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Armstrong's axioms

Armstrong's axioms are a set of references (or, more precisely, inference rules) used to infer all the functional dependencies on a relational database. They were developed by William W. Armstrong in his 1974 paper.[1] The axioms are sound in generating only functional dependencies in the closure of a set of functional dependencies (denoted as F^+) when applied to that set (denoted as F). They are also complete in that repeated application of these rules will generate all functional dependencies in the closure F^+.

More formally, let $\langle R(U), F\rangle$ denote a relational scheme over the set of attributes U with a set of functional dependencies F. We say that a functional dependency f is logically implied by F, and denote it with $F \models f$ if and only if for every instance r of R that satisfies the functional dependencies in F, r also satisfies f. We denote by F^+ the set of all functional dependencies that are logically implied by F.

Furthermore, with respect to a set of inference rules A, we say that a functional dependency f is derivable from the functional dependencies in F by the set of inference rules A, and we denote it by $F \vdash_A f$ if and only if f is obtainable by means of repeatedly applying the inference rules in A to functional dependencies in F. We denote by F^*_A the set of all functional dependencies that are derivable from F by inference rules in A.

Then, a set of inference rules A is sound if and only if the following holds:

$$F^*_A \subseteq F^+$$

that is to say, we cannot derive by means of A functional dependencies that are not logically implied by F. The set of inference rules A is said to be complete if the following holds:

$$F^+ \subseteq F^*_A$$

more simply put, we are able to derive by A all the functional dependencies that are logically implied by F.

Axioms (primary rules)

Let $R(U)$ be a relation scheme over the set of attributes U. Henceforth we will denote by letters X, Y, Z any subset of U and, for short, the union of two sets of attributes X and Y by XY instead of the usual $X \cup Y$; this notation is rather standard in database theory when dealing with sets of attributes.

Axiom of reflexivity

If X is a set of attributes and Y is a subset of X, then X holds Y. Hereby, X holds Y [$X \rightarrow Y$] means that X functionally determines Y.

If $Y \subseteq X$ then $X \to Y$.

Axiom of augmentation

If X holds Y and Z is a set of attributes, then XZ holds YZ. It means that attribute in dependencies does not change the basic dependencies.

If $X \to Y$, then $XZ \to YZ$ for any Z.

Axiom of transitivity

If X holds Y and Y holds Z, then X holds Z.

If $X \to Y$ and $Y \to Z$, then $X \to Z$.

Additional rules (Secondary Rules)

These rules can be derived from the above axioms.

Decomposition

If $X \to YZ$ then $X \to Y$ and $X \to Z$.

Proof

1. $X \to YZ$ (Given)
2. $YZ \to Y$ (Reflexivity)
3. $X \to Y$ (Transitivity of 1 & 2)

Composition

If $X \to Y$ and $A \to B$ then $XA \to YB$.

Proof

1. $X \to Y$ (Given)
2. $A \to B$ (Given)
3. $XA \to YA$ (Augmentation of 1 & A)
4. $XA \to Y$ (Decomposition of 3)
5. $XA \to XB$ (Augmentation 2 & X)
6. $XA \to B$ (Decomposition of 5)
7. $XA \to YB$ (Union 4 & 6)

Union (Notation)

If $X \to Y$ and $X \to Z$ then $X \to YZ$.

Proof

1. $X \to Y$ (Given)
2. $X \to Z$ (Given)
3. $X \to XZ$ (Augmentation of 2 & X)
4. $XZ \to YZ$ (Augmentation of 1 & Z)
5. $X \to YZ$ (Transitivity of 3 and 4)

Pseudo transitivity

If $X \to Y$ and $YZ \to W$ then $XZ \to W$.

Proof

1. $X \to Y$ (Given)
2. $YZ \to W$ (Given)
3. $XZ \to YZ$ (Augmentation of 1 & Z)
4. $XZ \to W$ (Transitivity of 3 and 2)

Self determination

$I \to I$ for any I. This follows directly from the axiom of reflexivity.

Extensivity

The following property is a special case of augmentation when $Z = X$.

> If $X \to Y$, then $X \to XY$.

Extensivity can replace augmentation as axiom in the sense that augmentation can be proved from extensivity together with the other axioms.

Proof

1. $XZ \to X$ (Reflexivity)
2. $X \to Y$ (Given)
3. $XZ \to Y$ (Transitivity of 1 & 2)
4. $XZ \to XYZ$ (Extensivity of 3)
5. $XYZ \to YZ$ (Reflexivity)
6. $XZ \to YZ$ (Transitivity of 4 & 5)

Armstrong relation

Given a set of functional dependencies F, an **Armstrong relation** is a relation which satisfies all the functional dependencies in the closure F^+ and only those dependencies. Unfortunately, the minimum-size Armstrong relation for a given set of dependencies can have a size which is an exponential function of the number of attributes in the dependencies considered.[2]

References

1. William Ward Armstrong: *Dependency Structures of Data Base Relationships (https://web.archive.org/web/20180126091352if_/https://ipfs.io/ipfs/QmWYWTGUZyTm2iRFTZY2pTr2x1vWkDiJr2CBp2PGVpSVSv)*, page 580-583. IFIP Congress, 1974.
2. Beeri, C.; Dowd, M.; Fagin, R.; Statman, R. (1984). "On the Structure of Armstrong Relations for Functional Dependencies" (https://web.archive.org/web/20180723025708/https://researcher.watson.ibm.com/researcher/files/us-fagin/jacm84.pdf) (PDF). *Journal of the ACM*. **31**: 30–46. CiteSeerX 10.1.1.68.9320 (https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.68.9320). doi:10.1145/2422.322414 (https://doi.org/10.1145%2F2422.322414). Archived from the original (https://researcher.watson.ibm.com/researcher/files/us-fagin/jacm84.pdf) (PDF) on 2018-07-23.

External links

- UMBC CMSC 461 Spring '99 (http://www.cs.umbc.edu/courses/461/current/burt/lectures/lec14/)
- CS345 Lecture Notes from Stanford University (http://www-db.stanford.edu/~ullman/cs345notes/slides01-1.ps)