EE23010 NCERT Exemplar

Vishal A - EE22BTECH11057

Question 12.13.3.75
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability that exactly two of the three balls were red, the first ball being red is
Solution:

Random variable	Value	Definition
X_1	0	drawing a blue ball in the first pick
	1	drawing a red ball in the first pick
X_2	0	drawing a blue ball in the second pick
	1	drawing a red ball in the second pick
X_3	0	drawing a blue ball in the third pick
	1	drawing a red ball in the third pick

The probability that the first ball is red.

$$= \Pr\left(X_1 = 1\right) \tag{1}$$
$$= \frac{5}{8} \tag{2}$$
$$\tag{3}$$

The probability that two of the three balls are red and the first ball being red,

$$= \Pr\left(X_1 = 1, X_2 = 1, X_3 = 0\right) + \Pr\left(X_1 = 1, X_2 = 0, X_3 = 1\right) \tag{4}$$
$$= \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \tag{5}$$
$$= \frac{120}{336} \tag{6}$$

Thus, the probability that exactly two of the three balls are red given the first ball being red is,

$$= \frac{\Pr\left(X_1 = 1, X_2 = 1, X_3 = 0\right) + \Pr\left(X_1 = 1, X_2 = 0, X_3 = 1\right)}{\Pr\left(X_1 = 1\right)} \tag{7}$$
$$= \frac{\frac{120}{336}}{\frac{5}{8}} \tag{8}$$
$$= \frac{4}{7} \tag{9}$$