Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -3 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 5 \\ 4 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -4 \\ -8 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} -1 \\ 4 \end{pmatrix}$	CA
$\|\|A-B\|\|$	6.4	length of AB
$\|\|B-C\|\|$	8.94	length of BC
$\|\|C-A\|\|$	4.12	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 4 \\ -5 \end{pmatrix}$	AB
c_1	-21	
$\mathbf{n_2}$	$\begin{pmatrix} -8 \\ 4 \end{pmatrix}$	BC
c_2	12	
$\mathbf{n_3}$	$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	CA
c_3	-15	
Area	12	Area of Triangle
$\angle A$	114.62°	Angles
$\angle B$	24.77°	
$\angle C$	40.6°	

TABLE 1: Vectors.

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -3.5, -1 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} -1.5 \\ 3 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	
c_4	-3	
$\mathbf{m_5}$	$\begin{pmatrix} -4.5 \\ -6 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -6 \\ 4.5 \end{pmatrix}$	
c_5	16.5	
$\mathbf{m_6}$	$\begin{pmatrix} 1.5 \\ 6 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 6 \\ -1.5 \end{pmatrix}$	
c_6	-13.5	
$\mathbf{G}$	$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

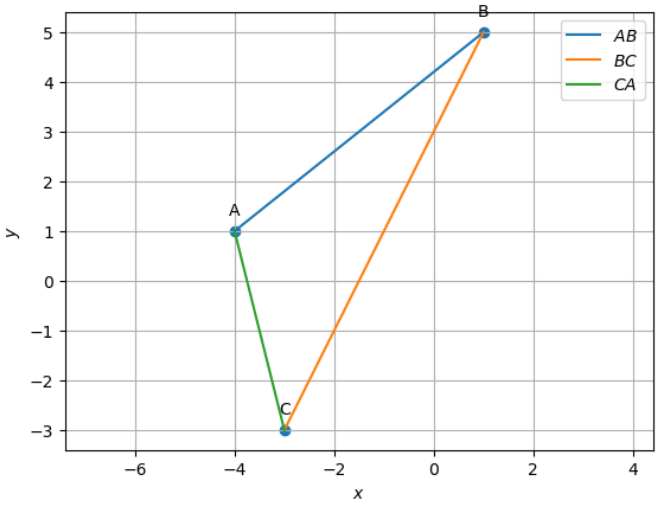

Fig. 1: triangle plotted using python

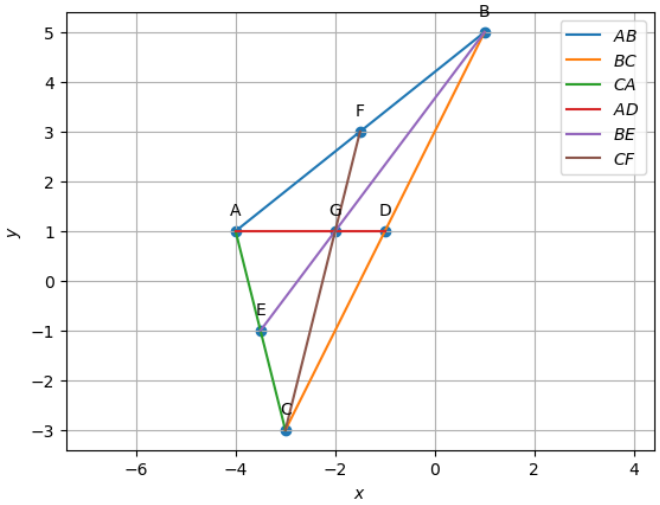

Fig. 2: medians plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}-1.6\\-0.2\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$(-4.65, 3.58)$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-5.34\\-0.07\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}2.4\\-1.2\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-1.2\\-2.4\end{pmatrix}$	
c_7	2.4	
$\mathbf{m_8}$	$\begin{pmatrix}-5.65\\-1.41\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}-1.41\\5.65\end{pmatrix}$	
c_8	26.8	
$\mathbf{m_9}$	$\begin{pmatrix}-2.3\\2.9\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}2.9\\2.3\end{pmatrix}$	
c_9	-15.8	
H	$\begin{pmatrix}-7.6\\2.8\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

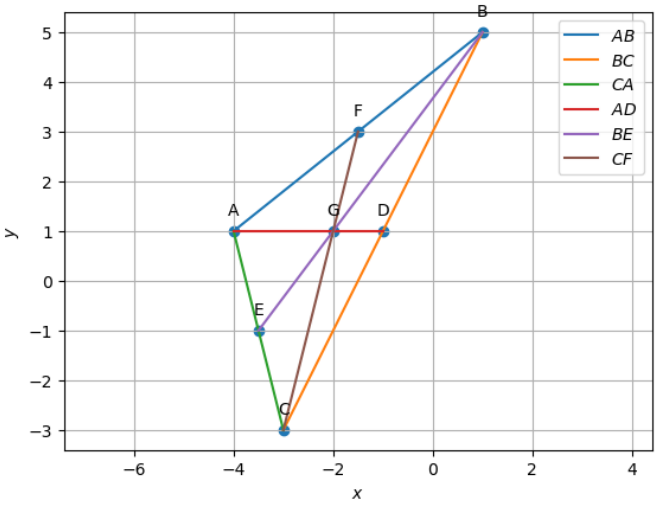

Fig. 3: altitudes plotted using python

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-8\\4\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}-4\\-8\end{pmatrix}$	
c_{10}	-4	
$\mathbf{m_{11}}$	$\begin{pmatrix}-4\\-1\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}1\\-4\end{pmatrix}$	
c_{11}	0.5	
$\mathbf{m_{12}}$	$\begin{pmatrix}-4\\5\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-5\\-4\end{pmatrix}$	
c_{12}	-4.5	
O	$\begin{pmatrix}0.83\\0.08\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	4.19	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	130.75°	$\angle BOC = 2\angle BAC$
$\angle BAC$	114.62°	
$\angle AOC$	49.55°	$\angle AOC = 2\angle ABC$
$\angle ABC$	24.77°	
$\angle AOB$	278.79°	$\angle AOB = 2\angle BCA$
$\angle BCA$	40.6°	

TABLE 4: Perpendicular Bisector.

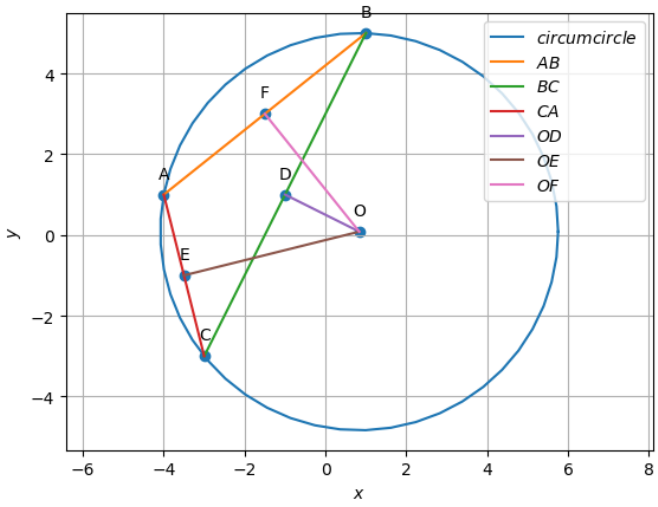

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

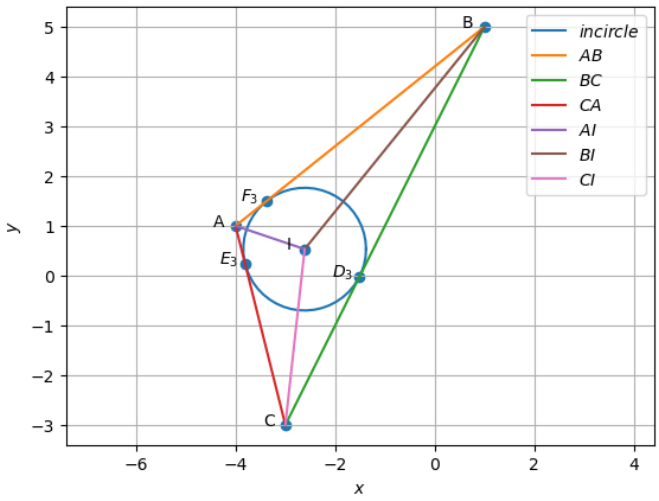

Fig. 5: Angle bisectors plotted using python

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-1.02\\0.34\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}0.34\\1.02\end{pmatrix}$	
c_{13}	-0.35	
$\mathbf{m_{14}}$	$\begin{pmatrix}-1.22\\-1.52\end{pmatrix}$	BI
$\mathbf{m_{14}}$	$\begin{pmatrix}1.51\\-1.22\end{pmatrix}$	
c_{14}	-4.62	
$\mathbf{m_{15}}$	$\begin{pmatrix}-0.2\\-1.86\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}1.86\\-0.2\end{pmatrix}$	
c_{15}	-4.97	
$\mathbf{I}$	$\begin{pmatrix}-2.6\\0.53\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}-1.5\\-0.01\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-3.8\\0.23\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}-3.3\\1.5\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	1.23	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	57.31°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	12.38°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	20.3°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.