Probability Assignment 1 (12.13.5.2)

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Question

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Solution

Let X denote the number of doublets/successes obtained after the 4 trials. Clearly, X has the binomial distribution with $n = 4$ and

$$\begin{aligned} p &= \text{probability of getting a doublet with two dice} \\ &= \frac{1}{6} \end{aligned}$$

Now, since X has the binomial distribution, the probability mass function is given by

$$P(\text{exactly } r \text{ successes}) = {}^{n}C_{r}\left(\frac{1}{6}\right)^{r}\left(\frac{5}{6}\right)^{n-r}$$

$$P_X(r) = {}^{n}C_{r}\left(\frac{1}{6}\right)^{r}\left(\frac{5}{6}\right)^{n-r}$$

Hence, the probability of two successes is

$$\begin{aligned} P_X(2) &= {}^{4}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{2} \\ &= \frac{25}{216} \end{aligned}$$