Construction of Reduced Grammar

Step 1:

For every context free Grammar $G(V_N, \Sigma, P, S)$ we can have another CFG $G'(V_N', \Sigma, P', S)$ such that every variable in G' will derive some terminal string.

a) **Construction of V_N'**

$W_i = \{ A \mid A \in V_N,$ there exists a production $A \rightarrow w$, where $w \in \Sigma^* \}$

$W_{i+1} = \{ W_i \cup A \mid A \rightarrow \alpha$ Rathy $A \in V_N$, $\alpha \in (W_i \cup \Sigma)^* \}$ preselected V_n.

$\boxed{W_i \subseteq W_{i+1} \text{ for all } i}$

b.) **Construction of P'**

$P' = \{ A \rightarrow \alpha \mid A \in V_N', \alpha \in (V_N' \cup \Sigma)^* \}$

Step 2: For every context free Grammar $G(V_N, \Sigma, P, S)$, we can have another CFG $G'(V_N', \Sigma, P', S)$ such that every symbol (variable) present in $(V_N' \cup \Sigma)$ must appear some sentential form $(V_N' \cup \Sigma')^*$

a) Construction of W_K

a) $W_i = \{S\}$

b) $W_{i+1} = \{ W_i \cup X \mid A \to \alpha,\ A \in W_i$ and α contains the symbol of $X \}$

$X \in (V_N \cup \Sigma)$

b) Construction of V_N', Σ', P'

$V_N' = W_K \cap V_N$

$\Sigma' = W_K \cap \Sigma$

$P_1 = \{ A \to \alpha \mid A \in W_K,$
$\alpha \in (V_N' \cup \Sigma')^* \}$

$W_{i+1} = \{ W_i \cup X \mid X \in (V_N \cup \Sigma)$ and there exists a production $A \to \alpha$, where $A \in W_i$ and α contains symbol X

Find the reduced grammar equivalent to CFG, whose production are given below:

$S \to AB \mid CA$

$B \to CB \mid AB$

$A \to a$

$C \to aB \mid b$

Sol:

CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A, B, C\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

CFG $G(V_N, \Sigma, P, S)$ to

CFG $G'(V_N', \Sigma, P', S)$

Step 1 Construction of V_N'

$W_1 = \{A \mid A \in V_N, A \to w, w \in \Sigma^*\}$.

$W_1 = \{A, C\}$ since $A \to a$

$C \to b$.

$W_2 = \{ W_1 \cup A \mid A \in V_n,\ A \to \alpha,$
$\alpha \in (W_1 \cup \Sigma)^* \}$

$W_2 : \{ \{A, C\} \cup \{S\}$ since $\underset{A}{S} \to \underset{\alpha}{\underline{CA}}$
$= \{S, A, C\}$.

$W_3 = \{ W_2 \cup A \mid A \in V_N,\ A \to \alpha$
$\alpha \in (W_2 \cup \Sigma)^* \}$

$= \{ \{S, A, C\} \cup \phi \}$

$= \{S, A, C\}$.

Here $W_3 = W_2$ Hence stop.

$V_N' = W_3 = \{S, A, C\}$

(b) Construction of P'

$P' = \{ A \to \alpha \mid A \in V_N',\ \alpha \in (V_N' \cup \Sigma)^* \}$.

$S \to CA$ Since $C, A \in (V_N' \cup \Sigma)^*$
$A \to a$ " $a \in$ "
$C \to b$ " $b \in$ "

Step 2: Step 2 will be applied on the o/p of step 1.

$S \to CA$

$A \to a$

$C \to b$

CFG (V_N', Σ, P', S)

$V_N' = \{S, A, C\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

CFG G (V_N, Σ, P, S) to

CFG G' (V_N', Σ, P', S)

a) Construction of W_x

$W_1 = \{S\}$

$W_{2} = \{ W_1 \cup X \mid A \to \alpha, A \in W_1$ &

α contain the symbol of X

$X \in (V_N \cup \Sigma)$

$= \{\{S\} \cup \{C, A\}\}$ since $S \to CA$, $A \to \alpha$

$= \{S, C, A\}$

$W_3 = \{W_2 \cup x \mid A \to \alpha,\ A \in W_2$ and α contains the symbols of x
$x \in (V_N, \Sigma)$

$W_3 = \{\{S, A, C\} \cup \{a, b\}\}$ since $A \to a$
$C \to b$.

$= \{S, A, C, a, b\}$

$W_4 = \{W_3 \cup x \mid A \to \alpha,\ A \in W_3$ and α contains the symbols of x
$x \in (V_N, \Sigma)\}$

$= \{\{S, A, C, a, b\} \cup \phi\}$.
$\{S, A, C, a, b\}$

$W_4 = W_3$ Hence stop
$W_4 = \{S, A, C, a, b\}$

b) Construction of V_N', Σ', P'.

$V_N' = W_K \cap V_N$
$= \{S, A, C, a, b\} \cap \{S, A, C\}$
$= \{S, A, C\}$.

$\Sigma' = W_4 \cap \Sigma$

$= \{S, A, C, a, b\} \cap \{a, b\}$

$= \{a, b\}$

$P' = \{A \to \alpha \mid \underline{A \in W_4}$
$\underline{\alpha \in (V_N' \cup \Sigma')^*}\}$

$S \to CA$

$A \to a$

$C \to b$

② Construct the reduced grammar equivalent to the grammar whose production is given below:

$S \rightarrow aAa$

$A \rightarrow Sb \mid bCC \mid DaA$

$C \rightarrow abb \mid DD$

$E \rightarrow aC$

$D \rightarrow aDA$

CFG G (V_N, Σ, P, S)

$V_N = \{S, A, C, E, D\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

Step 1 CFG G (V_N, Σ, P, S) to CFG G' (V_N', Σ, P', S)

a) Construction of V_N'

$W_1 = \{A \mid A \in V_N, A \rightarrow w, w \in \Sigma^*\}$

$= \{C\}$ since $C \rightarrow abb$

$W_2 = \{ W_1 \cup A \mid A \in V_N,\ A \to \alpha$
$\alpha \in (W_1 \cup \Sigma)^* \}$

$= \{C\} \cup \{A, E\}$ Since $A \to bCC$
$E \to aC$

$= \{A, C, E\}$

$W_3 = \{ W_2 \cup A \mid A \in V_n,\ A \to \alpha$
$\alpha \in (W_2 \cup \Sigma)^* \}$

$= \{\{A, C, E\} \cup S\}$ Since ~~$A \to bC$~~
$S \to aAa$

$\{S, A, C, E\}$

$W_4 = \{ W_3 \cup A \mid A \in V_N,\ A \to \alpha$
$\alpha \in (W_3 \cup \Sigma)^* \}$

$= \{\{S, A, C, E\} \cup \phi\}$

$= \{S, A, C, E\}$

$W_4 = V_3$ stop

$V_N = \{S, A, C, E\}$

b.) Construction of P'

$P' = \{ A \to \alpha \mid$

$A \in V_N'$, $\alpha \in (V_N' \cup \Sigma)^* \}$

$S \to aAa$ Since $aAa \in (V_N' \cup \Sigma)^*$

$A \to Sb \mid bCC$ "

$C \to abb$

$E \to aC$

Step 2 step 2 will be on the o/p of step 1.

$S \to aAa$

$A \to Sb \mid bCC$

$C \to abb$

$E \to aC$

CFG G (V_N, Σ, P, S)

$V_N = \{S, A, C, E\}$ $\Sigma = \{a, b\}$, $S = \{S\}$

CFG G (V_N, Σ, P, S) to

CFG G' (V_N', Σ', P', S)

2) Construction of W_k

$W_1 = \{S\}$

$W_2 = \{ W_1 \cup X \mid A \to \alpha,\ A \in W_1$ and α contains the symbol of X
$X \in (V_N \cup \Sigma).\}$

$W_2 = \{ S \cup \{a, A\}$
$\{S, a, A\}$

$W_3 = \{ W_2 \cup X \mid A \to \alpha,\ A \in W_2$ and α contains the symbol of X
$\alpha \in (V_N \cup \Sigma)\}$

$= \{\{S, a, A\} \cup \{b, C\}$ $\quad$ $A \to bCC$

$\{S, a, A, b, C\}$ $\quad$ ~~C→abb~~

$W_4 = \{S, a, A, b, C\} \cup \{\phi\}\}$

$\{S, A, C, b, a\}$ $\quad$ Since $C \to abb$.

$W_3 = W_4$ $\quad$ stop

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Construction of V_N', Σ', P

$V_N' = W_4 \cap V_N'$

$= \{S, A, C, a, b\} \cap \{S, A, C, E\}$

$= \{S, A, C\}$

$\Sigma' = W_4 \cap \Sigma$

$= \{S, A, C, a, b\} \quad \{a, b\}$

$= \{a, b\}$

$P' = \{A \to \alpha \mid A \in \underline{W}_4$
$\alpha \in (V_N' \cup \Sigma')^* \}$

$= \quad S \to aAa$

$A \to Sb \mid bCC$

$C \to abb.$

Elimination of useless productions:

Useful symbols:

A variable is said to be useful if and only if it satisfies the following conditions.

(i) It can derive a terminal string.

(ii) It can be reached from the start symbol.

Useless symbols:

* If the symbol is not useful then it is called useless symbol ie if any one of the conditions for the useful symbol fails, the symbol becomes useless.

* The production involving any useless symbol is called useless production.

eg.

$S \to A \mid C$

$A \to a \mid B$

$B \to b$

Useful symbols: S, B, A

Useless symbols: C

Strategy:

Step 1: Remove all the null and unit productions if any.

Step 2: For all the non-terminals, check if a terminal string can be generated directly or indirectly.

If no, then remove that symbol and its associated productions otherwise keep them.

Step 3: For all the left out non-terminals, check if they can be reached from the start symbol directly or indirectly.

If no, then remove that symbol and its associated productions otherwise keep them.

The resultant grammar will be the reduced grammar.

eg. Eliminate the useless symbols from the following grammar.

$S \rightarrow AB \mid C$

$A \rightarrow a$

$B \rightarrow b$

Solution:

Step 1: Clearly, the given grammar has no null and unit productions.

Step 2: - Non-terminals in the grammar

S, A, B, C

- All the non-terminals generate terminal string, but C is not deriving any terminal string since there are no productions for C.

∴ C is useless symbol and so we eliminate it.

Step 3: Leftout non-terminals are -

S, A, B

The variable A and B can be reached from the start symbols.

The variables S, A and B are useful symbols.

Thus, the reduced grammar is:

$S \to AB$

$A \to a$

$B \to b$

eg 2. Eliminate the useless symbols.

$S \to AB \mid CA$

$B \to BC \mid AB$

$A \to a$

$C \to aB \mid b$

Solution:

step 1: Clearly, there are no null and unit productions.

Step 2: non-terminals in the grammar are:

S, A, B, C

* The variables A and C generates a terminal string. But B cannot be replaced by any terminating string. It tends to form a never ending loop.

∴ B is useless symbol. so we remove B and its associated productions.

<u>step 3:</u> <u>Left out Non-terminals are:</u>

<u>S, A, C</u>

The variables A and C can be reached from the start symbol.

∴ The variables S, A, C are useful symbols.

Thus, the reduced grammar is:

$$\boxed{\begin{array}{l} S \to CA \\ A \to a \\ C \to b. \end{array}}$$

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