Solution 1

a) Partition on A is defined to be a set of non-empty subset A each of which is

1. pairwise disjoint
2. Union yield to original set A.

1. pairwise disjoint

$A_i \cap A_j = \phi$ for each pair $(i,j) \in I$, $i \neq j$

2. $\cup A_i = A$

b)

c) $\cancel{(3)}\ 1_A^* = (3)$

d) Mutation probability is a measure of likeness that random elements of chromosome will be flipped into different element. For example if your chromosome is encoded as a binary string of length 200, if you have 2% mutation probability it means that 4 out of 200 bits (on average) picked at random will be flipped.

d) To calulate fitness value of string for minimization problem and $f(x)=0$

$$\boxed{F(x) = 1/(1+f(x))}$$

if $f(x)=0$

Section B.

Solution 2)

a) 1) Fuzzy relation for Cartesian product of A and B. (2)

$$R = A \times B = \min[\mu_A(x), \mu_B(y)]$$

$$= \begin{array}{cc} & \begin{array}{ccc} \text{positive} & \text{zero} & \text{negative} \end{array} \\ \begin{array}{c} \text{Low} \\ \text{Medium} \\ \text{High} \end{array} & \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

2) (3) $C \circ R = [0.1 \;\; 0.2 \;\; 0.7] \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix}$

$$= [0.5 \;\; 0.4 \;\; 0.5]$$

for instance

$$\mu_{C \circ R}(x_1, y_1) = \max[\min(0.1, 0.9), \min(0.2, 0.2), \min(0.7, 0.5)]$$

$$= \max(0.1, 0.2, 0.5)$$

$$= 0.5$$

b) Obtain ~~Relation b/w R_{SE} & I_a~~.

$\cancel{R = R_{SE} \times I_a}$

2.b) Computing relation between R_{SE} and N

$$\cancel{R} = R_{SE} \times N = \begin{array}{c|cccc} & 500 & 1000 & 1500 & 1800 \\ \hline 30 & .35 & .4 & .4 & .25 \\ 60 & .35 & .6 & .6 & .25 \\ 100 & .35 & .67 & .97 & .25 \\ 120 & .1 & .1 & .1 & .1 \end{array}$$

Solution 3

a) (5) $X = \{a, b, c\}$

$Y = \{1, 2\}$

$A = \{(a, 0.4), (b, 0.2), (c, 0.6)\}$

$B = \{(1, 1)\ (2, 0.4)\}$

$A' = \{(a, 0.6)\ (b, 0.9)\ (c, 0.7)\}$

$$A \times B = \begin{array}{c|cc} & 1 & 2 \\ \hline a & .4 & .4 \\ b & .2 & .2 \\ c & .6 & .4 \end{array} \Rightarrow \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))$$

$$\tilde{A} = \{(a,0.6)\ (b,0.8)\ (c,0.4)\}$$

$$Y = \{(1,1)\ (2,1)\}$$

$$\tilde{A} \times Y = \min[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)]$$

Y is universe of discourse with membership value $\forall y \in Y$ ie

$$\mu_Y(Y) = 1, \forall y \in Y$$

$$= \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} .6 & .6 \\ .8 & .8 \\ .4 & .4 \end{bmatrix} \end{array}$$

$$R(x,y) = \max(\tilde{A} \times \tilde{B}, \tilde{A} \times Y) = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} .6 & .6 \\ .8 & .8 \\ .6 & .4 \end{bmatrix} \end{array}$$

$$B' = A' \circ R(x,y)$$

$$\mu_{B'}(y) \Rightarrow \max(\min(\mu_{A'}(x), \mu_R(x,y)))$$

$$= [.6\ \ .9\ \ .7] \times \begin{bmatrix} .6 & .6 \\ .8 & .8 \\ .6 & .4 \end{bmatrix}$$

$$= \max(\min(.6,.6), \min(.9,.8), \min(.7,.6))$$

$$= \max(.6, .8, .6)$$

$$= .8$$

$$\Rightarrow \max[\min(.6,.6), \min(.9,.8), \min(.7,.4)]$$

$$\max(.6,.8,.4) = .8$$

$$.8$$

$$\Rightarrow [.8\ \ \ .8]$$

3.b) why fuzzy representation is always better than crisp representation for real world problem (2)

In real world life quantities that we consider may posses uncertainity within themselve due to vagueness and impreecision. So we need variable that is not crisp (0,1) i.e fuzzy. fuzzy variables are represented by membership funtions.

Also for real world problems we talk in terms of linguistic variables. for eg if temperature is 9°c, we convert it into linguistic variable such as cold or warm according to knowledge & decision about wearing jacket is made

fuzzy representation helps in making decisions, if fuzzification is not done properly, error decision may be reached.

Plotting membership function for weight of people using intution — (3)

Universe of discourse is weight of people. Let weight be in kg. Let linguistic variables be the following

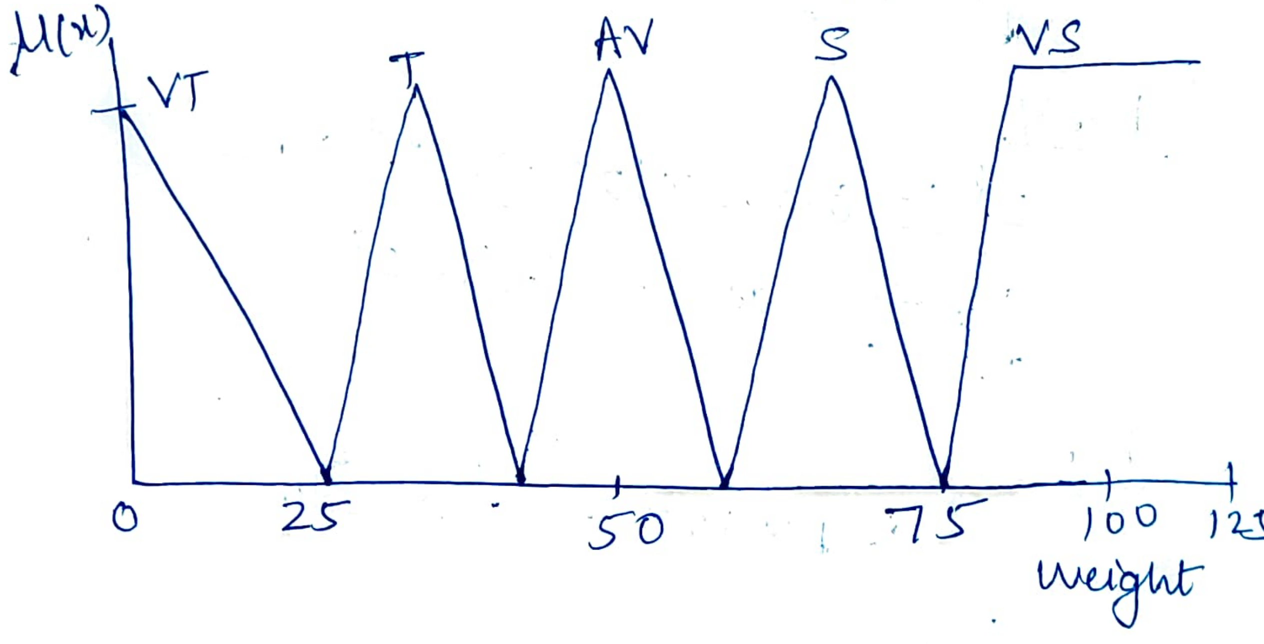
Very thin (VT): $w \leq 25$

Thin (T) : $25 < w \leq 45$

Average (AV): $45 < w \leq 60$

Stout (S) : $60 < w \leq 75$

Very stout (VS): $w > 75$



Membership function for weight of people

Solution 4 a)

a) Range of $x = 12-(-6) = 18$

Range of $y = 0.004 - 0.002 = 0.002$

Range of $z = 105 - 104 = 1$

To calulate string length for two significant digits use formula

$$2^l \geq \text{Range} \times 10^2 > 2^{l-1}$$

For x

$$2^l \geq 18 \times 10^2 \Rightarrow 2^l \geq 1800$$

$\boxed{l = 11}$ as $2^{10} < 1800 \leq 2^{11}$

For y

$$2^l \geq 0.002 \times 10^2 \Rightarrow 2^l \geq .2$$

$\boxed{l = 0}$ as $2^{-1}(.2) < 2^0$

For z

$$2^l \geq 1 \times 10^2 \Rightarrow 2^l \geq 100$$

$\boxed{l = 6}$

Solution 4 b)

Tournament Selection

String No	String	f_i	Tournament (size = 2)	Selected individual (most fit)	New mating Pool.
1	001100	8	2 and 4	2	010101
2	010101	12	3 and 6	6	100000
3	101011	6	7 and 6	7	010100
4	110001	2	1 and 3	1	001100
5	000100	18	5 and 6	5	000100
6	100000	9	2 and 6	2	010101
7	010100	10	5 and 4	5	000100

We have made 7 tournaments of size 2 (as popsize = 7).

eg → Select individual 2 and 4 at random

f_2 f_4
12 2

2 is winner, so 2 is selected

→ Select individual 3 & 6 at random

f_3 f_6
6 9

fitness value of 6 is higher, so 6 is selected.

Similarly we can find for others

From table we can see

Expected count of string (1, 6) = 1

" " " string (2, 5) = 2

" " " string (3, 4) = 0

" " " string 7 = 1

Expected no. of copies of best string is 2

Result of students may vary according to combination of tournaments taken & tournament size

Solution 5 a)

R is a relation on $P \times D$

$$R = \begin{array}{c|cccc} & D_1 & D_2 & D_3 & D_4 \\ \hline P_1 & 0.6 & 0.6 & 0.9 & 0.8 \\ P_2 & 0.1 & 0.2 & .9 & .8 \\ P_3 & 0.9 & 0.3 & .4 & .8 \\ P_4 & 0.9 & .8 & .4 & .8 \end{array}$$

$$T = \begin{array}{c|cccc} & S_1 & S_2 & S_3 & S_4 \\ \hline D_1 & .1 & .2 & .7 & .9 \\ D_2 & 1.0 & 1 & .4 & .6 \\ D_3 & 0.0 & 0 & .5 & .9 \\ D_4 & 0.9 & 1 & .8 & .2 \end{array}$$

Fuzzy relation

Association of plants with different symptom of disease can be calculated using composition between relation R & T

$\therefore\ S = R \circ T$

From a given relation R & T

$$\mu_S(P_1, S_1) = \max(\min(\mu_R(P_1, D_1), \mu_T(D_1, S_1)), \min(\mu_R(P_1, D_2), \mu_T(D_2, S_1)), \min(\mu_R(P_1, D_3), \mu_T(D_3, S_1)), \min(\mu_R(P_1, D_4), \mu_T(D_4, S_1)))$$

$$= \max(\min(0.6, 0.1), \min(.6, 1), \min(.9, 0), \min(.8, .9))$$

$$= \max(.1, .6, 0, .8)$$

$$= .8$$

Similarly we can calculate

$\mu_S(P_1, S_2) = .8$

$\mu_S(P_1, S_3) = .8$

$\mu_S(P_1, S_4) = .9$

$\mu_S(P_2, S_1) = .8$

$\mu_S(P_2, S_2) = .8$

$\mu_S(P_2, S_3) = .8$

$\mu_S(P_2, S_4) = .9$

$\mu_S(P_3, S_1) = .8$

$\mu_S(P_3, S_2) = .8$

$\mu_S(P_3, S_3) = .8$

$\mu_S(P_3, S_4) = .9$

$\mu_S(P_4, S_1) = .8$

$\mu_S(P_4, S_2) = .8$

$\mu_S(P_4, S_3) = .7$

$\mu_S(P_4, S_4) = .9$

$$\therefore S = R \circ T = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} .8 & .8 & .8 & .9 \\ .8 & .8 & .8 & .9 \\ .8 & .8 & .8 & .9 \\ .8 & .8 & .7 & .9 \end{bmatrix} \end{array}$$

Solution ~~5~~(b)

Fuzzy relation R on a single universe X is also a relation from X to X. It is a fuzzy equivalence relation if all 3 of the following properties of matrix relation defines

1. Reflexivity $\mu_R(x_i, x_i) = 1$
2. Symmetry $\mu_R(x_i, x_j) = \mu_R(x_j, x_i)$
3. Transitivity $\mu_R(x_i, x_j) = \lambda_1$ and

$\mu_R(x_j, x_k) = \lambda_2$

then $\mu_R(x_i, x_k) = \lambda$

where $\lambda \geq \min[\lambda_1, \lambda_2]$

Here the given relation is reflexive & symmetric but it is not transitive

eg $\mu_R(x_1, x_2) = 0.6 \quad \mu_R(x_2, x_5) = 0.$

$\mu_R(x_1, x_5)\ 0.3 \leq \min(0.8, 0.6)$

Doing one composition we get following relation

$$R^2 = R \circ R = \begin{bmatrix} 1 & .6 & .4 & .3 & .6 \\ .6 & 1 & .4 & .5 & .8 \\ .4 & .4 & 1 & 0 & .4 \\ .3 & .5 & 0 & 1 & .5 \\ .6 & .8 & .4 & .5 & 1 \end{bmatrix}$$

Here also transitivity doesn't hold.

$\mu_{R^2}(x_1, x_2) = .6 \quad \mu_{R^2}(x_2, x_4) = .5$

$\mu_{R^2}(x_1, x_4) = 0.3 \leq \min(0.6, 0.5)$

Finding R^3

$$R^3 = \begin{bmatrix} 1 & .6 & .4 & .5 & .6 \\ .6 & 1 & .4 & .5 & .8 \\ .4 & .4 & 1 & .4 & .4 \\ .5 & .5 & .4 & 1 & .5 \\ .6 & .8 & .4 & .5 & 1 \end{bmatrix}$$

$\mu_{R^3}(x_1, x_2) = .6, \quad \mu_{R^3}(x_2, x_4) = .5$

$\mu_{R^3}(x_1, x_4) = .5 \geq \min(.6, .5)$

Transitivity is satisfied hence equivalance relation is also satisfied.

Ques 6 a)

1. If x is A then y is B else y is C

$$= (A \times B) \cup (A' \times C)$$

$$= \max(\min(\mu_A(x), \mu_B(\cancel{x}y)), \min((1-\mu_A(x)), \mu_C(y)))$$

$$A \times B = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ .2 & .6 & .6 & 0 \\ .2 & .8 & .8 & 0 \\ .2 & 1 & .8 & 0 \end{bmatrix} \end{array}$$

$$A' = \cancel{\{(a, x)}\ [1 - A]$$

$$\mu_{A'}(x) = 1 - \mu_A(x)$$

$$A' = \{(a,1)\ (b,0.4)\ (c,0.2)\ (d,0)\}$$

$$\therefore A' \times C = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & .4 & 1 & .8 \\ 0 & .4 & .4 & .4 \\ 0 & .2 & .2 & .2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

now finding $(A \times B) \cup (A' \times C)$

$$= \max(\mu_{(A \times B)}, \mu_{(A' \times C)})$$

$$= \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & .4 & 1 & .6 \\ .2 & .6 & .6 & .4 \\ .2 & .8 & .8 & .2 \\ .2 & 1 & .8 & 0 \end{bmatrix} \end{array}$$

2) If x is A then y is B

$$= (A \times B) \cup (A' \times Y)$$

$$A \times B = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ .2 & .6 & .6 & 0 \\ .2 & .8 & .8 & 0 \\ .2 & 1 & .8 & 0 \end{bmatrix} \end{array}$$

$$A' = \{(a,1)\ (b,.4)\ (c,.2)\ (d,0)\}$$

$$Y = \{(1,1)\ (2,1)\ (3,1)\ (4,1)\}$$ Universal set

$$A' \times Y = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ .4 & .4 & .4 & .4 \\ .2 & .2 & .2 & .2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Evaluating $(A \times B) \cup (A' \times Y) = \max(\mu_{(A \times B)}, \mu_{(A' \times Y)})$

$$= \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ .4 & .6 & .6 & .4 \\ .2 & .8 & .8 & .3 \\ .2 & 1 & .8 & 0 \end{bmatrix} \end{array}$$

Solution 6 b)

1. If temperature is high rotation is slow.

$$R = (H \times S) \cup (\bar{H} \times Y)$$

2. Temperature is very high (VH)

Thus to deduce rotation is ~~slow~~ Quite slow, we make use of Composition rule

$$\boxed{QS = VH \circ R}$$

To calculate R we are given

$$H = \{(70,1)\ (80,1)\ (90,.3)\}$$

$$VH = \{(80,.6)\ (90,.9)\ (100,1)\}$$

$$S = \{(30,.8)\ (40,1)\ (50,.6)\}$$

$$H \times S = \begin{array}{c} \\ 70 \\ 80 \\ 90 \end{array} \begin{array}{c} \begin{array}{ccc} 30 & 40 & 50 \end{array} \\ \begin{bmatrix} .8 & 1 & .6 \\ .8 & 1 & .6 \\ .3 & .3 & .3 \end{bmatrix} \end{array}$$

$$\bar{H} = \{(70,0)\ (80,0)\ (90,0.7)\}$$

$$\bar{H} \times Y = \begin{array}{c} \\ 70 \\ 80 \\ 90 \end{array} \begin{array}{c} \begin{array}{ccc} 30 & 40 & 50 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ .7 & .7 & .7 \end{bmatrix} \end{array}$$

$$R = \max(H \times S, \bar{H} \times Y)$$

$$= \begin{bmatrix} .8 & 1 & .6 \\ .8 & 1 & .6 \\ .7 & .7 & .7 \end{bmatrix}$$

$QS = VH \circ R$. (using min max composition)

$$= [.6 \quad .9 \quad 1] \circ \begin{bmatrix} .8 & 1 & .6 \\ .8 & 1 & .6 \\ .7 & .7 & .7 \end{bmatrix}$$

$$= [.8 \quad .9 \quad .7]$$

ie

$$QS = [(10, .8)\ (20, .9)\ (30, .7)]$$

Solution 7a)

1. Max Membership

not applicable for the given figure, only applicable to peak membership functions.

2. Weighted Average Method

$$x^* = \frac{2(.7) + 4(1)}{.7 + 1} = 3.176$$

Mean - Max Method

$$x^* = \frac{a+b}{2} = \frac{2.5+3.5}{2} = 3$$

Center of Sum Method

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{c_j}(x)\,d(x)}{\int_x \sum_{i=1}^{n} \mu_{c_j}(x)\,d(x)}$$

$$= \frac{\left[\int_0^6 \left[\frac{1}{2} \times .7 \times (3+2) \times 2 + \frac{1}{2} \times 1 \times (2+4) \times 4\right] dx\right]}{\left[\int_0^6 \left[\frac{1}{2} \times .7 \times (3+2) + \frac{1}{2} \times 1 \times (2+4) \times 4\right] dx\right]}$$

$$= \frac{\int_0^6 (3.5+12)\,dx}{\int_0^6 (1.75+3)\,dx} = 2.84$$

First of maxima :- defuzzified output value is

$$x^* = 3$$

Last of maxima

$$x^* = 4$$

Solution 7 b)

Let U - Universe of discourse is

$\{U : x = 75^\circ \geq y = 60^\circ \geq z = 45^\circ; x+y+z = 180^\circ\}$

1. Calculating membership of Isoceles triangle

$$\mu_I(U) = 1 - \frac{1}{60^\circ} \min(x-y, y-z)$$

$$= 1 - \frac{1}{60^\circ} \min(15^\circ, 15^\circ)$$

$$= 1 - \frac{1}{60^\circ} \times 15^\circ$$

$$\mu_I(u) = 1 - 0.25 = .75$$

2. Calculating membership of right angle triangle

$$\mu_R(U) = 1 - \frac{1}{90^\circ} |\cancel{x} - 90^\circ|$$

$$= 1 - \frac{1}{90^\circ} |75 - 90^\circ|$$

$$= 1 - \frac{1}{90} \times 15 = 1 - .166$$

$$= .833$$

3. Membership value of Isoceles Right angle triangle

$$\mu_{IR}(U) = \min[\mu_I, \mu_R]$$

$$= \min[.75, .833] = .75$$

4. Membership value of equilateral triangle

$$\mu_E(U) = 1 - \frac{1}{180^\circ}(X - Z) = 1 - \frac{1}{180^\circ}(75^\circ - 45^\circ)$$

$$= 1 - \frac{1}{18\cancel{0}} \times 3\cancel{0} = 1 - .1666 = .833$$

5. Membership value of other triangles

$$\mu_O(U) = \min[(1 - .75), (1 - .833), (1 - .833)]$$

$$= \min[.25, .167, .167]$$

$$= .167$$

Ans 8 b

Problem ⇒

For function optimization, fitness function can be used in following ways

$F(x) = f(x)$ for maximization problem

$F(x) = 1/f(x)$ for minimization problem if $f(x) \neq 0$

$f(x) = \frac{1}{1+f(x)}$ for minimization problem if $f(x) = 0$

$F(x)$ is first derived from objective functions & is used in successive genetic operators.

Problem $y = x^2$, x is in integer interval (0,31)

Assume we are using Binary representation & popsize = 4

length of chromosome = ~~2^5~~ 5 bits.

Selection Method - Roulette wheel.

String NO	Initial pop	x value	x^2	Prob.	Expected count	Actual count
1	01101	13	169	.14	.58	1
2	11000	24	576	.49	1.07	2
3	01000	8	64	.06	0.22	0
4	10011	19	361	.31	1.23	1
Sum			1170	1.00	4.00	4
Average			293	.25	1.00	1
Max			576	.49	1.97	2

Crossover

String NO	Mating Pool	Crossover point	offspring after Crossover	x value	fitness $f(x)=x^2$
1	01101	4	01100	12	144
2	11000	4	11001	25	625
3	11000	2	11011	27	729
4	10011	2	10000	16	256
Sum					1754
Average					439
Max					729

Mutation

String No.	Offspring after crossover	Offspring after Mutation	x value	Fitness $f(x)=x^2$
1	01100	11100	26	676
2	11001	11001	25	625
3	11011	11011	27	729
4	10000	10100	18	324
Sum				2354
Average				588.5
Max				729

Solution 9a

$$f(x_1, x_2) = 21.5 + x_1 \cdot \sin(4\pi x_1) + x_2 \cdot \sin(20\pi x_2)$$

precision = 4 decimal places.

Calculation of length of x_1

$$2^{l_1} \geq \text{Range of } x_1 \times 10^{\text{precision}}$$

$$2^{l_1} \geq (12.1 - (-3.0)) \times 10^4$$

$$2^{l_1} \geq 151000$$

$$l_1 = 18 \text{ as } 2^{17} < 15000 \leq 2^{18}$$

Calculation of length of x_2

$$2^{l_2} \geq (5.8 - 4.1) \times 10^4$$

$$2^{l_2} \geq 17000$$

$$l_2 = 15 \quad \text{as } 2^{14} < 17000 \leq 2^{15}$$

Length of Chromoscome of 33 bit

$$\underbrace{010001001011010000}_{x_1}\ \underbrace{111110010100010}_{x_2}$$

Applying approximation f'n we get

$$x_1 = -3.0 + \text{decimal}(010001001011010000) \times \frac{12.1-(-3.0)}{2^{18}-1}$$

$$= -3.0 + \frac{70352 \times 15.1}{262143} = 1.052426$$

$$x_2 = 4.1 + \text{decimal}(111110010100010) \times \frac{5.8-4.1}{2^{15}-1}$$

$$= 4.1 + \frac{31906 \times 1.7}{32767} = 5.75530$$

Fitness value $= f(x_1, x_2)$

$$= f(1.052426, 5.755330)$$

$$= 20.2526$$

Solution 9 b)

Roulettee wheel

String No.	String	fitness	Pi	A	B	C	D	E	Pop(new
1	01101	5	.0413	.2480	.0413	.887	6	1	01101
2	11000	2	.0165	.0992	.0578	.1298	4	0	00111
3	10110	1	.0082	.0496	.066	.2397	6	0	00010
4	00111	10	.0826	.4960	.1486	.1899	6	1	00010
5	10101	3	.0247	.1488	.1733	.0313	1	0	0001(
6	00010	100	.8264	4.96	.9997	.772	6	4	0001(

Sum 121

Average 20.16

Here

P_i = Probability of chromosome being selected

$$\boxed{P_i = \frac{f_i}{\sum_{j=1}^{n} f_j}}$$

f_i = fitness value of string

n = popsize

$P_1 = \frac{5}{121}$, $P_2 = \frac{2}{121}$, $P_3 = \frac{1}{121}$, $P_4 = \frac{10}{121}$

$P_5 = \frac{3}{121}$ $\quad P_6 = \frac{100}{121}$

A = Expected Count

$A_i = \frac{f_i}{\bar{F}}$ where $\bar{F} = \sum_{j=1}^{n} f_j / n$

Here $\bar{F} = \frac{121}{6} = 20.16$

$A_1 = \frac{5}{20.16}$, $A_2 = \frac{2}{20.16}$, $A_3 = \frac{1}{20.16}$, $A_4 = \frac{10}{20.16}$

$A_5 = \frac{3}{20.16}$, $A_6 = \frac{100}{20.16}$

B = Commulative probability
C = Random no. b/w 0 & 1
D = String no.
E = Count in mating pool.

Expected count of string having maximum fitness value is 4.

Solution 8a)

Minimize

$$(x_1 - 2.5)^2 + (x_2 - 5)^2$$

Subject to

$$5.5x_1 + 2x_2^2 - 18 \leq 0$$

$$0 <= x_1, x_2 \leq 5$$

Range of x_1

$$= 5 - 0 = 5$$

Range of x_2

$$= 5 - 0 = 5$$

For 3 decimal places

Calculation of length of x_1

$2^{l_1} \geq$ Range of $x_1 \times 10^{\text{precision}}$

$2^{l_1} \geq 5 \times 10^3$

$2^{l_1} \geq 5000$

$\boxed{l_1 = 13}$

Calculation of length of x_2

$\boxed{l_2 = 13}$

For 2 decimal places

$2^{l_1} \geq 5 \times 10^2$

$l_1 = 9$

Similarly

$l_2 = 9$

1. No. of bits required for Coding variable

For 2 decimal places

$9 + 9 = 18$

For 3 decimal places

$13 + 13 = 26$

2. Fitness function required is $= \frac{1}{1 + f(x)}$ $\because f(x)$ can be 0