0519 & 0.0423 & 0.9058 \end{bmatrix}$

Using the estimate of the transition probabilities, we can predict the populations at the different locations after announcing the surprise event, given the initial populations at the locations.

[1] Reference: `https://help.llama.ai/release/platform/doc-center/demand_topics/dem_modeler_engine_forecasting_metrics_wape.htm`

- After estimating the populations, we calculate the estimated profits. These are shown in the graph. (fig. 2)
- Using the estimated profits, we can choose the optimal time to announce and conduct the surprise event.

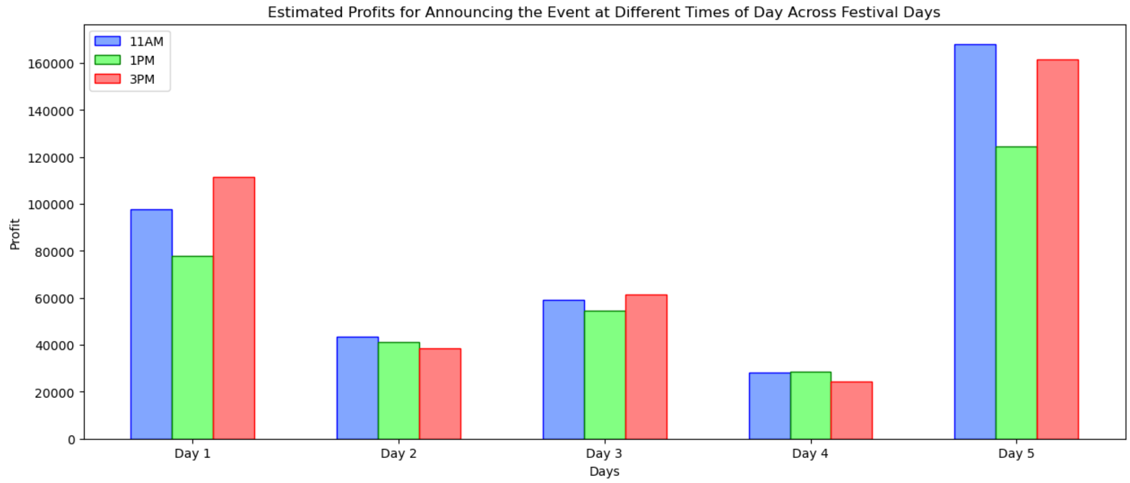

Figure 2: Estimated profits for announcing and conducting the event at different times for values of the initial populations

5 Advantages of Using a Markov-Based Model

- Easy to visualize and understand how people generally transition between locations based on the predicted outputs. It allows us to reason out why the model predicts certain outcomes based on the real-life situation (explainability)
- Provides a model that has been thoroughly researched and validated in the past.

6 Disadvantages of Using a Markov-Based Model

- During the learning process, if the data points are similar (considering each before and after population to be a single data point), the system of equations may become under-determined. This makes it difficult to learn the exact values for the transition probabilities.
- To learn transition probabilities effectively, the starting populations must be sufficiently random. This is not always the case with real-world data.
- Makes strong assumptions about population behavior, which may introduce errors.

7 Future Scope and Further Improvements

- Perform tests on real-world data to validate the models.
- Develop more robust modeling methods based on the data nature.
- Use deep learning models instead of parameterizing and assuming specific models.
- Test the models in more complex scenarios, such as larger numbers of states, and more complicated profit modeling, to demonstrate the computational utility of such analysis in decision-making.
- Create individual Markov models for each person, taking into account individual behavior when making decisions. This avoids relying on an average population behavior, which may be susceptible to errors from outliers.

8 Related Research

The following are research papers on similar topics that may be applied to improve the project.

- **Urban Mobility Analysis Using Hidden Markov Models**
- **Mining User Mobility Features for Next Place Prediction in Location-Based Services**: This paper extends Markov models for predicting an individual's next location based on their past mobility patterns.
- **Modeling User Mobility Using Personalized Markov Chains**: This study personalizes Markov chains by learning individual-specific transition probabilities, improving predictions for highly variable mobility patterns.