EE23010 NCERT Exemplar

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Question 12.13.3.75
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability that exactly two of the three balls were red, the first ball being red is
Solution:

Random variable	Value	Definition
X_1	0	drawing a blue ball in the first pick
	1	drawing a red ball in the first pick
X_2	0	drawing a blue ball in the second pick
	1	drawing a red ball in the second pick
X_3	0	drawing a blue ball in the third pick
	1	drawing a red ball in the third pick

Let M be the event that first ball is red.
Let N be the event that two of the three balls are red.

$$\Pr(M) = \Pr(X_1 = 1) \tag{1}$$
$$= \frac{5}{8} \tag{2}$$
$$\Pr(MN) = p_{X_1}(1)\, p_{X_2}(1)\, p_{X_3}(0) + p_{X_1}(1)\, p_{X_2}(0)\, p_{X_3}(1) \tag{3}$$
$$= \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \tag{4}$$
$$= \frac{120}{336} \tag{5}$$
$$\Pr(N|M) = \frac{\Pr(MN)}{\Pr(M)} \tag{6}$$
$$= \frac{\frac{120}{336}}{\frac{5}{8}} \tag{7}$$
$$= \frac{4}{7} \tag{8}$$