EE23010 Assignment 1

Vishal A - EE22BTECH11057

Question 1.5.11

Obtain m, n, p in terms of $a, b, c,$ the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution.

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$
$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$
$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The length of the vectors **AB**, **BC** and **CA** are c, a and b respectively.

$$a = \sqrt{122} \tag{4}$$
$$b = \sqrt{32} \tag{5}$$
$$c = \sqrt{74} \tag{6}$$

By observing the values of D_3, E_3 and F_3 from the previous question, we see that the points D_3, E_3 and F_3 are contained in the line segment BC, CA and AB respectively which means,

$$||\mathbf{D_3} - \mathbf{B}|| + ||\mathbf{C} - \mathbf{D_3}|| = ||\mathbf{C} - \mathbf{B}|| \tag{7}$$
$$||\mathbf{E_3} - \mathbf{C}|| + ||\mathbf{A} - \mathbf{E_3}|| = ||\mathbf{A} - \mathbf{C}|| \tag{8}$$
$$||\mathbf{F_3} - \mathbf{A}|| + ||\mathbf{B} - \mathbf{F_3}|| = ||\mathbf{B} - \mathbf{A}|| \tag{9}$$

Therefore

$$a = n + p \tag{10}$$
$$b = p + m \tag{11}$$
$$c = m + n \tag{12}$$

The linear equations can be written in a matrix equation which is given by

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{13}$$

By solving using row reduction,

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \\ 1 & 1 & 0 & c \end{array}\right) \tag{14}$$

$\xleftrightarrow{R_1 \leftrightarrow R_2}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \\ 1 & 1 & 0 & c \end{array}\right) \tag{15}$$

$\xleftrightarrow{R_1 \leftrightarrow R_3}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{array}\right) \tag{16}$$

$\xleftrightarrow{R_3 \leftarrow R_3 - R_1 + R_2}$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array}\right) \tag{17}$$

$\xleftrightarrow{R_1 \leftarrow R_1 - R_2}$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & c - a \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array}\right) \tag{18}$$

$\xleftrightarrow{R_1 \leftarrow 2R_1 + R_3}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array}\right) \tag{19}$$

$\xleftrightarrow{R_2 \leftarrow 2R_2 - R_3}$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 2 & 0 & a - b + c \\ 0 & 0 & 2 & b - c + a \end{array}\right) \tag{20}$$

we get

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-a+b+c}{2} \\ 0 & 1 & 0 & \frac{a-b+c}{2} \\ 0 & 0 & 1 & \frac{a+b-c}{2} \end{array}\right) = \begin{pmatrix} m \\ n \\ p \end{pmatrix} \tag{21}$$

The values of m, n and p in terms of a, b and c are,

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \tag{22}$$

The numerical values of m, n and p are,

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-\sqrt{122}+\sqrt{32}+\sqrt{74}}{2} \\ \frac{\sqrt{122}-\sqrt{32}+\sqrt{74}}{2} \\ \frac{\sqrt{122}+\sqrt{32}-\sqrt{74}}{2} \end{pmatrix} \tag{23}$$

By simplification, we get

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1.606 \\ 6.995 \\ 4.045 \end{pmatrix} \tag{24}$$