Removal of Unit production

A production of the form $A \rightarrow B$ where $A, B \in V_N$ is called unit production, it means one non terminal derives another non-terminal is called unit production. Unit production is also called the chain Rule. Unit production must be removed (using subsititution method) from the CFG.

A production whose RHS include single variable is called, unit production.

eg.

$S \rightarrow A$ $A \rightarrow T$ $T \rightarrow M$ $M \rightarrow K$ $K \rightarrow a$	$S \rightarrow a$.

Both CFG are same

A production in which one non-terminal derives another non-terminal is called unit production.

$$A \rightarrow B$$

Strategy:

Step 1: Ensure that the given grammar have no null productions. If any, first eliminate them.

Step 2: Find out all the unit productions.

Step 3: Eliminate the unit productions by substitution method.

Steps to Remove Unit production

Step 1: Construction of set of variables derivable from variable A through unit production

$$W_1(A) = \{A\}$$

$$W_{i+1}(A) = \{ W_i(A) \cup X \mid Y \rightarrow X,\ Y \in W_i(A),\ X, Y \in V_N \}$$

Step 2: Construction of Y variable production.

a) Write all non-unit production of 'A' variable

b) $P = \{ Y \rightarrow \alpha \mid \text{if } X \rightarrow \alpha,\ X \in W(Y),\ \alpha \notin V_N \}$

Q. Eliminate the Unit production from the CFG whose production is given below.

$E \rightarrow E+T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid a$

CFG $(G) = (V_N, \Sigma, P, S)$

$V_N = \{E, T, F\}$

$\Sigma = \{+, *, (,), a\}$

$S = \{E\}$

$Y \rightarrow X$

$E \rightarrow T$

Step 1 $W_1(E) = E$

$W_2(E) = \{W_1(E) \cup X \mid Y \rightarrow X,\ Y \in W_1(E),\ X, Y \in V_N\}$

$= \{\{E\} \cup \{T\}\}$

$= \{E, T\}$

$W_3(E) = \{W_2(E) \cup X \mid Y \rightarrow X,\ Y \in W_2(E),\ X, Y \in V_N\}$

$= \{\{E, T\} \cup F\}\}$

$= \{E, T, F\}$

$W_4(E) = W_3(E) \cup X \mid Y \rightarrow X,\ Y \in W_3(E),\ X, Y \in V_N\}$

$= \{E, T, F\} \cup \emptyset\} = \boxed{\{E, T, F\}}$ Stop

$W_1(T) = \{T\}$

$W_2(T) = \{W_1(T) \cup X \mid Y \to X,\ Y \in W_1(T),\ Y X \in V_N\}$

$= \{T\} \cup \{F\}$

$= \{T, F\}$

$W_3(T) = \{W_2(T) \cup X,\ Y \to X,\ Y \in W_2(T),\ Y, X \in V_N\}$

$= \{T, F\} \cup \{\phi\}$

$$\boxed{W_3(T) = \{T, F\}}$$

$W_1(F) = \{F\}$

$W_2(F) = \{W_1(F) \cup X \mid Y \to X,\ Y \in W_1(F),\ Y, X \in V_N\}$

$= \{\{F\} \cup \phi\}$

$= \{F\}$

Step 2: construction of production

a) write all non-unit production

$E \to E + T$

$T \to T * F$

$F \to (E) \mid a$

b) $W(E) = \{E, T, F\}$

$W(T) = \{T, F\}$

$W(F) = \{F\}$

$P = \{ Y \to \alpha \mid \text{if } X \to \alpha,\ X \in W(Y),\ \alpha \notin V_N \}$

$\{ E \to T*F \mid \text{if } T \to T*F,$
$T \in W(E),$
$T*F \notin V_N.$

$$\boxed{\begin{array}{l} E \to T \\ T \to F \\ F \to a \end{array}}$$

$\{ E \to (E) \mid \text{if } F \to (E),\ F \in W(E),$
$(E) \notin V_N. \}$

$\{ E \to a \mid \text{if } F \to a,\ F \in W(E),\ a \notin V_N \}$

$\{ T \to (E) \mid \text{if } F \to (E),\ F \in W(T),$
$(E) \notin V_N.)$

$\{ T \to a \mid \text{if } F \to a,\ F \in W(T),\ a \notin V_N. \}$

$E \to E+T$
$T \to T*F$
$F \to (E) \mid a$
$E \to T*F$
$E \to (E)$
$E \to a$
$T \to (E)$
$T \to a$

$E \to E+T \mid T*F \mid (E) \mid a$
$T \to T*F \mid (E) \mid a$
$F \to (E) \mid a$

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eg. Remove the unit production from CFG, whose production is given below

$S \to AB$

$A \to a$

$B \to C \mid b$

$C \to D$

$D \to E$

$E \to a$

Sol. CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A, B, C, D, E\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

Step 1: $W_1(S) = \{S\}$

$W_2(S) = \{W_1(S) \cup X \mid Y \to X, Y \in W_1(S), X, Y \in V_N\}$

$= \{S\} \cup \phi$

$W_2(S) = W_1(S)$ Hence stop.

$\boxed{W(S) = \{S\}}$

Now, $W_1(A) = \{A\}$

$$W_2(A) = \{ W_1(A) \cup X \mid Y \rightarrow X,\ y \in W_1(A)$$
$$X, Y \in V_N \}$$

$$= \{\{A\} \cup \phi\}$$

$$= \{A\}$$

$W_2(A) = W_1(A)$ Hence stop.

Now $W_1(B) = \{B\}$

$$W_2(B) = \{ W_1(B) \cup X \mid Y \rightarrow X,\ Y \in W_1(B)$$
$$X, Y \in V_N \}$$

$$= \{B\} \cup \{C\}$$ Since $B \rightarrow C$

$$= \{B, C\}.$$

$$W_3(B) = \{ W_2(B) \cup X \mid Y \rightarrow X,$$
$$Y \in W_2(B)$$
$$X, y \in V_N \}$$

$$= \{B, C\} \cup \{D\}$$

$$= \{B, C, D\}$$

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$W_4(B) = \{B, C, D\} \cup \{E\}$

$= \{B, C, D, E\}$ Since $D \to E$

$W_5(B) = \{B, C, D, E\} \cup \phi$

$\boxed{W_5(B) = \{B, C, D, E\}}$ stop $W_5(B) = W_4(B)$

Now $W_1(C) = \{C\}$

$W_2(C) = \{W_1(C) \cup X \mid Y \to X,\ Y \in W_1(C)$
$X, Y \in V_N\}$

$\{\{C\} \cup \{D\}\}$

$= \{C, D\}$

$W_3(C) = \{W_1(C) \cup X \mid Y \to X$
$Y \in W_2(C)$
$X, Y \in V_N\}$

$= \{C, D\} \cup \{E\}$

$= \{C, D, E\}$

$W_4(C) = \{C, D, E\} \cup \phi$

$\boxed{W_4(C) = \{C, D, E\}}$

$W_1(D) = \{D\}$

$W_2(D) = \{ W_1(D) \cup X \mid Y \to X,\ Y \in W_1(D),\ X, Y \in V_N \}$

$= \{ \{D\} \cup \{E\} \}$

$= \{D, E\}$

$W_3(D) = \{ W_2(D) \cup X \mid Y \to X,\ Y \in W_2(D),\ X, Y \in V_N \}$

$= \{D, E\} \cup \phi$

$= \boxed{\{D, E\}}$

$W_1(E) = \{E\}$

$W_2(E) = \{ W_1(E) \cup X \mid Y \to X \mid Y \in W_1(E),\ X, Y \in V_N \}$

$= \{ \{E\} \cup \phi \}$

$= \{E\}$

$W_2 = W_1$ Hence stop.

Step 2: Construction of production

a) Write all non-unit production

$S \to AB$

$A \to a$ $E \to a$

$B \to b$

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b.) $W(B) = \{B, C, D, E\}$

$W(C) = \{C, D, E\}$

$W(D) = \{D, E\}$.

$P = \{ y \to \alpha \mid \text{if } x \to \alpha, \; x \in W(y), \alpha \notin V_N \}$.

$B \to a \mid \text{if } E \to a, \; E \in W(B), a \notin V_N \}$.

$C \to a \mid \text{if } E \to a, \; E \in W(C), a \notin V_N \}$.

$D \to a \mid \text{if } E \to a, \; E \in W(D), a \notin V_N \}$.

$$\boxed{\begin{array}{l} S \to AB \\ A \to a \\ B \to b \\ E \to a \\ B \to a \\ C \to a \\ D \to a \end{array}}$$

Unit production

1. $S \to ABA \mid BA \mid AA \mid AB \mid A \mid B$

   $A \to aA \mid a$

   $B \to bB \mid b$

2. $E \to E + T \mid T$

   $T \to T * F \mid F$

   $F \to (E) \mid I$

   $I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

Solution

$W_1(S) = \{S\}$

$W_2(S) = \{W_1(S) \cup X \mid Y \to X,\ Y \in W_1(S)\}$

$W_2(S) = \{S\} \cup \{A, B\}$ $\qquad S \to A$, $S \to B$

$W_2(S) = \{S, A, B\}$

$\boxed{W_3(S) = \{S, A, B\}}$

$W_1(A) = \{A\}$

$W_2(A) = \{W_1(A) \cup X \mid Y \to X,\ Y \in W_1(A)\}$

$= \{A\} \cup \emptyset = \{A\}$

$\underline{W_1(B) = \{B\}}$

$W_2(B) = W_1(B) \cup \{X \mid Y \to X, Y \in W_1(B)\}$.

$= \{B\} \cup \phi$

$\boxed{W_2(B) = \{B\}}$

$\underline{\text{Step2:}}$ $\underline{\text{write all non-unit production}}$

(i)

$S \to ABA \mid BA \mid AA \mid AB$

$A \to aA \mid a$

$B \to bB \mid b$

(ii) $W(S) = \{S, A, B\}$

$P = \{y \to \alpha \mid \text{if } X \to \alpha, X \in W(y), \alpha \notin V_N\}$.

$P_1 = \{S \to aA \mid \text{if } A \to aA, A \in W(S)$
$A \notin V_N\}$

$P_2 = \{S \to a \mid \text{if } A \to a, A \in W(S),$
$a \notin V_N\}$

$P_3 = \{S \to bB \mid \text{if } B \to bB, B \in W(S),$
$B \notin V_N\}$

$P_4 = \{S \to b \mid \text{if } B \to b, B \in W(S)$
$B \notin V_N\}$

final productions are:

$S \rightarrow ABA \mid BA \mid AA \mid AB \mid aA \mid a \mid bB \mid b$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

(ii) $E \rightarrow E+T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid I$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

CFG $G(V_N, \Sigma, P, S)$

$V_N = \{E, T, F, I\}$

$\Sigma = \{+, *, (,), a, b, Ia, Ib, I0, I1\}$

Step 1:

$W_1(E) = \{E\}$

$W_2(E) = W_1(E) \cup \{T\}$

$= \{E, T\}$

$W_3(E) = \{E, T\} \cup \{F\}$

$= \{E, T, F\}$

$W_4(E) = \{E, T, F\} \cup \{I\}$

$= \{E, T, F, I\}$.

$W_5(E) = \{E, T, F, I\} \cup \phi$

$= \{E, T, F, I\}$

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$w_1(T) = \{T\}$

$w_2(T) = \{T\} \cup \{F\}$

$= \{T, F\}$

$w_3(T) = \{T, F\} \cup I$

$= \{T, F, I\}$

$w_4(T) = \{T, F, I\} \cup \{\phi\}$

$= \{T, F, I\}$

$w_1(F) = \{F\}$

$w_2(F) = \{F\} \cup \{I\}$

$= \{F, I\}$

$w_3(F) = \{F, I\} \cup \phi$

$= \{F, I\}$

$w_1(I) = \{I\}$

$w_2\{I\} = \{I\} \cup \phi$

$= \{I\}$

Step 2

(a) write all non- unit production

$E \rightarrow E + T$

$T \rightarrow T * F$

$F \rightarrow (E)$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

(b) $W(E) = \{E, T, F, I\}$

$W(T) = \{T, F, I\}$

$W(F) = \{F, I\}$

$W_I = \{I\}$

$P = \{y \rightarrow \alpha \mid \text{if } x \rightarrow \alpha,\ x \in W(y),\ \alpha \notin V_N\}$

$P_1 = \{E \rightarrow T * F \mid \text{if } T \rightarrow T * F,\ T \in W(E),$
$T * F \notin V_N\}$

$P_2 = \{E \rightarrow (E) \mid \text{if } F \rightarrow (E),\ F \in W(E),$
$(E) \notin V_N\}$

$P_3 = \{E \rightarrow a \mid \text{if } I \rightarrow a,\ I \in W(E),$
$(a) \notin V_N\}$

$P_4 = \{E \rightarrow b \mid \text{if } I \rightarrow b,\ I \in W(E),$
$b \notin V_N\}$

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$P_5 = \{E \to Ia \mid \text{if } I \to Ia,\ I \in W(E),\ Ia \notin V_N\}$

$P_6 : \{E \to Ib \mid \text{if } I \to Ib,\ I \in W(E),\ Ib \notin V_N\}$.

$P_7 = \{E \to I0 \mid \text{if } I \to I0,\ I \in W(E),\ I0 \notin V_N\}$

$P_8 = \{E \to I1 \mid \text{if } I \to I1,\ I \in W(E),\ I1 \notin V_N\}$

eg. Eliminate the unit productions from the following grammar.

$S \to AB$

$A \to a$

$B \to C$

$C \to D$

$D \to b$

Solution:

Step 1: Clearly, there are no null productions.

Step 2: The unit productions are:

$B \to C$

$C \to D$

Step 3: Eliminate unit production by Substitution:

$C \to D$ can be substituted by $C \to b$

$B \to C$ can be substituted /replaced by $B \to b$.

Hence the grammar becomes

$S \to AB$ $\quad$ $B \to b$

$A \to a$ $\quad$ $C \to b$

$D \to b$

eg 2. Eliminate the unit productions

$S \to AB$

$A \to a$

$B \to C \mid b$

$C \to D$

$D \to E \mid bC$

$E \to d \mid Ab$

Solution:

Step 1: clearly, there are no ε-productions.

Step 2: The unit productions are:

$B \to C$

$C \to D$

$D \to E$

Step 3: Eliminate unit production by substitution:

$D \to E$ can be replaced by $D \to d \mid Ab$

$C \to D$ can be replaced by $C \to d \mid Ab \mid bC$

$B \to C$ can be replaced by $B \to d \mid Ab \mid bC$.

Hence, the grammar becomes

$S \to AB$

$A \to a$

$B \to d \mid Ab \mid bC \mid b$

$C \to d \mid Ab \mid bC$.

$D \to d \mid Ab$

$E \to d \mid Ab$

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eg.3 Eliminate the unit production from the following grammar;

$$S \to Aa \mid B$$
$$B \to A \mid bb$$
$$A \to a \mid bc \mid B$$

Solution:

Step 1: Clearly, there are no null productions.

Step 2: The unit productions are:

$$S \to B$$
$$B \to A$$
$$A \to B$$

Step 3: Eliminate of unit production by substitution

$A \to B$ can be substituted by

$$A \to A \mid bb \implies \boxed{A \to bb}$$

$B \to A$ can be replaced by:

$$B \to a \mid bc \mid B \implies \boxed{B \to a \mid bc}$$

$S \to B$ can be replaced by:

$$S \to A \mid bb$$

$S \to A$ can be replaced

$$\boxed{S \to a \mid bc \mid bb}$$

Hence, the grammar becomes:

$S \to Aa \mid a \mid bc \mid bb$

$B \to a \mid bc \mid bb$

$A \to a \mid bc \mid bb$