Probability Assignment 3 (12.13.5.7)

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Question

In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it fails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution

Let X be the number of questions the student answers correctly. Clearly, X has the binomial distribution with $n = 20$ and $p = 0.5$ (assuming non-biased coin).

$$X = Bin(n, p) \tag{1}$$
$$= Bin(20, 0.5) \tag{2}$$

The mean of X,

$$\mu = n \times p \tag{3}$$
$$= 10 \tag{4}$$

The Variance of X,

$$\sigma^2 = n \times p \times (1 - p) \tag{5}$$
$$= 5 \tag{6}$$

Applying the normal approximation to X for large values of n,

$$\Pr(a \leq X \leq b) = \int_{a-0.5}^{b+0.5} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-\mu)^2}{2\sigma^2}} \, dk \tag{7}$$

$$\Pr(12 \leq X \leq 20) = \int_{11.5}^{20.5} \frac{1}{\sqrt{10\pi}} e^{-\frac{(k-10)^2}{10}} \, dk \tag{8}$$

$$\tag{9}$$

On computation,

$$\Pr(12 \leq X \leq 20) \approx 0.2512 \tag{10}$$