

EE23010 Assignment 1

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Question 1.5.11

Obtain m, n, p in terms of a, b, c , the sides of the triangle using a matrix equation. Obtain the numerical values.

Solution.

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

The length of a line segment given the two vectors \mathbf{A} and \mathbf{B} is given as

$$\sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})}$$

The length of the vectors \mathbf{AB} , \mathbf{BC} and \mathbf{CA} are c, a and b respectively.

Calculation of side length a

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -3 + 4 \\ -5 - 6 \end{pmatrix} \quad (4)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \quad (5)$$

$$a = \sqrt{(\mathbf{C} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B})} \quad (6)$$

$$a = \sqrt{\begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 1 \\ -11 \end{pmatrix}} \quad (7)$$

$$= \sqrt{122} \quad (8)$$

Calculation of side length b

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 1 + 3 \\ -1 + 5 \end{pmatrix} \quad (9)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (10)$$

$$b = \sqrt{(\mathbf{A} - \mathbf{C})^\top (\mathbf{A} - \mathbf{C})} \quad (11)$$

$$b = \sqrt{\begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}} \quad (12)$$

$$= \sqrt{32} \quad (13)$$

Calculation of side length c

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 - 1 \\ 6 + 1 \end{pmatrix} \quad (14)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (15)$$

$$c = \sqrt{(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})} \quad (16)$$

$$c = \sqrt{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -5 \\ 7 \end{pmatrix}} \quad (17)$$

$$= \sqrt{74} \quad (18)$$

By observing the values of D_3 , E_3 and F_3 from the previous question, we see that the points D_3 , E_3 and F_3 are contained in the line segment BC , CA and AB respectively which means,

$$\|\mathbf{D}_3 - \mathbf{B}\| + \|\mathbf{C} - \mathbf{D}_3\| = \|\mathbf{C} - \mathbf{B}\| \quad (19)$$

$$\|\mathbf{E}_3 - \mathbf{C}\| + \|\mathbf{A} - \mathbf{E}_3\| = \|\mathbf{A} - \mathbf{C}\| \quad (20)$$

$$\|\mathbf{F}_3 - \mathbf{A}\| + \|\mathbf{B} - \mathbf{F}_3\| = \|\mathbf{B} - \mathbf{A}\| \quad (21)$$

where

$$\|\mathbf{C} - \mathbf{D}_3\| = p \quad (22)$$

$$\|\mathbf{E}_3 - \mathbf{C}\| = p \quad (23)$$

$$\|\mathbf{A} - \mathbf{E}_3\| = m \quad (24)$$

$$\|\mathbf{F}_3 - \mathbf{A}\| = m \quad (25)$$

$$\|\mathbf{B} - \mathbf{F}_3\| = n \quad (26)$$

$$\|\mathbf{D}_3 - \mathbf{B}\| = n \quad (27)$$

Therefore

$$a = n + p \quad (28)$$

$$b = p + m \quad (29)$$

$$c = m + n \quad (30)$$

The linear equations can be written in a matrix equation which is given by

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (31)$$

Swapping R_1 and R_2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & b \\ 0 & 1 & 1 & a \\ 1 & 1 & 0 & c \end{array} \right) \quad (32)$$

Swapping R_1 and R_3

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{array} \right) \quad (33)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 1 & 0 & 1 & b \end{array} \right) \xleftrightarrow{R_3 \leftarrow R_3 - R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \quad (34)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & c \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & c - a \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \quad (35)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & c - a \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftrightarrow{R_1 \leftarrow 2R_1 + R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \quad (36)$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 1 & 1 & a \\ 0 & 0 & 2 & b - c + a \end{array} \right) \xleftrightarrow{R_2 \leftarrow 2R_2 - R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & c - a + b \\ 0 & 2 & 0 & a - b + c \\ 0 & 0 & 2 & b - c + a \end{array} \right) \quad (37)$$

we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} \frac{-a+b+c}{2} \\ \frac{a-b+c}{2} \\ \frac{a+b-c}{2} \end{pmatrix} \quad (38)$$

The values of m, n and p in terms of a, b and c are,

$$m = \frac{-a+b+c}{2} \quad (39)$$

$$n = \frac{a-b+c}{2} \quad (40)$$

$$p = \frac{a+b-c}{2} \quad (41)$$

The numerical values of m, n and p are,

$$m = \frac{-\sqrt{122} + \sqrt{32} + \sqrt{74}}{2} \quad (42)$$

$$n = \frac{\sqrt{122} - \sqrt{32} + \sqrt{74}}{2} \quad (43)$$

$$p = \frac{\sqrt{122} + \sqrt{32} - \sqrt{74}}{2} \quad (44)$$

By simplification, we get

$$\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 1.606 \\ 6.995 \\ 4.045 \end{pmatrix} \quad (45)$$