EE23010 NCERT Exemplar

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Question 12.13.3.49
Let X be a discrete random variable whose probability distribution is defined as follows:

$$\Pr(X = x) = \begin{cases} k(x+1) & , x = 1, 2, 3, 4 \\ 2kx & , x = 5, 6, 7 \\ 0 & , otherwise \end{cases}$$

where k is a constant. Calculate

(i) the value of k
(ii) E(X)
(iii) Standard deviation of X

Solution:

x_i	1	2	3	4	5	6	7	Otherwise
$p_x(i)$	2k	3k	4k	5k	10k	12k	14k	0
$x_i p_x(i)$	2k	6k	12k	20k	50k	72k	98k	0
$x_i^2 p_x(i)$	2k	12k	36k	80k	250k	432k	686k	0

TABLE I
PROBABILITY DISTRIBUTION

(i) the value of k:
Using the third axiom of probability,

$$\sum_{i=1}^{n} p_X(i) = 1 \tag{1}$$

We get,

$$2k + 3k + 4k + 5k + 10k + 12k + 14k = 1 \tag{2}$$
$$50k = 1 \tag{3}$$
$$k = 0.02 \tag{4}$$

(ii) E(X):

$$E(X) = \sum_{i=1}^{n} x_i p_X(i) \tag{5}$$
$$E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k \tag{6}$$
$$= 260k \tag{7}$$
$$= 260 \times 0.02 \tag{8}$$
$$= 5.2 \tag{9}$$

(iii) Standard deviation of X:

$$Var(X) = E(X^2) - [E(X)]^2 \tag{10}$$
$$= \sum_{i=1}^{n} x_i^2 p_X(i) - [\sum_{i=1}^{n} x_i p_X(i)]^2 \tag{11}$$
$$= 1498k - (5.2)^2 \tag{12}$$
$$= 1498 \times 0.02 - 27.04 \tag{13}$$
$$= 2.92 \tag{14}$$

Standard deviation of X is

$$X = \sqrt{Var(X)} \tag{15}$$
$$= \sqrt{2.92} \tag{16}$$
$$= 1.7 \tag{17}$$