EE23010 Assignment 1

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Question 1.5.11

Obtain m, n, p in terms of $a, b, c,$ the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution.

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The length of a line segment given the two vectors $\mathbf{A}$ and $\mathbf{B}$ is given as

$$\sqrt{(\mathbf{A} - \mathbf{B})^\top(\mathbf{A} - \mathbf{B})}$$

The length of the vectors $\mathbf{AB}$, $\mathbf{BC}$ and $\mathbf{CA}$ are c, a and b respectively.

Calculation of side length a

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -3+4 \\ -5-6 \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{5}$$

$$a = \sqrt{(\mathbf{C} - \mathbf{B})^\top(\mathbf{C} - \mathbf{B})} \tag{6}$$

$$a = \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix}\begin{pmatrix} 1 \\ -11 \end{pmatrix}} \tag{7}$$

$$= \sqrt{122} \tag{8}$$

Calculation of side length b

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1+3 \\ -1+5 \end{pmatrix} \tag{9}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{10}$$

$$a = \sqrt{(\mathbf{A} - \mathbf{C})^\top(\mathbf{A} - \mathbf{C})} \tag{11}$$

$$a = \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix}} \tag{12}$$

$$= \sqrt{32} \tag{13}$$

Calculation of side length c

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4-1 \\ 6+1 \end{pmatrix} \tag{14}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{15}$$

$$a = \sqrt{(\mathbf{B} - \mathbf{A})^\top(\mathbf{B} - \mathbf{A})} \tag{16}$$

$$a = \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix}\begin{pmatrix} -5 \\ 7 \end{pmatrix}} \tag{17}$$

$$= \sqrt{74} \tag{18}$$

By observing the values of $\mathbf{D_3}$, $\mathbf{E_3}$ and $\mathbf{F_3}$ from the previous question, we see that the points $\mathbf{D_3}$, $\mathbf{E_3}$ and $\mathbf{F_3}$ are contained in the line segment $\mathbf{BC}$, $\mathbf{CA}$ and $\mathbf{AB}$ respectively which means,

$$\mathbf{BD_3} + \mathbf{D_3C} = \mathbf{BC} \tag{19}$$

$$\mathbf{CE_3} + \mathbf{E_3A} = \mathbf{CA} \tag{20}$$

$$\mathbf{AC_3} + \mathbf{C_3B} = \mathbf{AB} \tag{21}$$

where

$$|\mathbf{D_3C}| = n \tag{22}$$

$$|\mathbf{CE_3}| = p \tag{23}$$

$$|\mathbf{E_3A}| = m \tag{24}$$

$$|\mathbf{AC_3}| = m \tag{25}$$

$$|\mathbf{C_3B}| = n \tag{26}$$

$$|\mathbf{BD_3}| = n \tag{27}$$

Therefore

$$a = n + p \tag{28}$$
$$b = p + m \tag{29}$$
$$c = m + n \tag{30}$$

The linear equations (25), (26) and (27) can be written in a matrix equation which is given by

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{31}$$

Using the elimination using row transformation method,

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + R_2 + R_3} \left(\begin{array}{ccc|c} 2 & 2 & 2 & a+b+c \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \tag{32}$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & a+b+c \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \tag{33}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & -1 & 0 & \frac{b-a-c}{2} \\ 1 & 1 & 0 & c \end{array}\right) \tag{34}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & -1 & 0 & \frac{b-a-c}{2} \\ 1 & 1 & 0 & c \end{array}\right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 1 & 1 & 0 & c \end{array}\right) \tag{35}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 1 & 1 & 0 & c \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & -1 & \frac{c-a-b}{2} \end{array}\right) \tag{36}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & -1 & \frac{c-a-b}{2} \end{array}\right) \xleftrightarrow{R_3 \leftarrow -R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{array}\right) \tag{37}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & \frac{a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 - R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{array}\right) \tag{38}$$

we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \tag{39}$$

The values of m, n and p in terms of a, b and c are,

$$m = \frac{-a+b+c}{2} \tag{40}$$
$$n = \frac{a-b+c}{2} \tag{41}$$
$$p = \frac{a+b-c}{2} \tag{42}$$

The numerical values of m, n and p are,

$$m = \frac{-\sqrt{122} + \sqrt{32} + \sqrt{74}}{2} \tag{43}$$
$$n = \frac{\sqrt{122} - \sqrt{32} + \sqrt{74}}{2} \tag{44}$$
$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \tag{45}$$

By simplification, we get

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1.606 \\ 6.995 \\ 4.045 \end{pmatrix} \tag{46}$$