Simultaneous Equation Models
Unit 8

Prepared by

Aditya Korekallu Srinivasa

Scientist (Senior Scale)

ICAR-Indian Agricultural Research Institute
New Delhi

Contents

1 Introduction to Simultaneous Equation Models

1.1 Motivation

In many economic contexts, variables are determined together, not in isolation. For example, in the market for a good, both price and quantity are determined simultaneously by the interaction of supply and demand. In such settings, the standard single-equation regression approach is inadequate because it assumes unidirectional causality and exogeneity of regressors.

1.2 What is a Simultaneous Equation Model?

A **simultaneous equation model** (SEM) consists of two or more equations, each representing an economic relationship, where at least some variables are determined jointly (i.e., are *endogenous*). In SEMs, some variables appear as dependent variables in one equation and as explanatory variables in others.

1.3 Examples

- **Supply and Demand:**

$$Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 Y_t + u_{1t} \quad \text{(Demand)}$$
$$Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 W_t + u_{2t} \quad \text{(Supply)}$$

  Here, P_t (price) and Q_t (quantity) are endogenous; Y_t (income) and W_t (wage) are exogenous.

- **Macroeconomic Models:** Income, consumption, and investment are often modeled as jointly determined.

1.4 Why Not Use OLS?

Applying OLS separately to each equation in a simultaneous system is generally inappropriate because endogenous regressors are correlated with the error terms, leading to biased and inconsistent estimates.

2 The Identification Problem

2.1 What is Identification?

Identification refers to the ability to obtain unique numerical estimates of the structural parameters of an equation from the observed data. If an equation is not identified, its parameters cannot be estimated, regardless of the estimation technique or the amount of data.

2.2 Why Does Identification Matter?

Without identification, it is impossible to distinguish between alternative structural relationships that are observationally equivalent. For example, in a market, if only price and

quantity are observed, it is not possible to distinguish between the demand and supply equations unless additional information or restrictions are imposed.

2.3 Types of Identification

- **Unidentified (Underidentified):** Not enough information to estimate parameters.
- **Exactly Identified:** Just enough information to estimate parameters uniquely.
- **Overidentified:** More than enough information; multiple ways to estimate parameters.

2.4 Illustrative Example: Supply and Demand

Suppose we have:

$$Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 Y_t + u_{1t}$$
$$Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 Z_t + u_{2t}$$

If we only observe Q_t and P_t, we cannot tell whether a regression of Q on P estimates demand or supply. Identification requires exclusion restrictions (e.g., Y_t excluded from supply, Z_t from demand).

3 Conditions for Identification

3.1 Order Condition (Necessary)

- In a system with M endogenous variables, an equation is identified if the number of variables (endogenous and exogenous) excluded from the equation is at least $M-1$.
- **Just Identified:** Exactly $M-1$ variables excluded.
- **Overidentified:** More than $M-1$ variables excluded.
- **Underidentified:** Fewer than $M-1$ variables excluded.

3.2 Rank Condition (Necessary and Sufficient)

- The rank of a matrix constructed from the coefficients of the excluded variables must be at least $M-1$.
- In practice, if the order condition is not met, the rank condition will not be met. If the order condition is met, the rank condition should be checked for sufficiency.

3.3 Summary Table: Identification Status

Exclusions	Identification Status	Estimation Possible?
$< M-1$	Underidentified	No
$= M-1$	Just Identified	Yes (unique estimates)
$> M-1$	Overidentified	Yes (multiple estimates possible)

3.4 Practical Example

Suppose a three-equation system ($M = 3$). For an equation to be identified, it must exclude at least 2 variables from the system.

4 How Simultaneous Equation Models Solve Endogeneity

4.1 Endogeneity in Single-Equation Models

In single-equation models, if an explanatory variable is endogenous (correlated with the error term), OLS estimates are biased and inconsistent.

4.2 Simultaneous Equations Approach

- By modeling all endogenous variables jointly, SEMs account for feedback and mutual causation.
- Estimation methods (such as Two-Stage Least Squares, 2SLS) use exogenous variables (instruments) to purge endogeneity, yielding consistent parameter estimates.

4.3 Example:

In the supply and demand model, using income (Y) and wage (W) as instruments for price (P) allows us to estimate the demand and supply equations consistently.

5 Summary and Key Points

- Simultaneous equation models are essential when variables are jointly determined.
- The identification problem must be resolved before estimation; otherwise, structural parameters cannot be uniquely estimated.
- The order and rank conditions provide practical tools for checking identification.
- SEMs address endogeneity by modeling the system as a whole and using appropriate estimation techniques.

Glossary

- **Simultaneous Equation Model (SEM)**: A system of equations with mutually dependent variables.
- **Endogenous Variable**: A variable determined within the system of equations.
- **Exogenous Variable**: A variable determined outside the system.
- **Identification**: The ability to obtain unique estimates of structural parameters from observed data.

- **Order Condition**: A necessary condition for identification based on the number of excluded variables.
- **Rank Condition**: A necessary and sufficient condition for identification based on matrix rank.
- **Just Identified**: An equation with just enough information for unique estimation.
- **Overidentified**: An equation with more than enough information; multiple estimation methods possible.
- **Underidentified**: An equation with insufficient information for estimation.
- **Instrumental Variable**: An exogenous variable used as an instrument to address endogeneity.

Practice Questions

1. What is a simultaneous equation model? Give two economic examples.
2. Explain the identification problem in your own words.
3. What is the order condition for identification? Illustrate with an example.
4. How does the rank condition differ from the order condition?
5. Why is OLS generally inappropriate in simultaneous equation models?
6. How do simultaneous equation models address the issue of endogeneity?
7. What is the difference between just identified and overidentified equations?
8. In a system with three endogenous variables, how many variables must be excluded from an equation for it to be identified?
9. Describe the steps for checking identification in a simultaneous equation system.
10. Discuss the importance of structural and reduced-form equations in SEMs.

References

- Gujarati, D. N., & Porter, D. C. (2010). *Basic Econometrics* (5th ed.). McGraw-Hill. [See especially Chapters 18, 19, and 20]