Number system

❖ 📘 Topic 1: Introduction

1. Natural Numbers :-

- The numbers used for counting are called natural numbers.
- Example: 1, 2, 3, 4, 5, ...
- These are denoted by the symbol $\mathbb{N}$.

2. Whole Numbers :-

- Natural numbers along with 0 are called whole numbers.
- Example: 0, 1, 2, 3, 4, ...
- These are denoted by the symbol $\mathbb{W}$.

3. Integers :-

- All whole numbers and their negative counterparts are called integers.
- Example: -3, -2, -1, 0, 1, 2, 3, ...
- These are denoted by the symbol $\mathbb{Z}$.

4. Rational Numbers :-

- Numbers that can be written in the form of p/q, where p and q are integers and $q \neq 0$, are called rational numbers.
- Example: 1/2, -4/5, 0.75, -2, 3
- These are denoted by the symbol $\mathbb{Q}$.
- All integers and fractions are rational numbers.
- Rational numbers can be terminating or non-terminating but repeating.

5. Irrational Numbers :-

- Numbers that cannot be written as p/q are called irrational numbers.
- Example: $\sqrt{2}$, $\sqrt{3}$, π (pi), 0.1010010001...
- These are non-terminating and non-repeating decimals.
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6. Real Numbers :-

- The collection of rational and irrational numbers is called real numbers.
- Example: 2, -5, $\sqrt{3}$, 1.5, π
- These are denoted by the symbol $\mathbb{R}$.
- All the numbers we commonly use in real life are real numbers.

⭐ **Important Points :-**

- ***Every natural number is a whole number but every whole number is not a natural number.***
- ***Every whole number is an integer but every integer is not a whole number.***
- ***Every integer is a rational number but every rational number is not an integer.***

Every rational and irrational number is a real number but a real number may not be rational.

❖ 📘 Topic 2: Irrational Numbers

1. Irrational Numbers :-

- Numbers that cannot be expressed in the form p/q, where p and q are integers and q ≠ 0, are called irrational numbers.
- These numbers have non-terminating and non-repeating decimal expansions.
  :- Example: $\sqrt{2}$ = 1.41421356... (goes on forever and doesn't repeat)
  :- Example: π = 3.141592653... (never ends, never repeats)
- Irrational numbers are not rational and cannot be written as fractions.
- Some common irrational numbers are:
  :- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, π, e, etc.
- Perfect squares like $\sqrt{4} = 2$ are not irrational.Only non-perfect square roots are irrational.
- Sum or product of a rational and an irrational number is usually irrational.
  :- Example: $2 + \sqrt{3}$ is irrational and $3 \times \sqrt{2}$ is irrational, etc.
- Irrational numbers are part of real numbers.They exist on the number line just like rational numbers.

⭐ **Important Points :-**

- ***Decimal form of irrational numbers is non-terminating and non-repeating. This is the main identity of an irrational number.***
- ***Irrational numbers cannot be written as fractions (p/q). If you can write a number as p/q, it is not irrational.***
- ***All non-perfect square roots are irrational. Example: $\sqrt{2}$ is irrational but $\sqrt{4} = 2$ is rational.***

❖ 📘 Topic 3: Real Numbers

1. Real Numbers :-

- All rational and irrational numbers together form the set of real numbers.

- Real numbers include:
  👉 Natural numbers (1, 2, 3, ...)
  👉 Whole numbers (0, 1, 2, ...)
  👉 Integers (-3, -2, -1, 0, 1, ...)
  👉 Rational numbers (1/2, -3/4, 0.75, 5)
  👉 Irrational numbers (√2, π, etc.)

- Symbol: Real numbers are denoted by the letter ℝ.
- Every point on the number line represents a unique real number. Real numbers completely fill the number line.
- All arithmetic operations like addition, subtraction, multiplication, and division (except by zero) are valid for real numbers.
- Decimal representation of real numbers can be of two types:
  - Terminating or non-terminating repeating (rational numbers)
  - Non-terminating non-repeating (irrational numbers)

⭐ Important Points :-

- ***ℝ = Rational Numbers + Irrational Numbers. Real numbers include both rational and irrational numbers.***
- ***Every real number has a decimal representation. This helps in placing it on the number line.***
- ***All numbers used in real life are real numbers. Example: Distance, weight, money, time, etc.***

📘Topic 4: Operations on Real Numbers

- Real numbers can be added, subtracted, multiplied, and divided (except by 0).
- These operations follow certain algebraic properties or laws.

1. Closure Property

- If we perform an operation on any two real numbers, the result is also a real number.
  👉 Example: 2 + √3 = 2 + 1.732 = 3.732 (which is a real number)
- ✔️ Real numbers are closed under addition, subtraction, multiplication, and division (except division by zero).

2. Commutative Property

- Changing the order of numbers does not change the result.
  👉 $a + b = b + a$
  👉 $a \times b = b \times a$

👉 Example: $4 + 5 = 5 + 4 = 9$

- ✔️ Real numbers are commutative under addition and multiplication.

3. Associative Property

- Grouping of numbers does not change the result.
  👉 $(a + b) + c = a + (b + c)$
  👉 $(a \times b) \times c = a \times (b \times c)$
  👉 Example: $(2 + 3) + 4 = 2 + (3 + 4) = 9$
- ✔️ Real numbers are associative under addition and multiplication.

4. Distributive Property

- Multiplication distributes over addition.
  👉 $a \times (b + c) = a \times b + a \times c$
  👉 Example: $2 \times (3 + 4) = 2\times3 + 2\times4 = 6 + 8 = 14$
- ✔️ This property is very useful in algebraic simplification.

5. Identity Elements

- Additive identity: 0
  👉 $a + 0 = a$
- Multiplicative identity: 1
  👉 $a \times 1 = a$
- ✔️ 0 and 1 are identity elements for addition and multiplication.

6. Inverse Elements

- Additive inverse: For every real number a, there exists -a such that:
  👉 $a + (-a) = 0$
- Multiplicative inverse: For every non-zero real number a, there exists 1/a such that:
  👉 $a \times (1/a) = 1$
- ✔️ Every non-zero real number has a multiplicative inverse.

⭐ Important Points :-

- ***All basic operations are valid for real numbers except division by zero. You can never divide by 0.***
- ***Real numbers follow algebraic properties like closure, commutativity, associativity, distributivity. These make calculation rules easy.***
- ***Identity and inverse elements are useful for solving equations. Additive identity = 0; Multiplicative identity = 1.***

📘 Topic 5: Decimal Expansion of Real Numbers

- Every real number has a decimal expansion.
  👉 Example: 2 = 2.000..., 1/2 = 0.5, √2 = 1.4142135...
- There are two types of decimal expansions of real numbers:
  (i) Terminating
  (ii) Non-terminating

1. Terminating Decimal

- A decimal which comes to an end after a finite number of digits is called a terminating decimal.
  👉 Example: 1/2 = 0.5, 3/4 = 0.75, 7/8 = 0.875
- All terminating decimals are rational numbers.

2. Non-Terminating Recurring Decimal

- A decimal which does not end but repeats a pattern is called a non-terminating recurring decimal.
  👉 Example: 1/3 = 0.333..., 22/7 = 3.142857142857...
  👉 Written as: 0.\overline{3}, 3.\overline{142857}
- These are also rational numbers.

3. Non-Terminating Non-Recurring Decimal

- A decimal which neither ends nor repeats is called a non-terminating non-recurring decimal.
  👉 Example: √2 = 1.41421356..., π = 3.14159265...
- These are irrational numbers.

⭐ Important Points :-

- ***A decimal terminates if its denominator (in lowest form) has only 2 and/or 5 as prime factors.***
  👉 Example: 3/8 → 0.375 → denominator is 8 = 2^3
- ***If the decimal is non-terminating but repeating, then the number is rational. Rational numbers can be written as fractions.***
- ***If the decimal is non-terminating and non-repeating, then the number is irrational. Irrational numbers cannot be written as fractions.***
- ***All real numbers have a decimal form, either terminating or non-terminating.***

📘 Topic 6: Representation of √x on Number Line

- Irrational numbers like √2, √3, etc. can be shown on the number line using geometry.
- We use the Pythagoras Theorem to represent square roots on the number line.

✅ Steps to Represent √2 on Number Line :-

- Draw a number line and mark a point O at 0 and A at 1 unit from O.
  👉 So, OA = 1 unit
- From point A, draw a perpendicular line AB of 1 unit.
- Join OB. Now, triangle OAB is a right triangle with sides 1 and 1.
  👉 By Pythagoras Theorem:
  👉 $OB = \sqrt{(1^2 + 1^2)} = \sqrt{2}$
- Now take OB as radius and O as center, draw an arc to cut the number line.
  👉 That point on number line is √2.

✅ Similarly :-

1. To represent √3:
   - Make right triangle with base 1 unit and height √2 unit.
2. To represent √5:
   - Make triangle with sides 2 and 1, because $\sqrt{(2^2 + 1^2)} = \sqrt{5}$

⭐ Important Points :-

- ***√x can be drawn using right triangle and compass method. Use of Pythagoras theorem is necessary.***
- ***This method helps us to place irrational numbers on number line. Though they can't be written as fractions, they do have positions on number line.***
- ***Compass and ruler construction is used for accurate placement.***

📘 Topic 7: Laws of Exponents for Real Numbers

- Exponents are used to express repeated multiplication of the same number.
  👉 Example: $2 \times 2 \times 2 = 2^3$
- Exponents are also called powers.
  👉 a^n means "a raised to the power n"
- The number a is the base, and n is the exponent or index.

✅ Laws of Exponents :-

- Law 1:
  - 👉 Example: $2^3 \times 2^2 = 2^5 = 32$
  - ✔️ Add the powers when the base is same and multiplying.
- Law 2: $(a \neq 0)$
  - 👉 Example: $5^4 \div 5^2 = 5^2 = 25$
  - ✔️ Subtract the powers when dividing same bases.

- Law 3:
    - 👉 Example: $(3^2)^3 = 3^6 = 729$
    - ✔️ Multiply the powers when raising a power to another power.
- Law 4: $(a \neq 0)$
    - 👉 Example: $7^0 = 1$, $100^0 = 1$
    - ✔️ Any non-zero number raised to power 0 is always 1.
- Law 5: $(a \neq 0)$
    - 👉 Example: $2^{-3} = 1/8$
    - ✔️ Negative power means reciprocal.
- Law 6:
    - 👉 Example: $(2 \times 3)^2 = 2^2 \times 3^2 = 4 \times 9 = 36$
- Law 7: $(b \neq 0)$
    - 👉 Example: $(2/3)^2 = 2^2/3^2 = 4/9$

⭐ **Important Points :-**

- ***Laws of exponents apply to real numbers, including rational and irrational numbers. These laws help simplify expressions.***
- ***Never apply exponent laws if the bases are different. These rules work only when the bases are same.***
- ***Negative exponents give reciprocal values. Useful in writing very small or large numbers.***

📘 Topic 8: Rationalisation

- Rationalisation is the process of removing irrational numbers from the denominator of a fraction.
- If the denominator contains a surd (like √2, √3), we multiply numerator and denominator by a suitable number to make the denominator rational.
- This suitable number is called the conjugate (if the denominator is of the form a + √b or a - √b).

✅ Case 1: When Denominator is a Simple Surd
👉 Example:

1. Multiply numerator and denominator by √2:
   👉Now denominator is rational.

✅ Case 2: When Denominator is in the form (a + √b)
👉 Example:

1. Multiply numerator and denominator by conjugate (3 - √2):

Now apply identity:
So, Denominator is now rational.

⭐ Important Points :-

- ***Rationalisation is used to simplify expressions and make denominators rational. Helps in solving and comparing values easily.***
- ***Use identity:***
- ***for rationalising complex denominators. This is the key formula.***
- ***Always multiply with same surd or its conjugate to remove irrationality. Conjugate of a + √b is a - √b and vice versa.***