

EE23010 NCERT Exemplar

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Question 12.13.3.49

Let X be a discrete random variable whose probability distribution is defined as follows:

$$\Pr(X = x) = \begin{cases} k(x+1) & , x = 1, 2, 3, 4 \\ 2kx & , x = 5, 6, 7 \\ 0 & , \text{otherwise} \end{cases}$$

where k is a constant. Calculate

- (i) the value of k
- (ii) $E(X)$
- (iii) Standard deviation of X

Solution:

- (i) the value of k :

Using the third axiom of probability,

$$\sum_{i=1}^n p_X(i) = 1 \quad (1)$$

We get,

$$2k + 3k + 4k + 5k + 10k + 12k + 14k = 1 \quad (2)$$

$$50k = 1 \quad (3)$$

$$k = 0.02 \quad (4)$$

- (ii) $E(X)$:

$$E(X) = \sum_{i=1}^n x_i p_X(i) \quad (5)$$

$$E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k \quad (6)$$

$$= 260k \quad (7)$$

$$= 260 \times 0.02 \quad (8)$$

$$= 5.2 \quad (9)$$

- (iii) Standard deviation of X :

$$\text{Var}(X) = E(X^2) - [E(X)]^2 \quad (10)$$

$$= \sum_{i=1}^n x_i^2 p_X(i) - \left[\sum_{i=1}^n x_i p_X(i) \right]^2 \quad (11)$$

$$= 1498k - (5.2)^2 \quad (12)$$

$$= 1498 \times 0.02 - 27.04 \quad (13)$$

$$= 2.92 \quad (14)$$

Standard deviation of X is

$$X = \sqrt{\text{Var}(X)} \quad (15)$$

$$= \sqrt{2.92} \quad (16)$$

$$= 1.7 \quad (17)$$