# Optimum extraction of renewable resources

Lecture Notes

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# 1 Introduction

Renewable resources, such as forests, fisheries, and groundwater, differ fundamentally from exhaustible resources because they can replenish themselves through natural growth or regeneration. This regenerative property means that, if managed properly, renewable resources can provide a continuous flow of benefits to society over time, rather than being depleted after a single use.

The management of renewable resources centers on balancing current use with future availability. The goal is to achieve sustainable yields-harvest levels that do not compromise the resource's ability to regenerate-while also considering economic, social, and ecological objectives. Effective management ensures that both present and future generations can benefit from the resource.

Key questions that arise in the economics and management of renewable resources include:

- **What is the optimal rate of use for a renewable resource?** This involves determining how much of the resource can be harvested without reducing its stock below a level that can sustain future use.
- **How does the biological growth process affect economic decision-making?** Since the rate at which a resource regenerates depends on its current stock, understanding the biological dynamics is crucial for setting harvest policies.
- **What are the implications of property rights and common-property regimes?** The way in which access to the resource is governed-whether through private ownership, common property, or open access-has significant effects on incentives and outcomes.
- **How do the economic optimum and the biological optimum (maximum sustainable yield) compare?** While biology may suggest a certain harvest level is sustainable, economic considerations (such as costs, prices, and discounting) may lead to a different, often more conservative, optimum.

# 2 Biological Growth and Resource Dynamics

The defining feature of a renewable resource is its ability to regenerate or replenish itself over time. The stock of the resource at any time, denoted $X(t)$, changes according to the interplay between its natural growth and the rate at which it is harvested or extracted by humans.

Mathematically, the evolution of the resource stock can be described by a differential equation:

$$\frac{dX}{dt} = G(X) - H(t)$$

where:

- $G(X)$ is the natural growth function, representing the net increase in resource stock due to biological processes such as reproduction, recruitment, or regrowth. The form of $G(X)$ depends on the resource but is often assumed to be logistic for simplicity.
- $H(t)$ is the harvest or extraction rate at time $t$, representing the amount of resource removed by human activity.

**Typical growth curve:**

- At low stock sizes, growth is slow because there are few individuals to reproduce or regenerate the resource (e.g., few fish to spawn).
- As the stock increases, growth accelerates due to more individuals contributing to reproduction, reaching a maximum at an intermediate stock size.
- As the stock approaches the carrying capacity $K$ (the maximum population the environment can sustain), growth slows down and eventually stops due to limiting factors such as competition for food, space, or other resources.

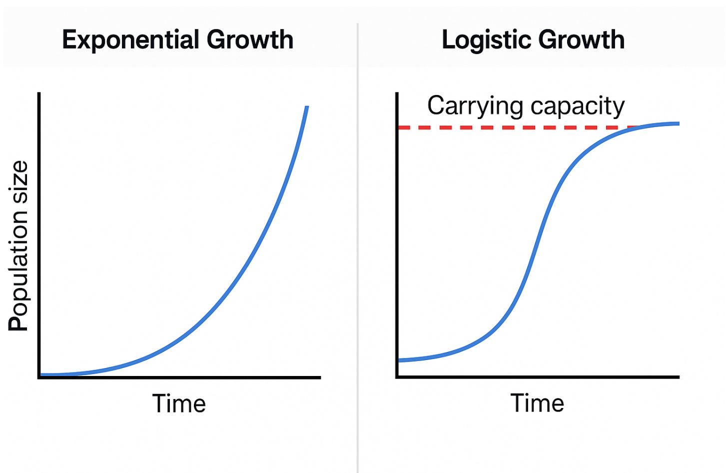


Figure 1: Biological growth law: $G(X)$ as a function of stock size $X$. The curve is typically hump-shaped, peaking at an intermediate stock size.

A common functional form for the growth function is the logistic growth model:

$$G(X) = rX\left(1 - \frac{X}{K}\right)$$

where:

- $r$ is the intrinsic growth rate, representing the maximum per capita rate of increase,
- $K$ is the carrying capacity, the maximum stock size the environment can support.

This model captures the essential features of renewable resource growth: slow growth at low and high stock sizes, and maximum growth at an intermediate stock.

# 3 A Model of Optimal Use

The central economic problem in renewable resource management is to determine the harvest path $H(t)$ that maximizes the present value of net benefits from the resource over time, while respecting the biological growth constraint. This is a dynamic optimization problem, as decisions made today affect the future stock and thus future harvest possibilities.

## Planner's Problem

The resource manager (or planner) seeks to:

$$\max_{H(t)} \int_0^\infty e^{-rt}[pH(t) - c(H(t), X(t))]\, dt$$

subject to:

$$\frac{dX}{dt} = G(X) - H(t), \qquad X(0) = X_0, \qquad X(t) \geq 0$$

where:

- $p$ is the price received per unit of harvest,
- $c(H, X)$ is the cost function, which may depend on both the harvest rate and the current stock (e.g., it is typically more costly to harvest when the stock is low),
- $r$ is the discount rate, reflecting the preference for current benefits over future benefits,
- $X_0$ is the initial stock size.

This formulation captures the trade-off between current extraction (which provides immediate benefits) and conserving the resource for future use (which allows for continued benefits over time).

## Current-Value Hamiltonian

To solve the planner's problem, we use the calculus of variations or optimal control theory. The current-value Hamiltonian is constructed as:

$$\mathcal{H} = pH(t) - c(H(t), X(t)) + \mu(t)[G(X(t)) - H(t)]$$

where:

- $\mu(t)$ is the co-state variable or shadow price, representing the marginal value of having an additional unit of the resource stock at time $t$. It reflects the opportunity cost of depleting the resource.

### First-Order Conditions

The necessary conditions for an optimal solution (the Pontryagin Maximum Principle) are:

1. **Harvest condition:**
$$\frac{\partial \mathcal{H}}{\partial H} = p - \frac{\partial c}{\partial H} - \mu = 0$$
This condition states that the marginal benefit of harvesting (price minus marginal cost) should equal the shadow price of the resource. In other words, the planner should harvest up to the point where the gain from extracting one more unit equals the value of leaving it in the resource stock for future growth.

2. **Costate (shadow price) evolution:**
$$\frac{d\mu}{dt} = r\mu - \frac{\partial \mathcal{H}}{\partial X} = r\mu - \left(-\frac{\partial c}{\partial X} + \mu\frac{dG}{dX}\right)$$
This equation describes how the shadow price changes over time, depending on the discount rate, the marginal effect of stock on costs, and the marginal contribution of stock to natural growth.

3. **State equation:**
$$\frac{dX}{dt} = G(X) - H$$
This is simply the biological law of motion for the resource stock.

**Interpretation:**

- The optimal harvest policy equates the marginal benefit from harvesting with the marginal opportunity cost of reducing the future stock.
- The shadow price $\mu$ captures the future value of the resource: if the resource is expected to be more valuable in the future (due to scarcity or higher prices), the optimal policy will be more conservative today.
- The model ensures that the resource is not depleted too quickly, preserving its ability to generate benefits in the long run.

## 4 Maximum Sustainable Yield (MSY) and the Biological Optimum

**Maximum Sustainable Yield (MSY):**

- The MSY is a key concept in renewable resource management. It represents the largest constant harvest that can be sustained indefinitely without depleting the resource.

- MSY is achieved at the stock size $X_{MSY}$ where the natural growth function $G(X)$ reaches its maximum. At this point, the resource is growing as fast as possible, and the entire growth can be harvested each period without reducing the stock.
- For the logistic growth model, the maximum growth occurs at $X_{MSY} = K/2$, and the maximum sustainable yield is $MSY = G(K/2) = rK/4$.

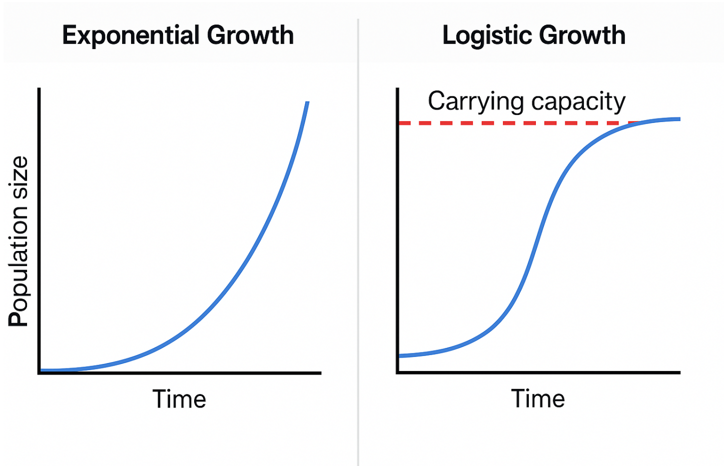


Figure 2: Growth function $G(X)$ and Maximum Sustainable Yield. The peak of the curve corresponds to MSY.

**Biological vs Economic Optimum:**

- The MSY is a purely biological concept. It does not consider the economic costs of harvesting, the price of the resource, or the time value of money (discounting).
- In reality, harvesting at MSY may not be economically optimal. As the stock decreases, the cost of harvesting usually increases (e.g., it takes more effort to catch fish when they are scarce).
- The economic optimum typically involves maintaining a higher stock than $X_{MSY}$ and harvesting less than MSY. This reduces costs and can increase the present value of net benefits.
- The economic optimum also accounts for discounting: if future benefits are valued less (high discount rate), the optimal policy may favor higher current harvests and lower future stocks.

# 5 The Economic Optimum

**Key points:**

- The economic optimum is found by maximizing the present value of net benefits from the resource, taking into account prices, costs, and the discount rate.

- The optimal steady-state stock $X^*$ is generally greater than $X_{MSY}$, because maintaining a higher stock reduces harvesting costs and provides a buffer against uncertainty or shocks.
- The optimal sustainable harvest under economic management is less than the MSY, reflecting the trade-off between current profits and future resource availability.
- The shadow price $\mu$ represents the marginal value of the resource in situ-that is, the value of leaving one more unit in the stock rather than harvesting it now.
- If the discount rate is high (society is impatient), the economic optimum shifts toward lower stocks and higher harvests, resembling the exploitation of a non-renewable resource (mining).

**Comparison Table:**

Table 1: Biological vs Economic Optimum

| | **MSY (Biological)** | **Economic Optimum** |
|---|---|---|
| Objective | Maximize sustainable yield | Maximize present value of net benefits |
| Stock size | $K/2$ (for logistic growth) | $> K/2$ (depends on costs, price, $r$) |
| Harvest | $rK/4$ | $< rK/4$ |
| Considers costs/price? | No | Yes |
| Considers discounting? | No | Yes |

This table summarizes the key differences between the biological and economic approaches to resource management. The economic optimum is generally more conservative, ensuring both economic efficiency and resource sustainability.

# 6 The Common-Property Problem

**Open Access and the Tragedy of the Commons:**

- When property rights over a renewable resource are not well-defined or enforced, the resource is subject to "open access" exploitation. This means that anyone can harvest from the resource, with little or no restriction.
- In an open access regime, each user acts independently and does not take into account the negative impact of their harvest on the resource stock and on other users. The result is a classic externality: overuse of the resource, leading to depletion and reduced benefits for all.
- This situation is known as the "tragedy of the commons," a term coined by Garrett Hardin. It describes the tendency for common-property or open-access resources to be over-exploited and degraded.

**Economic Consequences:**

- **Rent dissipation:** In open access, economic profits (resource rents) are driven to zero as more users enter the resource until the average cost of harvesting equals the average revenue. No one has an incentive to conserve the resource for the future.
- **Over-exploitation:** Resource stocks are driven below the economic optimum, and often even below the level that can sustain MSY. This reduces both current and future yields.
- **Resource depletion or extinction:** In extreme cases, the resource may be harvested faster than it can regenerate, leading to collapse or even extinction (as has happened with some fisheries and wildlife populations).

**Policy Responses:**

- **Establishing property rights:** Assigning individual or community rights to resource use (such as individual transferable quotas in fisheries, or community forest management) can align incentives with sustainable management.
- **Regulatory measures:** Governments may impose catch limits, seasonal closures, gear restrictions, or licensing systems to control access and harvest levels.
- **Economic instruments:** Tools such as taxes on harvest, tradable permits, or subsidies for conservation can help internalize the externality and promote more efficient and sustainable use.

**Summary:** The common-property problem illustrates the importance of institutional arrangements in resource management. Without effective governance, renewable resources are vulnerable to over-exploitation and degradation, with negative consequences for both the economy and the environment. Sustainable management requires aligning individual incentives with the collective interest in conserving the resource for the long term.

# 7 Optimum Harvest in Fisheries: Harvest Cost, Effort, and MSY

## Harvest Function and Fishing Effort

In fisheries economics, the **harvest function** is typically modeled as:

$$H = qEX$$

where

- $H$ is the harvest rate (catch per unit time),

- $q$ is the catchability coefficient (efficiency of fishing effort),
- $E$ is the fishing effort (e.g., number of boats, hours fished),
- $X$ is the fish stock biomass.

**Fishing effort** ($E$) represents the intensity of fishing activities. It is a crucial control variable in fisheries management and can be regulated through quotas, gear restrictions, or seasonal closures*arXiv*[3].

## Harvest Cost

The **cost of harvest** is typically an increasing function of effort and a decreasing function of stock size, reflecting that it is easier and cheaper to catch fish when stocks are abundant. A simple linear cost function is:

$$C(E, X) = cE$$

where $c$ is the cost per unit effort. In practice, costs may rise as stocks decline, since more effort is required to catch the same amount of fish.

## Maximum Sustainable Yield (MSY)

**Maximum Sustainable Yield (MSY)** is a central concept in fisheries management. It is defined as the largest constant harvest that can be sustained indefinitely without depleting the stock*ScienceDirect*[2]. For a logistic growth model:

$$G(X) = rX\left(1 - \frac{X}{K}\right)$$

the MSY is obtained at the stock size $X_{MSY}$ that maximizes $G(X)$:

$$X_{MSY} = \frac{K}{2}$$

$$MSY = G\left(\frac{K}{2}\right) = r\frac{K}{2}\left(1 - \frac{1}{2}\right) = \frac{rK}{4}$$

At this point, the surplus production of the fishery is at its maximum, and harvesting at this rate can, in theory, be continued indefinitely*ScienceDirect*[2]*Stockholm University*[6].

## Effort at MSY

The level of fishing effort that achieves MSY, denoted $E_{MSY}$, can be derived by setting the harvest function equal to MSY:

$$H = qE_{MSY}X_{MSY} = MSY$$

$$E_{MSY} = \frac{MSY}{qX_{MSY}} = \frac{rK/4}{q(K/2)} = \frac{r}{2q}$$

This is the effort level that, if maintained, would produce the maximum sustainable yield.

### Economic Optimum vs. MSY

While MSY maximizes the biological yield, it does not account for economic factors such as costs and prices. The **economic optimum** typically occurs at a lower level of effort and a higher stock size than MSY, because the marginal cost of harvesting rises as the stock declines. The **maximum economic yield** (MEY) is the level of catch that maximizes the net economic benefit (profit) from the fishery, and is generally less than MSY

Table 2: MSY vs. MEY

| | **MSY** | **MEY (Economic Optimum)** |
|---|---|---|
| Objective | Maximize sustainable yield | Maximize profit |
| Effort | $E_{MSY}$ | $E_{MEY} < E_{MSY}$ |
| Stock size | $K/2$ | $> K/2$ |
| Harvest | $rK/4$ | $< rK/4$ |

## 8 Common Property Fisheries and Over-Exploitation

### The Tragedy of the Commons

Fisheries are often managed as **common property resources**, meaning access is open to all, and no individual has exclusive rightsİn such a regime:

- Each fisher seeks to maximize their own gain, ignoring the impact on the resource and on others.
- The result is excessive entry and effort, leading to over-exploitation of the fishery.
- Economic profits are dissipated as more fishers enter until profits fall to zero.

This phenomenon is known as the **tragedy of the commons**. Each user has the incentive to increase their own harvest, since any restraint only benefits others who continue to exploit the resource. The equilibrium is reached where average revenue equals average cost, which is at a higher level of effort and lower stock than is socially optimal.

### Consequences of Open Access

- **Over-capitalization:** Too much investment in boats and gear relative to the sustainable yield.
- **Stock depletion:** Fish stocks fall below the level that would support MSY or MEY, sometimes to the point of collapse.

- **Economic waste:** The same catch could be obtained with less effort and cost if the fishery were managed optimally.
- **Loss of resource rent:** The potential economic surplus (rent) from the fishery is lost due to excessive competition.
- **Risk of extinction:** In extreme cases, the resource may be driven to extinction.

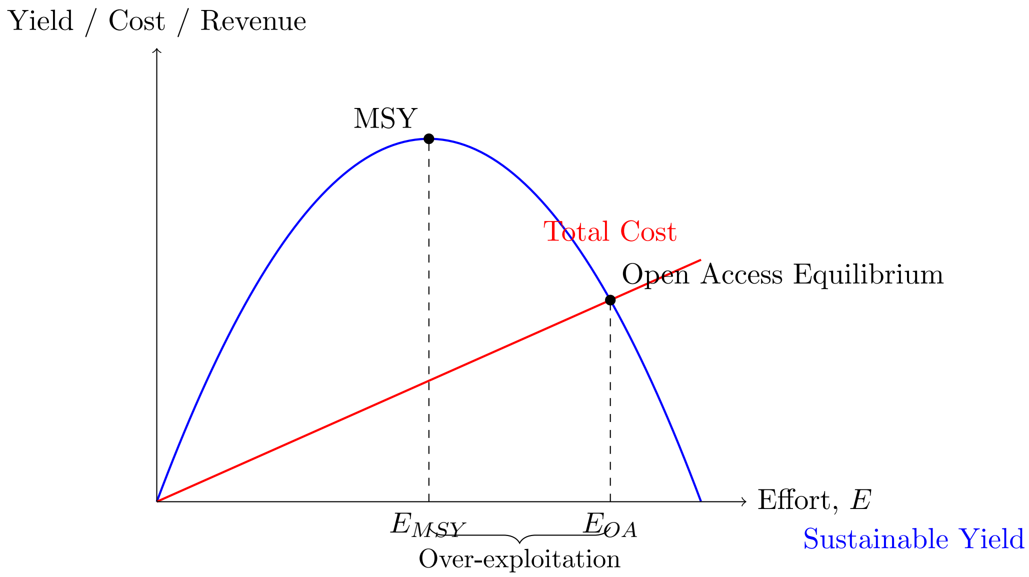


Figure 3: Open access equilibrium ($E_{OA}$) occurs at higher effort and lower yield than the economic optimum ($E_{MSY}$). Under open access, profits are dissipated and the fishery is over-exploited.

## Policy Responses

To address the common property problem and prevent over-exploitation, various management strategies are used:

- **Establishing property rights:** Individual transferable quotas (ITQs), territorial use rights in fisheries (TURFs).
- **Regulation:** Limiting entry, effort, or catch through licenses, quotas, or seasonal closures.
- **Economic instruments:** Taxes, tradable permits, or subsidies for conservation.

**Summary:** Without effective management or property rights, common property fisheries are prone to over-exploitation, resulting in depleted stocks, economic inefficiency, and loss of long-term benefits for all users

# 9 Concluding Remarks

- Renewable resources require management strategies that balance current use with future regeneration.
- The biological optimum (MSY) is not generally the economic optimum.
- Open access leads to over-exploitation and loss of economic and ecological value.
- Effective management often requires well-defined property rights or regulatory interventions.

# 10 Summary Table

Table 3: Comparison of Resource Management Regimes

| **Regime** | **Objective** | **Steady-State Stock** | **Long-Run Outcome** |
|---|---|---|---|
| Economic Optimum | Maximize present value | High (¿$X_{MSY}$) | Sustainable, with rent |
| MSY | Maximize yield | $K/2$ | Sustainable, no rent |
| Open Access | Maximize individual gain | Low ($< K/2$) | Over-exploited, zero rent |