Reduction of CFG

Reduction of the grammar simply means the simplification of a grammar by elimination of unnecessary symbols (terminals or non-terminals)

Condition for a reduced grammar

A reduced grammar is one which satisfies the following conditions:

(i) It does not contain null productions i.e

$A \rightarrow \epsilon$

(2) It does not contain unit productions i.e $A \rightarrow B$. where A and B are non-termina

(3) It does n't contain useless symbols.

Removal of NULL Productions

A production of the form
$A \to \lambda$ is called NULL production.

A variable 'A' $\boxed{A \in V_N}$ is said to be Nullable variable

if $A \xrightarrow{*} \lambda$, A is said to be nullable, if it derives ϵ in zero or more steps.

Steps for Removal of NULL productions

1) Find all nullable variables which directly or indirectly derives ϵ.
2) Write all the production whose RHS doesn't include NULLable variable.
3) Now, consider the production whose RHS include the nullable variable.

New production will be formed either by.
(i) Not erasing the nullable variable on the RHS.
(ii) By erasing one or more nullable variable from the RHS provided that some symbol must appear on the RHS

(1) Consider the following grammar, construct CFG without Null production. $L(G) - \{\lambda\}$

$A \rightarrow \lambda$

$S \rightarrow aS \mid AB$

$B \rightarrow \lambda$

$D \rightarrow b$

Sol. find all nullable variable

$W = \{A \mid A \xrightarrow{*} \lambda,\ A \in V_N\}$

$= \{A, B, S\}$ Since $A \rightarrow \lambda$

$B \rightarrow \lambda$

$S \rightarrow AB$

Step 2 construction of P'

(a) Now consider the production whose RHS doesn't include nullable variable

$D \rightarrow b$.

(b) Now consider the production whose RHS include the nullable variable

New production will be formed either

(i) by not erasing the nullable variable on the RHS

or

(ii) by erasing one or more nullable variable from the RHS provided that some symbol must appear on the RHS.

(i) $S \to aS$ gives $S \to aS$, $S \to a$

(ii) $S \to AB$ gives $S \to AB$, $S \to A$, $S \to B$

Now the production (P')

$D \to b$

$S \to aS \mid a$

$S \to AB \mid A \mid B$

(2)

② ~~Consider~~ Remove the NULL production from CFG whose production is given below:

$S \to XYX$

$X \to 0X \mid \lambda$

$Y \to 1Y \mid \lambda$

Sol.

CFG $G\,(V_N, \Sigma, P, S)$

$V_N = \{S, X, Y\}$

$\Sigma = \{0, 1\}$

$S = \{S\}$

Step 1: find all nullable variable

$W = \{A \mid A \xrightarrow{*} \lambda,\ A \in V_N\}$

$= \{X, Y, S\}$.

since $X \to \lambda$

$Y \to \lambda$

$S \to XYX$

Step 2. (a) Write all the production whose RHS doesn't include nullable variable

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(b) Now consider the production, whose RHS contain the nullable variable.

(i) New production will be formed by either by not erasing the nullable variable on the RHS

or

(ii) by erasing one or more nullable variable from the RHS provided that some symbol must appear on the RHS.

$S \to XYX$ gives $S \to XYX$, $S \to YX$, $S \to XX$, $S \to XY$; $S \to X$, $S \to Y$

$X \to 0X$ gives $X \to 0X$, $X \to 0$

$Y \to 1Y$ gives $Y \to 1Y$, $Y \to 1$

Now the production P'

$S \to XYX \mid YX \mid XX \mid XY \mid X \mid Y$

$X \to 0X \mid 0$

$Y \to 1Y \mid 1$

(3)

$S \to ABC$

$A \to \epsilon$

$B \to \epsilon$

$C \to c$

Sol. CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A, B, C\}$

$\Sigma = \{c\}$

$S = \{S\}$

Step 1: Find all nullable variables:

$W = \{A \mid A \xrightarrow{*} \lambda \mid A \in V_N\}$

$= \{A, B\}$

Step 2: write all the production whose

a) RHS doesn't include nullable variable.

$C \to c$

b) Now consider the production whose RHS include the nullable variable.

$S \to ABC$ gives $S \to ABC$, $S \to AC$, $S \to BC$
$S \to C$

Now the production P'

$S \to ABC \mid AC \mid BC \mid c$
$C \to c$

eg. (4) $S \rightarrow aS \mid bS \mid \epsilon$

Eliminate ϵ-productions

Ans. $S \rightarrow aS \mid bS \mid a \mid b$

(5). $S \rightarrow AB$

$A \rightarrow aAA \mid \epsilon$

$B \rightarrow bBB \mid \epsilon$

Eliminate ϵ-productions

Ans.

$S \rightarrow AB \mid A \mid B$

$A \rightarrow aAA \mid aA \mid a$

$B \rightarrow bBB \mid bB \mid b$

(6) $S \rightarrow ABAC$

$A \rightarrow aA \mid \epsilon$

$B \rightarrow bB \mid \epsilon$

$C \rightarrow c$

Ans. $S \rightarrow ABAC \mid BAC \mid ABC \mid AC \mid BC \mid AAC \mid C$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$C \rightarrow c$

7) $S \to a \mid Ab \mid aBa$

$A \to A \mid \lambda$

$B \to b \mid A$

Sol. CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A, B\}$.

$\Sigma = \{a, b\}$

$S = \{S\}$.

Step 1: Find all nullable variable

$W = \{A \mid A \xrightarrow{*} \lambda, A \in V_N\}$

$= \{A, B\}$

Since : $A \to \lambda$

$B \to A$ (indirectly)

Step 2. Now, consider the production whose RHS. ~~write all the variable~~ doesn't include all nullable variable.

$S \to a$

$B \to b$

Step 2 (iii) Now consider the production, whose RHS include nullable variable.

$S \to aBa$ gives $S \to aBa$, $S \to aa$,

$S \to Ab$ gives $S \to Ab$, $S \to b$

$A \to A$ gives $A \to A$

$B \to A$ gives $B \to A$

final production

$S \to a \mid aBa \mid aa \mid Ab \mid b$

$A \to A$

$B \to A \mid b$

eg. $S \rightarrow aS \mid bS \mid \epsilon$

Eliminate ϵ productions, if any.

Solution

The given grammar is:

$S \rightarrow aS$

$S \rightarrow bS$

$S \rightarrow \epsilon$

Step 1: Identify the nullable variables.

The nullable variable is S.

Step 2: Create two versions of all productions having nullable variables.

for production $S \rightarrow aS$:

$$\left.\begin{array}{ll} S \rightarrow aS & \text{(with S)} \\ S \rightarrow a & \text{(without S)} \end{array}\right\} \quad S \rightarrow aS \mid a$$

for production $S \rightarrow bS$

$$\left.\begin{array}{l} S \rightarrow bS \\ S \rightarrow b \end{array}\right\} \quad S \rightarrow bS \mid b$$

Thus, the complete production becomes

$$\boxed{S \rightarrow aS \mid bS \mid a \mid b}$$

This is the required grammar.

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② Consider the following grammar

$S \to AB$

$A \to aAA \mid \epsilon$

$B \to bBB \mid \epsilon$

Eliminate ϵ productions, if any.

Solution:

The given grammar is:

$S \to AB$

$A \to aAA$

$A \to \epsilon$

$B \to bBB$

$B \to \epsilon$

Step 1. Identify the nullable variable

Nullable variables : S, A, B

Step 2: Create two versions of all productions having nullable variables

(i) for production : $S \to AB$

Considering A & $S \to AB$

$\cancel{S \to A}$

$S \to B$

$S \to AB \mid B$

Considering B

Two versions of this production are :

$$\left.\begin{array}{l} S \rightarrow AB \\ S \rightarrow A \end{array}\right\} \quad S \rightarrow AB \mid A$$

Considering both A and B

Two versions of this production are:

$$\left.\begin{array}{l} S \rightarrow AB \\ S \rightarrow \epsilon \end{array}\right\} \quad S \rightarrow AB$$

Thus, complete production becomes :

$$S \rightarrow AB \mid B \mid AB \mid A \mid AB$$

$$\boxed{S \rightarrow AB \mid A \mid B}$$

* For production $A \rightarrow aAA$

Considering first A

Two versions of this production are.

$$\left.\begin{array}{ll} (i) & A \rightarrow aAA \\ (ii) & A \rightarrow aA \end{array}\right\} \rightarrow A \rightarrow aAA \mid aA$$

Considering second A

$$\left.\begin{array}{ll} (i) & A \rightarrow aAA \\ (ii) & A \rightarrow aA \end{array}\right\} \rightarrow A \rightarrow aAA \mid aA$$

Considering both A

$$\left.\begin{array}{l} A \to aAA \\ A \to a \end{array}\right\} \quad A \to aAA \mid a$$

Thus,

Complete production becomes.

$A \to aAA \mid aA \mid aAA \mid aA \mid aAA \mid a$

$$\boxed{A \to aAA \mid aA \mid a}$$

* for production

$B \to bBB$.

Considering first B

$$\left.\begin{array}{l} B \to bBB \\ B \to bB \end{array}\right\} \quad B \to bBB \mid bB$$

Considering second B $\quad B \to bBB \mid bB$

Considering both $\quad B \to bBB \mid b$

Thus. complete production becomes

$B \to bBB \mid bB \mid bBB \mid bB \mid bBB \mid b$

$$\boxed{B \to bBB \mid bB \mid b}$$

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Thus, grammar after elimination of ϵ productions are

$S \to AB \mid A \mid B$

$A \to aAA \mid aA \mid a$

$B \to bBB \mid bB \mid b$

eg3. Consider the following grammar

$S \to aS \mid AB$

$A \to \epsilon$

$B \to \epsilon$

$D \to b$

Eliminate ϵ productions, if any.

Solution

$W = \{A \mid A \xrightarrow{*} \lambda, A \in V_n\}$

The nullable variables are: $S \to AB$.

A, B, S (indirectly)

Since - $A \to \lambda$, $B \to \lambda$, $S \to AB$.

The grammar after eliminating ϵ - production

$S \to aS$

$S \to a$

$D \to b$

$S \to AB \mid A \mid B \mid aS \mid a$

$D \to b$

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eg. 4. Consider the following grammar

$S \to ABAC$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$C \to c$

Eliminate ϵ productions

Solution

The nullable variables are :

A, B

The grammar after eleminate ϵ-productions is.

$S \to ABAC \mid BAC \mid AAC \mid ABC \mid BC \mid C \mid AC$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to c$