EE23010 NCERT Exemplar

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Question 10.13.3.35
Box A contains 25 slips of which 19 are marked Rs 1 and others are marked Rs 5 each. Box B contains 50 slips of which 45 are marked Rs 1 and others are marked Rs 13 each. Slips of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than Rs 1?
Solution:

Random variable	Value	Definition
X	0	Slips of Rs 1
	1	Slips of Rs 5
	2	Slips of Rs 13
Y	0	Box A
	1	Box B

TABLE I
DISTRIBUTION

The PMF of Y (the box that the slip came from) is given as.

$$p_Y(k) = \begin{cases} \frac{1}{3} & \text{if k=0} \\ \frac{2}{3} & \text{if k=1} \end{cases} \tag{1}$$

Conditional Probabilty,

$$\begin{cases} \Pr(Y=0|X=0) = \frac{19}{25} \\ \Pr(Y=0|X=1) = \frac{6}{25} \\ \Pr(Y=1|X=0) = \frac{45}{50} \\ \Pr(Y=1|X=2) = \frac{5}{50} \end{cases} \tag{2}$$

The desired probability is the probability that a slip drawn at random is marked other than Rs 1,

$$= 1 - p_X(0) \tag{3}$$
$$= p_X(1) + p_X(2) \tag{4}$$

Using Bayes theorem,

$$= p_Y(0) \times \Pr(Y=0|X=1) + p_Y(1) \times \Pr(Y=1|X=2) \tag{5}$$
$$= \frac{1}{3} \times \frac{6}{25} + \frac{2}{3} \times \frac{5}{50} \tag{6}$$
$$= \frac{11}{75} \tag{7}$$