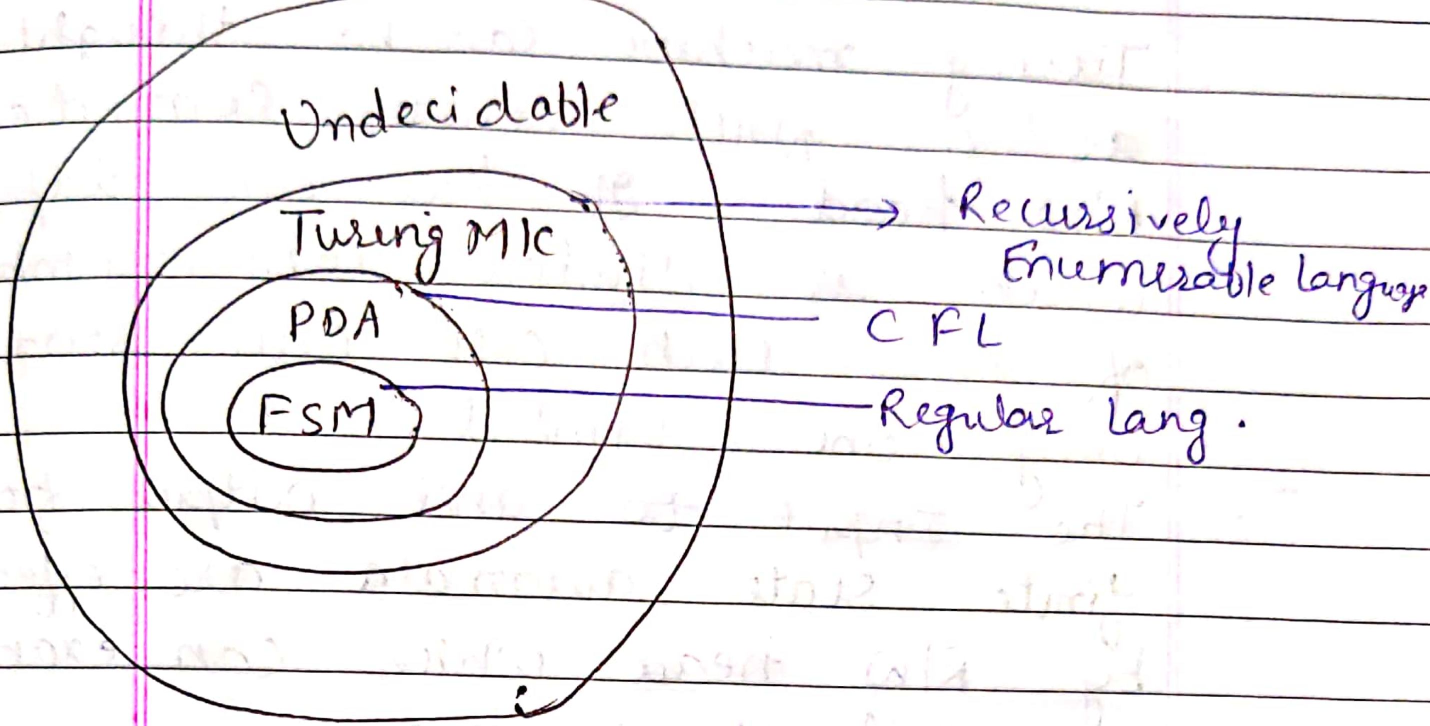
Turing machine

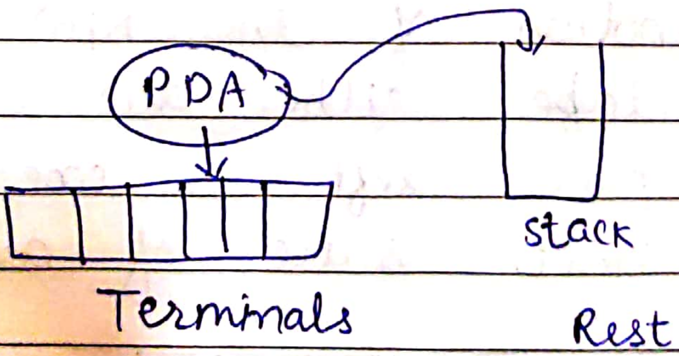


FSM: The I/p string.

a	a	a	a	b	a	b

Control – moves only on one direction
finite string.

PDA: DS – I/p string
Stack



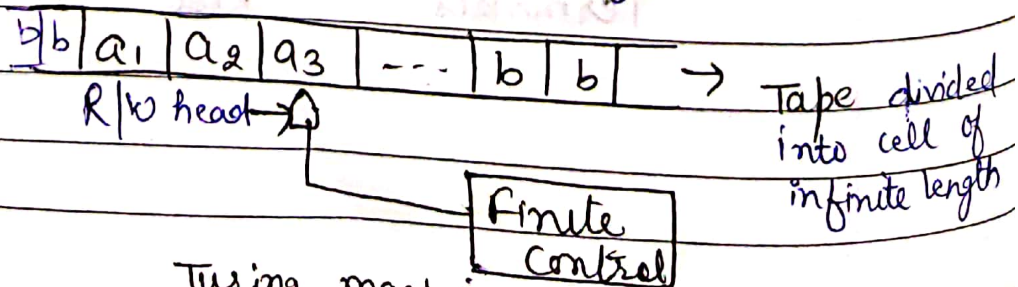
Turing machine

Turing machine can be thought of as a finite control connected to R/W head. It has one tape which is divided into number of cells. Each cell can store only one symbol.

The Input to and output from finite state automata are effected by R/W head which can examine one cell at a time.

In one move, the m/c examines the present symbol under the R/W head on the tape and the present state of automata to determine,

1) A new symbol to be written on the tape in the cell under R/W head.
2) A motion of the R/W head along the tape either the head moves one cell left or one cell right.
3) The next state of automata



Turing machine model

A Turing machine M can be described by 7 tuples

$(Q, \Sigma, \Gamma, \delta, q_0, b, F)$ where

Q :- Set of states
Σ : Set of I/p symbols $(b \notin \Sigma)$
Γ : Set of tape symbols $(\Sigma \subseteq \Gamma)$
δ : Transition mapping function
q_0 : Initial state
b : Blank symbol $(b \in \Gamma)$
F : Set of final state (Accept state, Reject state)

The blank is a special symbol, used to fill the infinite tape.

Operations on the tape

→ Read / Scan symbol below the tape head.
→ Update / write a symbol below the tape head.
→ Move the tape head one step left.
→ Move the tape head one step right.

$$Q \times \Sigma \rightarrow \Gamma \times (R/L) \times Q$$

Transition Mapping function

$\delta(q_0, x) = (q_1, y, D)$ or (y, D, q_1)
Here D is either L or R

Language Accepted by Turing Machine

Consider TM $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$ is a string
$w \in \Sigma^*$ is accepted by

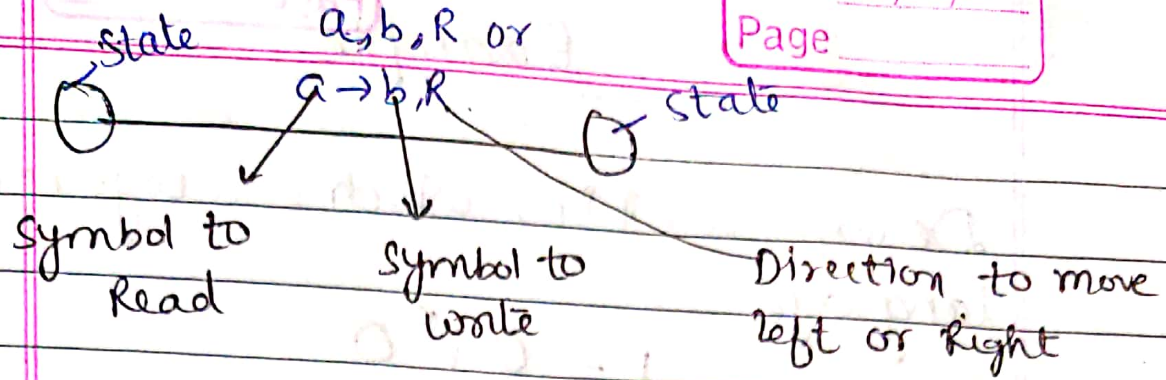
$$q_0 w \overset{*}{\vdash} \alpha_1 p \alpha_2$$

$p \in F$
$\alpha_1, \alpha_2 \in \Gamma^*$

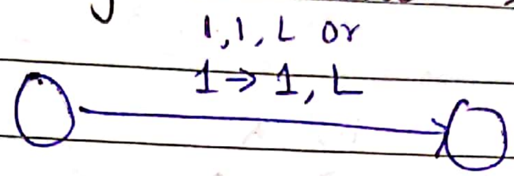
Rule of operation 1.

→ At each step of the computation Read the current symbol
→ Update (write) the same cell
→ Move exactly one cell either left or right.

If we are at the left end of the tape, and trying to move left, then do not move. Stay at the left end of the tape.



If you don't want to update the cell, just write the same symbol.



Rule of operation - 2

→ Control is with a sort of FSM
→ Initial state
→ Final states: (there are two final states)
  1) The Accept state
  2) The Reject state

Computation can either

1) Halt and Accept
2) Halt and Reject
3) Loop (the machine fails to Halt)

A machine M doesnot accept w if the Machine M either halts in a non accepting state or does not halt.

Transition

TM = $\{\{q_1, q_2, q_3, q_4, q_5\}, \{0,1\}, \{0,1,b\}, \delta, q_1, b, \{q_5\}\}$

Tape symbol

	b	0	1
$\rightarrow q_1$	$1Lq_2$	$0Rq_1$	
q_2	bRq_3	$0Lq_2$	$1Lq_2$
q_3		bRq_4	bRq_5
q_4	$0Rq_5$	$0Rq_4$	$1Rq_4$
q_5	$0Lq_2$		

Draw the computation sequence of the I/p string 00.

Sol. If the string in the tape is $a_1 a_2 \dots a_j a_{j+1} \dots a_m$ and the TM is in state q is to read a_{j+1} then, we write $a_1 a_2 \dots a_j\, q\, a_{j+1} \dots a_m$

Computation sequence $\quad w = 00$

$q_1 00b \vdash 0q_1 0b$

$\vdash 00q_1 b$

$\vdash 0q_2 01$

$\vdash q_2 001$

$\vdash q_2 b001$

$\vdash bq_3 001$

$\vdash bbq_4 01$

$\vdash bb0q_4 1$

$\vdash bb01q_4 bb$

$\vdash bb010q_5b$

$\vdash bb01q_200$

$\vdash bb0q_2100$

$\vdash bbq_20100$

$\vdash bq_2b0100$

$\vdash bbq_30100$

$\vdash bbbq_4100$

$\vdash bbb1q_400$

$\vdash bbb10q_40$

$\vdash bbb100q_4b$

$\vdash bbb1000q_5$

$\vdash bbb100q_200$

$\vdash bbb10q_2000$

$\vdash bbb1q_20000$

$\vdash bbbq_210000$

$\vdash bbq_2b10000$

$\vdash bbbq_310000$

$\vdash bbbbq_50000$

$q_0w \overset{*}{\vdash} \alpha_1 p \alpha_2$

$p \in F$

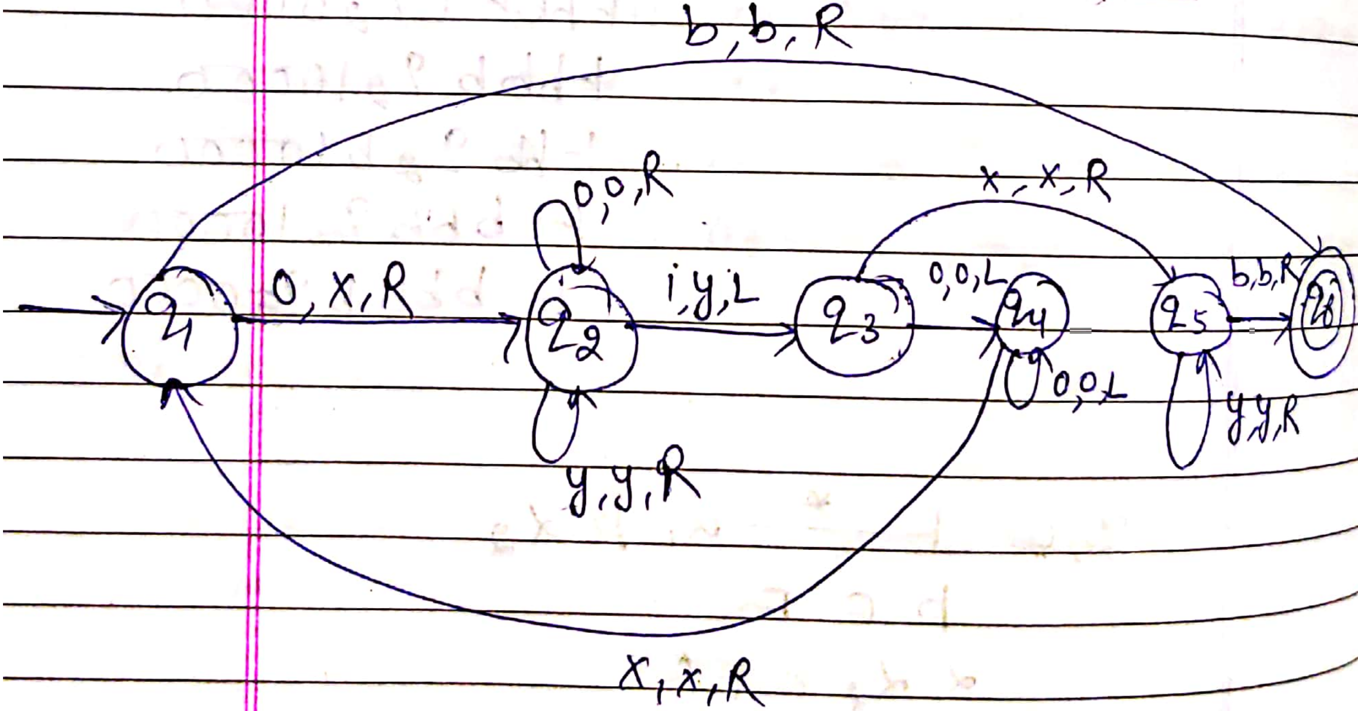
$\alpha_1, \alpha_2 \in \Gamma^*$

String Accepted

Turing Machine

States	0	Tape symbols	x	b	y
$\rightarrow q_1$	$(X, R, q_2$			b, R, q_6	
q_2	$0, R, q_2$	y, L, q_3			y, R, q_2
q_3	$0, L, q_4$		x, R, q_5		y, L, q_3
q_4	$0, L, q_4$		x, R, q_1		
q_5				b, R, q_6	y, R, q_5
q_6					

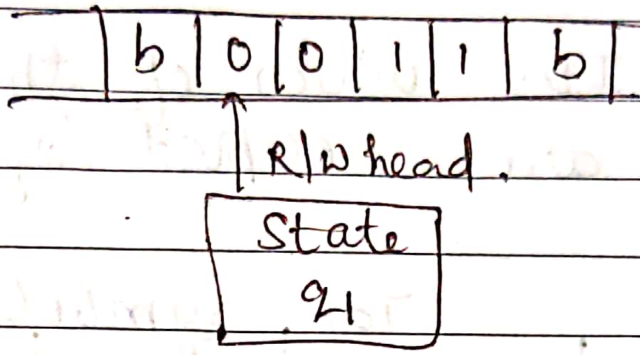
Transition diagram of above table



Obtain the computation sequence of M for processing the I/P string 0011.

Computation sequence :

$q_1, 0011b \vdash$



$q_1, b0011b \vdash bxq_2011b$

$\vdash bx0q_211b$

$\vdash bxq_30y1b$

$\vdash bq_4x0y1b$

$\vdash bxq_10y1b$

$\vdash bxxq_2y1b$

$\vdash bxxyq_21b$

$\vdash bxxq_3yyb$

$\vdash bxq_3xyyb$

$\vdash \cancel{bq_3xxyyb}$

$\vdash bxxq_5yyb$

$\vdash bxxyq_5b$

$\vdash bxxyyq_5b$

$\vdash bxxyybq_6b.$

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Q. Describe the processing of
a) 011
b) 0011
c) 001
using ID's. which of the above strings are accepted by M.

Present state	Tape symbols 0	1	x	y	b
$\rightarrow q_1$	xRq_2				bRq_5
q_2	$0Rq_2$	yLq_3		yRq_2	
q_3	$0Lq_4$		xRq_5	yLq_3	
q_4	$0Lq_4$		xRq_1		
q_5				yRq_5	bRq_6
q_6					

a) $q_1011 \vdash xq_211$
$\vdash q_3xy1$
$\vdash xq_5y1$
$\vdash xyq_51$

As $\delta(q_5, 1)$ is not defined. M halts, so the i/p string 011 is not accepted.

b) $q_10011 \vdash xq_2011$
$\vdash x0q_211$
$\vdash xq_30y1$
$\vdash q_4x0y1$

$q_4 x 0 y 1 \vdash x q_1 0 y 1$
$\vdash x x q_2 y 1$
$\vdash x x y q_2 1$
$\vdash x x q_3 y y$
$\vdash x q_3 x y y$
$\vdash x x q_5 y y$
$\vdash x x y q_5 y$
$\vdash x x y y q_5 b$
$\vdash x x y y b q_6$

M halts, As
q_6 is an accepting state, the I/p string 0011 is accepted by M.

c.) $q_1 001 \vdash x q_2 01$
$\vdash x 0 q_2 1$
$\vdash x q_3 0 y$
$\vdash q_4 x 0 y$
$\vdash x q_1 0 y$
$\vdash x x q_2 y$
$\vdash x x y q_2$

M halts,
As q_2 is not an accepting state.
So string 001 is not accepted by M.