EE23010 NCERT Exemplar

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Question 12.13.3.75
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability that exactly two of the three balls were red, the first ball being red is
Solution:

$$X = \begin{cases} 0 & \text{, Blue ball} \\ 1 & \text{, Red ball} \end{cases} \tag{1}$$

Let M be the event that first ball is red.
Let N be the event that two of the three balls are red.

$$\Pr(M) = p_X(1)\,p_X(1)\,p_X(1) + p_X(1)\,p_X(1)\,p_X(0) + p_X(1)\,p_X(0)\,p_X(0) + p_X(1)\,p_X(0)\,p_X(1) \tag{2}$$

$$= \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} + \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} + \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6} + \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6} \tag{3}$$

$$= \frac{210}{336} \tag{4}$$

$$\Pr(MN) = p_X(1)\,p_X(1)\,p_X(0) + p_X(1)\,p_X(0)\,p_X(1) \tag{5}$$

$$= \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} + \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6} \tag{6}$$

$$= \frac{120}{336} \tag{7}$$

$$\Pr(N|M) = \frac{\Pr(MN)}{\Pr(M)} \tag{8}$$

$$= \frac{\frac{120}{336}}{\frac{210}{336}} \tag{9}$$

$$= \frac{4}{7} \tag{10}$$