Greibach Normal Form (GNF)

A CFG is said to be in GNF if every production of the grammar is of the form.

$A \rightarrow a\alpha$ where $A \rightarrow a$

$A \in V_N$ $| A \rightarrow aC_1C_2$

$a \in \Sigma$

$\alpha \in V_N^*$ (α may by empty)

The R.H.S. of the production into GNF will contain single terminal followed by zero or more variables.

$\boxed{\text{A CFG generating the set accepted by a PDA is in GNF}}$

$S \rightarrow \Lambda$ is in GNF if

$\Lambda \in L(G)$

when $\Lambda \in L(G)$, we assume that S doesn't appear on the R.H.S of any production.

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Theorem 1:

$A \rightarrow Bab$

$B \rightarrow aA \mid bB \mid aa \mid AB$

Here A production will start with B and none of the B production start with A then we can write the productions as follows:

$A \rightarrow aAab \mid bBab \mid aaab \mid ABab$

Theorem 2

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid A\alpha_3 \mid \beta_1 \mid \beta_2 \mid \beta_3 \mid \beta_4$

where β_1, β_2, β_3, β_4 doesn't start with A, then we can introduce a new variable 'Z' to write the production as follows.

$A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \beta_4 \mid \beta_1 Z \mid \beta_2 Z \mid \beta_3 Z \mid \beta_4 Z$

$Z \rightarrow \alpha_1 Z \mid \alpha_2 Z \mid \alpha_3 Z \mid \alpha_1 \mid \alpha_2 \mid \alpha_3$

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eg. $A \rightarrow aBD \mid bDB \mid C$

$A \rightarrow AB \mid AD.$

$$A \rightarrow \underbrace{AB}_{\alpha_1} \mid \underbrace{AD}_{\alpha_2} \mid \underbrace{aBD}_{\beta_1} \mid \underbrace{bDB}_{\beta_2} \mid \underbrace{c}_{\beta_3}$$

$A \rightarrow aBD \mid bDB \mid c \mid aBDZ \mid bDBZ \mid cZ$

$Z \rightarrow BZ \mid DZ \mid B \mid D.$

Steps to Convert CFG into GNF

① Check if the given CFG has any unit productions, null production. Remove if there are any unit & null pro.

② Check whether the CFG is already in CNF and convert it into CNF if it is not.

③ Change the names of Non-Terminal symbols into some A_i in ascending order of i.

Ⓠ Convert the CFG into GNF whose productions are given below:

$S \to AB$

$A \to BS \mid b$

$B \to SA \mid a$

① Remove the null & unit productions if any

② Check the Grammar is in CNF

③ change the name of non-Terminal

Let $S = A_1$

$A = A_2$

$B = A_3$

$A_1 \rightarrow A_2 A_3$ —(i)

$A_2 \rightarrow A_3 A_1 \mid b$ —(ii)

$A_3 \rightarrow A_1 A_2 \mid a$ —(iii)

Using (iii) and (i) apply theorem (1)

$A_1 \rightarrow A_2 A_3$

$A_3 \rightarrow A_1 A_2 \mid a$

$$\boxed{A_3 \rightarrow A_2 A_3 A_2 \mid a} \quad \text{—(iv)}$$

using (iv) & (II) apply therem 1

$A_2 \rightarrow A_3 A_1 \mid b$ $\quad$ $A_3 \rightarrow A_2 A_3 A_2 \mid a$

$$\boxed{A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a} \quad \text{—(v)}$$

Using equation (5) Apply theorem (2)

$A_3 \rightarrow A_3 \underbrace{A_1 A_3 A_2}_{\alpha_1} \mid \underbrace{b A_3 A_2}_{\beta_1} \mid \underbrace{a}_{\beta_2}$

$A_3 \rightarrow b A_3 A_2 \mid a \mid b A_3 A_2 Z \mid aZ$ —(vi)

$Z \rightarrow A_1 A_3 A_2 Z \mid A_1 A_3 A_2$ (vii)

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equation (ii) and (vi) apply theorem 1

$A_2 \to A_3 A_1 \mid b$ — (ii)

$\boxed{A_3 \to bA_3A_2 \mid a \mid bA_3A_2Z \mid aZ}$ — (vi)

$\boxed{A_2 \to bA_3A_2A_1 \mid aA_1 \mid bA_3A_2ZA_1 \mid aZA_1 \mid b}$ (8)

equation (i) and (8) apply theorem 1

$A_1 \to A_2 A_3$

$\boxed{A_1 \to bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2ZA_1A_3 \mid bA_3}$ — (9)

put (9) in (7) by theorem 1

$Z \to A_1A_3A_2Z \mid A_1A_3A_2$ — (vii)

$$\boxed{\begin{aligned} Z \to\ & bA_3A_2A_1A_3A_3A_2Z \mid aA_1A_3A_3A_2Z \mid \\ & bA_3A_2ZA_1A_3A_3A_2Z \mid bA_3A_3A_2Z \mid \\ & bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid \\ & bA_3A_2ZA_1A_3A_3A_2 \mid bA_3A_3A_2 \end{aligned}}$$

Q. Convert the CFG into GNF, whose productions are:

$S \rightarrow AA \mid a$

$A \rightarrow SS \mid b$

Sol. Step 1: Remove all null and unit productions.

There is no null and unit productions in the given grammar.

Step 2: Let $S = A_1$

$A = A_2$

$\therefore A_1 \rightarrow A_2 A_2 \mid a$ — (i)

$A_2 \rightarrow A_1 A_1 \mid b$ — (ii)

In equation (ii) $i > j$, so we substitute the value of A_1

$i > j$ means $\underset{i}{A_2} \rightarrow \underset{j}{A_1} A_1$

$A_2 \rightarrow \underline{A_1} A_1 \mid b$

$$\boxed{A_2 \rightarrow A_2 A_2 A_1 \mid a A_1 \mid b \quad \text{— (iii)}}$$

Applying theorem (ii) on (iii) equation

$$A_2 \rightarrow \underbrace{A_2 A_2 A_1}_{\alpha_1} \mid \underbrace{aA_1}_{\beta_1} \mid \underbrace{b}_{\beta_2}$$

$$\boxed{A_2 \rightarrow aA_1 \mid b \mid aA_1Z \mid bZ \quad \text{—(iv)}}$$

$$Z \rightarrow A_2A_1Z \mid A_2A_1 \quad \text{—(v)}$$

Using (iv) and (v)

$$Z \rightarrow \underline{A_2} A_1 Z \mid \underline{A_2} A_1 \quad \text{(Put value } A_2\text{, equa iv)}$$

$$\boxed{\begin{aligned} Z \rightarrow\ & aA_1A_1Z \mid bA_1Z \mid aA_1ZA_1Z \mid bZA_1Z \mid \\ & aA_1A_1 \mid bA_1 \mid aA_1ZA_1 \mid bZA_1 \end{aligned}}$$

Applying (i) and (iv)

$$A_1 \rightarrow \underline{A_2} A_2 \mid a$$

$$\boxed{A_1 \rightarrow aA_1A_2 \mid bA_2 \mid aA_1ZA_2 \mid bZA_2 \mid a}$$

So, final productions

$A_1 \rightarrow aA_1A_2 \mid bA_2 \mid aA_1ZA_2 \mid bZA_2 \mid a$

$A_2 \rightarrow aA_1 \mid b \mid aA_1Z \mid bZ$

$Z \rightarrow aA_1A_1Z \mid bA_1Z \mid aA_1ZA_1Z \mid bZA_1Z \mid$
$aA_1A_1 \mid bA_1 \mid aA_1ZA_1 \mid bZA_1$

CFG to GNF

eg. $S \rightarrow CA \mid BB$

$B \rightarrow b \mid SB$

$C \rightarrow b$

$A \rightarrow a$

Sol.

Let $S = A_1$

$C = A_2$

$A = A_3$

$B = A_4$

we get

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$ — (i)

$A_4 \rightarrow b \mid A_1 A_4$ — (ii) $A_4 > A_1$

$A_2 \rightarrow b$ (iii)

$A_3 \rightarrow a$ (iv)

Step 4 Alter the rules so that the non-Terminals are in ascending order such that if the production is of the form $A_i \rightarrow A_j x$, then $i < j$ and should never be $i \geq j$

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So Resolve $A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow b \mid A_1 A_4$

Put the value of equ. (i) into eq. (iv)

$A_4 \rightarrow b \mid \underline{\underline{A_2}} A_3 A_4 \mid \cancel{A_4} A_4 A_4 A_4$ — (v)

Replace the value of A_2 by eq. (iii)

$A_2 \rightarrow b$

$A_4 \rightarrow b \mid b A_3 A_4 \mid \underline{A_4} A_4 A_4$

Left Recursion

Step 5: Remove Left Recursion.

Introduce a new variable to remove th Left Recursion.

$$A_4 \rightarrow \underbrace{b}_{\beta_2} \mid \underbrace{b A_3 A_4}_{\beta_1} \mid A_4 \underbrace{A_4 A_4}_{\alpha}$$

$$Z \rightarrow A_4 A_4 \cancel{A_4} Z \mid A_4 A_4$$

$$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z$$

Now the grammar is

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z$

$Z \rightarrow A_4 A_4 \mid A_4 A_4 Z$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

<u>Now</u> $A_1 \rightarrow b A_3 \mid b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid$

$b A_3 A_4 Z A_4$

$A_4 \rightarrow b \mid b A_3 A_4 \mid b Z \mid b A_3 A_4 Z$

$Z \rightarrow b A_4 \mid b A_3 A_4 A_4 \mid b Z A_4 \mid b A_3 A_4 Z A_4 \mid$

$b A_4 Z \mid b A_3 A_4 A_4 Z \mid b Z A_4 Z \mid b A_3 A_4 Z A_4 Z$

$A_2 \rightarrow b$

$A_3 \rightarrow a$