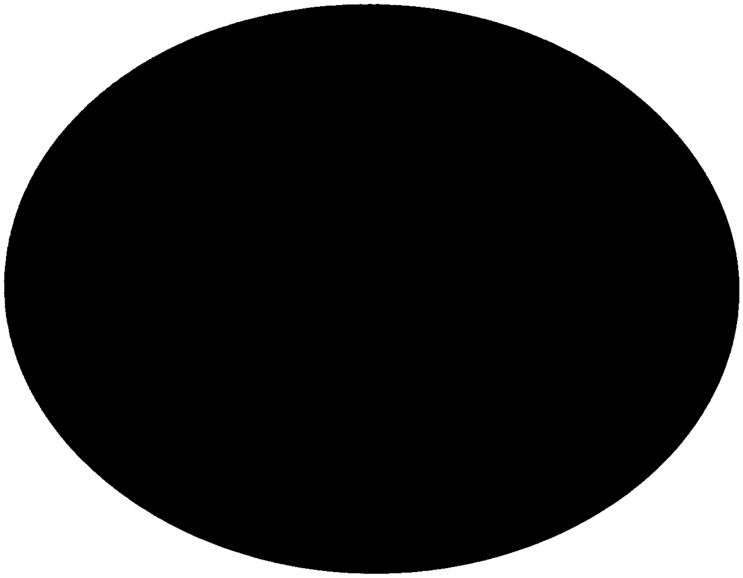

Stationarity of the EEG Series

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An electroencephalogram (EEG) may be considered as a time series measured on a dynamical system that represents brain activity. This subject has attracted various researchers in this field who have noted that the variability of the EEG signals is not noise and presents an attractor [1, 2, 3].

The treatment of EEG series using the approach of nonlinear dynamic systems has opened new possibilities to the dynamic knowledge of the brain. With this approach, new techniques can be applied for quantifying differences in the EEG series with brain activity.

The methods usually employed in nonlinear dynamical analysis are based on distances, and assume the stationarity of the data sets. Distances between points in appropriate embeddings of the data are used to compute a set of metric properties (i.e., correlation dimensions, entropies, Lyapunov exponents, etc.). These quantities are difficult to compute [4], require large data sets and degrade rapidly with additive noise.

Mayer-Kress and Layne [2] used reconstruction techniques on the EEG time series to obtain a phase portrait. These diagrams suggest chaotic attractors with divergent trajectories, which inducates that the EEG series is non-stationary. Thus, the average position of a series defined over one interval will be different in another interval.

Layne, et al. [5], conclude that the EEG series is non-stationary and presents high dimentionality. In such a case, the concepts of attractor and fractal dimension would not be applied because they are asintotic or stationary properties of a dynamical system. However, Babloyantz and Destexhe [6] focus their attention on the fact that this non-stationarity is strictly true for awake states but could be different for states of the sleep cycle or for patients with certain pathologies.

This problem has not been well defined and thus has brought forth a variety of concepts, as expresed by different researchers [3]. Due to the long time segment of the EEG series that is necessary for the nonlinear metric treatment (satisfying all the mathematics hypothesis) the criteria of stationarity is almost impossible to satisfy in practice. Consequently, if the time series are non-stationary, the metric algorithms can not be used, in other way the calculated magnitudes will be wrong.

In this article, we introduce a routine for the analysis of stationarity, which is based on the weak stationarity criteria. From the application of the weak stationarity criteria, we see that only in some cases or in some epochs of a series will the information obtained from these algorithms be realiable. Therefore, we can only obtain dynamic information in those cases. As a complement to this finding, the correlation dimension and the Lyapunov exponents were calculated for the different EEG series analyzed.

Experimental Set Up

Scalp EEGs were acquired using an 8-bit analog-to-digital (A/D) converter that digitalized the signals from 20 monopolar electrodes. The International Federation 10-20 EEG reference system [7] with a bimastoidean reference point was

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used. The data were sampled at 256 Hz, and filtered with a high-pass filter at 0.5 Hz and a low-pass filter at 32 Hz.

The EEGs analyzed were from normal patients and from patients with different pathological diagnoses. The subjects were awake but in a quiet state with eyes closed.

Clinical Data

We analyzed data from 6 patients with ages ranging between 55 to 70 years, selected randomly from a group of 200 submited to the neurophysiology lab for EEG studies.

Three studies were selected from the normal EEG results, denoted as N1, N2, and N3. The other three cases were selected for the abnormal group and were denoted as A1, A2, and A3.

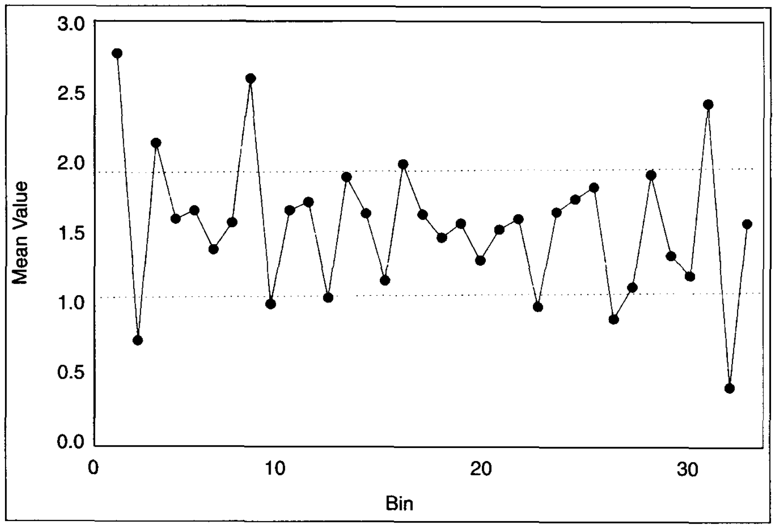
The main abnormalities observed in the A patients were: slow waves, increase in the theta rythm, very low alpha reactivity, and a significant decrease in the alpha/theta ratio.

Artifact Rejection

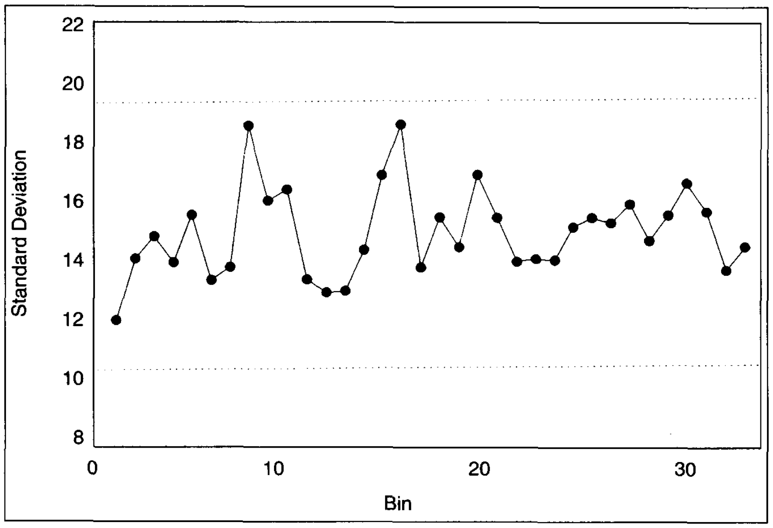
Effective elimination of artifact (head and body movement, perspiration, and low frequency instrument artifacts, under 1 Hz; high frequency artifact including gross muscle potentials, 35-50 Hz; and eye movements, under 3 Hz, in the frontal channels) from the collected data is an essential step in preparation of data for analysis. It is neither sensible nor correct to apply elaborate computer analysis to data contaminated with artifacts.

The traditional practice has been to select visually a "representative" segment of artifact-free data for computer analysis. These procedures obviously introduce an element of subjective bias in data selection.

In this work, we analyzed the data in artifact free segments, without previous filtering [8], and without interruption. We used only the temporal series of the central electrodes, as being least contaminated with artifact. However, the presence of artifact produces significant distortion and does not allow the dynamical analysis of large portions of the series. In some cases, owing to the contamination of a few seconds, one would be tempted to decimate the entire series. We must then verify that this decimation does not modify the information contained in the series. A way to evaluate this possibility is by means of the observation of the changes in the correlation dimension when we decimate the series. We realized this test for EEG sections which satisfy the requirements for dynamic analysis given below. In consequence, if more than 5 percent of the data must be discarded, we reject all of the series.



1. Mean values for bins of 1000 data points for EEG series N1.



2. Standard deviation for bins of 1000 data points for EEG series N1.

Stationarity Analysis

The terms "non-stationarity" or "time-varying" mean that characteristic of a time series, such as mean, variance, and spectral characteristics, change with time. Statistical tests of stationarity have revealed a variety of results, depending on conditions, with estimates of the amount of time during which the EEG is stationary ranging from several seconds to several minutes [9-11].

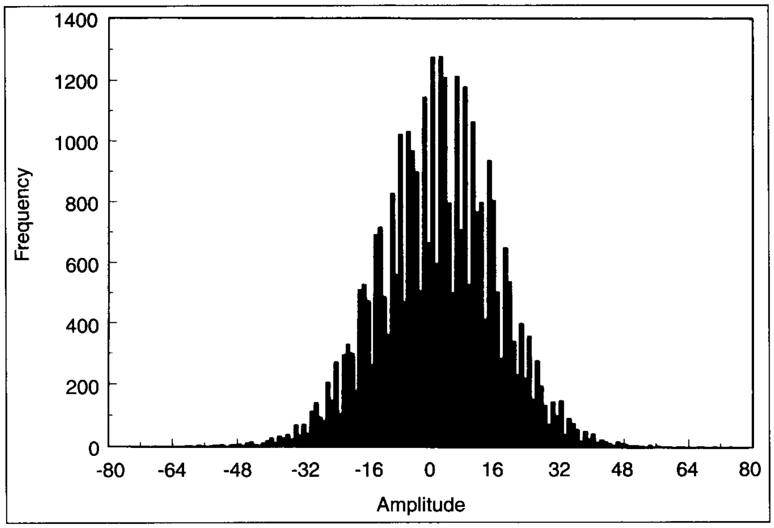
However, as a practical matter, whether or not the same data segment is considered stationary depends on the problem being studied, the type of analysis being performed, and the measured (features) used to characterize the data.

If the probability distribution f(x) associated with the measurements is normal or Gaussian, it can be completely characterized by its mean (m) and its variance (σ^2). So, if we have a multivariate normal distribution, the existence of a fixed mean and variance should be enough to ensure the stationarity of a Gaussian process.

A less restrictive requirement, called "weak stationarity" of order n, is that the

Table 1: Total length N_0 of the EEG records analized and work length N_d employed for calculations. Embedding dimension D_e, Correlation dimension D_c and first Lyapunov exponent λ_1, respectively.

EEG	N_0	N_d	D_e	D_c	λ_1
N1	34000	15000	13	5.5-6.0	0.26
N2	32000	13000	13	5.0-6.0	0.23
N1	35000	14000	13	4.5-5.5	0.25
A1	35000	15000	13	4.5-5.5	0.28
A2	34000	14000	13	5.0-5.5	0.26
A3	36000	15000	13	4.5-5.0	0.27



3. Histogram for EEG series N1, in the selected zone (see text).

moments up to some order, n, depend only on time differences. Then the second order stationarity (n = 2), plus an assumption of normality, is enough to produce complete stationarity [12].

Then, in order to assure the stationarity of the EEG zones to be used for the dynamical analysis, we use the following procedure, based in the weak stationarity criteria defined above.

We have chosen to take bins with a length of 1000 data points (about 4 s of digitalized EEG signal). This number is large enough to give reliable statistics in the mean and variance calculations. We have also verified that sometimes, with less data, the bins do not have normal distribution. The selection of the bin length must be verified anyway for all the series to be analyzed.

a) The mean and variance were evaluated for each bin, and we looked for zones where these values did not change significatively for at least five consecutive bins.

b) We constructed the corresponding histogram for this zone, and verified the normality of the obtained distribution.

c) We compared the statistical parameters of the total time series with those of the selected zone that satisfied the requirements the two steps above. If the differences between these statistical parameters were significant, we rejected the corresponding zone. This procedure was followed to ensure the selected zone corresponded to the dynamics of all the series.

In Figs. 1 and 2 we show the mean and variance for the time series denoted by N1. From these figures, we conclude that the weak stationariry criteria is satisfied between bins 9 and 26. Note that in this case, the variance does not give any restriction about selected bins to be used in the forthcoming analysis.

In Fig. 3, we show the histograms realized for the selected portion of N1, which satisfies the weak stationary criteria; observing that, in fact, these histograms indeed have a normal distribution.

Babloyantz [13] and other authors employed the graphical tool introduced by Eckman and Ruelle [14], called a "recurrence plot," as an alternative method for the diagnosis of the stationarity. From our point of view, the recurrence plot is a very useful tool for visualizing the homogeneity of the attractor, but is a subjective tool for the analysis of stationarity. However, recurrence plots can be used to find an optimal value for the embedding dimension that must be used in the algorithm for the calculating the correlation dimension, as we will explain below. In Table 1, we present the total length, as well as the work lengths, for the EEG series analyzed in the present work.

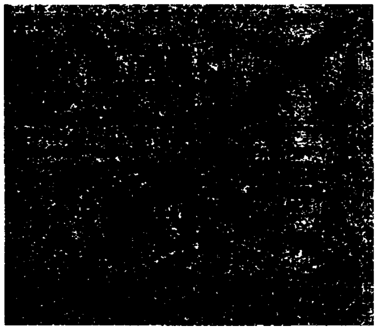
In order to illustrate the use of the described stationarity criteria, we employed the EEG series, calculating the zones that could give realiable results, and in those zones we calculated the correlation dimension and the Lyapunov exponents.

Dynamical Analysis

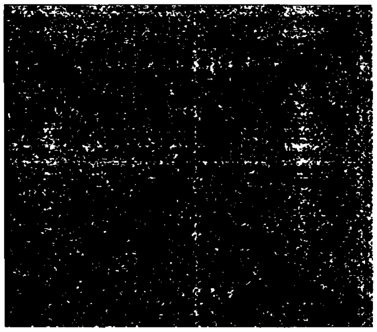
The dynamics behavior of most deterministic systems is generally described by a finite set of differential equations. These equations can be constructed when precise knowledge is available about the elements of the system and about the type of interactions that take place between these elements. When the equations can be solved (either analytically or by numerical methods) the solution describes the systems behavior as a function of time under the conditions defined by the value of the parameters.

The state of such a system at any point in time can be completely characterized by the set of observables $x_1, \dots, x_n$. Furthermore, the evolution of this system is determined by $x_1(t), \dots, x_n(t)$, that is to say, by a trace in an n-dimensional space spanned by $x_1, \dots, x_n$ (where n is the number of degree of freedom of the system).

Such a system is called deterministic because its evolution is always defined by its current state. The laws governing this evolution are given by the equations (set of differential equations) which express the rate of change of the state in terms of the observables.



4. Recurrence plot for EEG series N1 in the selected zone, with embedding dimension $D_e = 6$ and time delay $\tau = 5$.



5. Recurrence plot for EEG series N1 in the selected zone, with embedding dimension $D_e = 12$ and time delay $\tau = 5$.

Attractors

One gains a good understanding of the global behavior of the system by plotting the evolution of ($x_1(t), \ldots, x_n(t)$) in n-dimensional space. The set of all possible states of the system is called the state space. In general, the state space consist of a curved hyper surface (manifold). The dynamics of a system is represented as a vector field, showing the direction of the evolution. An integral curve of a vector field is called a trajectory.

The set of trajectories that represents the behavior of a system in dynamic equilibrium (stationarity), after transients have died out is called an attractor of the system. The basin of attraction for an attractor is that part of the phase space formed by trajectories that have the attractor as their limit set. Some different concepts have been defined to describe properties of basins of attractors, attractors, and other possible limit sets (not corresponding with an equilibrium) in a state space [15-19].

The insight gained by the concept of deterministic chaos for the EEG is that this seemingly disordered process may be governed by a relatively few simple laws, which could be determined.

Correlation dimension

An upper bound for the "complexity" of the system is the smallest (topological) dimension, n, of the space in which the attractor can be embedded. This topological dimension (embedding dimension), however, is in general only a very rough description of the "amount of space" an attractor occupies. Several types of dimensions have been introduced in order to give a more precise description [19]. The correlation dimension has become the most widely used measure in the literature. It can be computed on basis of experimental data by the method described by Grassberger and Procaccia [20-21].

When a dynamics system does not approach either equilibrium or a periodic state, the correlation dimension can be used as a measure by means of which it is possible to distinguish whether the system's apparently ``random" behavior can be ascribed to the existence of a strange attractor or whether it may be due to (external) noise.

Whether a system behaves periodically or not can be determined by applying Fourier analysis. However, if the system does not behave periodically, Fourier analysis cannot identify the cause of the irregular behavior. By calculating the correlation dimension, it is possible to distinguish between irregular, random, motion on an attractor blurred by noise, from also irregular, but strictly deterministic, motion on a strange attractor.

The algorithms for calculation of the correlation dimension assumes stationarity of the time series. In particular, the algorithm of Grassberger and Procaccia assumed that the time series is noise free and needs an estimation of the embedding dimension. To estimate the embedding dimension, we can consider the Takens criteria [22], which tells that the embedding dimension must be at least $2\nu + 1$, where ν is the correlation dimension.

If the attactor is in a space of embedding dimension larger than the real space, the distances between its points is not altered, and the points which are first neighbors will continue to be such. A way to see this kind of invariance (which uses the property mentioned above) is by means of the recurrence plot, calculated for successive embedding dimensions until we reach an invariant pattern.

As an example of this procedure, in Figs. 4 and 5, we show the corresponding recurrence plots of the selected part of time series N1, with embedding dimensions of 6 and 12, respectively. For embedding dimensions greter than 12, we observed that the pattern of recurrence plots do not present significative changes. Then, by Takens criteria, we say that the corresponding correlation dimension for this time series is less than 7.

As previously mentioned, this algorithm for the calculation of correlation dimension assumes stationarity and noise free time series. Since the EEG is contamined by noise, this method is not adequate for the treatment of this type of time series. So, it is convenient to calculate the correlation dimension by using the method called singular value decomposition (SVD), developed by Albano, et al., [23]. This method combines the Broomhead and King [24] decomposition in singular values method with the Grassberger and Procaccia algorithm. The SVD has the advantage of reducing the embedding dimension of the system and also of eliminating much of the noise.

With this method, we have evaluated the corresponding correlation dimensions for the EEG time series, after the criterion of weak stationarity was applied for selecting the data to be used. The values obtained are given in Table 1.

Lyapunov exponents

One important way to describe attractors is to use the concept of Lyapunov exponents [19]. These exponents measure the extent at which nearby points on an attractor diverge or converge with respect to each other while moving along any trajectory of the attractor.

If the biggest Lyapunov exponent is grater than zero, we are in presence of deterministic chaos. If the exponent is less than or equal to zero, we are in presence of a periodic or quasiperiodic motion, respectively. In Table 1, we present the largest Lyapunov exponents for the EEG signals analyzed. For the determination of these exponents, the method of Eckman and Ruelle [25] was employed. The embedding dimension, as in the previous case, was estimated using the recurrence plots.

We must emphasize that these indicators (correlation dimension, Lyapunov exponent) are defined only if the series are stationary or weakly stationary.

Conclusions

In this work, we have presented a practical and objective method for the determination of the stationarity of a EEG series. This method is based on the criteria of weak stationarity. The condition of stationarity is a requirement necessary to allow the metric algorithms of the nonlinear dynamics to be used as a tool in of the analysis of EEG series. In this way, we may obtain reliable indicators of some aspects of brain dynamics. As an EEG record can show different states of stationarity, we conclude that the only ones that could be characterized with metric methods are those that present a behavior comparable at least with the weak stationarity criteria.

Acknowledgment

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