Stock Portfolio Optimisation

(Final year project)

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Stock Portfolio Optimization

Optimizing Portfolio returns using Statistics & Computers Science.

The Problem

Problem

How to allocate M amount of money into N assets ?

Creating Portfolio is a very critically aspect of investment management. Even after active research and multiple mathematical frameworks

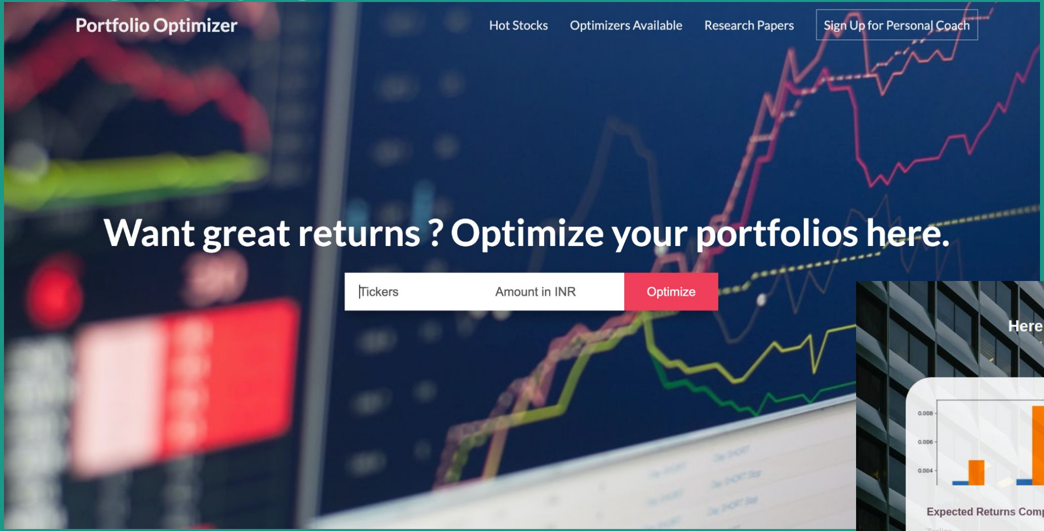
Small investors aren’t able to decide how much to allocate to each asset in order to improve on the returns per unit risk.

What investors do today

<u>Principle : Diversification of assets</u>

- **Identify Potential Stocks**
- **Allocate stocks** *by Human Understanding*
- **Volatility estimation** *mostly ignored*
- *Leads to* **Risky Portfolio**
- *Results in* **Average returns/ Losses**

Solution



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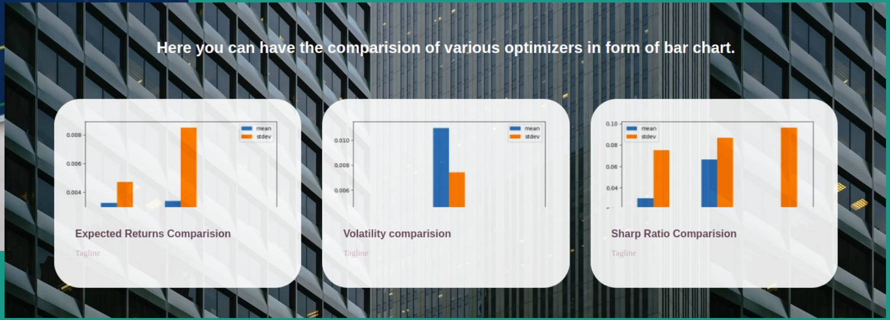
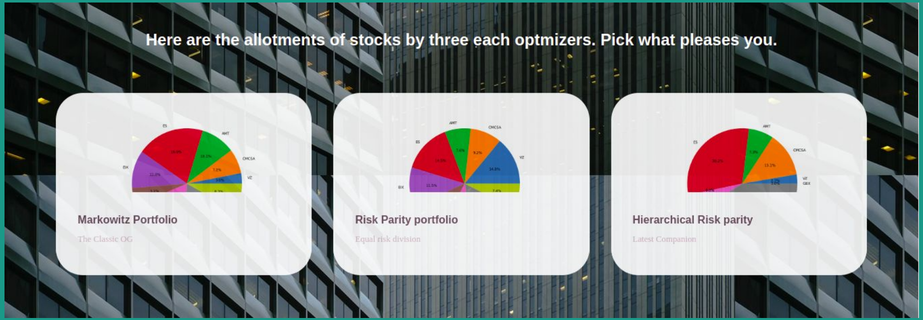
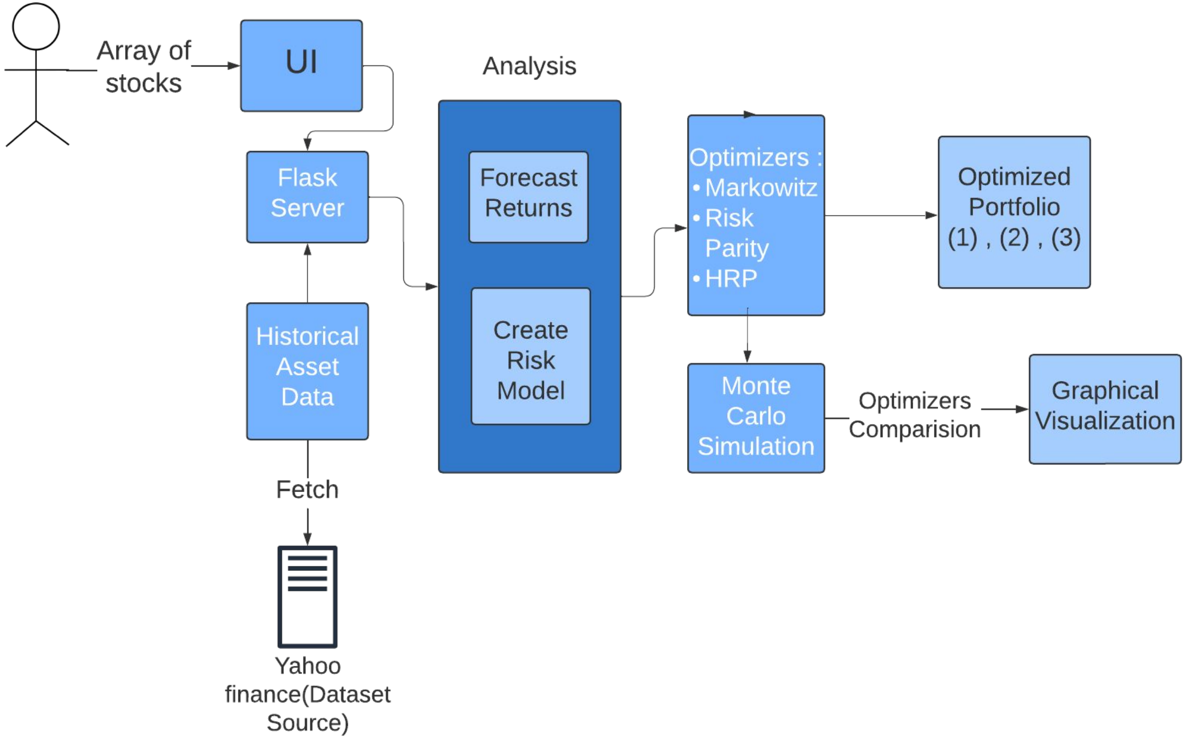


Here you can have the comparision of various optimizers in form of bar chart.
Expected Returns Comparision
Tagline
Volatility comparision
Tagline
Sharp Ratio Comparision
Tagline



Here are the allotments of stocks by three each optmizers. Pick what pleases you.
Markowitz Portfolio
The Classic OG
Risk Parity portfolio
Equal risk division
Hierarchical Risk parity
Latest Companion

Solution Diagram



Prerequisite

Expected Returns of assets :

- Percent returns given by each of the assets
- Arr => [return(i) | i < N]
- Come up with estimates, for example by extrapolating historical data.
- Expected Portfolio returns : $\mathrm{E}(R_p) = \sum_i w_i \, \mathrm{E}(R_i)$ where R*i* is the return on asset i and and *wi* is the weighting of component asset i.

Prerequisite

Risk Model (Covariance Matrix) :

- Formula

$$\operatorname{cov}(r^A, r^B) = \sigma_{r^A, r^B} = \frac{1}{n} \sum_{i=1}^{n} \left(r_i^A - \mu^A\right)\left(r_i^B - \mu^B\right)$$

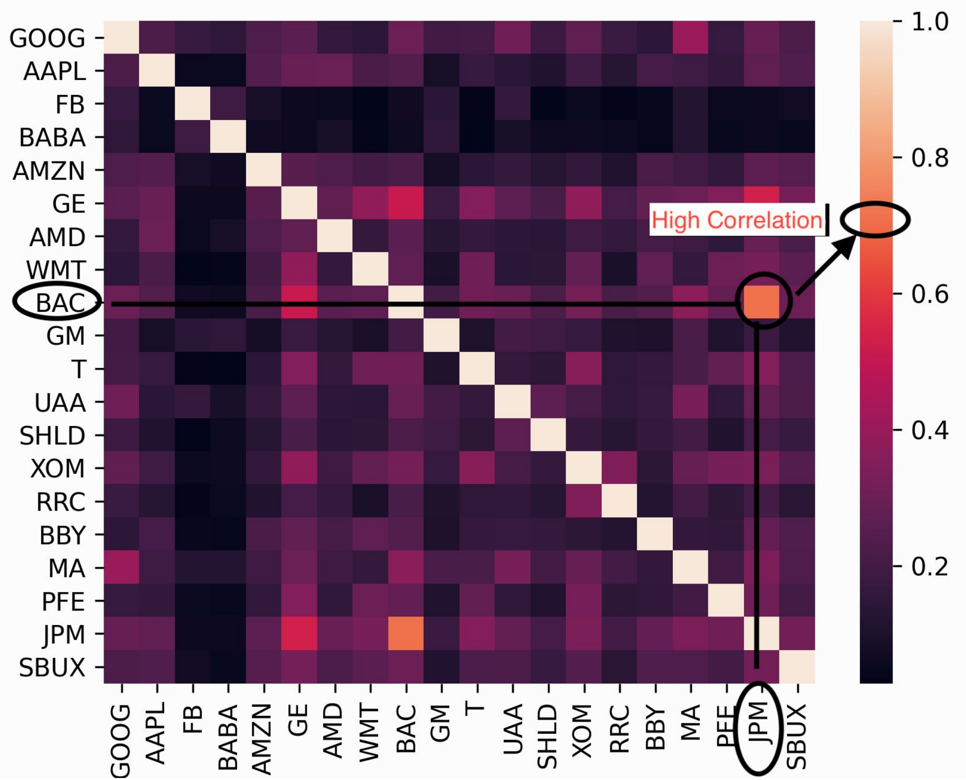
where μ corresponds to the average return and ri corresponds to return on i-th day.

- Portfolio return variance

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_{ij}$$

- Portfolio Return Volatility (Risk)

$$\sigma_p = \sqrt{\sigma_p^2}$$

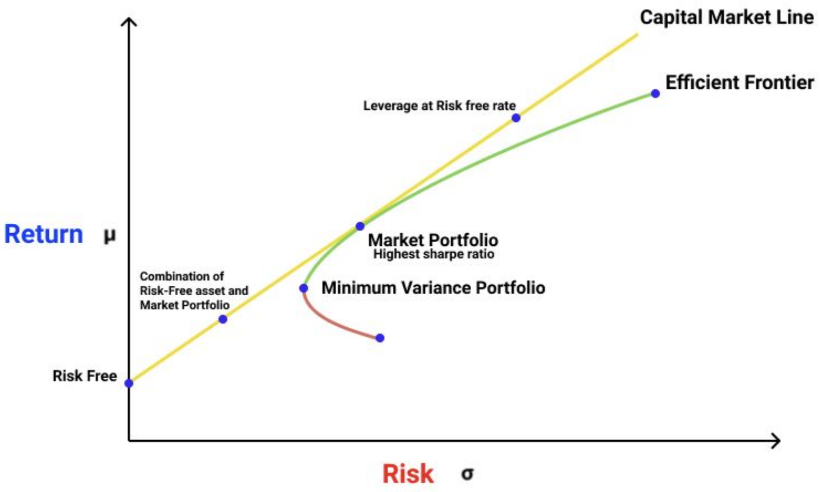


Modern Portfolio Theory[5]

- **Basic Idea** : Classical Mean-Variance Optimizer (Efficient Frontier)[4]
    - Input ← (Expected Returns , Risk model)
    - Output → Optimal Weights distribution.

- Maximize Sharpe Ratio among portfolios on efficient frontier
    - Slope of capital allocation line

- Dr. Harry Markowitz (1952)
    - 1990 Nobel Memorial Prize in Economic Sciences



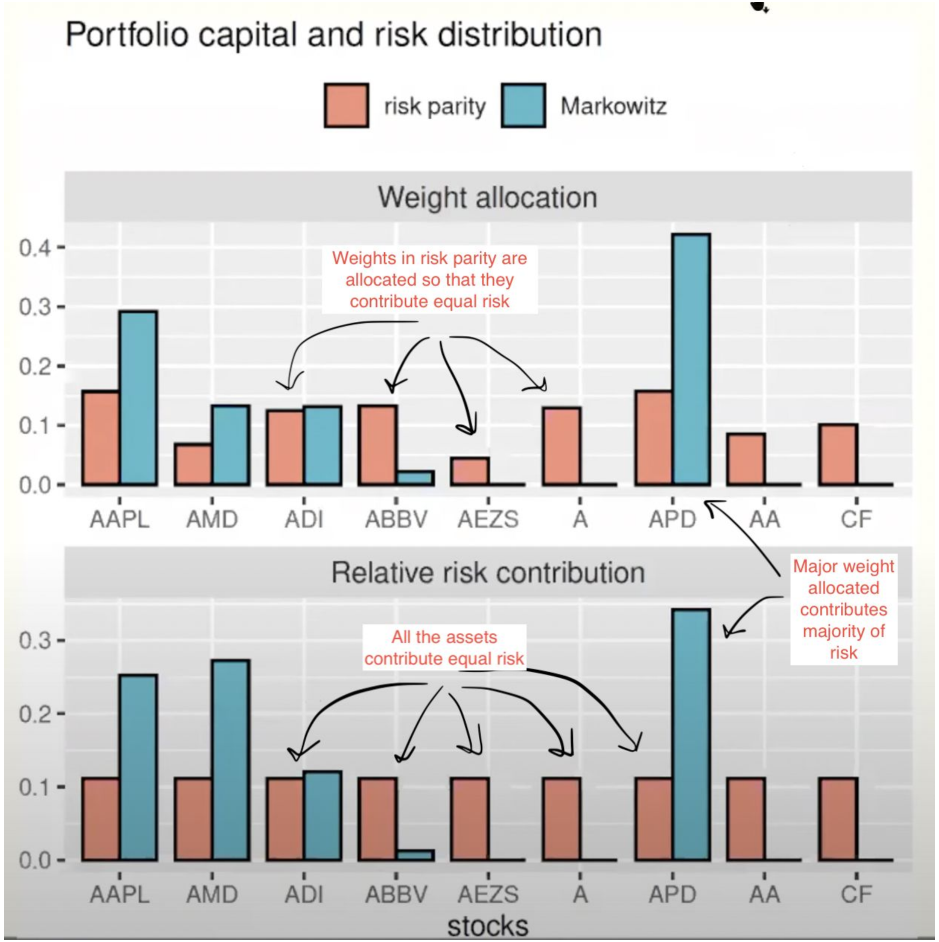
[EXTENSION] Black-Litterman - Allocation Inputs from investor → Improves Expected Returns

Use Semantic Analysis on twitter/investment forums in order to estimate drops/increase in stocks → input to Black-Litterman Allocation

Risk Parity[4]

- **Basic Idea** : Design a Portfolio such that the risk is equally distributed among asset.

- Volatility is the measure of risk, the volatility of each asset is expected to be 1/len(assets)
- Multiple ways to formulate the problem, one below
  - Iterative Algorithm (No. of Iterations ∝ closer to optimal)
  - Start with equal weighted portfolio (Each asset ←Budget/len(assets))
  - Each iteration reduce weights of assets with high risk ratings and increase weights of low risk ratings using a predefined convergence speed.



Hierarchical Risk Parity (HRP)[2],[3]

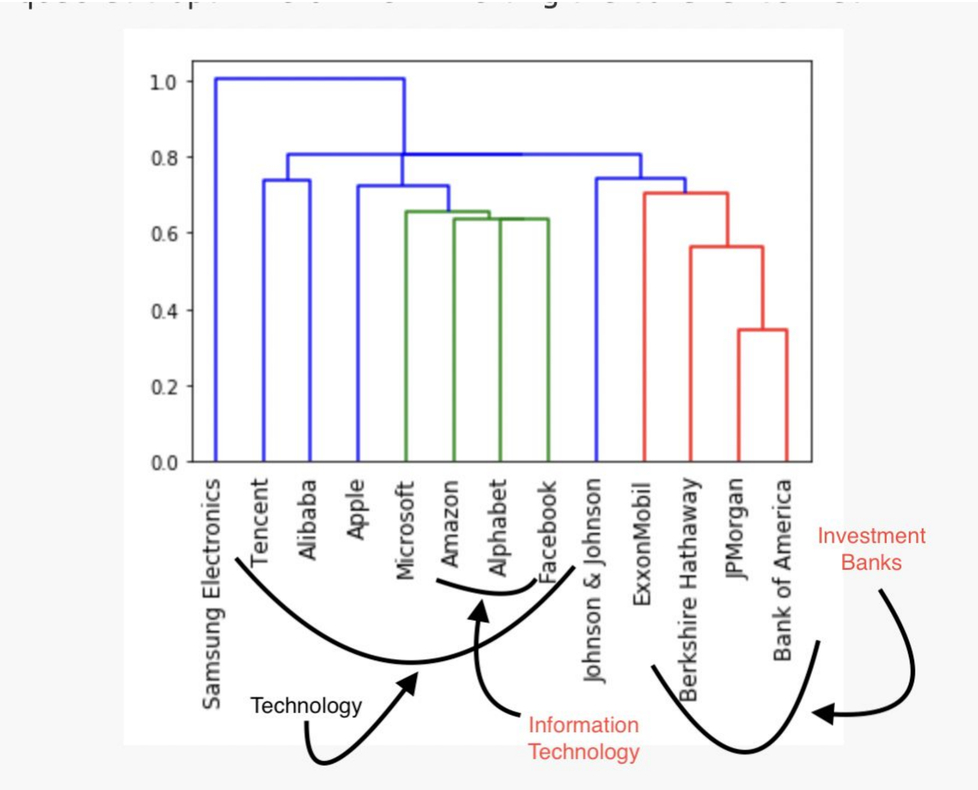
- **<u>Basic Idea</u>** : Exploit the intrinsic hierarchy of the correlation matrix.
- This algorithm was proposed by Lopez de Prado in 2016.

- Stage 1: Tree Clustering
  - Build a distance matrix ← Correlation Matrix where, $d(i,j) = \sqrt{1/2\,(1 - \varrho\,(i,j))}$
  - Build euclidian distance matrix D*(i,j) = sqrt(summation (d(k,i) ^ 2 - d(k,j) ^ 2) for all k)
  - Followed by clustering using some heuristic over Euclidean distance
- Stage 2: Quasi Diagonalisation (using clusters created above)
  - Reorganizes rows & cols of covariance matrix → largest values lie along the diagonal.
  - Renders property: Similar investments ⟷ together ; dissimilar ⟷ far apart.
- Stage 3: Recursive Bisection (Bottom Up and Top Down)
  - **Distribute the allocation through recursive bisection based on cluster covariance.**

Hierarchical Risk Parity (HRP)[2],[3]

Clustering in HRP produces a tree like structure where similar assets come closer to each other in the Hierarchical structure.

Dendrogram on right presents how assets would be allocated in a top down fashion splitting into two parts at each section



Monte Carlo Simulation[1]

Why & What ?

- A fit-all solution doesn't exist.
  - Although mathematically correct, CLA is known to be a poor estimator of the optimal solution out-of-sample.

- System with N random variables, where the expected value of draws $\rightarrow \mu$, and the variance of these draws $\rightarrow V$,the covariance matrix. Calculate $\rightarrow \omega$
- The input variables V and μ are typically unknown.
- Lead to unstable solutions
  - solutions where a small change in the inputs $\rightarrow$extreme changes in $\hat{\omega}$.

Monte Carlo Simulation[1]

How ?

- Derives the simulated pair $\{\hat{\mu}, \hat{V}\}$ from original $\{\mu, V\}$.
- De-noise the covariance matrix .
- Estimate $\hat{\omega}*$ from $\{\hat{\mu}, \hat{V}\}$ according to various alternative methods. (Here - Markowitz , Risk Parity , Hierarchical Risk Parity)

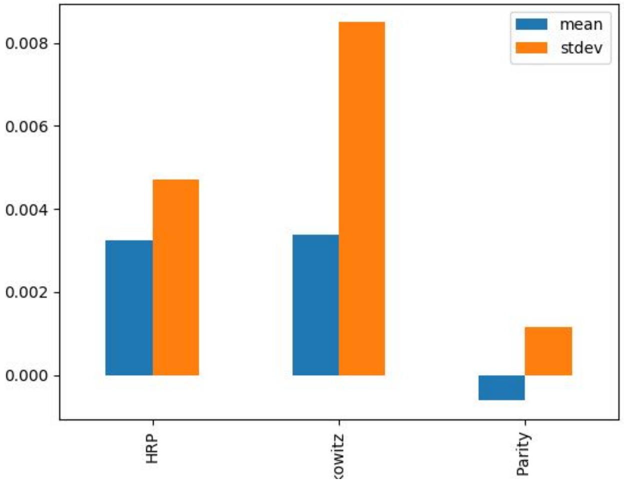
- Combine all previous steps into a Monte Carlo experiment, whereby optimal allocations $\hat{\omega}*$ are computed on a large number of simulated pairs $\{\hat{\mu}, \hat{V}\}$
- Computes the true optimal allocation $\omega*$ from the pair $\{\mu, V\}$, and compares that result with the estimated $\hat{\omega}*$.
  - computes the standard deviation of the differences between $\hat{\omega}*$ and $\omega*$

- Pick the method which suits best.

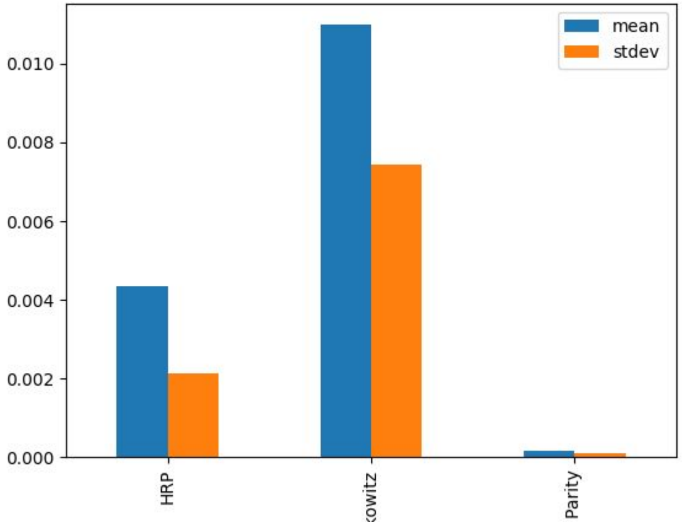
Analysis performed

Portfolio : [VZ,CMCSA,AMT,ES,EIX,TSN,GLPI,WMT,GBX]
Analysis Month : 05/2020
Amount to invest : $ 10,000

Expected Outcome Error estimate



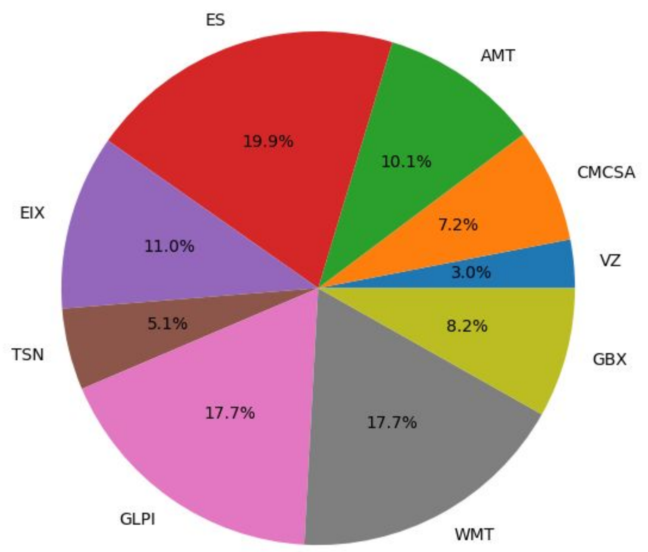
Variance Error Estimate



Analysis performed

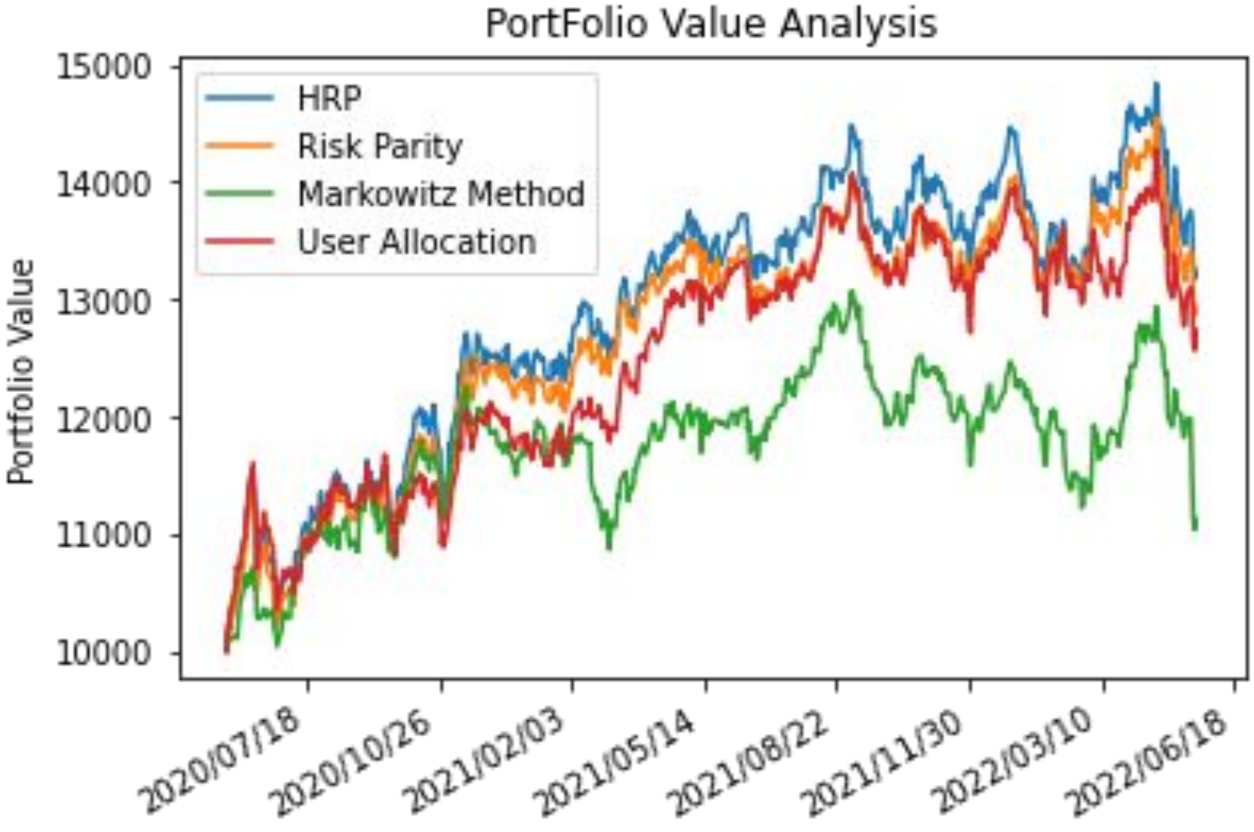
1. Annualized mean return by HRP : 14.84%
2. Annualized mean return by Markowitz : 14.05%
3. Annualized mean return by Risk Parity : 14.18%

Allocate w.r.t HRP.



Analysis performed

Watching how different strategies work over a period of 2 years after analysis is performed.



Methodology & Technology to be used

- Optimization based on
  - Modern Portfolio Theory
  - Risk Parity
  - Hierarchical Risk Parity
- Analysis
  - Risk Analysis : Covariance between stocks , etc
  - Portfolio Forecast : Mean, Weighted mean, etc
  - Monte Carlo Simulation.
- Python for analysis
- Modules : pandas, numpy, matplotlib, seabourn, scikit-learn, PyPortfolioOpt , mcos
- Flask for development of server.
- HTML/CSS & Javascript for UI development.

Questions?

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