EE23010 Assignment 1

Vishal A - EE22BTECH11057

Question 1.5.11

Obtain m, n, p in terms of a, b, c, the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution.

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The length of a line segment given the two vectors $\mathbf{A}$ and $\mathbf{B}$ is given as

$$\sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})}$$

The length of the vectors $\mathbf{AB}$, $\mathbf{BC}$ and $\mathbf{CA}$ are c, a and b respectively.

Calculation of side length a

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -3 + 4 \\ -5 - 6 \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \tag{5}$$

$$a = \sqrt{(\mathbf{C} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B})} \tag{6}$$

$$a = \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \tag{7}$$

$$= \sqrt{122} \tag{8}$$

Calculation of side length b

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 + 3 \\ -1 + 5 \end{pmatrix} \tag{9}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{10}$$

$$b = \sqrt{(\mathbf{A} - \mathbf{C})^\top (\mathbf{A} - \mathbf{C})} \tag{11}$$

$$b = \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \tag{12}$$

$$= \sqrt{32} \tag{13}$$

Calculation of side length c

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 - 1 \\ 6 + 1 \end{pmatrix} \tag{14}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{15}$$

$$a = \sqrt{(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})} \tag{16}$$

$$a = \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \tag{17}$$

$$= \sqrt{74} \tag{18}$$

By observing the values of D_3, E_3 and F_3 from the previous question, we see that the points D_3, E_3 and F_3 are contained in the line segment BC, CA and AB respectively which means,

$$BD_3 + D_3C = BC \tag{19}$$
$$CE_3 + E_3A = CA \tag{20}$$
$$AC_3 + C_3B = AB \tag{21}$$

where

$$D_3C = p \tag{22}$$
$$CE_3 = p \tag{23}$$
$$E_3A = m \tag{24}$$
$$AC_3 = m \tag{25}$$
$$C_3B = n \tag{26}$$
$$BD_3 = n \tag{27}$$

Therefore

$$a = n + p \tag{28}$$

$$b = p + m \tag{29}$$

$$c = m + n \tag{30}$$

The linear equations can be written in a matrix equation which is given by

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{31}$$

Swapping R_1 and R_2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \\ 1 & 1 & 0 & c \end{array} \right) \tag{32}$$

Swapping R_1 and R_3

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{array} \right) \tag{33}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{array} \right) \xleftarrow{R_3 \leftarrow R_3 - R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \tag{34}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & c - a \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \tag{35}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & c - a \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftarrow{R_1 \leftarrow 2R_1 + R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \tag{36}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftarrow{R_2 \leftarrow 2R_2 - R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 2 & 0 & a - b + c \\ 0 & 0 & 2 & b - c + a \end{array} \right) \tag{37}$$

we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \tag{38}$$

The values of m, n and p in terms of a, b and c are,

$$m = \frac{-a + b + c}{2} \tag{39}$$

$$n = \frac{a - b + c}{2} \tag{40}$$

$$p = \frac{a + b - c}{2} \tag{41}$$

The numerical values of m, n and p are,

$$m = \frac{-\sqrt{122} + \sqrt{32} + \sqrt{74}}{2} \tag{42}$$

$$n = \frac{\sqrt{122} - \sqrt{32} + \sqrt{74}}{2} \tag{43}$$

$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \tag{44}$$

By simplification, we get

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1.606 \\ 6.995 \\ 4.045 \end{pmatrix} \tag{45}$$