From PDA to CFG

PDA $A(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ can be converted into CFG $G(V_N, \Sigma, P, S)$ as follows.

(i) The V_n symbol is calculated as follows:

$V_N = S \cup \{[q, Z, q']\}$

where

$q, q' \in Q$

$Z \in \Gamma$

(ii) The production in P is calculated by transitions present into the PDA as follows:

a) S production is given by

$S \to [q_0, Z_0, q]$ for every q in Q.

b) Each transition erasing the push down symbol

$\delta(q, a, Z) = (q', \lambda)$ gives the production

$[q, Z, q'] \to a$

c) Each transition not erasing the push down symbol

$\delta(q, a, Z) = (q_1, Z_1 Z_2 \ldots Z_m)$ gives the production

$[q, Z, q'] \to a [q_1, Z_1, q_2] [q_2, Z_2, q_3] \ldots [q_m, Z_m, q']$

where $q', q_2, \ldots q_m$ can be any state in Q.

Q. Construct the CFG, which accepts the PDA given below.

PDA A $\{\{q_0, q_1\}, \{a, b\} \{Z_0, Z\}, \delta, q_0, Z_0, \phi\}$

δ is defined by

$\delta(q_0, b, Z_0) = (q_0, ZZ_0)$

$\delta(q_0, \lambda, Z_0) = (q_0, \lambda)$

$\delta(q_0, b, Z) = (q_0, ZZ)$

$\delta(q_0, a, Z) = (q_1, Z)$

$\delta(q_0, b, Z) = (q_1, \lambda)$

$\delta(q_1, a, Z_0) = (q_0, Z_0)$

Solution: PDA A is defined

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

$\Gamma = \{Z_0, Z\}$

$q_0 = \{q_0\}$

$Z_0 = \{Z_0\}$

$F = \phi$

δ production is given as follows:

(i)

P_1: $S \rightarrow [q_0, Z_0, q_0]$

P_2. $S \rightarrow [q_0, Z_0, q_1]$

(ii) Transitions erasing the pushdown symbol:

a) $\delta(q_0, \lambda, Z_0) = (q_0, \lambda)$

b) $\delta(q_1, b, Z) = (q_1, \lambda)$

a) $\delta(q_0, \lambda, Z_0) = (q_0, \lambda)$ gives.

$[q_0, Z_0, q_0] \rightarrow \lambda$ (P3)

b) $\delta(q_1, b, Z) = (q_1, \lambda)$

$[q_1, Z, q_1] \rightarrow b$ (P4)

(iii) Transitions not erasing the pushdown symbol

a) $\delta(q_0, b, Z_0) = (q_0, ZZ_0)$

$(q_0, Z_0, q_0) \rightarrow b[q_0, Z, q_0]\ [q_0, Z_0, q_0]$ – (P5)

$(q_0, Z_0, q_0] \rightarrow b[q_0, Z, q_1]\ [q_1, Z_0, q_0]$ (P6)

$(q_0, Z_0, q_1] \rightarrow b\ [q_0, Z, q_0]\ [q_0, Z_0, q_1]$ (P7)

$[q_0, Z_0, q_1] \rightarrow b[q_0, Z, q_1]\ [q_1, Z_0, q_1]$ (P8)

b) $\delta(q_0, b, Z) = (q_0, ZZ)$ gives

$(q_0, Z, q_0) \rightarrow b[q_0, Z, q_0]\ [q_0, Z, q_0]$ – P9

$(q_0, Z, q_0) \rightarrow b[q_0, Z, q_1]\ [q_1, Z, q_0]$ – P10

$(q_0, Z, q_1) \rightarrow b\ [q_0, Z, q_0]\ [q_0, Z, q_1]$ – P11

$(q_0, Z, q_1) \rightarrow b\ [q_0, Z, q_1]\ [q_1, Z, q_1]$ – P12 c.) $\delta(q_0, a, Z) = (q_1, Z)$ gives

$[q_0, Z, q_0] \rightarrow a[q_1, Z, q_0]$ - P13

$[q_0, Z, q_1] \rightarrow a[q_1, Z, q_1]$ P14.

d.) $\delta(q_1, a, Z_0) = (q_0, Z_0)$ gives

$[q_1, Z_0, q_0] \rightarrow a[q_0, Z_0, q_0]$ P15

$[q_1, Z_0, q_1] \rightarrow a[q_0, Z_0, q_1]$ P16

$V_N = \{ S, [q_0, Z_0, q_0], [q_1, Z, q_1], [q_0, Z_0, q_1],$
$[q_0, Z, q_0], [q_0, Z, q_1], [q_1, Z_0, q_0],$
$[q_1, Z_0, q_1] \}$

Scanned with CamScanner

Q. Find the CFG for the PDA, which is given below

PDA $A(\{q_0, q_1, q_2\}, \{a,b\}, \{A, Z_0\}, \delta, q_0, Z_0, \phi\}$

$\delta(q_0, a, Z_0) = (q_0, AZ_0)$

$\delta(q_0, a, A) = (q_0, AA)$

$\delta(q_0, b, A) = (q_1, A)$

$\delta(q_1, a, A) = (q_1, \lambda)$

$\delta(q_1, \lambda, Z_0) = (q_2, \lambda)$

Solution PDA A is defined

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{A, Z_0\}$

$q_0 = q_0$

$Z_0 = Z_0$

$F = \phi$

a) S productions is given by:

P_1 : $S \rightarrow [q_0, Z_0, q_0]$

P_2 : $S \rightarrow [q_0, Z_0, q_1]$

P_3 : $S \rightarrow [q_0, Z_0, q_2]$

b) Erasing the pushdown symbols

$\delta(q_1, a, A) = (q_1, \lambda)$ gives

$[q_1, A, q_1] \rightarrow a$ P_4

$\delta(q_1, \lambda, z_0) = (q_2, \lambda)$ gives

$[q_1, z_0, q_2] \rightarrow \lambda$ (P5)

Not erasing the pushdown symbols

$\delta(q_0, a, z_0) = (q_0, Az_0)$ gives

$[q_0, z_0, q_0] \rightarrow a[q_0, A, q_0]\ [q_0, z_0, q_0]$ – P6

$[q_0, z_0, q_0] \rightarrow a[q_0, A, q_1]\ [q_1, z_0, q_0]$ – P7

$[q_0, z_0, q_0] \rightarrow a[q_0, A, q_2]\ [q_2, z_0, q_0]$ – P8

$[q_0, z_0, q_1] \rightarrow a[q_0, A, q_0]\ [q_0, z_0, q_1]$ – P9

$[q_0, z_0, q_1] \rightarrow a[q_0, A, q_1]\ [q_1, z_0, q_1]$ – P10

$[q_0, z_0, q_1] \rightarrow a[q_0, A, q_2]\ [q_2, z_0, q_2]$ – P11

$[q_0, z_0, q_2] \rightarrow a[q_0, A, q_0]\ [q_0, z_0, q_2]$ – P12

$[q_0, z_0, q_2] \rightarrow a[q_0, A, q_1]\ [q_1, z_0, q_2]$ – P13

$[q_0, z_0, q_2] \rightarrow a[q_0, A, q_2]\ [q_2, z_0, q_2]$ – P14

$\delta(q_0, a, A) = (q_0, AA)$ gives

$[q_0, A, q_0] \rightarrow a[q_0, A, q_0]\ [q_0, A, q_0]$ P15

$[q_0, A, q_0] \rightarrow a[q_0, A, q_1]\ [q_1, A, q_0]$ P16

$[q_0, A, q_0] \rightarrow a[q_0, A, q_2]\ [q_2, A, q_0]$ P17

$[q_0, A, q_1] \rightarrow a\,[q_0, A, q_0]\,[q_0, A, q_1]$ P_{18}

$[q_0, A, q_1] \rightarrow a\,[q_0, A, q_1]\,[q_1, A, q_1]$ P_{19}

$[q_0, A, q_1] \rightarrow a\,[q_0, A, q_2]\,[q_2, A, q_1]$ P_{20}

$[q_0, A, q_2] \rightarrow a\,[q_0, A, q_0]\,[q_0, A, q_2]$ P_{21}

$[q_0, A, q_2] \rightarrow a\,[q_0, A, q_1]\,[q_1, A, q_2]$ P_{22}

$[q_0, A, q_2] \rightarrow a\,[q_0, A, q_2]\,[q_2, A, q_2]$ P_{23}

$\delta(q_0, b, A) = (q_1, A)$ gives

$[q_0, A, q_0] \rightarrow b\,[q_1, A, q_0]$ P_{24}

$[q_0, A, q_1] \rightarrow b\,[q_1, A, q_1]$ P_{25}

$[q_0, A, q_2] \rightarrow b\,[q_1, A, q_2]$ P_{26}

CFG $G\,(V_N, \Sigma, P, S)$

$V_N = \{\ S, [$

Q Construct the CFG for the PDA

PDA $A(\{q_0, q_1\}, \{0,1\}, \{x, z_0\}, \delta, q_0, z_0, \{q_1\})$

$\delta(q_0, 1, z_0) = (q_0, x z_0)$

$\delta(q_0, 1, x) = (q_0, xx)$

$\delta(q_0, 0, x) = (q_0, x)$

$\delta(q_0, \lambda, x) = (q_1, \lambda)$

$\delta(q_1, \lambda, x) = (q_1, \lambda)$

$\delta(q_0, 0, x) = (q_1, xx)$

$\delta(q_1, 0, z_0) = (q_1, \lambda)$

Solution: PDA A is defined.

$Q = \{q_0, q_1\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{x, z_0\}$

$q_0 = q_0$

$z_0 = z_0$

$F = \{q_1\}$

a) S productions is given by:

$P_1: \quad S \rightarrow [q_0, z_0, q_0]$

$P_2: \quad S \rightarrow [q_0, z_0, q_1]$

Erasing transitions are given by

$\delta(q_0, \lambda, X) = (q_1, \lambda)$ gives

$[q_0, X, q_1] \rightarrow \lambda$

$\delta(q_1, \lambda, X) = (q_1, \lambda)$ Gives

$[q_1, X, q_1] \rightarrow \lambda$

$\delta(q_1, 0, Z_0) = (q_1, \lambda)$ gives

$[q_1, Z_0, q_1] \rightarrow 0$

Non-erasing transitions are given by

$\delta(q_0, 1, Z_0) = (q_0, XZ_0)$ gives

$[q_0, Z_0, q_0] \rightarrow 1 [q_0, X, q_0] [q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \rightarrow 1 [q_0, X, q_1] [q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \rightarrow 1 [q_0, X, q_0] [q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] \rightarrow 1 [q_0, X, q_1] [q_1, Z_0, q_1]$

$\delta(q_0, 1, X) = (q_0, XX)$ gives

$[q_0, X, q_0] \rightarrow 1 [q_0, X, q_0] [q_0, X, q_0]$

$[q_0, X, q_0] \rightarrow 1 [q_0, X, q_1] [q_1, X, q_0]$

$[q_0, X, q_1] \rightarrow 1 [q_0, X, q_0] [q_0, X, q_1]$

$[q_0, X, q_1] \rightarrow 1 [q_0, X, q_1] [q_1, X, q_1]$

$\delta(q_0, 0, X) = (q_0, X)$ gives

$[q_0, X, q_0] \rightarrow 0\,[q_0, X, q_0]$

$[q_0, X, q_1] \rightarrow 0\,[q_0, X, q_1]$

$\delta(q_0, 0, X) = (q_1, X\,X)$ gives

$[q_0, X, q_0] \rightarrow 0\,[q_1, X, q_0]\,[q_0, X, q_0]$

$[q_0, X, q_0] \rightarrow 0\,[q_1, X, q_1]\,[q_1, X, q_0]$

$[q_0, X, q_1] \rightarrow 0\,[q_1, X, q_0]\,[q_0, X, q_1]$

$[q_0, X, q_1] \rightarrow 0\,[q_1, X, q_1]\,[q_1, X, q_1]$