Optimum rotation

Lecture Notes on Natural Resource Economics (AEC- 608)

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May 23, 2025

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1 Introduction

Forests and fisheries are both renewable resources, but their optimal management differs due to underlying biological and economic principles. While the economic logic of maximizing net benefits applies to both, the forest problem is fundamentally one of **divestment**: determining the optimal time to harvest the entire stock and begin a new rotation. In contrast, fisheries are often managed as a problem of continuous harvest.

Key difference: In forestry, the decision is about when to cut and replant, whereas in fisheries, it is about how much to harvest each period.

2 Theory and Objective

The central objective in forest economics is to determine the **optimal rotation period**—the age at which a stand should be harvested to maximize net economic benefits for society or a private owner.

Optimal rotation: The age at which the present value of net returns from timber production (and, where relevant, other forest services) is maximized.

3 Assumptions

- **Even-aged management:** All trees in the stand are of the same age.
- **Certainty:** The growth function of the stand is known with certainty.
- **Perfect capital markets:** Unlimited access to borrowing or lending at a constant market interest rate r.
- **Fixed net stumpage price:** The price per unit volume of timber is constant; non-use values are ignored due to lack of markets.
- **Regeneration costs:** Denoted by WE, where w is the cost per unit of silvicultural effort and E is the effort; incurred immediately after harvest.
- **Timber yield as a function of time:** $Q = f(\text{Time})$.

4 Forest Growth Concepts

4.1 Growth Pattern

The growth of a single tree (or stand) typically follows an **inverse S-shaped curve** over time.

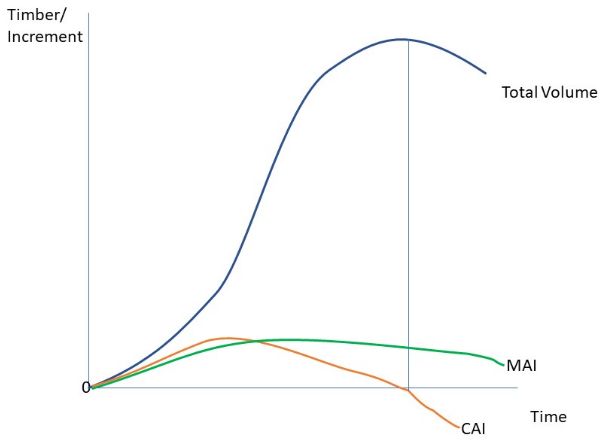

Figure 1: CAI and MAI curves

4.2 Current Annual Increment (CAI)

Current Annual Increment (CAI) is the incremental change in volume year by year—analogous to the marginal product curve.

$$\text{CAI} = \frac{dQ}{dt} = f'(t)$$

4.3 Mean Annual Increment (MAI)

Mean Annual Increment (MAI) is the average annual increase in volume up to time t:

$$\text{MAI}(t) = \frac{f(t)}{t}$$

It is analogous to the average product curve.

5 Criteria for Optimal Rotation

Several criteria have been proposed for determining the optimal rotation period in forestry:

5.1 1. Maximization of Gross Yield

Criterion: Harvest when the stand volume is maximized.

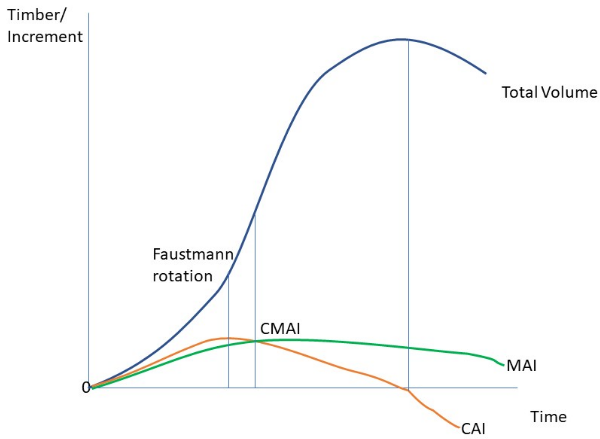

Figure 2: CAI and MAI curves. The intersection is the culmination of MAI (CMAI).

$$\max Q = f(t)$$
$$\frac{dQ}{dt} = \text{CAI} = 0$$

Limitation: Ignores costs, interest on capital, and land value. Not theoretically supported.

5.2 2. Maximization of Annual Yield (Culmination of MAI)

Criterion: Harvest at the age when MAI is maximized, i.e., at the culmination of mean annual increment (CMAI), also known as the biologically maximum sustained yield (MSY).

$$\max \left(\frac{Q_t}{t}\right)$$
$$\frac{d}{dt}\left(\frac{Q_t}{t}\right) = 0$$
$$\Rightarrow \text{CAI} = \text{MAI}$$

Interpretation: The age where the current annual increment equals the mean annual increment.

Limitation: Ignores economic considerations, such as costs, prices, and discounting.

5.3 3. Maximization of Discounted Net Revenues from a Single Rotation

Criterion: Maximize the present net worth (PNW) of a single rotation, accounting for the time value of money.

$$\text{PNW} = p \cdot V_t \cdot e^{-rt} - C_0$$

where

- p: fixed net stumpage price per unit volume,
- V_t: volume at age t,
- r: discount rate,
- C_0: regeneration cost (incurred at $t = 0$).

Optimality condition:

$$\frac{d}{dt}\left[pV_t e^{-rt}\right] = 0$$

$$p\frac{dV_t}{dt}e^{-rt} - pV_t r e^{-rt} = 0$$

$$\Rightarrow \frac{dV_t}{dt} = rV_t$$

$\Rightarrow$ Marginal value product of keeping the tree for one more year = opportunity cost of capital

Limitation: Does not account for future rotations or land opportunity cost.

5.4 4. Maximization of Discounted Net Revenues from Infinite Rotations (Faustmann Formula)

Criterion: Maximize the present value of an infinite series of rotations, accounting for both timber and land opportunity costs.

$$\text{Faustmann Formula:} \quad \text{SEV} = \frac{pV_t e^{-rt} - C_0}{1 - e^{-rt}}$$

where SEV is the soil expectation value (or land expectation value).

Optimality condition:

$$\frac{d}{dt}\left[\frac{pV_t e^{-rt} - C_0}{1 - e^{-rt}}\right] = 0$$

Interpretation: The optimal rotation age is when the rate of change in stumpage value equals the opportunity cost of holding timber plus the opportunity cost of land (discounted SEV).

$$\frac{dV_t}{dt} = rV_t + r \cdot \text{SEV}$$

Superiority: The Faustmann rotation is generally shorter than the single-rotation optimum, as it incorporates the opportunity cost of land.

6 Summary Table: Rotation Criteria

Table 1: Comparison of Rotation Criteria

Criterion	**Key Features**
Max Gross Yield	Harvest at maximum stand volume; ignores costs and time value.
Max MAI (CMAI)	Harvest at culmination of mean annual increment; maximizes long-term average yield.
Max Discounted Net Revenue (Single Rotation)	Maximizes present value from one harvest; ignores future rotations and land cost.
Faustmann (Infinite Rotations)	Maximizes present value from infinite rotations; incorporates land opportunity cost.

7 Key Insights

- The optimal rotation period depends on biological growth, timber price, costs, and especially the discount rate.
- Higher discount rates lead to earlier harvests.
- The Faustmann formula is the gold standard in forest economics, as it incorporates both timber and land values, and is widely used in policy and management.
- In practice, non-timber values (biodiversity, recreation, carbon) may alter the optimal rotation, but are often difficult to price.

8 Conclusion

Optimal forest rotation is a classic problem in resource economics, balancing biological growth with economic returns and the time value of money. The Faustmann approach remains the most comprehensive and widely accepted criterion, ensuring sustainable and economically efficient forest management.

9 Practice questions

- In case of forest, what are the biological rotation criteria's?
- In case of economic criteria used to determine the optimum rotation age, what additional components are considered in decision making?
- What is the difference between single rotation and infinite rotations?
- Elaborate on rotation criteria for a single rotation cycle.
- Why the optimum rotation time in case of Faustmann formula is shorter than that of single rotation?

Optimal forest rotation is a classic problem in resource economics, balancing biological growth with economic returns and the time value of money. The Faustmann approach remains the most comprehensive and widely accepted criterion, ensuring sustainable and economically efficient forest management.