

③ Design the PDA for the language  
 $L = \{ a^n b^n c^m d^m \mid n, m \geq 1 \}$

PDA  $A(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{ q_0, q_1, q_2, q_3 \}$

$\Sigma = \{ a, b, c, d \}$

$\Gamma = \{ a, b, c, d, z_0 \}$

$q_0 = \{ q_0 \}$

$z_0 = \{ z_0 \}$

$F = \emptyset$

$\delta$  is defined by

$\delta(q_0, a, z_0) \vdash (q_0, a z_0)$

$\delta(q_0, a, a) \vdash (q_0, a a)$

$\delta(q_0, b, a) \vdash (q_1, \lambda)$

$\delta(q_1, b, a) \vdash (q_1, \lambda)$

$\delta(q_1, c, z_0) \vdash (q_2, c z_0)$

$\delta(q_2, c, c) \vdash (q_2, c c)$

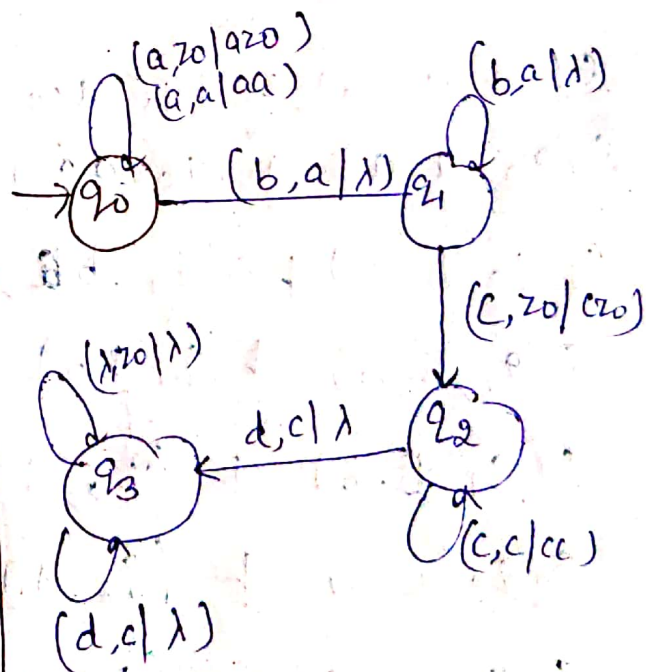
$\delta(q_2, d, c) \vdash (q_3, \lambda)$

$\delta(q_3, d, c) \vdash (q_3, \lambda)$

$\delta(q_3, \lambda, z_0) \vdash (q_3, \lambda)$

Acceptance by  
NULL store

State - transition diagram



④ Write the PDA for the language

$$L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$$

PDA  $A(Q, \Sigma, \Gamma, S, q_0, z_0, F)$ .

Acceptance by  
null store

is defined by

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b, c, d\}$$

$$\Gamma = \{a, b, z_0\}$$

$$q_0 = \{q_0\}$$

$$z_0 = \{z_0\}$$

$$F = \emptyset$$

S is defined

$$\delta(q_0, a, z_0) \vdash (q_0, a z_0)$$

$$\delta(q_0, a, a) \vdash (q_0, a a)$$

$$\delta(q_0, b, a) \vdash (q_1, b a)$$

$$\delta(q_1, b, b) \vdash (q_1, b b)$$

$$\delta(q_1, \epsilon, b) \vdash (q_2, \lambda)$$

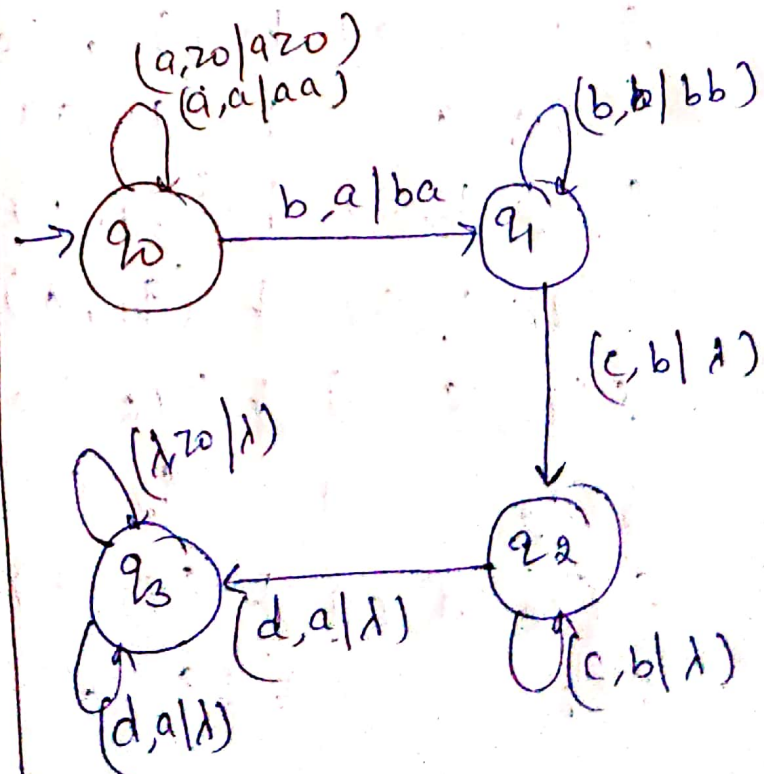
$$\delta(q_2, c, b) \vdash (q_2, \lambda)$$

$$\delta(q_2, d, a) \vdash (q_3, \lambda)$$

$$\delta(q_3, d, a) \vdash (q_3, \lambda)$$

$$\delta(q_3, \lambda, z_0) \vdash (q_3, \lambda)$$

state-trans. diagram



⑤ Design the PDA for the language

$$L = \{a^n c b^{2n} \mid n \geq 1\}$$

PDA  $A(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, z_0\}$$

$$q_0 = \{q_0\}$$

$$z_0 = \{z_0\}$$

$$F = \emptyset$$

instead of push a,

push 2a at one time.

or

instead of ~~pop~~ 2b pop  
1a

$\delta$  is defined by

$$\delta(q_0, a, z_0) \vdash (q_0, a z_0)$$

$$\delta(q_0, a, a) \vdash (q_0, a a a)$$

$$\delta(q_0, c, a) \vdash (q_1, a \text{ ~~aaa~~})$$

$$\delta(q_1, b, a) \vdash (q_2, \lambda)$$

$$\delta(q_2, b, a) \vdash (q_2, \lambda)$$

$$\delta(q_2, \lambda, z_0) \vdash (q_2, \lambda)$$

$$\begin{aligned} &\vdash q_0, c b b b b b b, a a a a a z_0 \\ &\vdash q_0, b b b b b b b, a a a a a z_0 \\ &\vdash q_2, b b b b b b, a a a a a z_0 \\ &\vdash q_2, b b b b b, a a a a z_0 \\ &\vdash q_2, b b b b, a a a z_0 \\ &\vdash q_2, b b b, a a a z_0 \\ &\vdash q_2, b b, a a z_0 \\ &\vdash q_2, b, a z_0 \\ &\vdash q_2, \lambda, z_0 \\ &\vdash q_2, \lambda \end{aligned}$$

check that string  $w = a a a c b b b b b b b$

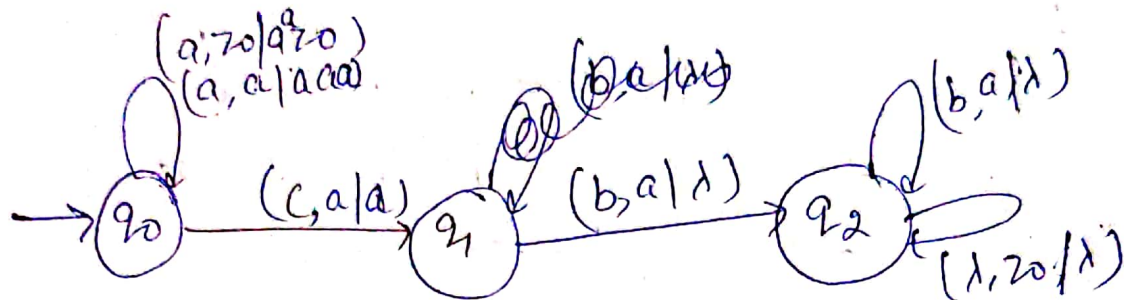
is accepted by above PDA

$$\delta(q_0, a a a c b b b b b b b, z_0) \vdash$$

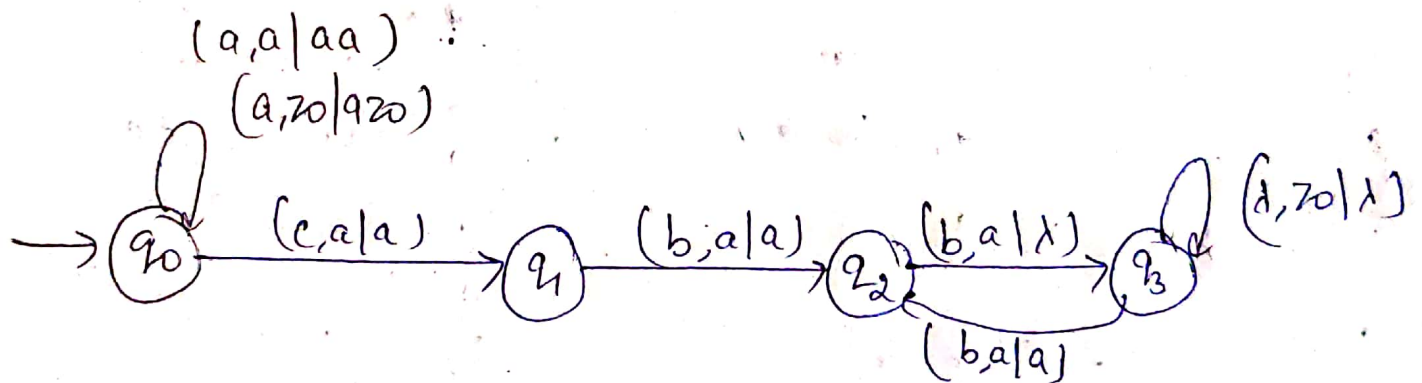
$$\vdash q_0, a a c b b b b b b b, a q z_0)$$

$$\vdash q_0, a c b b b b b b b, a a a q z_0)$$

# State transition diagram $a^n c b^{2n}$



or





⑥ Design the PDA for the language

$$L = \{w c w^R \mid w \in (a, b)^*\} \quad \text{or } w c w^R$$

PDA  $A (Q, \Sigma, \Gamma, q_0, z_0, F)$

$$Q = \{q_0, q_1\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, b, z_0\}$$

$$q_0 = \{q_0\}$$

$$z_0 = \{z_0\}$$

$$F = \emptyset$$

$\delta$  is defined by:

$$\delta(q_0, a, z_0) \vdash (q_0, a z_0)$$

$$\delta(q_0, b, z_0) \vdash (q_0, b z_0)$$

$$\delta(q_0, a, a) \vdash (q_0, a a)$$

$$\delta(q_0, b, b) \vdash (q_0, b b)$$

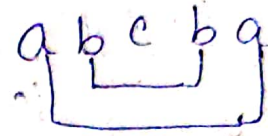
$$\delta(q_0, a, b) \vdash (q_0, a b)$$

$$\delta(q_0, b, a) \vdash (q_0, b a)$$

$$\delta(q_0, a, c) \vdash (q_1, a)$$

$$\delta(q_0, c, b) \vdash (q_1, b)$$

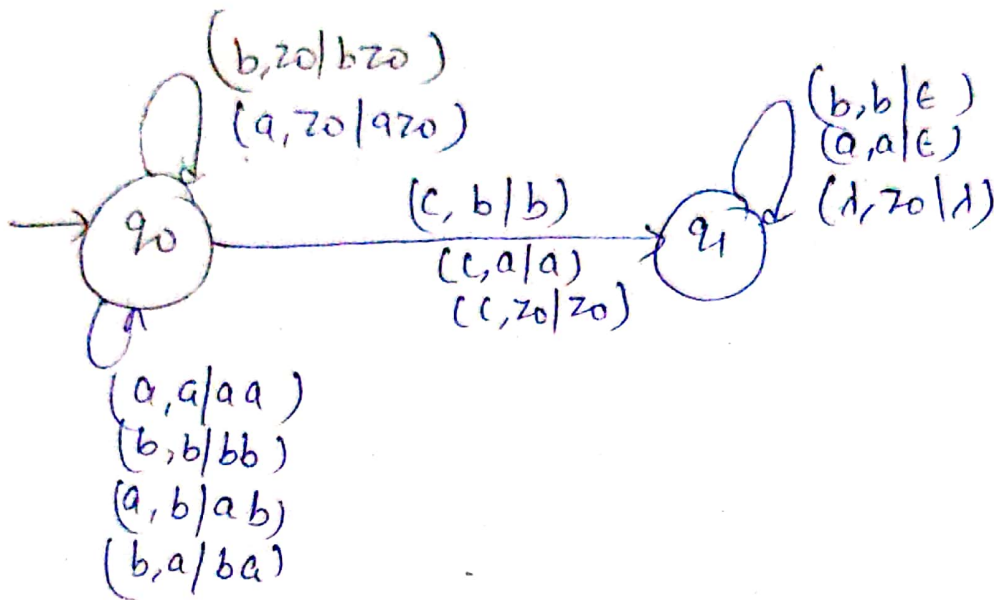
$$\delta(q_0, c, z_0) \vdash (q_1, z_0)$$



$$\delta(q_1, b, b) \vdash (q_1, \lambda)$$

$$\delta(q_1, a, a) \vdash (q_1, \lambda)$$

$$\delta(q_1, \lambda, z_0) \vdash (q_1, \lambda)$$

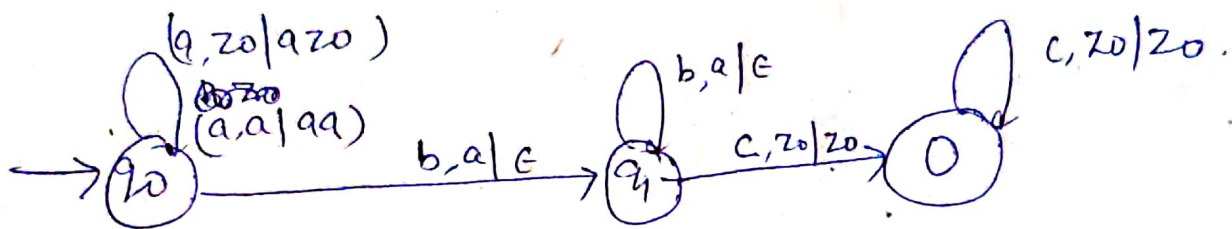


state-transition diagram of  

$$\frac{wcw^R}{w \in (a,b)^*}$$

⑦ state transition diagram of PDA for the language  $a^n b^n c^m \mid n, m \geq 1$

Acceptance by final state

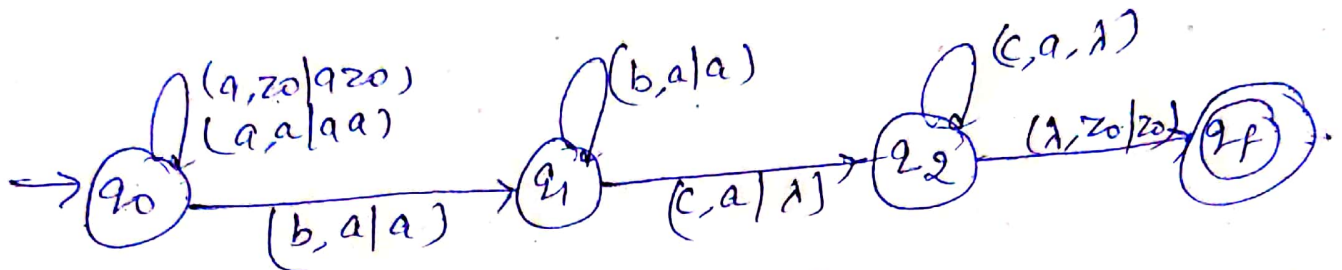


PDA

⑧

$$L = a^n b^m c^n \mid n, m \geq 1$$

Acceptance by final state

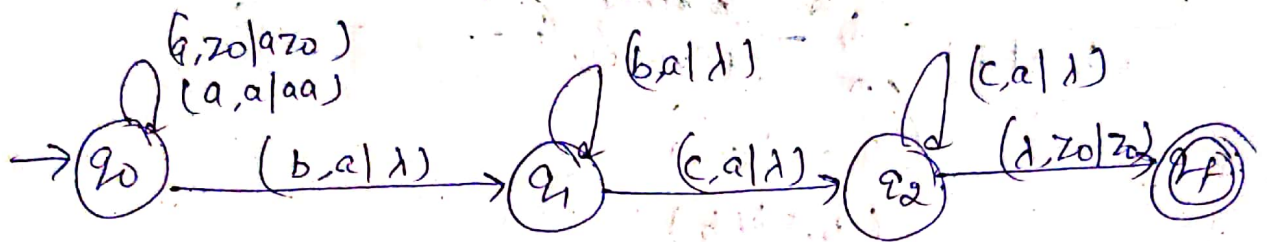




⑨  $a^{m+n} b^m c^n \mid m, n \geq 1$

so we break the string

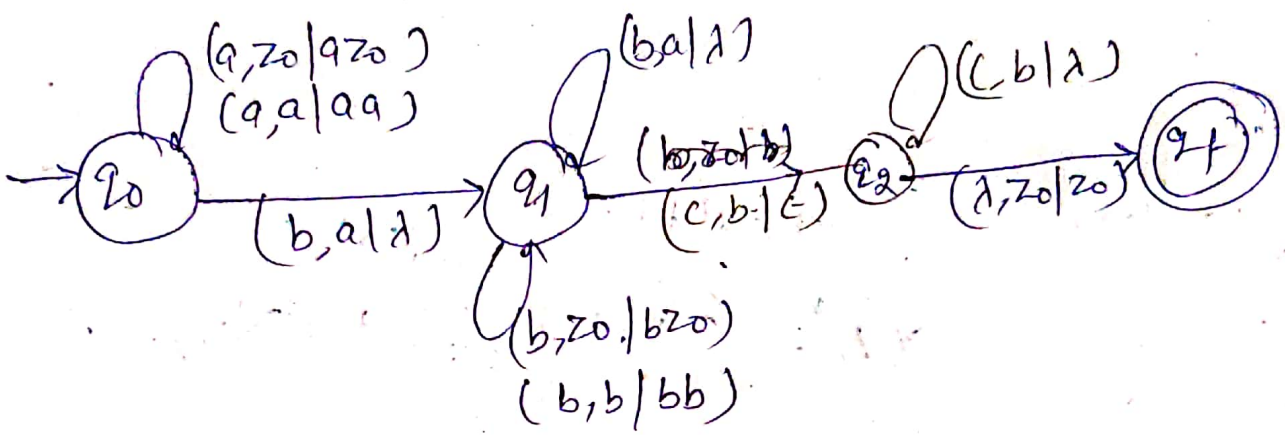
$$L = a^n a^m b^m c^n$$



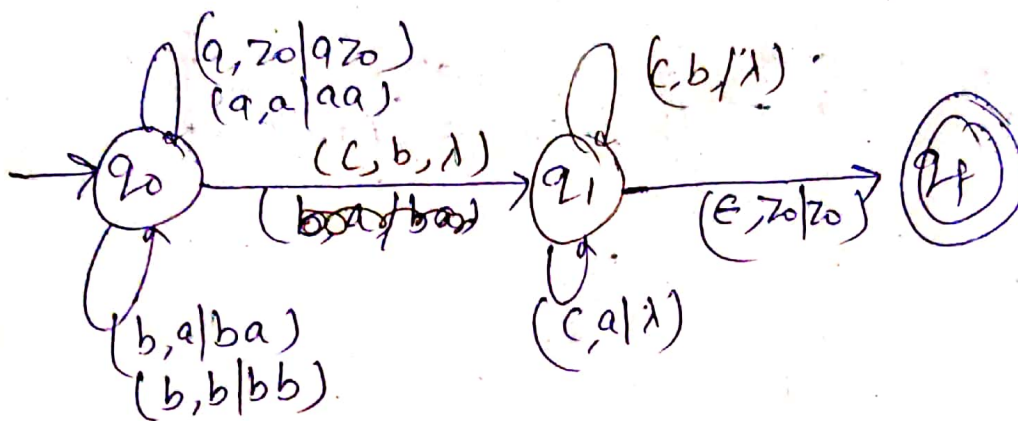
10)

$a^n b^{m+n} c^m \mid n, m \geq 1$   
or.

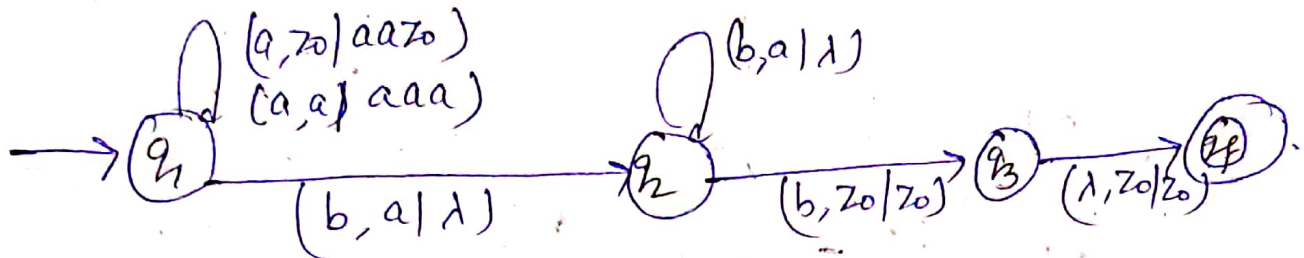
$a^n b^n b^m c^m \mid n, m \geq 1$



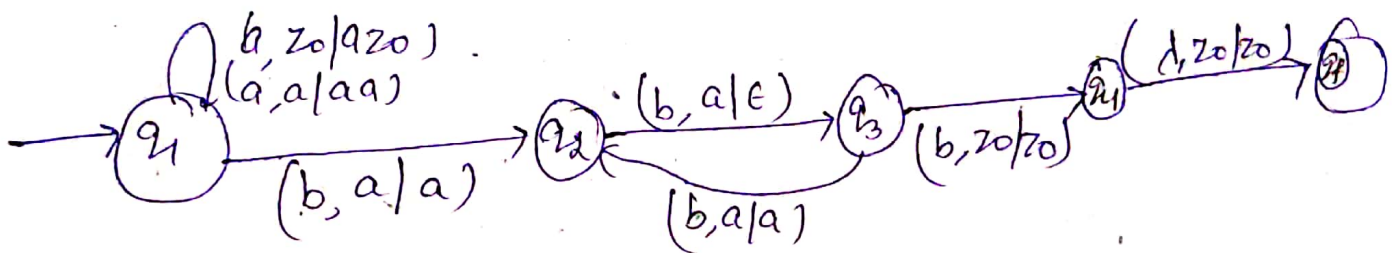
(11)  $a^n b^m c^{n+m} \mid n, m \geq 1$   
 or.  
 $a^n b^m c^m c^n \mid n, m \geq 1$



$$a^n b^{2n+1} \mid n \geq 1$$



or



$ww^R \mid w \in (a,b)^+$

NDPDA

Set of all even length Palindrome

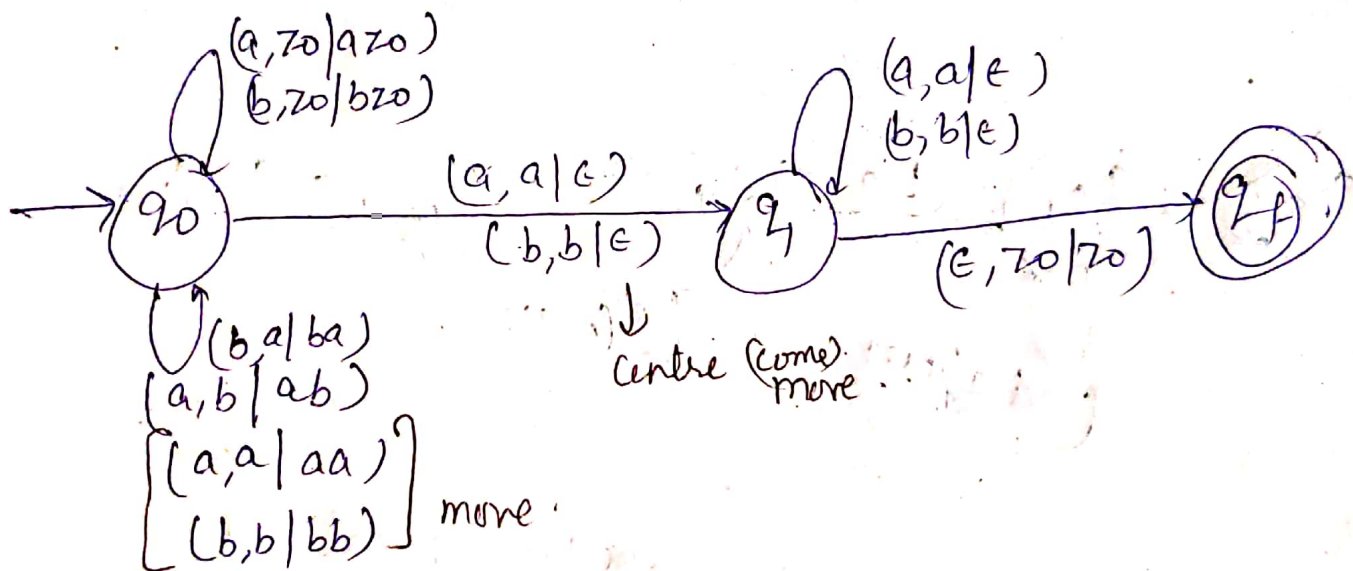
$\frac{aba}{w} \frac{aba}{w^R}$

$\frac{abba}{w} \frac{abba}{w^R}$

$\frac{ba}{w} \frac{ab}{w^R}$

$\frac{aaaa}{w} \frac{aaaa}{w^R}$

We need a non-deterministic machine, no centre.  
its a problem.



Centre hasn't come

check (aaaa)

$\delta(q_0, aaaa, z_0)$

$\delta(q_0, aaa, \underline{z_0})$

Centre  $\delta(q_0, aa, z_0)$

Dead  
(configuration)

No Centre

$\delta(q_0, aa, aaz_0)$

$(q_0, a, az_0)$

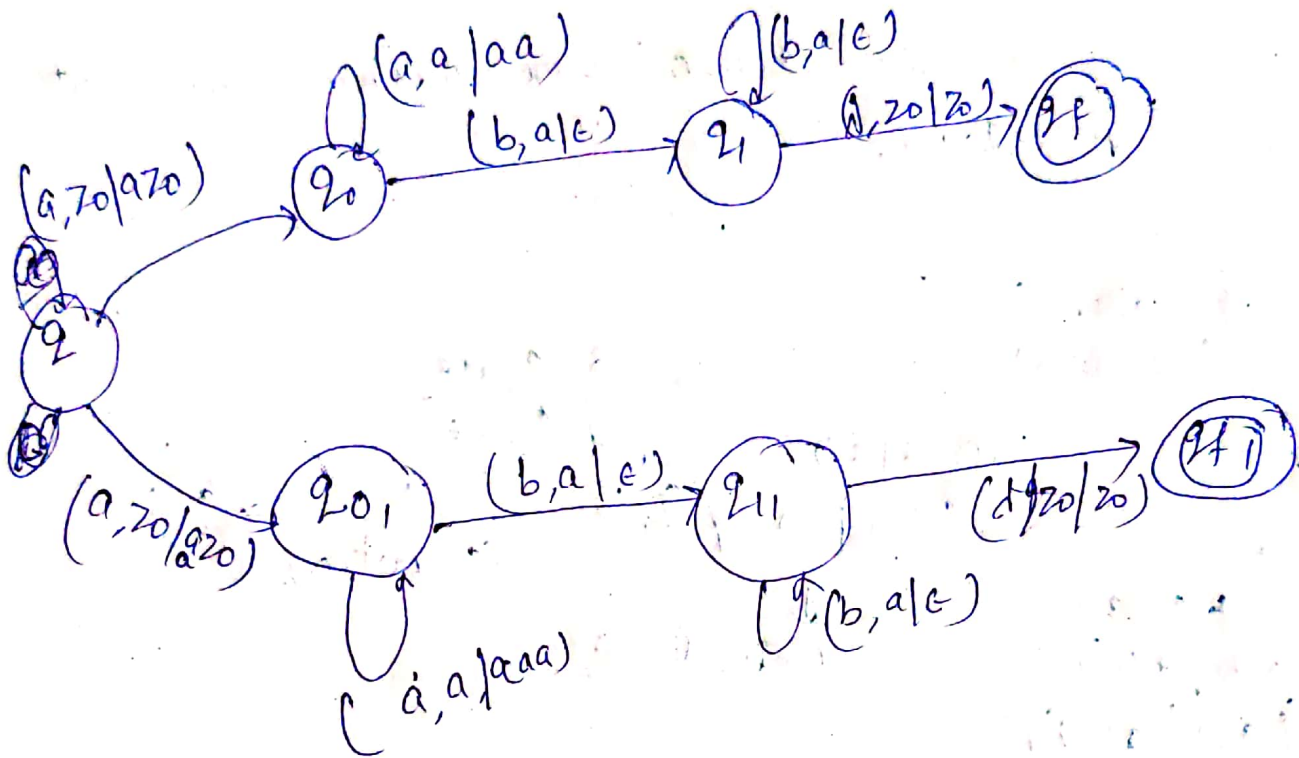
$(q_0, a, aaaa z_0)$

$(q_1, \lambda, z_0) \rightarrow (q_f, \lambda, z_0)$  Acceptance.



## Union

$$L = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$$



If at least one PDA goes to final state, we can say that the string is accepted.

NDPDA  $\subset$  DPDA

Accept  
More languages.

$w c w^R \mid w \in (a,b)^+$   
Set of all<sup>odd</sup> palindromes

- (1) a b b c b a a (odd length)  
 Push Pop.
- (2) b a c a b

