

EE23010 NCERT Exemplar

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Question 12.13.3.2

A fair die is thrown two times. Let A and B be the events, 'same number each time', and a 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

Solution:

Random Variable	Description
X	Number appearing on the first die
Y	Number appearing on the second die.

TABLE I
RANDOM VARIABLES

Thus,

$$\Pr(AB) = \frac{2}{36} \quad (10)$$

$$\Pr(A) \Pr(B) = \frac{6}{36} \times \frac{6}{36} \quad (11)$$

$$= \frac{1}{36} \quad (12)$$

$$\Pr(A) \Pr(B) \neq \Pr(AB) \quad (13)$$

$$\frac{1}{36} \neq \frac{2}{36} \quad (14)$$

The probability mass functions(PMFs) of X and Y are given by,

$$\Pr(X = i) = \frac{1}{6}, i = 1, 2, 3, 4, 5, 6 \quad (1)$$

$$\Pr(Y = i) = \frac{1}{6}, i = 1, 2, 3, 4, 5, 6 \quad (2)$$

Event A occurs if and only if $X = Y$

$$\Pr(A) = \Pr(X = Y) \quad (3)$$

$$= \frac{6}{36} \quad (4)$$

Event B occurs if and only if $X + Y \geq 10$

$$\Pr(B) = \Pr(X + Y \geq 10) \quad (5)$$

Let n be the sum of two numbers appearing on the dice,

$$\Pr(X + Y = n) = \begin{cases} 0, n < 1 \\ \frac{n-1}{36}, 2 \leq n \leq 7 \\ \frac{13-n}{36}, 7 \leq n \leq 12 \\ 0, n > 12 \end{cases} \quad (6)$$

In this case, n can be 10, 11 or 12

$$\Pr(B) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \quad (7)$$

$$= \frac{6}{36} \quad (8)$$

For two events A and B to be independent,

$$\Pr(A) \Pr(B) = \Pr(AB) \quad (9)$$

Hence A and B are not independent events.