

Solution 12.13.3.22

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Question 22 The Probability of a man hitting target is 0.25. He shoots 7 times. What is the probability of his hitting atleast twice?

Solution:

TABLE 0
RANDOM VARIABLE AND PROBABILITY TABLE

| Random independent variable | value of R.V | Description |
|-----------------------------|-------------------|---------------------------------|
| n | 7 | Total no. of trials |
| X | $0 \leq X \leq 7$ | no. of times he hits the target |

Let the probability of hitting the targets correctly be $p = \frac{1}{4} = 0.25$ (1)

CDF of binomial distribution is: (2)

$$F_X(k) = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} \quad k = 0, 1, 2, \dots, 7$$

(3)

Probability of Hitting the target atleast twice is $= p(X \geq 2)$ (4)

$$p(X \geq 2) = 1 - p(X \leq 1) \quad (5)$$

$$= F_X(7) - F_X(1) \quad (6)$$

$$= 1 - \left\{ \sum_{k=0}^1 \binom{7}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{7-k} \right\} \quad (7)$$

$$= 1 - \left\{ \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^7 + 7 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^6 \right\} \quad (8)$$

$$= 1 - \left(\frac{3}{4}\right)^6 \frac{10}{4} \quad (9)$$

$$= 1 - \left(\frac{729}{1096}\right) \frac{10}{4} \quad (10)$$

$$= 1 - \frac{7290}{16384} \quad (11)$$

$$= \frac{9094}{16384} \quad (12)$$

$$= 0.55505 \quad (13)$$

(14)

Hence, the probability of hitting the target atleast twice is 0.55505