Homework 1
Binomial Tree

I. Binomial Model Derivation

(20%) In the binomial model, suppose that the initial stock price is S_0, and the life of the option is T. S_0 can either move up from S_0 to a new level, $S_0 u$, where $u > 1$, or down to a new level, $S_0 d$, where $0 < d < 1$. Suppose the payoff from option is f_u in the up state, and is f_d in the down state. Denote the risk-free rate by *r*.

Please construct a riskless portfolio in a one-step tree and show **in detail** that $f = e^{-rT}[pf_u + (1-p)f_d]$ where $p = \dfrac{e^{rT} - d}{u - d}$

II. Binomial Trees in Practice

Consider a non-dividend-paying stock with current stock price S_0 =\$50, volatility σ=0.3, strike price K=\$52, time to maturity T=2 years, interest rate r=5%.

Please use binomial model to price European put options. You may refer to the materials in Section 21.1 of the textbook. Consider the following three alternative settings of time steps: Δt =1 month (12*T steps); 1 week (52*T steps); and 1 day (252*T steps).

(a) (10%) First compute the up step size *u*, the down step size *d*, and the probability of up move *p* under these three settings.
(b) (40%) Use binomial model to compute the put option prices under these three settings. Report your results and compare them with that of the Black-Scholes formula. Briefly explain your findings.
(c) (10%) Change the number of time steps from 1 to 2 to 3 all the way to 252. Plot your results as well as the Black-Scholes closed form solution. Briefly explain your findings.
(d) (20%) Modify your program in (b) to compute the American put option values. Report your result.

Bonus: (20%) For 6, 12, and 52 time steps, compute the terminal stock prices as well as their corresponding probabilities. Plot the terminal stock price distribution. Briefly explain your findings.

Matlab function and syntax:

1. zeros(): to create an matrix of all zeros. e.g. S = zeros(m,n)
2. sqrt(): square root
3. exp(): exponential function
4. max(): max function
5. for loop

    e.g.

```
for j=1:1:10
        statement
end
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*You have to submit the results of your homework in a word (or pdf) file as well as **programs by e3**. Your computer program is part of this assignment. You can use either C++ or Matlab for programming.