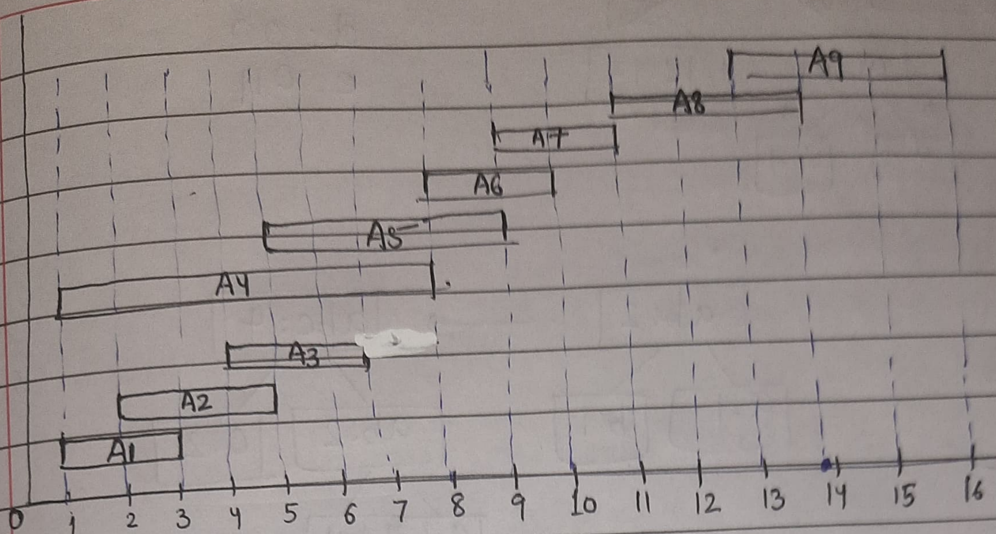


Optimal solution = {A1, A3, A6, A8}

Greedy Activity Selection (s, f)

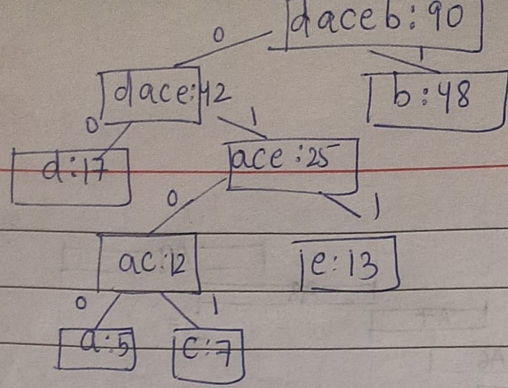
1. n ← length[s]
2. A ← {a_1}
3. i ← 1
4. for m ← 2 to n
5. do if $s_m \geq f_i$
6. then A ← A ∪ {a_m}
7. i ← m
8. return A

Hoffmann Coding (greedy).

Used in compression.

Code	Freq
a :	5
b :	48
c :	7
d :	17
e :	13

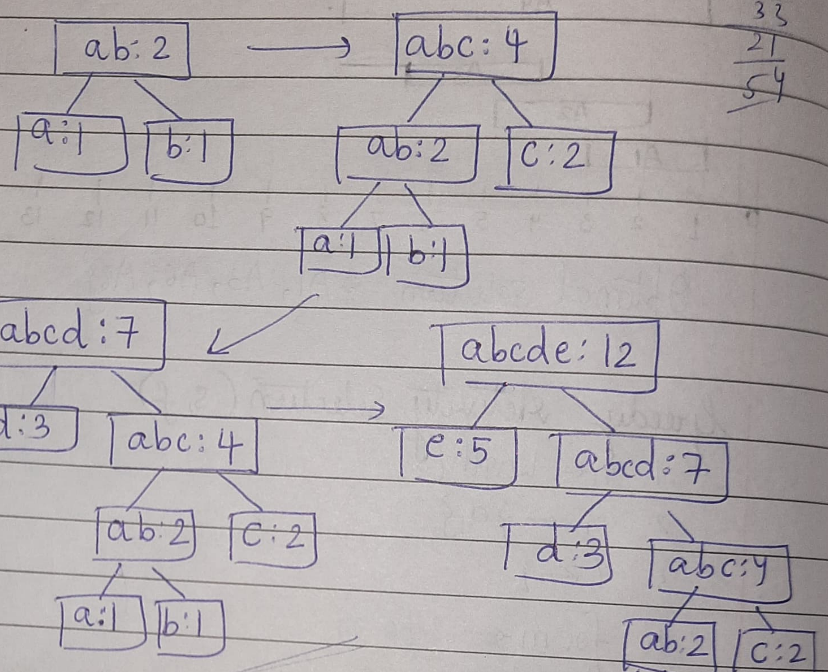
a : 5
c : 7
e : 13
d : 17
b : 48

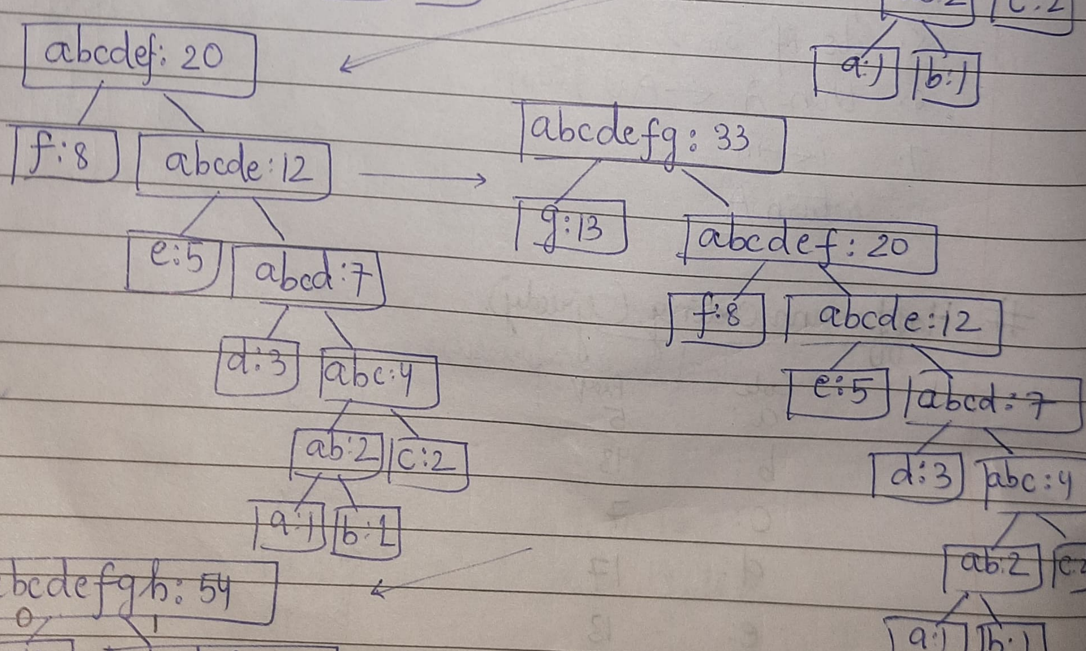


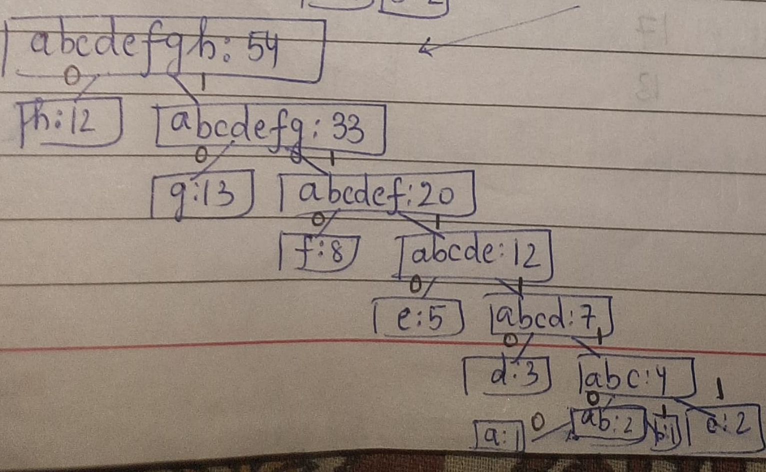
a: 0100
b: 1
c: 0101
d: 00
e: 011

20
13
33
21
54

a : 1
b : 1
c : 2
d : 3
e : 5
f : 8
g : 13
h : 21







a: 1111100
b: 1111101
c: 111111
d: 11110
e: 1110
f: 110
g: 10
h: 0

Date 21 Nov

Page

Matrix Chain Multiplication.

$M_1 = 10 * 20$

$M_2 = 20 * 50$

$M_3 = 50 * 1$

$M_4 = 1 * 100$

I $((M_1 \cdot M_2)(M_3 \cdot M_4))$

II $(M_1 (M_2 (M_3 M_4)))$

III $(M_1 \cdot (M_2 \cdot M_3)) \cdot M_4)$

IV $((M_1 \cdot M_2) M_3) \cdot M_4$

A matrix chain multiplication is used in compiler design for code optimisation.

$M_1 \cdot (M_2 \cdot (M_3 \cdot M_4))$

$50*1 \quad 1*100 \longrightarrow 50*1*100 = 5000$

$20*50 \quad 50*100$

$20*100 \longrightarrow 20*50*100 = 100000$

$10*20$

$10*100 \longrightarrow 10*20*100 = 20000$

Total Cost = 5000 + 100000 + 20000

= 1,25000.

$(M_1 \cdot (M_2 \cdot M_3)) M_4$

$20*50 \quad 50*1$

$20*1 \longrightarrow 20*50*1$

$= 2000$

$10*20$

$10*1 \longrightarrow 10*20*1$

200

$1*100$

$10*200 \longrightarrow 10*100*1$

$= 1000$

Total Cost = 1000 + 200 + 1000

= 2200

Complexity

Apply MM on the following array.

$A = <5, 4, 6, 2, 7>$

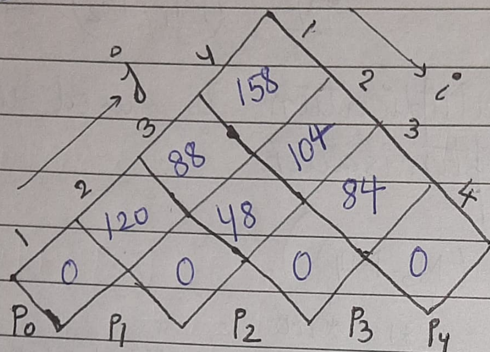
$P_0 \; P_1 \; P_2 \; P_3 \; P_4$

$A_1 = P_0 * P_1 = 5 * 4$

$A_2 = P_1 * P_2 = 4 * 6$

$A_3 = P_2 * P_3 = 6 * 2$

$A_4 = P_3 * P_4 = 2 * 7$



Cost Matrix

single Matrix so cost 0

$M[1,1]$ A_1

$M[2,2]$ A_2

$M[3,3]$ A_3

$M[4,4]$ A_4

$M[1,2]$ $A_1 . A_2 = M[1,1] + M[2,2] + P_0 . P_1 . P_2$

$A_1 = P_0 * P_1$

$A_2 = P_1 * P_2$

$0 + 0 + 5 * 4 * 6$

$= 120$

$M[2,3] = A_2 . A_3 = M[2,2] + M[3,3] + P_1 . P_2 . P_3$

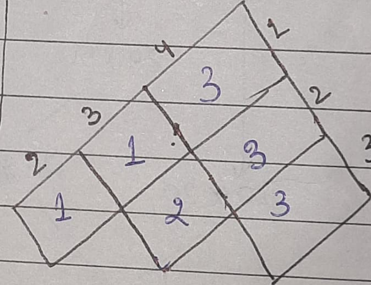
$= 0 + 0 + 4 * 6 * 2$

$= 48$

$M[3,4] = A_3 A_4 = 0 + 0 + P_2 * P_3 * P_4$

$= 0 + 0 + 6 * 2 * 7$

$= 84$



Pranathesis Matrix

$M[1,3] = A_1 (A_2 . A_3)$

OR

$(A_1 . A_2) . A_3$

$M[1,1] + M[2,3] + P_0 . P_1 . P_3$

$0 + 48 + 40$

$= 88$

$(A_1 . A_2) . A_3$

180

min

$M[2,4] = A_2 . (A_3 . A_4)$

$= M[2,2] + M[3,4] + P_1 P_2 P_4$

$= 0 + 84 + 4 * 6 * 7$

$= 252$

OR

$(A_2 . A_3) . A_4$

$= M[2,3] + M[4,4] + P_1 * P_3 * P_4$

$= 104$

$(A_1 . A_2) . (A_2 . A_3) (A_3 . A_4)$

Date
Page

$$P_0 \quad P_2$$

$M[1,4] = (A_1 . A_2) \; (A_3 . A_4)$ — ①

$M[1,2] + M[3,4] + P_0 P_2 P_4$

$= 414$

$P_0 \quad P_1 \quad P_2 \quad P_4$

$A_1 (A_2 . (A_3 . A_4))$ — ②

$M[1,1] + M[2,4] + \cancel{M[3,4]} + P_0 P_1 P_4$

$\cancel{0 + 84} = 0 + 104 + 5 * 4 * 7$

$= 104 + 140$

$= 244$

$(A_1 (A_2 . A_3)) A_4$ — ③

$M[1,3] + M[4,4] + P_0 P_3 P_4$

$= 158$

$((A_1 . A_2) . A_3) A_4$ — ④

$= 158$

$A_1 [A_2 \text{———} A_4)$

$(A \text{———} A_3) A_4$

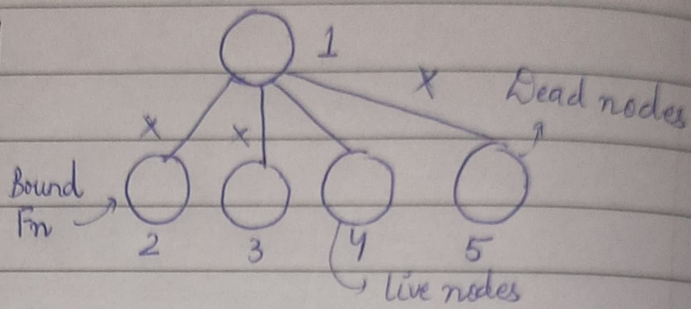
Travelling Sales Person Problem (Branch & Bound)

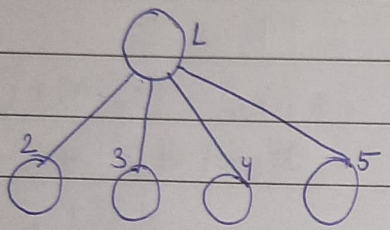
Cost Matrix

$$
\begin{array}{cc|ccccc}
 & & 1 & 2 & 3 & 4 & 5 \\
10 \leftarrow & 1 & \infty & 20 & 30 & 10 & 11 \\
2 & 2 & 15 & \infty & 16 & 4 & 2 \\
2 & 3 & 3 & 5 & \infty & 2 & 4 \\
3 & 4 & 19 & 6 & 18 & \infty & 3 \\
4 & 5 & 16 & 4 & 7 & 16 & \infty
\end{array}
$$

Row Wise

State Space Tree





Step 1: Compute Reduced Cost Matrix

$$
\begin{bmatrix}
\infty & 10 & 20 & 0 & 1 \\
13 & \infty & 14 & 2 & 0 \\
1 & 3 & \infty & 0 & 2 \\
16 & 3 & 15 & \infty & 0 \\
12 & 0 & 3 & 12 & \infty
\end{bmatrix}
$$

↓ Column wise

Column:- 1, 0, 3, 0, 0

Reduced Cost Matrix ←

$$
\begin{bmatrix}
\infty & 10 & 17 & 0 & 1 \\
12 & \infty & 11 & 2 & 0 \\
0 & 3 & \infty & 0 & 2 \\
15 & 3 & 12 & \infty & 0 \\
11 & 0 & 0 & 12 & \infty
\end{bmatrix}
$$

$10 + 2 + 2 + 3 + 4 + 1 + 3$

$= 25$

C[1:2] ① Make first row & second column ∞

② Make single entry $C_{21} = \infty$

③ Check each row & each column should contain atleast 1 zero → 1

$$
\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & 11 & 2 & 0 \\
0 & \infty & \infty & 0 & 2 \\
15 & \infty & 12 & \infty & 0 \\
11 & \infty & 0 & 12 & \infty
\end{bmatrix}
$$

$C[1:2]$ = Reduced Cost + C_{12} + zero entry

$= 25 + 10 + 0$

$= 35$

$$C[1:3] = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ \cancel{15}4 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$C[1:3] = 25 + 17 + 0$

$= 42 + 11 = 53.$

11

$$C[1:4] = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$C[1:4] = 25 + 0$

$= 25$

$= 25$

$C[1:5]$

$$= \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$C[1:5]$ + Reduced Cost

$= 1 + 25 + 2 + 3$

$= 31$

$C[1:4:2]$

first row | 4th row
fourth column | 2nd Column

$\Rightarrow \infty$

C_{41}, C_{24}, $C_{21} \Rightarrow \infty$

$C[1:4:2]$ = Reduced Cost ~~of~~ of $C[1:4]$ + C_{42} + any zero