Post correspondence problem

PCP is an undecidable decision problem that was introduced by Emil Leon Post in 1946.

1) A problem is said to be solvable if either you find a solution or you can prove mathematically problem is not solvable.

Problem

- Solvable
  - Decidable (Algorithm & procedure)
  - Undecidable (Only procedure)
- Unsolvable
  - Neither we can solve the problem nor we are in condition to say that problem cannot be solved.

Decidable: If the problem solves in a finite no. of steps and calculate the time complexity (How much time we need to solve the problem) of an algorithm.

Undecidable: Only know the no. of steps to solve a problem, but don't know how much time ~~will~~ it will take to solve a problem.

The undecidability of strings is determined with the help of PCP. Let us define the PCP.

The Post Correspondence problem consists of two lists of string that are at equal length over the input Σ. The Two lists are A & B.

A - $w_1, w_2, w_3 \ldots w_n$

B : $x_1, x_2, x_3 \ldots x_n$

Then there exists a non-empty set of integers $i_1, i_2, i_3 \ldots i_n$ such that

$w_1, w_2, w_3 \ldots w_n = x_1, x_2, x_3 \ldots x_n$

To Solve the PCP, we try all the combinations of $i_1, i_2, i_3 \ldots i_n$ to find the $w_i = x_i$. Then we can say that PCP has a solution.

eg.

Problem: Consider a Correspondence system given below.

A = (1 ; 0 ; 010 ; 11)

B = (10 ; 10 ; 01 ; 1)

The input set in $\Sigma = \{0, 1\}$

Find the Solution ?

Solution: A Solution in

1 : 2 : 1 : 3 : 3 : 4 That means

$w_1 w_2 w_1 w_3 w_3 w_4 = x_1 x_2 x_1 x_3 x_3 x_4$

The Constructed string from both list is

1 0 1 0 1 0 0 1 0 1 1 1

Set of dominoes

$$\begin{array}{c} W \\ X \end{array} \begin{bmatrix} 1 \\ \hline 10 \end{bmatrix} \begin{bmatrix} 0 \\ \hline 10 \end{bmatrix} \begin{bmatrix} 010 \\ \hline 01 \end{bmatrix} \begin{bmatrix} 11 \\ \hline 1 \end{bmatrix}$$

1 2 3 4

$$\begin{array}{c} W \\ X \end{array} \begin{bmatrix} 1 \\ \hline 10 \end{bmatrix} \begin{bmatrix} 0 \\ \hline 10 \end{bmatrix} \begin{bmatrix} 1 \\ \hline 10 \end{bmatrix} \begin{bmatrix} 010 \\ \hline 01 \end{bmatrix} \begin{bmatrix} 010 \\ \hline 01 \end{bmatrix} \begin{bmatrix} 11 \\ \hline 1 \end{bmatrix}$$

1 2 1 3 3 4

String is

A: 1 0 1 010 010 11

B: 10 10 10 010 11

Read from Left to Right

Hence, Solution is :

1 2 1 3 3 4

P. Obtain the Solution for the following Correspondence System:

A: { ba, ab, a, baa, b }

B = { bab, baa, ba, a, aba }

$\Sigma = \{a, b\}$

Find the Solution

Solut. <u>Solution is</u>

A: { ba, ab, a, baa, b }

$w_1 = ba$, $w_2 = ab$, $w_3 = a$, $w_4 = baa$, $w_5 = b$

B = { bab, baa, ba, a, aba }

$x_1 = bab$, $x_2 = baa$, $x_3 = ba$, $x_4 = a$, $x_5 = aba$

<u>Set of dominoes</u>

~~A~~ B

$$\overset{1}{\begin{bmatrix} ba \\ bab \end{bmatrix}} \overset{2}{\begin{bmatrix} ab \\ baa \end{bmatrix}} \overset{3}{\begin{bmatrix} a \\ ba \end{bmatrix}} \overset{4}{\begin{bmatrix} baa \\ a \end{bmatrix}} \overset{5}{\begin{bmatrix} b \\ aba \end{bmatrix}}$$

To obtain the correspondence system the one sequence can be chosen. Hence we get

$w_1 w_5 w_2 w_3 w_4 w_4 w_3 w_4 =$

$x_1 x_5 x_2 x_3 x_4 x_4 x_3 x_4$

So.

$$\underset{1}{\begin{bmatrix} ba \\ bab \end{bmatrix}} \underset{5}{\begin{bmatrix} b \\ aba \end{bmatrix}} \underset{2}{\begin{bmatrix} ab \\ baa \end{bmatrix}} \underset{3}{\begin{bmatrix} a \\ ba \end{bmatrix}} \underset{4}{\begin{bmatrix} baa \\ a \end{bmatrix}}$$

$$\underset{4}{\begin{bmatrix} baa \\ a \end{bmatrix}} \underset{3}{\begin{bmatrix} a \\ ba \end{bmatrix}} \underset{4}{\begin{bmatrix} baa \\ a \end{bmatrix}}$$

<u>String is</u>

A → b a b a b a b a a b a a a b a a

B → b a b a b a b a a b a a a b a a

P. Obtain the ~~correspondence~~ Solution for the following system of Post Correspondence Problem.

$A = \{100, 0, 1\}$
$B = \{1, 100, 00\}$

Solution is
$A = \{100, 0, 1\}$
$w_1 = 100, \ w_2 = 0, \ w_3 = 1$

$B = \{1, 100, 00\}$
$x_1 = 1, \ x_2 = 100, \ x_3 = 00$

Set of dominoes

$$\begin{array}{c} w \\ x \end{array} \quad \underset{1}{\left[\frac{100}{1}\right]} \quad \underset{2}{\left[\frac{0}{100}\right]} \quad \underset{3}{\left[\frac{1}{00}\right]}$$

To obtain the PCP, one solution is
$w_1 w_3 w_1 w_1 w_3 w_2 w_2 = x_1 x_3 x_1 x_1 x_3 x_2 x_2$

So,

$$\underset{1}{\left[\frac{100}{1}\right]} \underset{3}{\left[\frac{1}{00}\right]} \underset{1}{\left[\frac{100}{1}\right]} \underset{1}{\left[\frac{100}{1}\right]} \underset{3}{\left[\frac{1}{00}\right]} \underset{2}{\left[\frac{0}{100}\right]} \underset{2}{\left[\frac{0}{100}\right]}.$$

string is $\rightarrow$ A: 100 1 100 100 1 0 0
B: 1 00 1 100 100 1 00

The Solution is 1 3 1 1 3 2 2.

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Pro. Obtain the Solution for the System of PCP.

$A = \{ba, abb, bab\}$

$B = \{bab, bb, abb\}$

Solu. $A = \{ba, abb, bab\}$

$w_1 = \{ba\}, w_2 = \{abb\}, w_3 = \{bab\}$

$B = \{bab, bb, abb\}$

$x_1 = \{bab\}, x_2 = \{bb\}, x_3 = \{abb\}$

Set of dominoes

$$\begin{array}{cccc} w & \begin{bmatrix} ba \\ bab \end{bmatrix} & \begin{bmatrix} abb \\ bb \end{bmatrix} & \begin{bmatrix} bab \\ abb \end{bmatrix} \\ x & & & \\ & 1 & 2 & 3 \end{array}$$

To obtain the PCP, the one sequence can be chosen. Hence we get

Now to consider 1, 3, 2 the string
bababab b from set A
babab bbb from set B. Thus, the two strings obtained are not equal.
As we can try various combinations from both the sets to find the unique sequence, but we could not get such a sequence. Hence, there is no solution of

Problem. Does PCP with two list
$x = (b, bab^3, ba)$
$y = (b^3, ba, a)$ have a solution

Solution: $x = (b, bab^3, ba)$
$x_1 = b, \; x_2 = bab^3, \; x_3 = ba$

$y = (b^3, ba, a)$
$y_1 = b^3, \; y_2 = ba, \; y_3 = a$

Set of dominoes

$\frac{x}{y} \quad \left[\frac{b}{bbb}\right] \quad \left[\frac{babbb}{ba}\right] \quad \left[\frac{ba}{a}\right]$

To obtain the PCP, the one sequence can be chosen. Hence, we get
$x_2 x_1 x_1 x_3 = y_2 y_1 y_1 y_3$

So., $\left[\frac{babbb}{ba}\right] \left[\frac{b}{bbb}\right] \left[\frac{b}{bbb}\right] \left[\frac{ba}{a}\right]$

String is: x: babbb b b ba
y: babbb bbb a

The Solution is 2113.

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Pro. Explain how Post Correspondence problem can be treated as a game of dominoes.

So. As in the game of dominoes, the upper half corresponds to some strings and lower half corresponds to some strings.

$$\boxed{\frac{A_i}{B_i}} \qquad \frac{\text{Upper half}}{\text{Lower half}}$$

To win a game, the same string appear in upper and lower half. Wining of a game is equivalent to getting solution for post-correspondence problem.

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