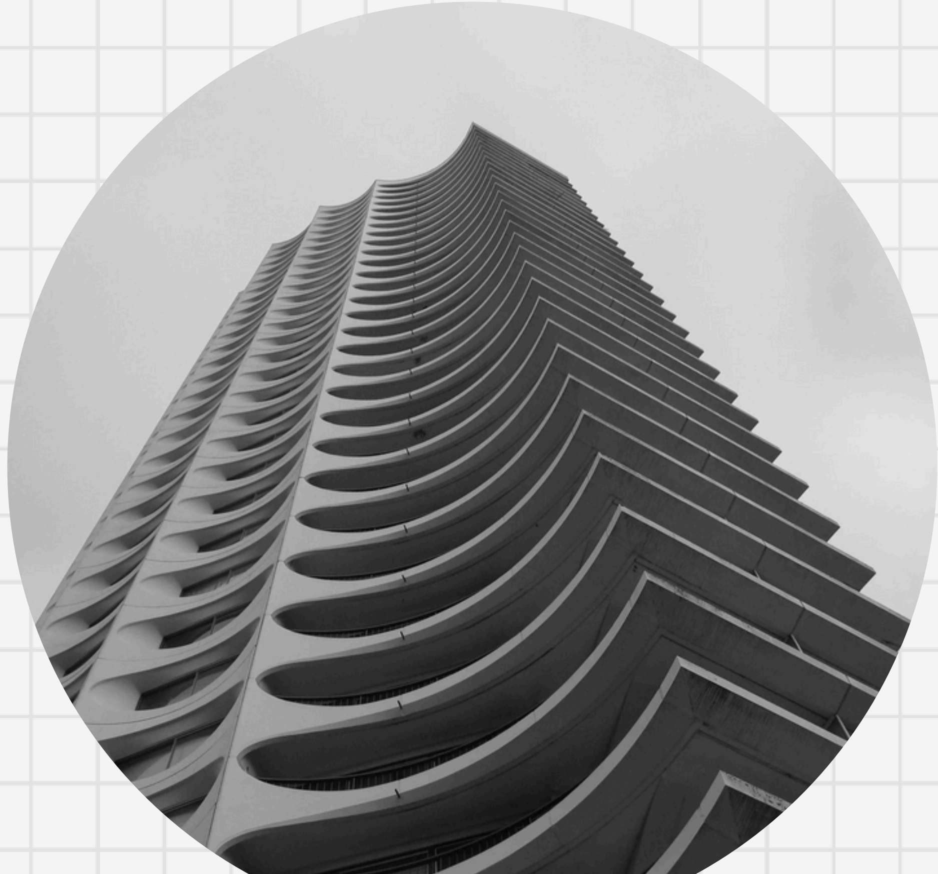
Regime Dependent Portfolio Optimisation: An Integrated Framework



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Overview

- Developed a regime-aware portfolio optimization framework that adapts to changing market conditions using Hidden Markov Models (HMMs).
- Benchmarked traditional methods — MVP, HRP, and Autoencoder-based portfolios — across 10 NSE thematic sectors.
- Proposed regime-specific strategies like Black–Litterman CVaR and Direct CVaR based on detected market states (bullish, bearish, neutral).
- Demonstrated improved Sharpe ratios and portfolio stability during both training (2018–2021) and out-of-sample testing (2022).

Motivation

- Financial markets frequently shift between regimes (bullish, bearish, volatile) due to economic, political, and structural factors.

- Traditional portfolio models assume stable return distributions, leading to poor performance during regime shifts.

- Static optimization strategies lack the flexibility to respond to sudden changes in volatility, correlation, or market trends.

- There is a clear need for adaptive portfolio models that detect market regimes and adjust allocations in real time to improve risk-adjusted returns and portfolio resilience.

- This thesis addresses that gap by integrating statistical regime detection and machine learning–based optimization, creating a dynamic and robust investment strategy.

Presentation Outline

- Introduction
- Mathematical Analysis: Static Portfolio Optimisation Methods
- Prerequisite Analysis for Adaptive Strategy
- Adaptive Regime Strategy
- Conclusion and Future Work

Introduction

Portfolio Optimisation

- Portfolio optimisation is the process of selecting asset weights that maximise return for a given level of risk, or vice versa.

- While the **Classical Markowitz** framework relies on the mean and variance of asset returns, modern enhancements incorporate risk-sensitive metrics, regularisation, and investor views.

- Markets are not static they operate on regimes such as Bearish, Bullish, Neutral or Volatile, identifying these regimes and shifting portfolio weights accordingly is the main idea behind this thesis.

- **Risk Parity** is the allocation strategy that equalises the contribution of each asset to the overall portfolio risk. Unlike MVP, which may over-allocate to low-volatility assets, risk parity ensures a more balanced risk distribution. Hence dynamically adjusting risk upon the market conditions become important.

- **Statistics** play important role in **return** and **risk** estimation. On the other hand auto-encoders are used in dimensionality reduction and capturing non-linear pattern among assets. Hence hybrid adaptive regime methods combines these two techniques to produce highest performing **risk adjusted portfolios.**

- So the aim of this Thesis is to propose a integrated **Regime Dependent Framework** that dynamically adjust weights on detected market states in order to enhance portfolio returns.

Mathematical Analysis: Static Portfolio Optimisation Methods

Mean Variance Portfolio

- The Mean-Variance Portfolio (MVP) model, introduced by Markowitz, allocates portfolio weights to minimise risk (variance) for a given expected return or to maximise return for a fixed risk level.

- Volatility is defined as the standard deviation of daily returns, annualised by scaling by a factor of root of 250.

- Covariance and Correlation matrices are then computed to quantify the relationships between stock returns to help diversify portfolio and minimise risk volatility.

- Efficient Frontier represents the optimal portfolio with highest returns with given amount of risk.

Algorithm 1 Mean-Variance Portfolio Optimisation

1: Compute daily returns $r_i(t)$ for each stock i

2: Compute expected return $\mu_i = \mathbb{E}[r_i]$ and covariance matrix Σ

3: Set target return μ_p

4: Solve quadratic program:

$$\min_w w^\top \Sigma w \quad \text{s.t.} \quad w^\top \mu = \mu_p, \quad \sum w_i = 1$$

5: Output optimal weights w^*

Hierarchical Risk Parity Methodology

- Constructs diverse portfolios, using the hierarchical risk parity method which involves formation of **formation of clusters**, **Quasi Diagonalisation** and **recursive bisection.**

- Clusters are grouped on how similar their return patterns are, which forms a tree like structure known as dendrogram.

- **Quasi Diagonalisation** the correlation matrix is reordered so that similar assets appear next to each other.

- Finally, we split the **clusters recursively** and assign weights inversely to their risk—so no group dominates the portfolio.

Algorithm 2 Hierarchical Risk Parity (HRP)

1: Compute correlation matrix ρ_{ij} and distance matrix $d_{ij} = \sqrt{0.5(1 - \rho_{ij})}$
2: Perform hierarchical clustering using Ward's linkage
3: Quasi-diagonalize the covariance matrix using the dendrogram order
4: Recursively:
   - Split clusters into two subsets
   - Compute inverse-variance weight of each cluster
   - Allocate capital inversely proportional to cluster variance
5: Output final portfolio weights w_i

Autoencoder Based Portfolio Optimisation

- Constructs data-driven portfolios using **unsupervised deep learning** to uncover hidden patterns in stock prices.

- An autoencoder compresses stock data into a lower-dimensional latent space, capturing the most relevant features for portfolio construction.

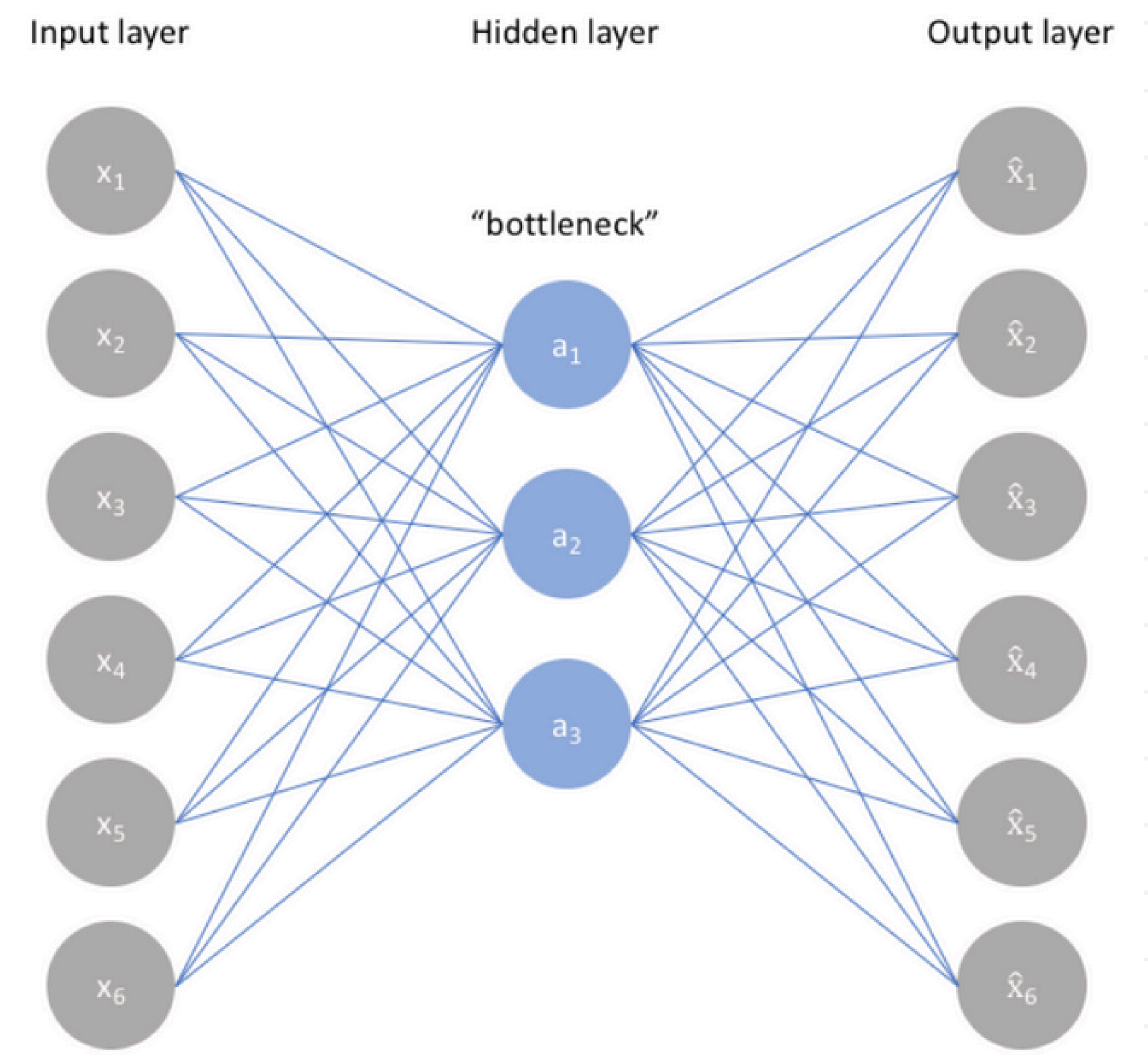
- It learns to reconstruct original price patterns by minimizing reconstruction error, ensuring the extracted features are meaningful.

- The final reconstructed output is normalized to derive portfolio weights, reflecting each asset's relative importance based on learned patterns.

- The model is trained to minimize the Mean Squared Error (LMSE) between the original and reconstructed data, ensuring the network captures only the most essential price patterns for accurate portfolio construction.

- Auto Encoders can be trained on historical stock price data to extract relevant features for asset returns. These weights reflect the relative importance of each asset and are used for portfolio construction, with the goal of capturing the latent structure in asset returns and allocating capital accordingly.

- Empirical studies show that autoencoder-based portfolios can outperform traditional models in terms of annual returns, though they may also exhibit higher volatility due to the nonlinear transformations involved.



Algorithm 3 Autoencoder-Based Portfolio Optimization

1: Train autoencoder on historical stock price matrix X
2: Encode: $c = f(X)$, Decode: $X' = g(c)$
3: Minimize reconstruction loss:

$$\mathcal{L} = \|X - X'\|^2$$

4: Normalize output layer: $w_i = \frac{x'_i}{\sum_j x'_j}$
5: Output weights w_i for portfolio allocation

Prerequisite Analysis for Adaptive Strategy

Choosing the Sectors

- The analysis focuses on ten thematic sectors of the National Stock Exchange (NSE), India. These sectors represent diverse investment themes such as infrastructure, manufacturing, consumption, and sustainability.
- For each sector, ten constituent stocks were selected based on their free-float market capitalisation as per NSE's report dated February 29, 2022.

Sector No.	NSE Thematic Sector
1	NIFTY Commodities
2	NIFTY Energy
3	NIFTY Manufacturing
4	NIFTY Services
5	NIFTY MNC
6	NIFTY Transportation and Logistics
7	NIFTY Infrastructure
8	NIFTY Housing
9	NIFTY Consumption
10	NIFTY 100 ESG (Environmental, Social, and Governance)

Acquiring the Data

- To construct and evaluate the portfolios, historical stock price data is collected using the DataReader function from the *pandas-datareader* Python library.
- The closing price data for each of the 100 selected stocks (10 per sector) is extracted from Yahoo Finance.
- Training period: January 1, 2018 – December 31, 2021 and Testing period: January 1, 2022 – December 31, 2022
- Only the daily closing prices are used in the analysis, as this is a univariate portfolio construction setup. From the raw prices, daily returns are computed and used to assess risk and return characteristics.

Python Implementation MVP Portfolio

- The Mean-Variance Portfolio (MVP) is constructed using the Markowitz framework, aiming to maximize returns for a given level of risk or minimize risk for a desired return.

- Monte Carlo simulations in Python are used to generate 10,000 random portfolios. The Dirichlet distribution ensures that portfolio weights sum to 1, providing diverse portfolio combinations.

- Annualized mean returns and covariance matrices are calculated using Python libraries such as NumPy and Pandas, and the Sharpe ratio is computed to evaluate performance.

- The portfolio with the highest Sharpe ratio is selected as the optimal MVP portfolio.

- This approach provides a visual representation of the efficient frontier and allows flexibility for experimentation with different asset sets and portfolio sizes using Matplotlib for plotting.

Python Implementation HRP Portfolio

- Pandas is used to compute the correlation matrix *(train_returns.corr()),* ensuring that asset return data is structured for analysis.

- Scipy's *scipy.cluster.hierarchy.linkage* function is used for agglomerative clustering, applying Ward's linkage to group assets based on correlation.

- The PyPortfolioOpt library is used to apply *quasi-diagonalization* through its HRPOpt class, which optimises asset allocation based on the hierarchical structure.

- Portfolio weights are computed with the HRP optimizer using *hrp.optimize(),* ensuring balanced risk allocation across asset clusters.

- NumPy/Pandas are used to clip correlation values to the range [-1, 1] and fill missing values with zeros, avoiding instability during the clustering process.

Python Implementation Autoencoder Portfolio

- *MinMaxScaler* from *Scikit-learn* Library is used to scale stock price data into the range [0, 1], ensuring uniform contribution from all features during training.
- The *Keras Sequential API* is used to define the autoencoder architecture, consisting of an input layer, hidden layer (with ReLU activation), and output layer (with linear activation).
- *Adam optimizer* and *Mean Squared Error (MSE)* loss function are employed for training the autoencoder to minimize the reconstruction error.
- After training, the final observation of the scaled dataset is passed through the autoencoder model to obtain the reconstructed output, which is used to determine the portfolio weights.
- The reconstructed output is normalized so that the portfolio weights sum to 1, ensuring a valid and balanced portfolio allocation.

NIFTY Commodities Sector

Table 3.5: NIFTY Commodities Sector – Portfolio Weights

Stock	MVP	HRP	ENC
RELIANCE	0.2144	0.1464	0.0807
ULTRACEMCO	0.2140	0.1524	0.1079
TATASTEEL	0.0064	0.0428	0.1038
NTPC	0.3286	0.1803	0.1037
JSWSTEEL	0.0071	0.0459	0.1381
ONGC	0.0044	0.0614	0.0780
GRASIM	0.0234	0.1127	0.0997
HINDALCO	0.0025	0.0667	0.1194
COALINDIA	0.1408	0.0889	0.0590
UPL	0.0584	0.1025	0.1098

The **NIFTY Commodities** index represents companies operating in the commodities space, including energy, metals, cement, and natural resources.

NIFTY Commodities Sector Returns

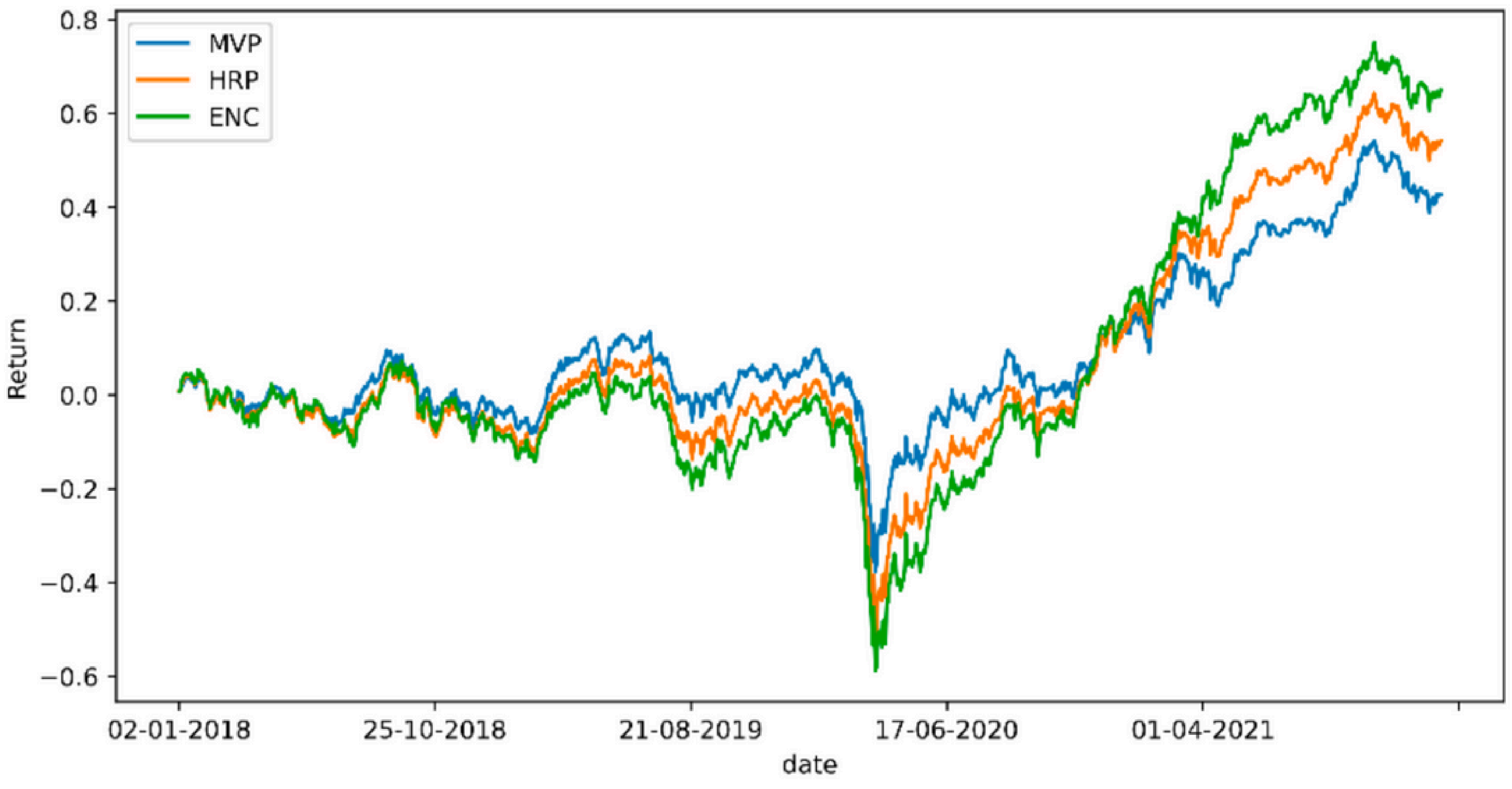


Figure 3.1: Cumulative Daily Returns (Training Period: 2018–2021)

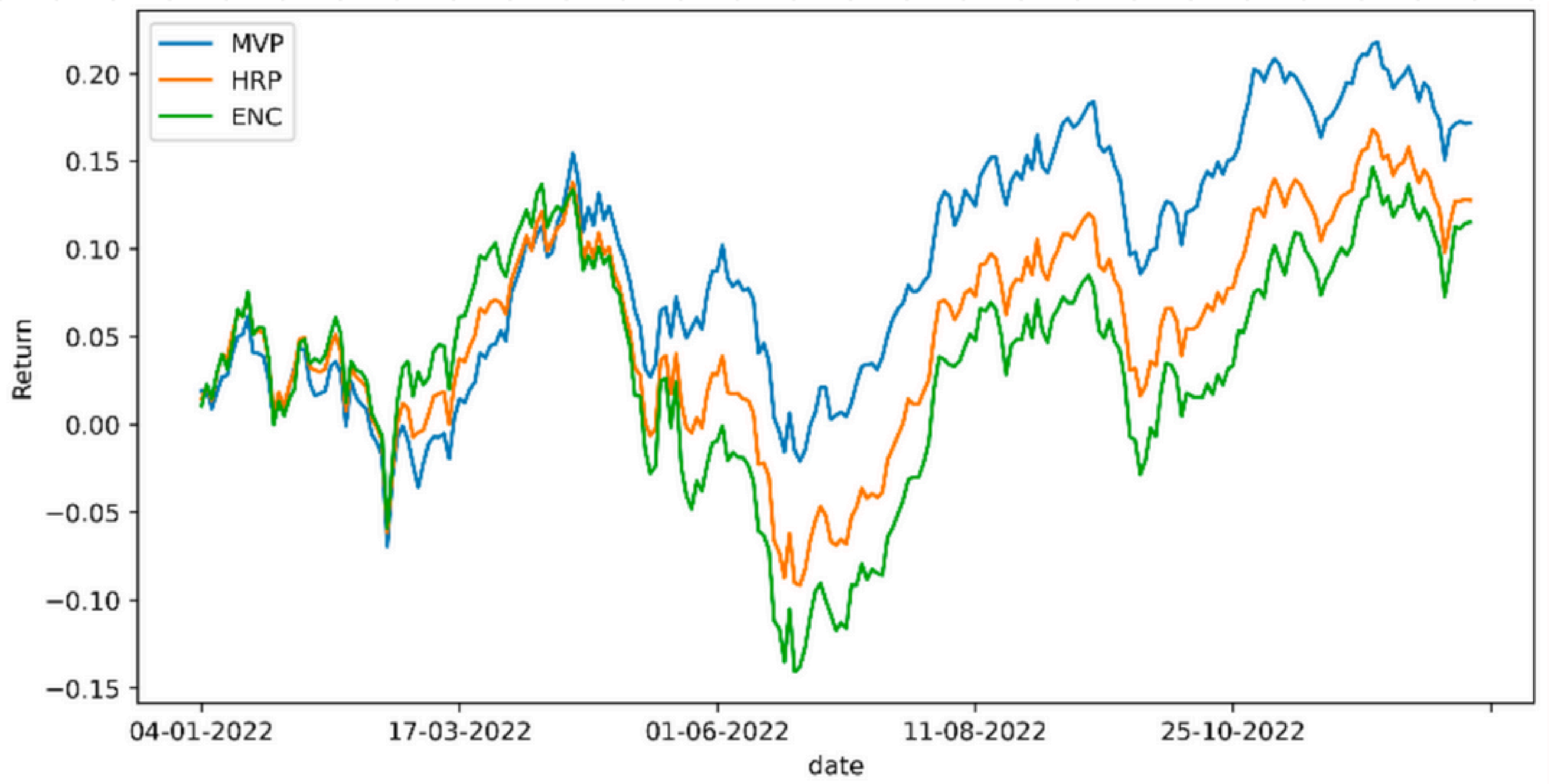


Figure 3.2: Cumulative Daily Returns (Test Period: 2022)

Table 3.6: Training and Test Performance of Portfolios

Portfolio	Training Performance			Test Performance		
	Annual Return	Annual Volatility	Sharpe Ratio	Annual Return	Annual Volatility	Sharpe Ratio
MVP	10.89	21.87	0.4978	17.51	19.49	0.8982
HRP	13.82	23.49	0.5883	13.01	20.48	0.6354
ENC	16.59	26.01	0.6375	11.74	23.32	0.5034

The above is the training and test performance metrics for the **NIFTY Commodities Sector.** Hence similarly we also have done analysis for remaining 9 sectors of the National Stock Exchange to derive a comprehensive analysis of sector wise performance.

Summary of Sector Wise Performance

Training Data Insights

1. The ENC portfolio achieved the highest annual returns in **5 out of 10** sectors.
2. The MVP portfolio yielded the highest Sharpe ratio in **6 sectors,** demonstrating superior risk-adjusted returns.
3. Most notably, the MVP portfolio had the lowest annual volatility across all ten sectors.

Test Data Insights

1. The ENC portfolio again produced the highest annual return in **5 sectors**, validating its consistency in out-of-sample performance.
2. The MVP portfolio recorded the highest Sharpe ratio in 6 sectors and lowest volatility in **8 out of 10 sectors.**

Overall, this comprehensive sectoral analysis supports a dual insight: while ENC captures hidden nonlinear return-driving features via deep learning, MVP continues to excel in producing balanced, low-volatility portfolios.

Adaptive Regime Portfolio Strategy

Data Acquisition and Preprocessing

- **Data Acquisition:** Use *yfinance* API to retrieve adjusted closing prices, accounting for corporate actions like dividends and stock splits.
- **Fallback Mechanism:** If yfinance is unavailable, data is retrieved from *Stooq API* or generated as a synthetic market index using averages of relevant indices.
- **Caching:** The data is cached locally to minimize redundant API calls, ensuring faster retrieval and avoiding rate-limit issues.
- **Alignment & Cleaning:** Data is aligned across assets and benchmark indices, filling missing values via forward filling or interpolation to ensure consistency.
- **Data Integrity:** Ensures high-quality data by validating and correcting any discrepancies to improve modeling accuracy.

Regime Detection Using Hidden Markov Models (HMMs)

- **Regime Identification**: HMMs are used to detect market regimes (bullish, bearish, neutral) based on historical asset returns.

- **Standardised Returns:** Standardisation of returns (log differences) ensures that each asset's volatility is accounted for, making the data comparable for model fitting.

- **Gaussian HMM:** A Gaussian HMM is trained on standardised return data, as financial returns often align with Gaussian distributions, representing different market conditions.

- **Model Selection:** The optimal number of hidden states (2-4 states for market conditions) is selected using the Bayesian Information Criterion (BIC), balancing model complexity and fit.

- **State Inference:** After training, the HMM predicts latent states (regimes) for each time period, providing insights into market transitions.

- **Re-fitting Mechanism**: A re-fitting guard ensures the model is updated with the latest data, maintaining accuracy during significant market changes or events.

Regime Specific Portfolio Construction

Mean Variance Portfolio (MVP)

- **Objective:** Allocates portfolio weights to maximize return for a given level of risk or minimize risk for a given return.
- **Optimisation Goal:** Maximises the Sharpe ratio to balance return and risk in the portfolio.
- **Method:** A Monte Carlo simulation is used to generate multiple random portfolios, and the one with the highest Sharpe ratio is selected.
- **Limitation:** The static nature of MVP makes it unsuitable for regime-switching scenarios where market conditions (volatility and correlations) change over time.

Black-Litterman CVaR (BL-CVaR)

- **Objective**: Combines **Black-Litterman** model with **Conditional Value-at-Risk (CVaR)** for downside risk minimisation.
- **Customization:** Incorporates investor views to adjust expected returns, allowing for tailored portfolio allocations.
- **Key Benefit:** Provides a robust allocation by combining market views and personalised opinions, especially during high volatility or market crashes.
- **Covariance Matrix Estimation:** Uses **Ledoit-Wolf** shrinkage to regularise covariance estimation, improving stability and reducing overfitting to noisy data.

Direct CVaR

- **Objective:** Focuses purely on **minimising downside risk** without relying on market equilibrium assumptions or investor views.
- **Method:** Directly minimises expected shortfall (average loss in the worst α-percent of cases) using historical return data.
- **Key Benefit:** Ideal for periods of high market stress, where **downside risk management** is the priority over maximising returns.
- **Simpler Approach:** Does not require Bayesian priors and focuses exclusively on **managing extreme risks.**

Summary:

- **MVP:** Best for stable market conditions, optimizing for return/risk trade-off but lacks adaptability to regime shifts.
- **Black-Litterman CVaR:** More flexible, blending market equilibrium with investor views and focusing on downside risk, ideal for volatile markets.
- **Direct CVaR**: A simpler, direct method for minimising extreme losses, focusing on downside risk without relying on prior assumptions.

Adaptive Rebalancing with Walk-Forward Discipline

- **Rebalancing Frequency:** Portfolio is rebalanced monthly, with 504-day warm-up period for the model to learn historical market dynamics and avoid overfitting.

- **Balance Between Responsiveness & Costs**: Monthly rebalancing ensures responsiveness to regime shifts while minimising transaction costs from excessive trading.

- **Walk-Forward Methodology**: Ensures robustness by training the model using only past data, preventing look-ahead bias and making the backtest more realistic.

- **Regime-Specific Portfolio Optimization**: HMM is retrained on the latest 504-day window to identify market regime (e.g., bullish, bearish, neutral) and apply regime-specific portfolio weights (e.g., MVP, Black-Litterman CVaR).

- **Performance Tracking:** Portfolio performance is tracked over time by applying optimised weights to the next period's returns and appending results to a cumulative return curve.

Sector Wise Performance

Table 4.1: Sector-Wise Performance during Training Period (2018–2021)

Sector	Annual Return (%)	Annual Volatility (%)	Sharpe Ratio
Commodities	14.2	22.7	0.62
Energy	25.4	23.5	1.08
Manufacturing	27.6	24.6	1.12
Services	26.1	22.2	1.17
MNC	18.8	19.4	0.97
Transportation	31.3	24.8	1.26
Infrastructure	21.2	21.5	0.99
Housing	21.7	22.0	0.99
Consumption	22.6	18.9	1.19
ESG	32.5	21.8	1.40

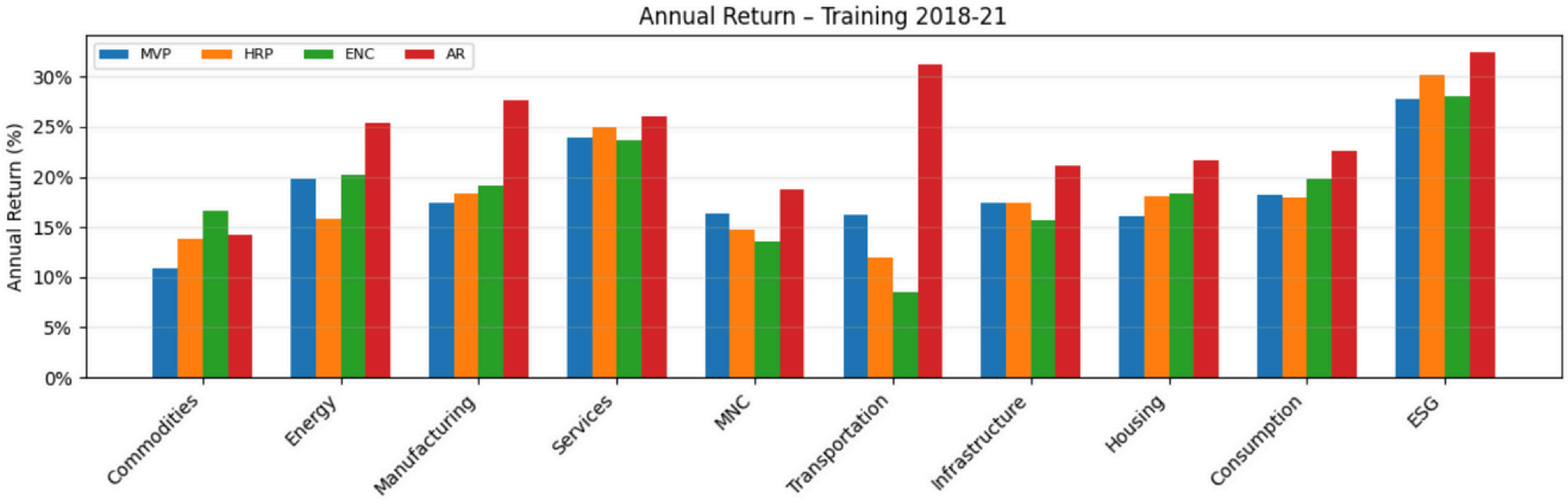


Figure 4.1: Annual Returns Comparison

Figure 4.3: Sharpe Ratio Comparison

Table 4.2: Sector-Wise Performance during Testing Period (2022)

Sector	Annual Return (%)	Annual Volatility (%)	Sharpe Ratio
Commodities	19.0	18.4	1.03
Energy	20.6	18.8	1.09
Manufacturing	13.8	15.2	0.82
Services	2.8	17.5	0.16
MNC	17.9	15.8	1.04
Transportation	33.7	20.6	1.55
Infrastructure	13.4	16.6	0.80
Housing	12.8	16.6	0.68
Consumption	18.3	14.8	1.19
ESG	1.2	17.1	0.07

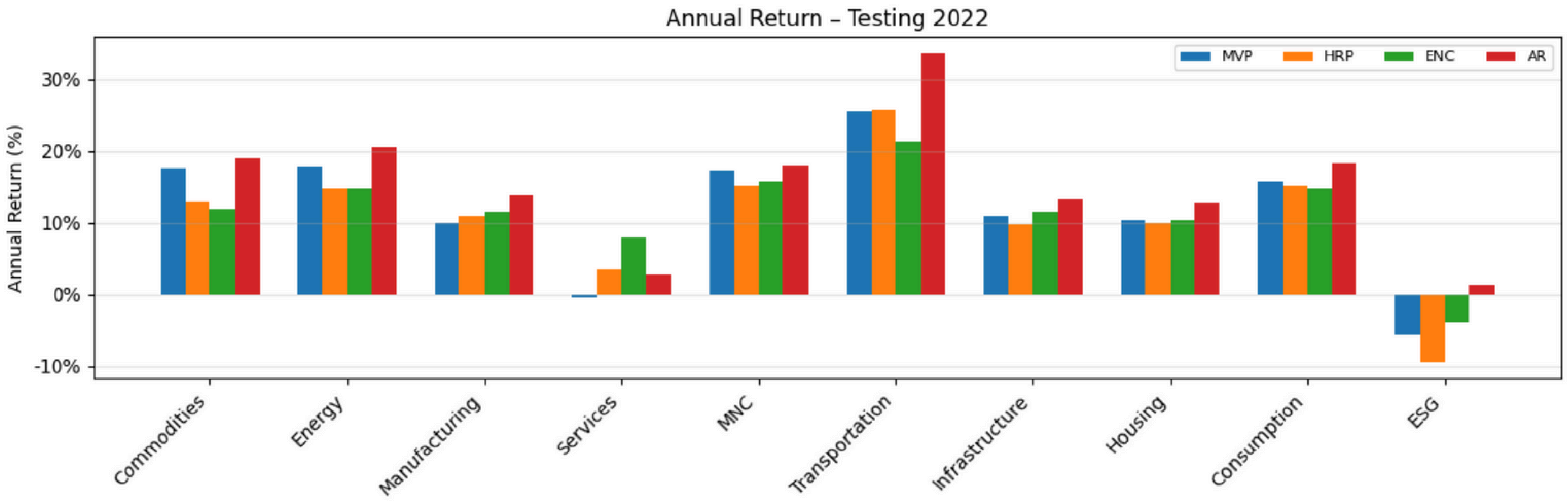
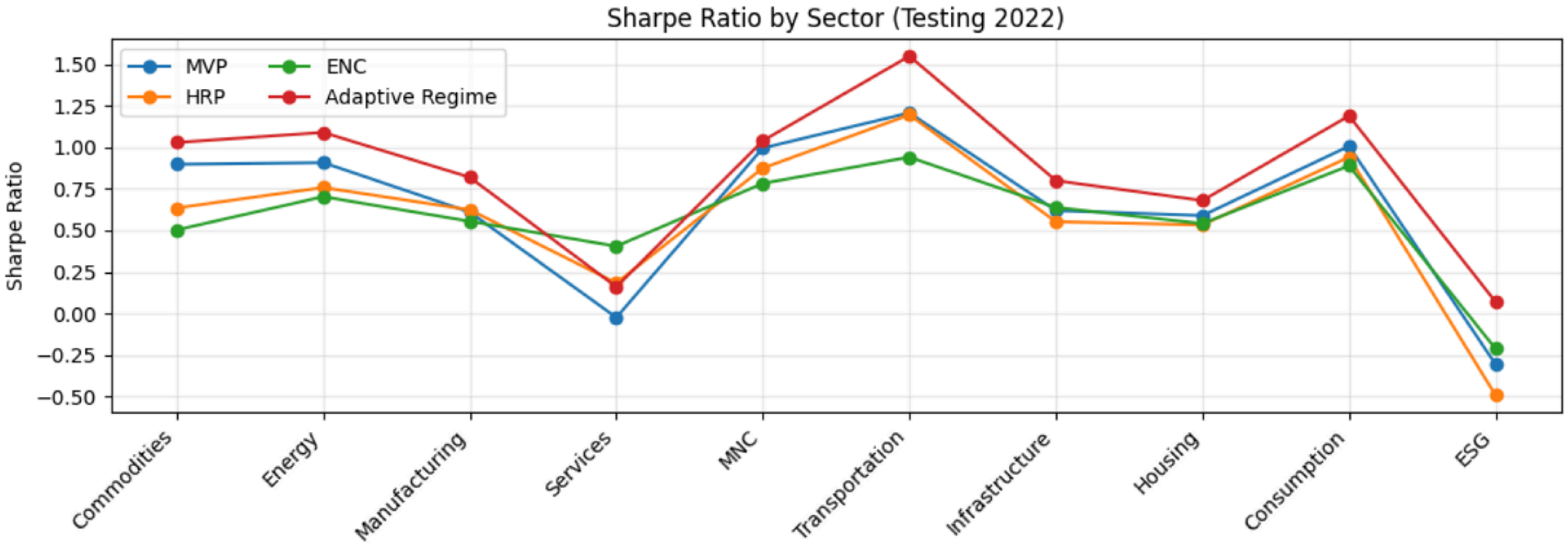


Figure 4.4: Annual Returns Comparison



Conclusion and Future Work

Conclusion

- Integration of HMM with Regime-Specific Optimization: The strategy combines **Hidden Markov Models (HMM)** for market regime detection with regime-specific portfolio optimisation to dynamically adjust allocations based on market conditions.

- Empirical Evaluation: Assessed using data from 10 NSE thematic indices during both the **training period (2018-2021)** and **out-of-sample testing period (2022).**

- **Superior Performance:** The adaptive strategy consistently outperformed or matched baseline models (e.g., MVP, HRP, Autoencoder):
  - **Sharpe ratio** improvements in key sectors like Commodities, Energy, and Manufacturing.
  - **Out-of-sample Sharpe ratio of 1.03 in testing, indicating strong generalization.**

- **Resilience and Robustness:** Demonstrated strong resilience to market volatility, particularly in sectors like Energy, where it maintained a Sharpe ratio of 1.09 during the testing period.

- **Promising Future:** The strategy's flexibility and regime-aware optimisation provide a forward-looking approach for resilient portfolio construction in dynamic financial markets.

Future Work

- **Macroeconomic Features**: Future models can improve regime detection by incorporating macroeconomic indicators (e.g., interest rates, inflation, GDP growth) for better regime transition accuracy.

- **Bayesian Model Averaging**: Integrating multiple HMM variants or priors using Bayesian model averaging to improve regime probabilities and reduce estimation risk.

- **Deep Learning Extensions:** Exploring the use of recurrent neural networks (RNNs) or transformer-based architectures for nonlinear regime inference to capture complex market dependencies.

- **Multi-Objective Optimisation**: Adding multi-objective criteria (e.g., ESG scores, drawdown limits) to better align the strategy with sustainable investing goals.

These are few of the areas where we can probably extend

Thank you!