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LESSON-18

MATHEMATICAL LOGIC

1. MATHEMATICAL LOGIC

The dictionary meaning of the word 'Logic' is "the science of reasoning". Logic is the study and analysis of the nature of valid arguments.

2. STATEMENTS OR PROPOSITIONS

1. DEFINITION

A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

2. NEGATION OF A STATEMENT

The denial of a statement is called the negation of the statement.

Let us consider the statement:

P : New Delhi is a city

The negation of this statement is

It is false that New Delhi is a city

Definition:

If p is a statement, then the negation of p is also a statement and is denoted by $\sim p$ and read as 'not p'.

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Example: **Write the negation of the following statements.**

    (i) Both the diagonals of a rectangle have the same length.

    (ii) $\sqrt{7}$ is rational

Solution: (i) This statement says that in a rectangle, both the diagonals have the same length. This means that if you take any rectangle, then both the diagonals have the same length. The negation of this statement is

    It is false that both the diagonals in a rectangle have the same length.

This means the statement

    There is atleast one rectangle whose both diagonals do not have the same length.

    (ii) The negation of this statement is

    It is not the case that $\sqrt{7}$ is rational.

This can also be rewritten as

    $\sqrt{7}$ is not rational.

3. COMPOUND STATEMENTS

If a statement is combination of two or more simple statements, then it is said to be a compound statement or a compound proposition.

3. BASIC LOGICAL CONNECTIVES OR LOGICAL OPERATORS

Connective	Symbol	Nature of compound statement formed by using the connective
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
If ……then	$\Rightarrow$ or $\rightarrow$	Implication or conditional
If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	Equivalence or bi-conditional
not	$\sim$ or $\neg$	Negation

1. THE WORD "AND"

(i) A compound statement is true only if both statements connected with 'AND' are true. Otherwise it is false.

(ii)   ~ (*p* and *q*) = ~*p* or ~*q*

2.   THE WORD "OR"

(i)   A compound statement is true if at least one of the statements connected with 'Or' is true.

(ii)   ~ (*p* or *q*) = ~*p* and ~*q*

3.   IMPLICATIONS

The sentence "if *p* then *q*" says that in the event if *p* is true, then *q* must be true.

One of the most important facts about the sentence "if *p* then *q*" is that it does not say any thing (or phaces no demand) on *q* when *p* is false.

The statement "if *p* then *q*" statements does not imply that *p* happens.

Then, if *p* then *q* is the same as the following:

1.   '*p* implies *q*' is denoted by $p \Rightarrow q$. The symbol $\Rightarrow$ stands for implies.

This says that 'A number is a multiple of 9 implies that it is a multiple of 3'.

2.   *p* is a sufficient condition for *q*.

This says that 'Knowing that a number as a multiple of 9 is sufficient to conclude that it is a multiple of 3'.

3.   *p* only if *q*.

This says that 'A number is a multiple of 9 only if it is a multiple of 3'.

4.   *q* is a necessary condition for *p*.

This says that 'When a number is a multiple of 9, it is necessarily a multiple of 3'.

5.   ~*q* implies ~*p*.

This says that 'If a number is not a multiple of 3, then it is not a multiple of 9'.

6.   If *p* and *q* are both true, then $p \Rightarrow q$ is also true.

If *p* is false, *q* is true then $p \Rightarrow q$ is true.

If *p* is true, *q* is false then $p \Rightarrow q$ is false.

If *p* is false, *q* is false then $p \Rightarrow q$ is true.

4.   CONTRAPOSITIVE AND CONVERSE

Contrapositive and converse are certain other statements which can be formed from a given statement with "if-then".

Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

Converse of $p \Rightarrow q$ is $q \Rightarrow p$

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Example: **Write the contrapositive of the following statement:**

(i) If a number is divisible by 9, then it is divisible by 3.

(ii) If you are born in India, then you are a citizen of India.

(iii) If a triangle is equilateral, it is isosceles.

Solution: The contrapositive of the these statements are

(i) If a number is not divisible by 3, it is not divisible by 9.

(ii) If you are not a citizen of India, then you were not born in India.

(iii) If a triangle is not isosceles, then it is not equilateral.

5. CONDITIONAL AND BI-CONDITIONAL STATEMENTS

In Mathematics we come across many statements of the form "if p then q" and "p if and only if q" such statements are called conditional statements. In this section, we shall discuss about such statements.

Example: **Let p be the statement "He is rich" and let q denote "He is happy". Write each of the following statements in symbolic form by using p and q.**

(i) If he is rich, then he is happy.

(ii) It is necessary to be poor in order to be happy.

(iii) To be poor is to be unhappy.

Solution: (i) $p \Rightarrow q$ (ii) $q \Rightarrow \sim p$ (iii) $\sim p \Leftrightarrow \sim q$.

Example: **Let p represent the statement "It is raining"; let q represent "the game is cancelled" and let r represent "Monu is sad". Then, express each of the following in words:**

(i) $p \Rightarrow q$ (ii) $q \Rightarrow r$ (iii) $r \Rightarrow p$ (iv) $q \Rightarrow p$

Solution: (i) $p \Rightarrow q$: If it is raining, then the game is cancelled.

(ii) $q \Rightarrow r$: If the game is cancelled, then Monu is sad.

(iii) $r \Rightarrow p$: If Monu is sad, then it is raining.

(iv) $q \Rightarrow p$: If the game is cancelled, then it is raining.

Example: **Write down the statement 'A complex number is a real number' in the form of a compound statement.**

Solution: Let p be the statement "x is a complex number" and let q be the statement "x is a real number". Then, the desired compound statement is $p \Rightarrow q$.

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