From CFG to PDA

Let a Context free Grammar CFG $G(V_N, \Sigma, P, S)$, which can be converted into PDA $A(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, which is accepted by null store as follows

$Q = \{q\}$

$\Sigma =$ same as in CFG

$\Gamma = V_N \cup \Sigma$

$q_0 = \{q\}$

$Z_0 = \{S\}$

$F = \phi$

δ is defined as follows:

Rule 1: $\delta(q, \lambda, A) = (q, \alpha)$.

if $A \rightarrow \alpha$ is in P

Rule 2: $\delta(q, a, a) = (q, \lambda)$

$\forall a \in \Sigma$ (for every a in Σ)

Q① Construct the PDA equivalent to the CFG, whose productions are

$S \rightarrow 0BB$

$B \rightarrow 0S \mid 1S \mid 0$

Sol.

CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, B\}$

$\Sigma = \{0, 1\}$

$S = \{S\}$

Equivalent PDA is

PDA $A(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q\}$

$\Sigma = \{0, 1\}$

$\Gamma = V_N \cup \Sigma = \{S, B, 0, 1\}$

$q_0 = \{q\}$

$Z_0 = \{S\}$

$F = \phi$

δ is defined by (Apply both Rules)

Rule 1. $\delta(q, \lambda, A) = (q, \alpha)$ if $A \rightarrow \alpha$ is in P.

Rule 2: $\delta(q, a, a) = (q, \lambda)$ for every a in Σ

$\delta(q, \lambda, S) \vdash (q, 0BB)$ Since. $S \to 0BB$

$\delta(q, \lambda, B) \vdash (q, 0S)$ $B \to 0S$

$\delta(q, \lambda, B) \vdash (q, 1S)$ $B \to 1S$

$\delta(q, \lambda, B) \vdash (q, 0)$ $B \to 0$

Using Rule 2

$\delta(q, 0, 0) \vdash (q, \lambda)$ since $0 \in \Sigma$

$\delta(q, 1, 1) \vdash (q, \lambda)$ Since $1 \in \Sigma$

Let us consider string $w = 010000$

$(q, 010000, S) \vdash (q, 010000, 0BB)$ $(S \to 0BB)$

$\vdash (q, 10000, BB)$ $(B \to 1S)$

$\vdash (q, 10000, 1SB)$

$\vdash (q, 0000, SB)$ $(S \to 0BB)$

$\vdash (q, 0000, 0BBB)$

$\vdash (q, 000, BBB)$ $B \to 0$

$\vdash (q, 000, 0BB)$

$\vdash (q, 00, BB)$ $B \to 0$

$\vdash (q, 00, 0B)$

$\vdash (q, 0, B)$ $B \to 0$

$\vdash (q, 0, 0)$

$\vdash (q, \lambda, \lambda)$

(2) Construct the PDA accepted by Null store, whose production is given

$$S \rightarrow aSb \mid A$$
$$A \rightarrow bSa \mid S \mid \lambda$$

Sol. CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A\}$

$\Sigma = \{a, b\}$

$S = \{S\}$

Equivalent PDA is

PDA $A(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q\}$

$\Sigma = \{a, b\}$

$\Gamma = \{S, A, a, b\}$

$q_0 = \{q\}$

$z_0 = \{S\}$

$F = \phi$

δ is defined by

$\delta(q, \lambda, S) = \delta(q, aSb)$ since $S \rightarrow aSb$

$\delta(q, \lambda, S) = \delta(q, A)$ " $S \rightarrow A$

$\delta(q, \lambda, A) = \delta(q, S)$ $A \rightarrow S$

$\delta(q, \lambda, A) \vdash (q, \overset{bSa}{\cancel{aSb}})$ $\quad A \to bSa$

$\delta(q, \lambda, A) \vdash (q, \lambda)$ $\quad A \to \lambda$

$\delta(q, a, a) \vdash (q, \lambda)$ $\quad a \in \Sigma$

$\delta(q, b, b) \vdash (q, \lambda).$ $\quad b \in \Sigma$

There are 3 transitions for A and 2 transitions for S..

So, It is a non-deterministic PDA.

(3) Show that language $L = \{\{0^n 1^n\} \cup \{0^n 1^{2n}\}$ where $n \geq 1\}$ is a CFL and it is not accepted by deterministic Pushdown Automata (DPDA)

Sol.

$S \rightarrow S_1 | S_2$

$S_1 \rightarrow 0S_11 | 01$ → (for L_1)

$S_2 \rightarrow 0S_211 | 011$ → (for L_2)

The given language is represented by above CFG. Hence, it is a CFL.

$V_N = \{S, S_1, S_2\}$

$\Sigma = \{0, 1\}$

$S = \{S\}$

Equivalent PDA is

PDA A $(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q = \{q\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{S, S_1, S_2, 0, 1\}$

$q_0 = \{q\}$

$z_0 = \{S\}$

$F = \phi$

δ is defined by

$$S \begin{cases} \delta(q, \lambda, S) = (q, S_1) \\ \delta(q, \lambda, S) = (q, S_2) \end{cases}$$

$$S_1 \begin{cases} \delta(q, \lambda, S_1) = (q, 0S_11) \\ \delta(q, \lambda, S_1) = (q, 01) \end{cases}$$

$$S_2 \begin{cases} \delta(q, \lambda, S_2) = (q, 0S_211) \\ \delta(q, \lambda, S_2) = (q, 011) \end{cases}$$

$$\delta(q, 0, 0) = (q, \lambda)$$
$$\delta(q, 1, 1) = (q, \lambda)$$

Since there are more than one transitions for S; S_1 and S_2.

Therefore $L = \{ \{0^n 1^n\} \cup \{0^n 1^{2n}\} \ \forall\ n \geq 1 \}$ is accepted by a non-deterministic Pushdown automata, not by a DPDA.

Here, from a particular state, reading a particular symbol and the particular top of the stack is giving more than one transition.

Hence, given language can't be accepted by DPDA.

Q. 4 Find the PDA for the language

$L = \{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$

Sol.

$S \to AB \mid XY$

$A \to aAb \mid R_1 \mid R_2$

$R_1 \to aR_1 \mid a$

$R_2 \to bR_1 \mid b$

$B \to cB \mid \lambda$

$Y \to bYc \mid R_3 \mid R_4$

$R_3 \to bR_3 \mid b$

$R_4 \to cR_4 \mid c$

$X \to aX \mid \lambda$

CFG $G(V_N, \Sigma, P, S)$

$V_N = \{S, A, B, X, Y, R_1, R_2, R_3, R_4\}$

$\Sigma = \{a, b, c\}$

$S = \{S\}$

Equivalent PDA is given by

PDA $A(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{S, A, B, X, Y, R_1, R_2, R_3, R_4, a, b, c\}$

$q_0 = \{q\}$

$Z_0 = \{S\}$

$F = \phi$

δ is defined by

$\delta(q, \lambda, S) = (q, AB)$
$\delta(q, \lambda, S) = (q, XY)$
$\delta(q, \lambda, A) = (q, aAb)$
$\delta(q, \lambda, A) = (q, R_1)$
$\delta(q, \lambda, A) = (q, R_2)$
$\delta(q, \lambda, R_1) = (q, aR_1)$
$\delta(q, \lambda, R_1) = (q, a)$
$\delta(q, \lambda, R_2) = (q, bR_2)$
$\delta(q, \lambda, R_2) = (q, b)$
$\delta(q, \lambda, B) = (q, cB)$
$\delta(q, \lambda, B) = (q, \lambda)$

$\delta(q, \lambda, Y) = (q, bYc)\ (q, R_3)\ (q, R_4)$
$\delta(q, \lambda, R_3) = (q, bR_3)\ (q, b)$
$\delta(q, \lambda, R_4) = (q, cR_4)\ (q, c)$
$\delta(q, \lambda, X) = (q, aX)\ (q, \lambda)$

Rule 2

$\delta(q, a, a) = (q, \lambda)$ since $a \in \Sigma$
$\delta(q, b, b) = (q, \lambda)$ $b \in \Sigma$
$\delta(q, c, c) = (q, \lambda)$ $c \in \Sigma$