Theory of Mine (Exhaustible Resources)

Natural Resource Economics

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Outline

Introduction

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- Exhaustible resources, such as fossil fuels and minerals, are finite and deplete with extraction
- Their finite nature necessitates strategies that balance current extraction with future availability
- Inter-generational equity? Profitability? Decision rule?

Exhaustible resources

How do exhaustible resources differ from other resources?

- **Limited stock:** The total quantity available is finite; once extracted and used, it cannot be replenished within a relevant timeframe.
- **Non-producibility:** These resources cannot be produced.

Key Point: There is an **opportunity cost** to current consumption: extracting one unit today means one less unit is available for the future.

Basic Concepts

- **Exhaustible resources:** These are natural assets available in limited quantities. Examples include fossil fuels (oil, coal, natural gas) and minerals (copper, gold).
- **Extraction Cost:** The expenses incurred in extracting the resource, including labor, equipment, and operational costs (Usual assumption -constant)
- **Rent or Opportunity Cost:** The scarcity rent, or the value of leaving the resource in the ground for future extraction.

Myopic Extraction and Opportunity Cost

Myopic Extraction and Opportunity Cost

Economic theory suggests that the optimal use of a resource occurs where Marginal Cost (MC) equals Marginal Return (Price), i.e., Price = MC.

However, for exhaustible resources:

- Applying this rule ignores the opportunity cost of depleting a finite resource.
- This can lead to **myopic extraction**: using up all the resource in the current period, without regard for future scarcity.
- The correct rule for exhaustible resources is: Price = MC + Opportunity Cost

Myopic Extraction: Illustration

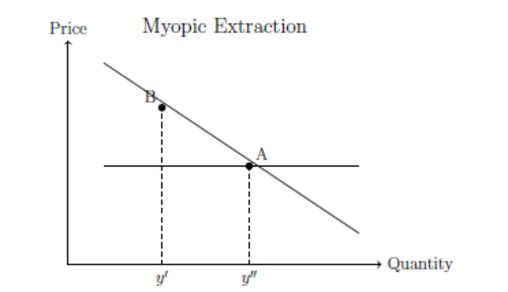

Figure: Myopic extraction y''. Point A is the equilibrium under myopic behavior

- **Myopic extraction** occurs at the point Y^{ii} (where only current costs are considered).
- **Optimum extraction** occurs at Y^{i} (where both current and future values are balanced)

Key Concepts- Rent

- **Resource Rent:** The difference between the market price and the marginal cost of extraction. This is the value of the unextracted resource (resource left in the ground)
- **Opportunity Cost (User Cost):** The value forgone by extracting and consuming a resource unit today rather than preserving it for future use.
- **Royalty:** The net social benefit derived from the resource, calculated as the total social benefit minus the cost of extraction.
- **Other names:** Shadow price or User cost

Key question: When should the resource be extracted to maximize the present value of royalty (or resource rent)? - Balancing immediate returns from extraction against the potential future value if extraction is delayed.

Resource Rent: An Illustration

A mineral deposit in situ is an asset. The value of such an asset can be decomposed into:

1. **Product Flow:** The output generated by utilizing the asset. For minerals still in the ground, this is zero until extraction occurs.
2. **Depreciation:** The decline in asset value over time. For most minerals, this is negligible while in situ.
3. **Appreciation:** The rate at which the value of the resource increases over time, often due to rising scarcity or prices.

$$\text{Net Value of Marginal Unit} = P - MC$$

This net value is known as the **asset price**, **resource rent**, or **royalty**. Used often interchangeably.

Gray's Model: Historical Context and Key Ideas

Gray’s Model: Historical Context and Key Ideas

Lewis Cecil Gray’s 1914 paper, “Rent Under the Assumption of Exhaustibility” provided decision rule focusing on a single mine owner.

Assumptions:

- Perfect competition (MR=AR, Horizontal price line)
- Homogeneous quality
- Identical cost curves across periods
- Known stock
- Usually presented as simplified two period model

Gray's Model: Key Insights and Decision Rules

Key insights:

- **Rent under Exhaustibility**: Scarcity value due to finite stocks.
- **Effect of Exhaustion**: Finite resources influence extraction rates.
- **Influence of Interest Rate**: Interest rates determine extraction timing.

Decision rules:

1. Extraction occurs where Price = MC + Rent.
2. Present value of resource rent is equalized across periods- Discounted present value of rent remain same:
$$P_t - MC = (P_{t+1} - MC)e^{-rt}$$
$$P_t = MC + (P_{t+1} - MC)e^{-rt}$$
3. Stock and terminal conditions: Extraction ceases when unprofitable. Sum of quantity extracted cannot exceed the stock.

Gray Model: Rent and Price over Time

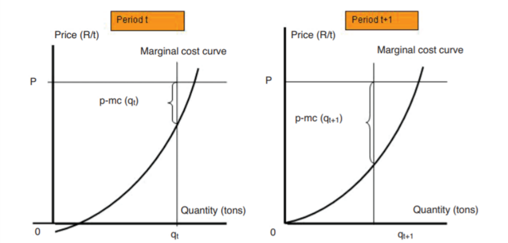

Figure: Gray two period model content...

- Each graph is for a period
- All the curves are for a period
- With each period, rent increases, q decreases.

Discount Rate and Sustainability

Discount Rate and Sustainability

The discount rate (interest rate) reflects the preference for current versus future income. A higher discount rate increases the attractiveness of immediate extraction, as future benefits are valued less.

Illustration: The present value of receiving ₹3,000 after 3 years:

- At 10% interest: Present value = ₹2,253
- At 20% interest: Present value = ₹1,736

Hotelling's Model

Hotelling's Model: Assumptions

Harold Hotelling's 1931 paper formalized the theory of exhaustible resources, extending Gray's ideas to competitive markets.

Assumptions:

- Perfectly competitive industry - Assumption can be relaxed to include monopoly or other market imperfections
- Homogeneous resource with known stock
- Constant unit extraction cost c
- No fixed costs or stock effects
- Rational expectations

Hotelling's Rule

The rule states that the net price $p(t) - c$ grows at the interest rate (assumption of constant extraction cost) r:

$$p(t) - c = (p(t+1) - c)e^{-rt}$$

For zero extraction cost, a special case, $(c = 0)$, the price itself grows at the interest rate: $p(t) = p(t+1)e^{-rt}$.
Or
$p(t+1) = p(t)e^{rt}$.

Hotelling's Model: Competitive Markets

Resource owner maximizes:

$$\max \int_0^\infty e^{-rt}[p(t)q(t) - c(q(t))]\,dt$$

Subject to:

$$\dot{S}(t) = -q(t), \quad S(0) = S_0, \quad S(t) \geq 0$$

Using the Hamiltonian, first-order conditions yield:

$$p(t) - c'(q(t)) = \mu(0)e^{rt}$$

For constant marginal cost c, $p(t) - c = \mu(0)e^{rt}$. Net price at period t equals compounded value of rent at period 0.

Hotelling's Model: Monopoly

For a monopolist: MR lies below AR, so here, Discounted Net MR is equalized across periods.

$$\max \int_0^\infty e^{-rt}[R(q(t)) - c(q(t))]\, dt$$

First-order conditions yield:

$$MR(q(t)) - c'(q(t)) = \mu(0)e^{rt}$$

If $c'(q(t)) = c$, then $MR(q(t)) - c$ grows at the interest rate, leading to slower initial extraction compared to competitive markets. **Monopolist is a friend of resources!** **NOTE** Here MR-c grows at market rate, not P-c as in case of perfect competition.

Hotelling Rule for a Firm

Assuming a constant price P and marginal cost $MC > 0$, the present value of rent across two periods must be equal:

$$\left(\frac{1}{1+r}\right)^{t}(P - MC(Q_t)) = \left(\frac{1}{1+r}\right)^{t+1}(P - MC(Q_{t+1}))$$

This leads to:

$$r = \frac{(P - MC(Q_{t+1})) - (P - MC(Q_t))}{P - MC(Q_t)}$$

This ensures that the resource rent grows at the rate of interest. Multiply both sides with hundred to get **Hotelling percent rule**

Backstop Technology

Backstop Technology

In planning the extraction of a non-renewable resource, the mine owner must consider the potential emergence of **backstop technology**—an alternative resource or technology that becomes economically viable when the price of the non-renewable resource exceeds a certain threshold.

- **Threshold Price:** The price at which the cost of producing the alternative is less than the current market price of the exhaustible resource.
- Once this threshold is reached, the alternative replaces the exhaustible resource, rendering further extraction unprofitable.

Backstop Technology: Example

Example: Solar energy vs. coal:

- Previously, solar energy cost ₹17 per unit, coal-based electricity cost ₹1 per unit.
- Now, solar costs about ₹4 per unit, coal-based electricity about ₹4.5 per unit.
- At this point, solar becomes a viable backstop technology, potentially replacing coal as the preferred energy source.

Backstop Technology: Illustration

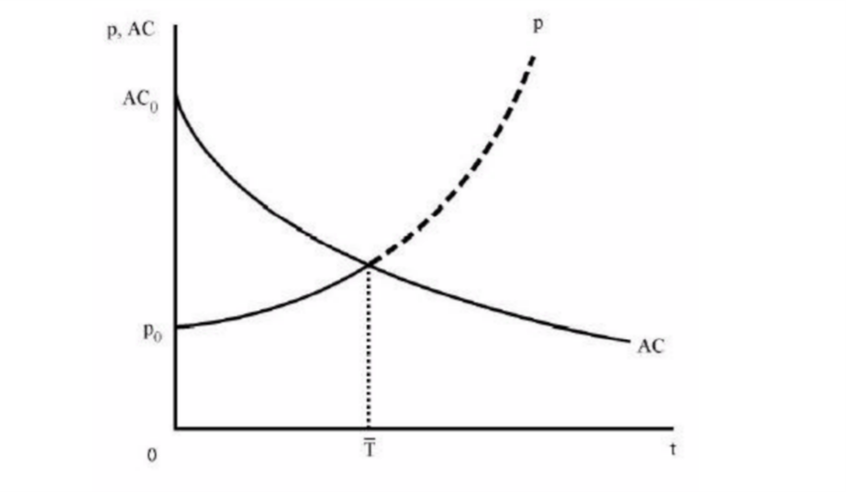

Figure: When the price of the exhaustible resource rises to the level of the backstop

Two-Period Model

Two-Period Model: Competitive Markets

Maximize the present value of profits:

$$\pi = p(q_1)q_1 + \delta p(q_2)q_2$$

where $\delta = \frac{1}{1+r}$, subject to $q_1 + q_2 = S$.

For competitive markets, prices adjust such that:

$$p_1 = \delta p_2 \Rightarrow p_2 = p_1(1 + r)$$

For demand $p_t = a - bq_t$, solve:

$$a - bq_1 = \delta(a - bq_2), \quad q_1 + q_2 = S$$

Problem Setup and derivation

Objective: maximize discounted net revenues

$$\pi = max_{q_1,q_2}\Big[p_1\, q_1 - C(q_1)\Big] \;+\; \frac{1}{1+r}\Big[p_2\, q_2 - C(q_2)\Big]$$

Resource constraint:

$$q_1 + q_2 = S$$

Lagrangian:

$$\mathcal{L} = p_1\, q_1 - C(q_1) + \frac{1}{1+r}\big(p_2\, q_2 - C(q_2)\big) + \lambda\,(S - q_1 - q_2)$$

First-Order Conditions & Hotelling Rule

FOCs:

$$\frac{\partial \mathcal{L}}{\partial q_1} : \; p_1 - C'(q_1) - \lambda = 0, \qquad \frac{\partial \mathcal{L}}{\partial q_2} : \; \frac{1}{1+r}\big(p_2 - C'(q_2)\big) - \lambda = 0$$

Eliminate λ to get the Euler (Hotelling) condition:

$$p_1 - C'(q_1) = \frac{1}{1+r}\big(p_2 - C'(q_2)\big) \quad \Longleftrightarrow \quad (1+r)\big[p_1 - C'(q_1)\big] = p_2 - C'(q_2)$$

Constant marginal cost c:

$$p_1 - c = \frac{1}{1+r}(p_2 - c) \quad \Longrightarrow \quad p_2 - c = (1+r)\,(p_1 - c)$$

Zero-cost special case ($c = 0$):

$$p_2 = (1+r)\,p_1$$

Two-Period Model: Monopoly

The monopolist maximizes:

$$\pi = p(q_1)q_1 + \delta p(q_2)q_2, \quad q_1 + q_2 = S$$

This yields:

$$MR(q_1) = \delta MR(q_2)$$

The monopolist equates marginal revenue across periods, adjusted for discounting, leading to slower initial extraction.

Note: δ is the discount factor.

N-Period Model

N-Period Model

The n-period model generalizes the two-period framework to multiple periods:

$$\max \sum_{t=0}^{T} \delta^t [p(q_t)q_t - c(q_t)]$$

subject to:

$$\sum_{t=0}^{T} q_t \leq S$$

Solved using dynamic programming or optimal control, extending Hotelling's rule to multiple periods.

Conclusion

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- Economics of exhaustible resources is critical for sustainable management.
- Gray's model: Scarcity rent and interest rates for a single mine owner.
- Hotelling's model: Framework for competitive and monopoly markets.
- Two-period and n-period models: Tools for short- and long-term planning.

Summary Table

Comparison of Models and Decision Rules

Model	Focus	Key Decision Rule	Derivation Method
Gray's Model	Single mine owner, rent	Balance rent, interest rate influence	Qualitative, 1914 paper
Hotelling's Model	Competitive, monopoly markets	Net price rises at interest rate	Hamiltonian, optimal control
Two-Period	Simplified trade-off	$p_1 = \delta p_2$ or $MR_1 = \delta MR_2$	Lagrange, calculus
N-Period	Long-term planning	Dynamic optimization	Dynamic programming