Natural Resource Economics: Theory of Mine

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Theory of mine- Extraction of non-renewable resources

2 Outline

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- **Basic Concepts**: Definitions of exhaustible resources, extraction costs, and opportunity costs.
- **Myopic Extraction**: Consequences of ignoring future value in resource extraction.
- **Resource Rent and Related Concepts**: Explanation of resource rent, opportunity cost, user cost, and royalty.
- **Gray's Model**: Historical context and key ideas from Lewis Cecil Gray's 1914 work.
- **Discount Rate and Sustainability**: Role of discount rates in resource management.
- **Hotelling's Model**: Formalization of resource economics by Harold Hotelling (1931).
- **Application to Firms**: How Hotelling's rule applies to individual firms.
- **Backstop Technology**: Alternatives that replace exhaustible resources at a threshold price.
- **Two-Period and N-Period Models**: Simplified and generalized frameworks for extraction planning.
- **Conclusion and Summary Table**: Synthesis of key models and decision rules.

This structure ensures a logical progression from foundational concepts to advanced models, suitable for classroom discussion.

3 Introduction

3.1 Overview

Exhaustible resources, such as fossil fuels (oil, coal, natural gas) and minerals (copper, gold), are finite and deplete with use. Their management poses unique economic challenges due to the trade-off between current extraction and future availability. This section introduces the need for strategies that balance economic benefits, environmental sustainability, and intergenerational equity.

3.2 Key Characteristics of Exhaustible Resources

- **Limited Stock**: The total quantity is fixed, and extraction reduces the available stock permanently.
- **Non-Producibility**: Unlike renewable resources (e.g., timber), exhaustible resources cannot be replenished within a human-relevant timeframe.

The **opportunity cost** of extracting a unit today is the forgone value of having that unit available in the future, a central theme in resource economics.

3.3 Classroom Discussion Points

- Why is managing exhaustible resources different from managing renewable resources?
- How do finite resources impact long-term economic planning?

4 Basic Concepts in Exhaustible Resource Extraction

4.1 Definition

Exhaustible resources are natural assets with finite stocks that cannot be renewed within a human timeframe. Examples include fossil fuels and minerals, which are critical to industrial economies but face eventual depletion.

4.2 Key Terms

- **Extraction Cost**: Includes labor, equipment, and operational expenses. Costs may be constant or vary with the quantity extracted or remaining stock (e.g., deeper deposits may be costlier to extract).
- **Rent or Opportunity Cost**: The *scarcity rent* is the value of leaving a resource in the ground for future use, reflecting the trade-off between immediate profits and future value.

4.3 Classroom Discussion Points

- How do extraction costs influence extraction rates?
- Why is scarcity rent a critical concept for exhaustible resources?

5 Myopic Extraction and Opportunity Cost

5.1 Concept

Traditional economic theory suggests that optimal resource use occurs where marginal cost (MC) equals marginal return (price), i.e., $P = MC$. However, for exhaustible resources, this leads to **myopic extraction**, where the resource is depleted too quickly by ignoring the opportunity cost of future scarcity.

5.2 Correct Rule

The optimal extraction rule accounts for opportunity cost:

$$P = MC + \text{Opportunity Cost}$$

- **Myopic Extraction (Y^n)**: Occurs when only current costs are considered, leading to over-extraction.
- **Optimal Extraction (Y^i)**: Balances current and future values, reducing extraction to preserve future availability.

5.3 Graph Explanation

The graph (referenced on page 9) illustrates myopic vs. optimal extraction:

- **X-axis**: Quantity extracted.
- **Y-axis**: Price and cost.
- **Marginal Cost Curve**: Slopes upward, reflecting increasing costs as more is extracted.
- **Price Line**: Horizontal, assuming constant market price.
- **Point Y^n**: Equilibrium under myopic behavior, where $P = MC$, leading to excessive extraction.
- **Point Y^i**: Optimal extraction, where $P = MC + \text{Opportunity Cost}$, reducing extraction to account for future value.

5.4 Classroom Discussion Points

- Why does myopic extraction lead to unsustainable outcomes?
- How can policymakers encourage optimal extraction?

6 Resource Rent, Opportunity Cost, User Cost, and Royalty

6.1 Definitions

- **Resource Rent**: The difference between market price and marginal extraction cost, i.e., $P - MC$. It represents the economic value of the resource due to its scarcity.
- **Opportunity Cost (User Cost)**: The value forgone by extracting a unit today instead of preserving it for future use.
- **Royalty**: The net social benefit from the resource, calculated as total social benefit minus extraction costs.

6.2 Key Question

When should extraction occur to maximize the present value of royalty or resource rent? This involves balancing immediate profits against the potential future value of delayed extraction.

6.3 Classroom Discussion Points

- How does resource rent differ from profit in traditional economics?
- Why is royalty important for public policy in resource-rich countries?

7 Resource Rent: An Illustration

7.1 Concept

A mineral deposit in situ is an asset with value derived from:

- **Product Flow**: Zero until extraction occurs, as the resource remains in the ground.
- **Depreciation**: Negligible for minerals in situ, as they do not degrade.
- **Appreciation**: The resource's value may increase due to rising scarcity or prices.

The **net value** of a marginal unit is:

$$\text{Net Value} = P - MC$$

This is termed the asset price, resource rent, or royalty.

7.2 Classroom Discussion Points

- How does the appreciation of resource value influence extraction timing?
- Why is the concept of an in-situ asset important for resource economics?

8 Gray's Model: Historical Context and Key Ideas

8.1 Background

Lewis Cecil Gray's 1914 paper, "Rent Under the Assumption of Exhaustibility," was a foundational work in exhaustible resource economics, focusing on a single mine owner under perfect competition.

8.2 Assumptions

- Perfect competition and constant price expectations.
- Homogeneous resource quality.
- Identical cost curves across periods.
- Known stock.

8.3 Key Insights

- **Rent under Exhaustibility**: Finite stocks create scarcity rent.
- **Effect of Exhaustion**: Finite resources limit extraction rates.
- **Influence of Interest Rate**: Higher interest rates favor earlier extraction.

8.4 Decision Rules

1. Extraction occurs where:
$$P = MC + \text{Rent}$$
2. The present value of resource rent is equalized Roubaix-type conditions:
$$P_t = MC + (P_0 - MC)e^{-rt}$$
3. Extraction ceases when unprofitable.

8.5 Derivation of Decision Rule 2

The goal is to maximize the present value of profits over time. For two periods, the rent in period t must equal the discounted rent from the initial period:

$$P_t - MC = (P_0 - MC)e^{rt}$$

This ensures that the rent grows at the interest rate r, balancing extraction across periods.

8.6 Graph Explanation

The graph (page 17) for Gray's two-period model shows:

- **X-axis**: Quantity extracted in period 1.
- **Y-axis**: Price and marginal cost.
- **Marginal Cost Curve**: Upward-sloping, indicating rising costs.
- **Price Line**: Adjusts to include rent, reflecting scarcity value.

8.7 Classroom Discussion Points

- How does Gray's model lay the groundwork for modern resource economics?
- Why does the interest rate play a critical role in extraction decisions?

9 Discount Rate and Sustainability

9.1 Concept

The discount rate (interest rate) reflects the preference for current vs. future income. A higher discount rate reduces the present value of future benefits, encouraging earlier extraction.

9.2 Illustration

For Rs 3,000 received after 3 years:

- At 10% interest:

$$PV = \frac{3000}{(1+0.1)^3} \approx Rs2,253$$

- At 20% interest:

$$PV = \frac{3000}{(1+0.2)^3} \approx Rs1,736$$

9.3 Graph Explanation

The graph (page 20) shows:

- **X-axis**: Discount rate.
- **Y-axis**: Present value of Rs 3,000.
- The curve declines as the discount rate increases, illustrating the reduced value of future income.

9.4 Classroom Discussion Points

- How do discount rates affect sustainability?
- Should governments use lower discount rates for resource policy?

10 Hotelling's Model: Formalization and Extensions

10.1 Background

Harold Hotelling's 1931 paper formalized exhaustible resource economics for competitive markets, building on Gray's work.

10.2 Assumptions

- Perfectly competitive industry.
- Homogeneous resource with known stock.
- Constant unit extraction cost c.
- No fixed costs or stock effects.
- Rational expectations.

10.3 Hotelling's Rule

The net price grows at the interest rate:

$$p(t) - c = (p(0) - c)e^{rt}$$

If $c = 0$, then:

$$p(t) = p(0)e^{rt}$$

10.4 Derivation (Competitive Markets)

Maximize:

$$\max \int_0^\infty e^{-rt}[p(t)q(t) - c(q(t))]\, dt$$

Subject to:

$$\dot{S}(t) = -q(t), \quad S(0) = S_0, \quad S(t) \geq 0$$

Using the Hamiltonian:

$$H = e^{-rt}[p(t)q(t) - c(q(t))] - \mu(t)q(t)$$

First-order conditions yield:

$$p(t) - c'(q(t)) = \mu(0)e^{rt}$$

For constant c, this simplifies to Hotelling's rule.

10.5 Monopoly Case

For a monopolist:

$$\max \int_0^\infty e^{-rt}[R(q(t)) - c(q(t))]\, dt$$

First-order conditions give:

$$MR(q(t)) - c'(q(t)) = \mu(0)e^{rt}$$

The monopolist extracts more slowly initially, as marginal revenue grows at the interest rate.

10.6 Classroom Discussion Points

- How does Hotelling's rule differ in competitive vs. monopoly markets?
- What are the limitations of Hotelling's assumptions?

11 Application of Hotelling Rule to an Individual Firm

11.1 Concept

For a firm with constant price P and marginal cost $MC > 0$, the present value of rent must be equal across periods:

$$\left(\frac{1}{1+r}\right)^{t}(P - MC(Q_t)) = \left(\frac{1}{1+r}\right)^{t+1}(P - MC(Q_{t+1}))$$

This yields:

$$r = \frac{(P - MC(Q_{t+1})) - (P - MC(Q_t))}{P - MC(Q_t)}$$

11.2 Derivation

Equate the discounted rent:

$$(P - MC(Q_t)) = (1 + r)(P - MC(Q_{t+1}))$$

Rearrange to solve for r, ensuring rent grows at the interest rate.

11.3 Classroom Discussion Points

- How does this rule guide firm-level extraction decisions?
- What happens if prices or costs are not constant?

12 Backstop Technology

12.1 Concept

A backstop technology is an alternative resource or technology that becomes viable when the price of the exhaustible resource exceeds a threshold.

12.2 Example

Solar energy vs. coal:

- Past: Solar Rs 17/unit, coal Rs 1/unit.
- Present: Solar Rs 4/unit, coal Rs 4.5/unit.

Solar is now a viable backstop, potentially replacing coal.

12.3 Graph Explanation

The graph (page 31) shows:

- **X-axis**: Time or quantity extracted.
- **Y-axis**: Price.
- The price of the exhaustible resource rises until it reaches the backstop's cost, at which point extraction ceases.

12.4 Classroom Discussion Points

- How do backstop technologies influence resource extraction strategies?
- What are examples of emerging backstop technologies today?

13 Two-Period Model

13.1 Competitive Markets

Maximize:

$$\pi = p(q_1)q_1 + \delta p(q_2)q_2, \quad \delta = \frac{1}{1+r}$$

Subject to:

$$q_1 + q_2 = S$$

Prices adjust such that:

$$p_1 = \delta p_2 \Rightarrow p_2 = p_1(1+r)$$

For demand $p_t = a - bq_t$:

$$a - bq_1 = \delta(a - bq_2), \quad q_1 + q_2 = S$$

13.2 Monopoly

Maximize:

$$\pi = p(q_1)q_1 + \delta p(q_2)q_2, \quad q_1 + q_2 = S$$

Yields:

$$MR(q_1) = \delta MR(q_2)$$

13.3 Classroom Discussion Points

- How does the two-period model simplify real-world extraction problems?
- Why does the monopolist extract more slowly?

14 N-Period Model

14.1 Concept

Generalizes the two-period model:

$$\max \sum_{t=0}^{T} \delta^t [p(q_t)q_t - c(q_t)]$$

Subject to:

$$\sum_{t=0}^{T} q_t \leq S$$

Solved using dynamic programming or optimal control.

14.2 Classroom Discussion Points

- How does the n-period model improve upon simpler models?
- What computational challenges arise in n-period models?

15 Conclusion

The economics of exhaustible resources is critical for sustainable management. Key contributions include:

- **Gray's Model**: Focus on scarcity rent for a single mine owner.
- **Hotelling's Model**: Framework for competitive and monopoly markets.
- **Two- and N-Period Models**: Tools for planning extraction over time.

16 Summary Table

Model	Focus	Key Decision Rule	Derivation M
Gray's Model	Single mine owner, rent	Balance rent, interest rate	Qualitative, p
Hotelling's Model	Competitive, monopoly markets	Net price rises at interest rate	Hamiltonian,
Two-Period Model	Simplified trade-off	$p_1 = \delta p_2$ or $MR_1 = \delta MR_2$	Lagrange, cal
N-Period Model	Long-term planning	Dynamic optimization	Dynamic prog

Table 1: Comparison of Models and Decision Rules

16.1 Classroom Discussion Points

- Which model is most applicable to real-world resource management?
- How can these models inform policy for sustainable resource use?