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COLLISION FREE MOTION CONTROL OF MULTIPLE MOBILE ROBOTS ON PATH NETWORK

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Abstract

This paper addressės collision avoidance problem among multiple mobile robots traveling over two dimensional path network. A local approach based on non-linear programming is given to solve this problem. The basic idea of this method is to find the desired velocity vector for robot given as the optimal solution of objective function under velocity vector constraint. Collision free navigation of robot is carried out by following the desired velocity vector. This method is implemented using experimental robot and simulated one. The feasibility and effectiveness of this method is discussed through the results.

1. Introduction

Autonomous navigation of mobile robot gains an increasing importance in industrial utilization as well as in exploration of space or deep sea. A great deal of research has been made on the navigation of individual mobile robot and has achieved various degrees of success including issues of sensing, path planning and motion control. Such works will extend the possibility that robots can move more freely and perform more complicated tasks. This possibility raises a new problem of robotic application: Coordinated operation of multiple mobile robots.

Collision avoidance problem has been considered as a essential element for such coordination problems of multiple robots. Several approaches for this problem have been presented, which focus on the collision free path or trajectory plannig[1],[2] in two dimensional free space.

In most existing workspace, however, robots are restricted to move along narrow corridors, specified guide ways and gap space among obstacles. Such environment can be regarded as a network of path with forks, joins and intersections, e.g., automated transfer systems in a building or factory shops. Even with this restricted topology, an effective planning and control of robots' motion is nontrivial.

The problem of group control of mobile robots on path network is usually formulated as a routing assignment problems[4],[5]. These approahes provides the collision free optimal routes for all mobile robots involved. Such global optimality, however, can be easily lost if any change of tasks imposed on robots happens. Furthermore, the problem becomes time consuming as the number of robots or the complexity of path network increases.

This paper deals with a collision free motion control of multiple mobile robots on path network, which intend to improve the efficiency and flexibility of mobile robots' utilization. A local approach based on a nonlinear programming is presented for solving the above problem. A distance function obtained from the state information of neighboring mobile robots is introduced to derive the velocity vector constraint for robots. The desired velocity vector, which enables a robot to navigate without collision, is given by the solution of local optimization technique.

In the approach, the collision avoidance procedure of a mobile robot includes following four steps:

1. Obtain the state information of neighboring mobile robots, position and velocity, using mutual communication or machine vision;
2. Set up the admissible velocity vector region derived from a distance function between neighboring mobile robots;
3. Find out the desired velocity vector given as the optimal solution of an objective function under the constraint of admissible velocity vector region;
4. Carry out the motion control of a mobile robot referring to its desired velocity vector.

In basic experiment, we examine the feasibility of our method on crossing and joining paths using two experimental robots. In simulation study, two different coordination strategies for collision avoidance are examined to clarify the effect on large number of robots on path network.

2. Basic assumptions

The basic features of a path network and a mobile robots are modeled as shown in Fig. 1(a),(b). The path network is defined as a graph that consists of nodes $p_i(i=1,\cdots,m)$ and directed arcs $l_{ij}(i=j)$ connecting node p_i and p_j. Here, the characteristics of the path network are assumed as follows:

- The direction of arcs are predetermined by a supervisory system so that the robots never come into a dead-end at any node. This indicates the numbers of inflow arcs and outflow arcs at the node p_i, which are denoted $In(p_i)$ and $Out(p_i)$, satisfies $In(p_i)> 1$, $Out(p_i)> 1$ and $\sum_i In(p_i)=\sum_i Out(p_i)$.
- All the arcs have enough length for multiple robots to move in a queue.

These conditions divide the path network into several closed routes containing fork, join and intersection nodes and allows robots to move along any arc without dead-lock from each other.
The characteristics of mobile robots are also assumed as follows:

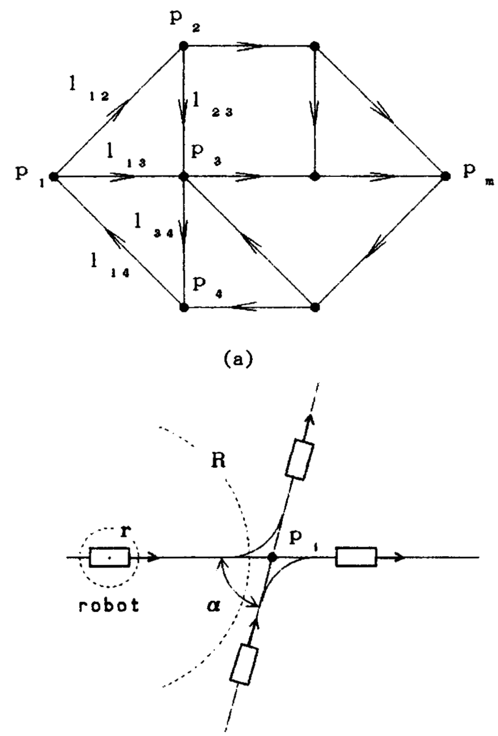
- All robots on the path network move individually along their routes at initially specified speeds in the case where there is no risk of collision.
- Each robot have its identical area and sensing area. The identical area is the area that contains robot body and the sensing area is the area in which a robot can obtain the position and velocity information of another robots. These areas, represented by closed circles of radius r and R, are equally assigned to all robots.

Throughout the paper, following notations and definitions are used: $\mathbf{R}^2$ denotes the two dimensional Euclidean space and a point $\mathbf{x}\in\mathbf{R}^2$ is shown with respect to a fixed global coordinate frame (x_1,x_2); $\mathbf{x}^t$ denotes the transpose of $\mathbf{x}$; $\|\mathbf{x}\|=\mathbf{x}^t\mathbf{x}$ denotes the norm on $\mathbf{R}^2$.

3. METHOD OF APPROACH

Velocity Vector Constraint

Given a set of individual mobile robots traveling over the path network, the major issue is how one can derive up some coordination strategies for the collision free motion control of each robot. In this section, we shall set up the velocity vector constraint for robots to derive simple coordination strategies, which assures robots of moving along path without collision. For such constraint, collision free and path following velocity vector regions can be considered.



(b)
Fig. 1 Basic feature of a path network(a) and a mobile robot(b)

In order for a robot to obtain collision free velocity vector region, we introduce a distance function[5] between neighboring robots using their state information. Let us assume that the i-th robot has m-number of other robots in its range of sensing at time k and can obtain their state information, position $\mathbf{x}_j(k)$ and velocity vector $\mathbf{v}_j(k)$ $(j=1,\cdots,m)$, at every discrete times $k,k+1,k+2,\cdots$. Then, the distance between the i-th robot and the j-th robot at time t from k can be estimated by the following function:

$$L_{ij}(t) \triangleq \|\mathbf{x}_{ij}(k) + \mathbf{v}_{ij}(k)t\|^2 \quad (1)$$

where $\mathbf{x}_{ij}(k)=\mathbf{x}_j(k)-\mathbf{x}_i(k)$, $\mathbf{v}_{ij}(k)=\mathbf{v}_j(k)-\mathbf{v}_i(k)$ are the relative position and the relative velocity vector of the i-th robot with respect to the j-th robot. This quadratic function of t gives its minimum value at time t_c:

$$L_{ij}(t_c) = \|\mathbf{x}_{ij}(k)\|^2 - \frac{(\mathbf{x}_{ij}(k)^t\mathbf{v}_{ij}(k))^2}{\|\mathbf{v}_{ij}(k)\|^2} \quad (2)$$

where $t_c = -\mathbf{x}_{ij}(k)^t\mathbf{v}_{ij}(k)/\|\mathbf{v}_{ij}(k)\|^2$.

If $t_c \geq 0$, $[L_{ij}(t)]_{min}$ is considered to give the distance at the closest point of approach between the two robots. Therefore, the collision free region for the velocity vector of the i-th robot at time k is given by solving the following inequality for $\mathbf{v}_i(k)$:

$$L_{ij}(t_c) \geq (2r) \quad (t_c \geq 0). \tag{3}$$

Since $\mathbf{x}_{ij}(k)$ and $\mathbf{v}_j(k)$ are known values, we have the solution of (4) by the inequality for the collision free velocity vector $\underline{\mathbf{v}}_i(k)$:

$$h_{ij} = \{\underline{\mathbf{v}}_i(k) \mid f_{ij}(\underline{\mathbf{v}}_i(k)) \leqq 0\} \tag{4}$$

where

$$f_{ij}(\underline{\mathbf{v}}_i(k)) = \mathbf{g}^t(\underline{\mathbf{v}}_i(k) - \mathbf{v}_j(k))$$

$$\mathbf{g} = \mathbf{x}_{ij}(k)^t \begin{bmatrix} 2r & a_{ij}\sqrt{\|\mathbf{x}_{ij}(k)\|^2 - 4r^2} \\ -a_{ij}\sqrt{\|\mathbf{x}_{ij}(k)\|^2 - 4r^2} & 2r \end{bmatrix}, \quad a_{ij} = \pm 1$$

(4) means two possible regions for collision free velocity vector of the i-th robot shown in Fig. 2. The sign of parameter a_{ij} in (4) determines these regions, i.e. the left side region($a_{ij} = +1$) or the right side region($a_{ij} = -1$) with respect to the j-th robot. For coordination of collision avoidance action between these robots, the sign of a_{ij} should correspond to that of a_{ji}. Thus, we can define the coordination strategy by setting a_{ij} fixedly or alternatively:

$$a_{ij} = 1 \text{ or } -1 \tag{5}$$

$$a_{ij} = \operatorname{sgn}(\mathbf{q}^t \mathbf{x}_{ij}(k)) = \begin{cases} 1 & (\mathbf{q}^t \mathbf{x}_{ij}(k) \geqq 0) \\ -1 & (\mathbf{q}^t \mathbf{x}_{ij}(k) < 0) \end{cases} \tag{6}$$

where $\mathbf{q}$ is a unit vector perpendicular to $\mathbf{x}_{ij}(k)$ as shown Fig. 2. In case (5) is applied to (4), collision avoidance between these robots is taken in a fixed rule: clockwise ($a_{ij} = 1$) or counterclockwise ($a_{ij} = -1$) with respect to the other robot. In case (6) is applied for (4), these robots avoid collision in accordance with their colliding situation. By applying (5) or (6) to (4), a unique region for velocity vector of the i-th robot is determined. In like manner, the collision free regions correspond to the other robots $(h_{i1}, \cdots, h_{im})$ can be obtained.

Next, we consider path following velocity vector region for the i-th robot. As shown in Fig. 3, let us define the virtual width of the current path as 2e, the offset of the i-th robot with respect to the virtual width at time k as $-\varepsilon_R(k)$ and $\varepsilon_L(k)$, the parallel and vertical unit vector with respect to the current path as $\mathbf{n}, \mathbf{n}'$. Then, the region for path following velocity vector $\underline{\mathbf{v}}_i(k)$ can be expressed as follows:

$$h_{io} = \{ \underline{\mathbf{v}}_i(k) \mid f_{io}(\underline{\mathbf{v}}_i(k)) \leqq 0 \}. \tag{7}$$

where

$$f_{io}(\underline{\mathbf{v}}_i(k)) = \mathbf{G}^t(\underline{\mathbf{v}}_i(k) - d_j\mathbf{n}')$$

$$\mathbf{G} = \begin{bmatrix} d_j\mathbf{n}' \\ -\mathbf{n} \end{bmatrix}, \quad \|\underline{\mathbf{v}}_i(k)\| \leqq v_{max}, \quad d_j = \varepsilon_L(k), -\varepsilon_R(k)$$

By collecting the above regions of $\underline{\mathbf{v}}_i(k)$ together, the velocity vector constraint of the i-th robot is given by the form:

$$H_i = \{ \underline{\mathbf{v}}_i(k) \mid \mathbf{f}_i(\underline{\mathbf{v}}_i(k)) \leqq 0 \} \tag{8}$$

where $\mathbf{f}_i(\underline{\mathbf{v}}_i(k)) = (f_{io} \ldots f_{im})^t$

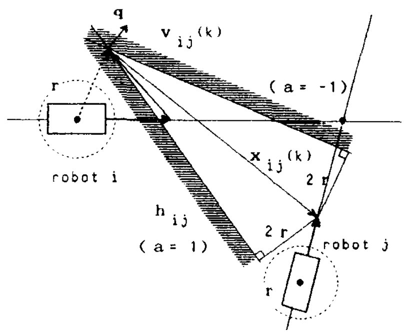


Fig. 2 Collision free velocity vector region for the i-th robot

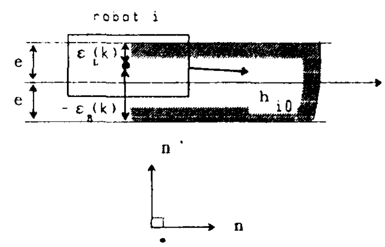


Fig. 3 Path following velocity vector region for the i-th robot

Local Optimization of Velocity Vector

Here, we establish the local optimization technique for the velocity vector of a mobile robot. Our objective is to determine the desired velocity vector which enables a robot to navigate without collision by following it.

Let us define the objective function for the i-th robot as follows:

$$J_i(\underline{\mathbf{v}}_i(k)) = W_1\|\mathbf{v}_i(k) - \underline{\mathbf{v}}_i(k)\|^2 + W_2\|v_o\mathbf{n} - \underline{\mathbf{v}}_i(k)\|^2 \tag{9}$$

where W_1 and W_2 are the positive weighting coefficients which satisfy $W_1 + W_2 = 1$. The first term is introduced to evaluate the cost with respect to the present velocity vector $\mathbf{v}_i(k)$, which restrains the change of velocity vector. The second term is also introduced to evaluate the cost with respect to the path following velocity vector $v_o\mathbf{n}$, which guide the i-th robot to its current path at speed v_o. Here, these coefficients are given in accordance with the following conditions of $\mathbf{v}_i(k)$ and $v_o\mathbf{n}$:

·In case $\mathbf{v}_i(k) \in H_i$, $v_o \mathbf{n} \notin H_i$
$W_1 = 1$, $w_2 = 0$
·In case $\mathbf{v}_i(k) \notin H_i$, $v_o \mathbf{n} \in H_i$
$W_1 = 0$, $w_2 = 1$
·In the other case of $\mathbf{v}_i(k)$, $v_o \mathbf{n}$
$W_1 = 0.5$, $w_2 = 0.5$

Now, consider the problem of finding $\underline{\mathbf{v}}_i(k)$ which minimize (11) under the constraint of (10):

$$\min J_i(\underline{\mathbf{v}}_i(k)) \text{ subj. to } \mathbf{f}_i(\underline{\mathbf{v}}_i(k)) \leqq 0 \qquad (10)$$

This problem is a type of nonlinear programming and its solution $\underline{\mathbf{v}}_i^*(k)$ gives the desired velocity vector of the i-th robot. Since $J_i(\underline{\mathbf{v}}_i(k))$ and $\mathbf{f}_i(\underline{\mathbf{v}}_i(k))$ are convex functions of $\underline{\mathbf{v}}_i(k)$, the solution of (13) can be found in uniquely[6]. Finally, robot navigation, avoiding collision and yet moving along the path, is carried out by following $\underline{\mathbf{v}}_i^*(k)$ at every sampling period.

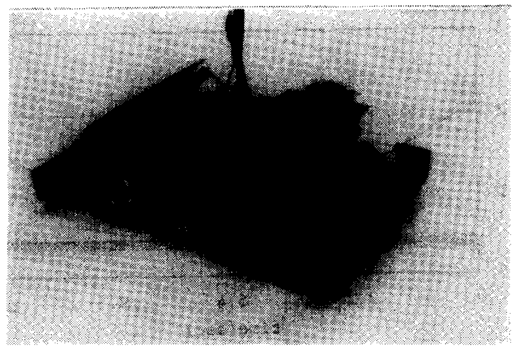
4. Basic experiments

we have developed an experimental system to examine our method, which consists of two self-contained mobile robots. Overview and specifications of this experimental robot are shown in Fig. 4. The on-board hardware of the robot is equipped with micro-computer (cpu:Z80A, 4MHz) and other three main components of driving, sensing and teaching unit. The robot has two independent drive wheels powered by DC motors, photo sensor alley for detecting the guide path and magnetic rotary encoders attached to both drive wheels for the measurement of position and velocity. Position and velocity information of the other robot is obtained by the mutual communication through 24bit PIO interface.

The schematic flow of the running control of the robot is shown in Fig. 5. The entire software of the running control is developed on VAX 2000 station, and be written into the 4k byte PROM. The cycling period of the running control is 128msec.

Here, we examine the basic performance of experimental robots on crossing and joining paths. In experiment, path layout for robots is given as shown in Fig. 6, where each path has 3m length and intersects, join in their middle points. G_1 and G_2,G_2 are the end points of robot 1 and robot 2. The parameters of robot, cruising speed, maximum speed, radius of identical area and sensing area, are given as follows:
$v_o = 0.2$m/sec, $v_{max} = 0.35$m/sec, $r = 0.25$m,
$R = 1 \sim 1.5$m.

The experimental results shown in Fig. 7,8 indicates the distance between two robots and the dynamical response of two robots when $R = 1.25$m, respectively. It can be seen From Fig. 7(a),(b) that the robots perform collision avoidance as close as possible both in case of crossing and joining, where the influence of range R on the collision avoidance is almost negligible. In addition, As seen in Fig. 8(a),(b), robots are controlled to move smoothly. It is also found that the lower limit of sensing area is $R = 0.98$m and robots come into a dead-lock if $R < 0.98$m.



Dimensions	372 (L) × 250 (W) × 120 (H)
Weight	3kg
Drive-motor	12vDC (Japan Servo co.)

Fig. 4 Overview and specifications of the experimental robot

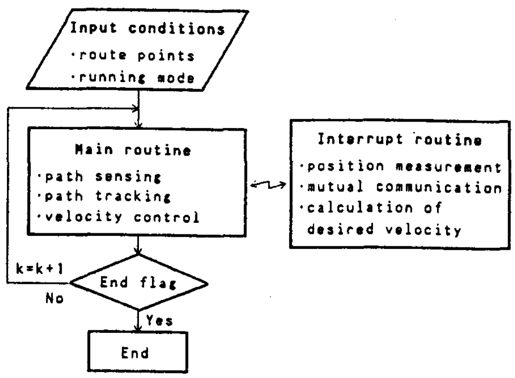


Fig. 5 Schematic flow of the running control

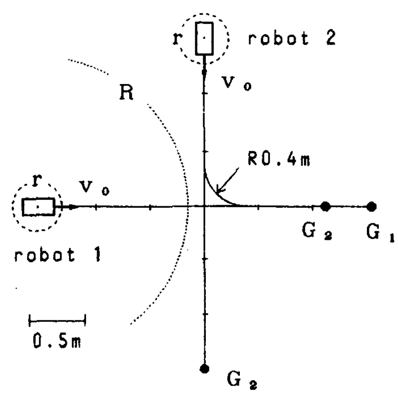
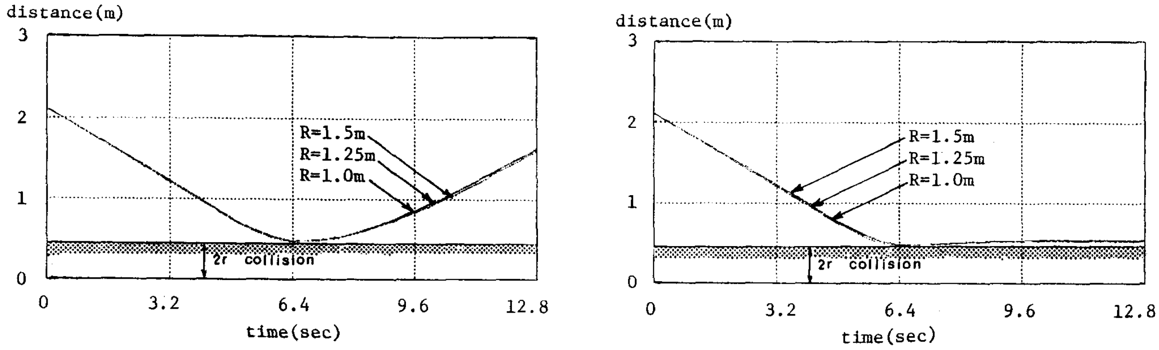
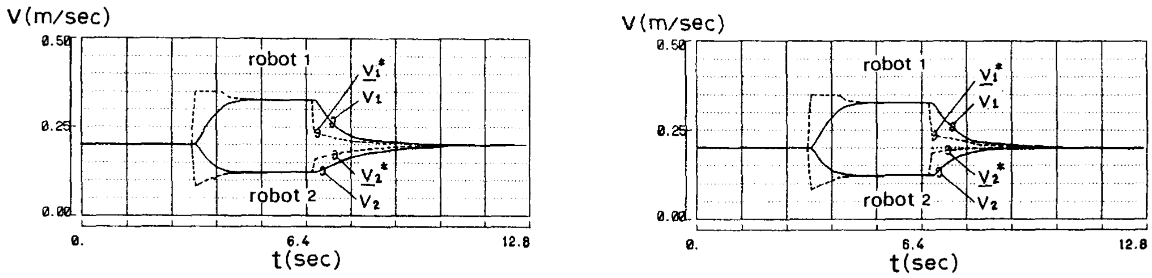


Fig. 6 Path layout for mobile robots



(a) (b)

Fig. 7 Distance between two mobile robots
((a)crossing, (b)joining)



(a) (b)

Fig. 8 Dynamical response of two mobile robots
((a)crossing, (b)joining)

5. Simulation study

We now examine the effectiveness of our method on large number of mobile robots traveling over path network. The main objective of simulation study is to clarify the effect on robots' traffic flow when the coordination strategies given in (5),(6) are applied for robots.

Here, we consider a simple dynamic model of a robot to design a. velocity vector following control system, which is described as

$$M\ddot{\mathbf{q}}(t) + C\dot{\mathbf{q}}(t) = K\mathbf{u}(t) \qquad (11)$$

Where the $\mathbf{q}(t)$ is the variable consists of robot yaw angle ϕ and robot position x , the $\mathbf{u}(t)$ is the input of right and left wheel drive unit. These variables are written as $\mathbf{q}(t)= (\phi\ x)^t$, $\mathbf{u}(t)= (u_r\ u_l)^t$. The M is the inertia matrix, the C is the friction coefficient matrix and the K is the parameter matrix of drive unit. For example, we use the following values for these matrixes:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \ C = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}, \ K = \begin{bmatrix} k_1 & k_2 \\ k_3 & k_4 \end{bmatrix}$$

where m_1= 0.028kgm, m_2= 3kg, c_1= 0.002kgm/s, c_2= 0.14kg/s, k_1= 0.05N/v, k_2= -0.05N/v, k_3= k_4= 0.2N/v.

The parameters of robots, v_0, v_{max}, r and R(= 1.25m) are given equally to these of the experimental robot described previously.

We also consider an example of path network shown in Fig. 8, which includes **21** nodes and **24** paths. The directions of paths are predetermined so that the path network contains five closed route with two forks (at p_3,p_7), three joins (at p_{11},p_{15},p_{20}) and intersection (at p_{20}). All robots are assumed to select these route randomly and travel from one node to another.

The simulation results shown in Fig. 9(a),(b) indicate the mean and scatters of traveling distance of robots within 2 minutes. Where (a) and (b) are the results that the coordination strategy (5) and (6) are respectively applied for robots.

In case (a), robots are allowed to avoid collision in the fixed rule. This condition is equivalent to giving priority levels for crossing and joining paths, where, likely to the traffic signals, robots on the higher priority path have the right of way . As a result, mean traveling distance of 20 robots decrease to 50% of its maximum value(24m). This suggests that traffic jam on lower priority paths becomes serious as the number of robots increases.

On the contrary, in case (b), robots avoid collision in alternative rule which correspond to their encounter situation. In this case, the resulting mean traveling distance up to 20 robots keeps more than 78% of its maximum value. As for scatter of traveling distance, it becomes about half as much as the result of case (a). This indicates that the smooth and high density traffic flow can be realized by applying our method.

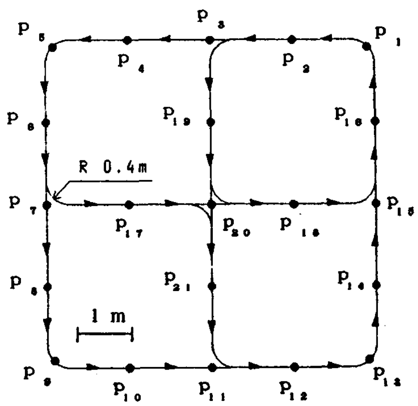
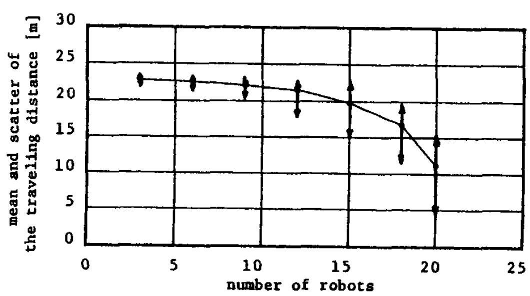
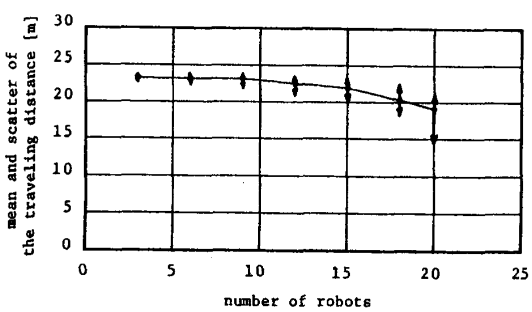


Fig. 9 An example of path network



(a)



(b)

Fig. 10 Simulation results
((a) and (b) are the result when Eq.(5) and Eq.(6) are applied for (4))

6. Conclusions

A local approach based on a nonlinear programming was given to solve the collision avoidance problem of multiple mobile robots on path network. In the approach, the problem is formulated as a motion control problem in terms of the velocity vector modification. A distance function obtained from the state information of neighboring mobile robots was introduced to set up the velocity vector constraint with simple coordination strategies. Under this constraint, the desired velocity vector, which enables robots to move without collision, was given as a optimal solution of the objective function. This method is implemented using experimental robot and simulated one.

From the basic experiment, it was shown that robots achieves acceptable performance both in case of crossing and joining. In simulation study, we have examined the effect of two different coordination strategies on robot traffic. The results shows that the alternative rule for collision avoidance allows robots smoother and higher density traffic flow than the fixed one.

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