Probability Assignment 3 (12.13.5.7)

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Question

In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it fails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Solution

Let X be the number of questions the student answers correctly. Clearly, X has the binomial distribution with $n = 20$ and $p = 0.5$ (assuming non-biased coin). Now, since X has the binomial distribution, the probability mass function is given by

$$\Pr(X = r) = {}^{20}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} \tag{1}$$

$$= \frac{{}^{20}C_r}{2^{20}} \tag{2}$$

Hence, the probability that he gets at least 12 correct:

$$\Pr(X \geq 12) = Cdf(X = 20) - Cdf(X = 11) \tag{3}$$

$$= 1 - Cdf(X = 11) \tag{4}$$

$$= 1 - \frac{\sum_{r=0}^{11} {}^{20}C_r}{2^{20}} \tag{5}$$

$$= \frac{\sum_{r=12}^{20} {}^{20}C_r}{2^{20}} \tag{6}$$