Probability Assignment 1 (12.13.5.2)

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Question

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Solution

Let X and Y be two random variables such that,

$$X = \begin{cases} 0, & \text{if student is resides in hostel} \\ 1, & \text{if student is a day scholar} \end{cases} \tag{1}$$

$$Y = \begin{cases} 0, & \text{if student does not attain A grade} \\ 1, & \text{if student attains A grade} \end{cases} \tag{2}$$

From the question, data given:

$$\Pr(X = 0) = \frac{3}{5} \tag{3}$$

$$\Pr(X = 1) = \frac{2}{5} \tag{4}$$

$$\Pr(Y = 1 \mid X = 0) = \frac{3}{10} \tag{5}$$

$$\Pr(Y = 1 \mid X = 1) = \frac{1}{5} \tag{6}$$

Now, from Bayes' theorem, the probability that the chosen student with the A grade is a hosteler is:

$$\Pr(X = 0 \mid Y = 1) = \frac{\Pr(Y = 1 \mid X = 0) \times \Pr(X = 0)}{\Pr(Y = 1 \mid X = 0) \times \Pr(X = 0) + \Pr(Y = 1 \mid X = 1) \times \Pr(X = 1)} \tag{7}$$

$$= \frac{\frac{3}{10} \times \frac{3}{5}}{\frac{3}{10} \times \frac{3}{5} + \frac{1}{5} \times \frac{2}{5}} \tag{8}$$

$$= \frac{9}{13} \tag{9}$$