

# EE23010 NCERT Exemplar

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## Question 12.13.3.2

A fair die is thrown two times. Let A and B be the events, 'same number each time', and a 'total score is 10 or more', respectively. Determine whether or not A and B are independent.

**Solution:**

Random Variable	Description
X	Number appearing on throwing the first time
Y	Number appearing on throwing the second time.

TABLE I  
RANDOM VARIABLES

The probability mass functions(PMFs) of  $X$  and  $Y$  are given by,

$$\Pr(X = i) = \frac{1}{6}, i = 1, 2, 3, 4, 5, 6 \quad (1)$$

$$\Pr(Y = i) = \frac{1}{6}, i = 1, 2, 3, 4, 5, 6 \quad (2)$$

Event A occurs if and only if  $X = Y$

$$\Pr(A) = \Pr(X = Y) \quad (3) \quad \text{Thus,}$$

Let  $n$  be the difference of two numbers appearing on the dice.

$$\Pr(X - Y = n) = \begin{cases} \frac{6-|n|}{36}, & -5 \leq n \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

In this case,  $n$  is 0 as  $X = Y$

$$\Pr(A) = \frac{6}{36} \quad (5)$$

Event B occurs if and only if  $X + Y \geq 10$

$$\Pr(B) = \Pr(X + Y \geq 10) \quad (6)$$

Let  $n$  be the sum of two numbers appearing on the dice,

$$\Pr(X + Y = n) = \begin{cases} 0, & n < 1 \\ \frac{n-1}{36}, & 2 \leq n \leq 7 \\ \frac{13-n}{36}, & 7 \leq n \leq 12 \\ 0, & n > 12 \end{cases} \quad (7)$$

In this case,  $n$  can be 10, 11 or 12

$$\Pr(B) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \quad (8)$$

$$= \frac{6}{36} \quad (9)$$

For two events A and B to be independent,

$$\Pr(A) \Pr(B) = \Pr(AB) \quad (10)$$

$$AB = ((5, 5), (6, 6)) \quad (11)$$

$$\Pr(AB) = \Pr(X = Y, X + Y \geq 10) \quad (12)$$

$$= \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \quad (13)$$

$$= \frac{2}{36} \quad (14)$$

$$\Pr(A) \Pr(B) = \frac{6}{36} \times \frac{6}{36} \quad (15)$$

$$= \frac{1}{36} \quad (16)$$

$$\Pr(A) \Pr(B) \neq \Pr(AB) \quad (17)$$

$$\frac{1}{36} \neq \frac{2}{36} \quad (18)$$

Hence A and B are not independent events.