SURFACE TENSION

Definition

Surface tension is a property of a liquid by which the free surface of a liquid behaves like a stretched elastic membrane, having contractive tendency. The surface tension is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of the liquid at rest. The direction of the force is at right angles to the line and tangential to the liquid surface.

$$\text{Surface tension} = \frac{Force}{Length}$$

$$T = \frac{F}{l}$$

Note: [To denote surface tension σ or T or γ (gamma) is used]
S.I. unit Nm^{-1} C.G.S. unit dyne cm^{-1}.
Dimensional formula of surface tension is $MLT^{-2}\,L^{-1} = [MT^{-2}]$
The surface tension acts perpendicular to the line AB and tangential to the liquid surface.

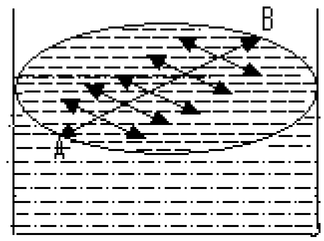
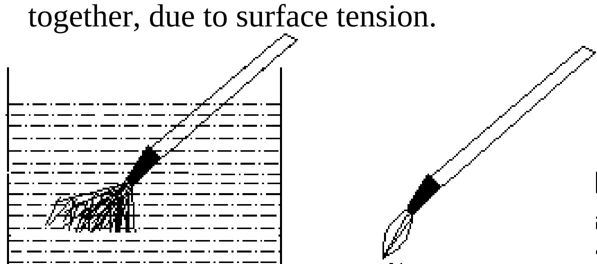
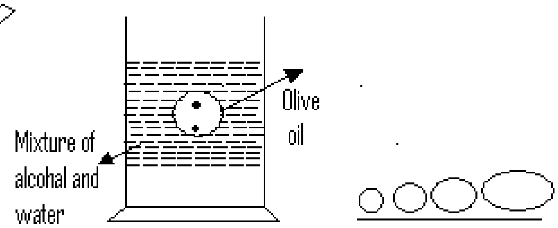


Illustration of surface Tension

(i) Dip an ordinary hair brush or shaving brush into water. When the brush is inside water its hair will be seen separately. But when taken out, the hairs are seen to cling together, due to surface tension.



(a) (b)

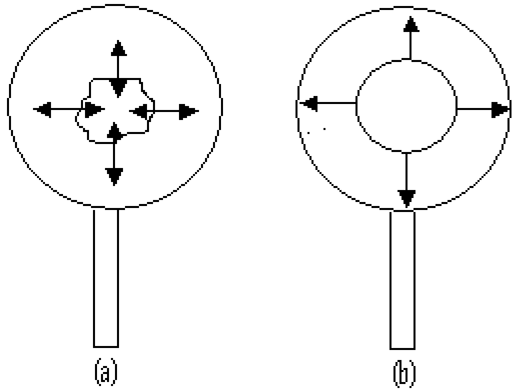


(a) (b)

(ii) In the absence of gravitational force a drop will take spherical shape. A mixture of alcohol and water if density equal to olive oil was taken in a beaker. A little oil was poured into the beaker, which sink into the mixture and remained as a spherical drop as shown.

(iii) If some mercury is poured on a clean glass plate, small drops are seen to be spherical, whereas large ones are ellipitcal or cylindrical. This is due to surface tension.

(v) A metallic ring is dipped in soap solution. A closed loop of cotton thread is gently placed over the film, as shown fig. (a) With a hot needle, the soap film within the



loop is touched and the film is broken. The thread experiences a force radially outwards and parallel to the film. The loop assumes a circular form, as shown fig. (b) This is because, for a given perimeter, a circle encloses the maximum area, and hence the film surroundings the loop tends to assume the least area.

(vi) Surface tension of oil is less than that of water so when it is sprayed it spreads and forms a thin film above water. This prevents mosquitos to breed on the surface of oily water.

Cohesive and Adhesive Force

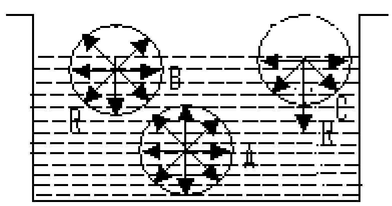
Force of attraction between molecules of the same body is called **cohesive force.** The definite shape and rigidity of a solid body is due to cohesion. It is strongest in solids.
Force of attraction between different kinds of molecules is called **adhesive force.** This property is called *adhesion*. A drop of water sticks to a due to adhesion.

Molecular Range and Sphere of Influence

The maximum distance upto which the cohesive force between two molecules can act is called their molecular range. A sphere drawn around a molecule as centre and with a radius equal to the molecular rage is called the sphere of influence of the molecule.

Surface Tension based on Molecular Theory

Consider three molecules A,B and C of a liquid. The sphere of influence of A is a fully inside the liquid. So it is attracted equally in all direction by other molecules lying within sphere of influence. So resultant cohesive on it is zero.

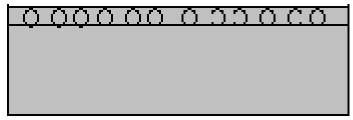


The sphere of influence of the molecule B lies partly outside. This molecule experiences less force upward and more downward by the molecules in its sphere of influence. So there is a net downward force (R).
For molecule C, half of the sphere of influence lies above surface, Thus the resultant downward force T in this case is maximum. Due to this net inward force on the molecules lying on the surface, the surface experiences tension called surface tension.
To bring a molecule to the surface of the liquid, work has to be done against the downward force. Hence, the molecules on the liquid surface have greater potential energy. For a system to be stable, the potential energy must be minimum. **A liquid will**

have minimum potential energy, if the number of molecules on the liquid surface is the least. This will happen if the surface area is minimum. Thus, a liquid tries to have minimum surface area.

Surface Film

If a plane is drawn parallel to the free surface layer inside the liquid and at a distance equal to the molecular range from it, the liquid lying in between the free surface and this plane is called the surface film.



Surface Energy

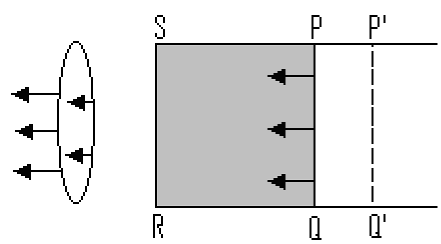
The potential energy per unit area of the surface film is called its surface energy. It is also equal to the work done to produce a fresh surface of liquid film of unit area.

$$Surface\ energy = \frac{Work\ done}{area} \quad S.I\ unit\ is\ Jm^{-2}$$

Surface energy is numerically equal to surface tension (means numbers are equal, but units are different).

Relation between surface tension and surface energy

Consider a rectangular metal frame PQRS . The side PQ is movable. Dip the frame in soap solution. A soap film is formed. Due to surface tension PQ is pulled towards SR. To keep the wire in equilibrium a force has to be applied in the opposite direction. The force to be applied on PQ = 2l x T. Here 2 appears because there are two surfaces. Suppose PQ is pulled through a small distance x to the position P′ Q′.



Then work done = force x displacement

= $2l$ x T x x

Area created = $2lx$

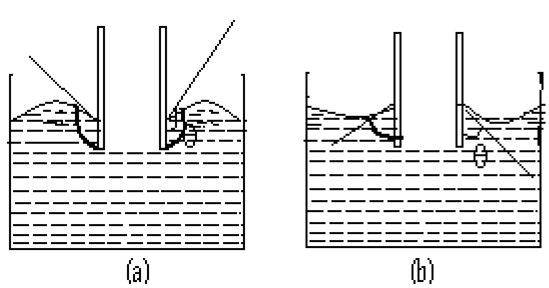
Surface energy =

$$\frac{Work\ done}{area} = \frac{2l\,Tx}{2lx} = T.$$

Surface energy is numerically equal to surface tension.

Angle of Contact

The angle between the tangent to the liquid surface and solid inside the liquid is called the angle of contact for that pair of solid and liquid. Liquid which wet the glass, the angle of contact is acute and for liquid which does not wet the glass it is obtuse.



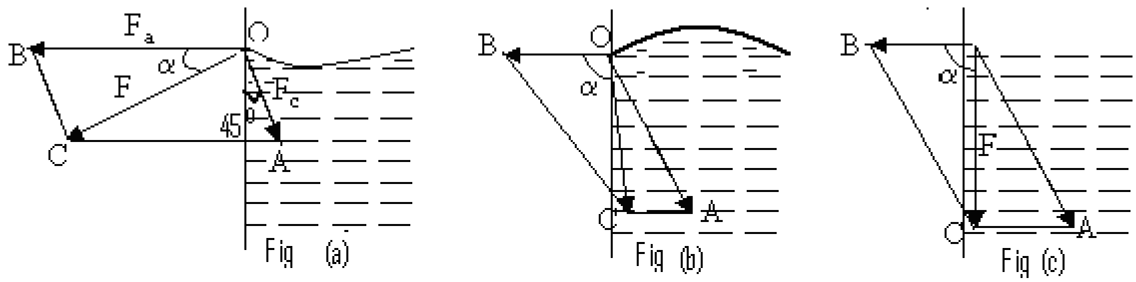
For pure water and clean glass, the angle of contact is 0^o. For ordinary water and glass it is about 8^o. For mercury and glass it is about 140^o. *The angle of contact depends on (i) the nature of the liquid (ii) the solid with which it is in contact (iii) the purity of solid and liquid (iv) to a certain extent on the nature of the vapour.*

Shape of Liquid Meniscus in a Tube

The shape of the liquids meniscus in a glass tube depends upon adhesive force F_a and cohesive force F_c.

Consider a liquid molecule O near the glass wall, as shown in fig. (a) This molecule is acted upon by a net cohesive force F_c.

Consider a liquid molecule O near the glass wall, as shown in fig. This molecule is acted upon by a net cohesive force F_c acting inwards along OA within the liquid. This molecule is acted upon by a net adhesive force F_a acting perpendicular to the glass surface, directed along OB as shown.



The cohesive force represented by OA makes an angle 45^o to the vertical wall of the tube. The two force F_a and F_c are inclined to each at an angle of 135^o. The resultant force will depend on the relative magnitudes of the two forces F_a and F_c. The direction of the resultant is given by α, where α is the angle between the resultant force and F_a.

$$\tan\alpha = \frac{F_c \sin 135^o}{F_a + F_c \cos 135^o}$$

$$\tan\alpha = \frac{F_c}{\sqrt{2}F_a - F_c}$$

Now there are three cases

(i) When $\sqrt{2}\ F_a > F_c$, tan α is positive. So the angle α is acute and the resultant OC is directed outside the liquid as shown in fig.(a). The liquid is at rest. So F must be perpendicular to be liquid surface. The force F will become perpendicular to the liquid surface only if the surface assumes a concave shape upward as shown in fig.(a)

(ii) When $\sqrt{2}\ F_a < F_c$, tan α is negative. So the angle α is obtuse . The resultant OC lies inside the liquid. The force F will become perpendicular to the liquid surface only if the surface assumes a convex shape upward as shown in fig.(b)

Consider a mercy molecule at O, as shown in fig. In this case the adhesive force $\sqrt{2}\ F_a$ is less than the cohesive force F_c. Hence the resultant force F lies within the mercury column. To make F perpendicular to the liquid surface, the assumes a

(iii) When $\sqrt{2}\ F_a = F_c$, tan α is infinity or α is 90^o. If $\sqrt{2}\ F_a = F_c$, The resultant force acts vertically downwards and parallel to the wall of the container. The force F will become perpendicular to the liquid surface. Hence liquid surface will horizontal.

Pressure-difference across a Liquid Surface

Suppose the free liquid surface is concave, as shown in fig(a). The surface tension acts tangential to the surface. It can be resolved into horizontal and vertical components add together and there is resultant force normal to the surface which is directed towards the centre of the curvature of the surface fig (a)

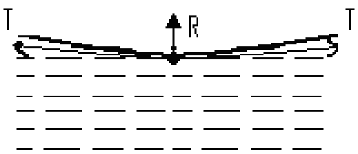


Fig (a)

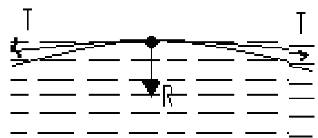


Fig (b)

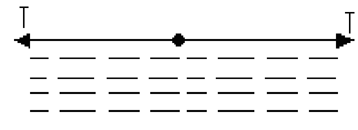


Fig (c)

In the case of a convex surface, as in fig(b), also there is a resultant force due to surface tension acting on a molecule on the surface of the liquid directed into the liquid towards the centre of curvature of the surface.

Thus, if the surface of a liquid is curved, there is a pressure towards the centre of the surface, due to tension. This pressure is greater on the concave side than on the convex side. Due to this, the liquid surface tries to contract. But liquid surface is in equilibrium. Hence, there must be an excess pressure on the concave side of the liquid surface so that the surface does not contract. This excess pressure depends on the surface tension and the radius of curvature of the meniscus.

Excess Pressure Inside a Drop

Consider a liquid drop of radius r and surface tension T. Due to its spherical shape the inside pressure will be greater than that of the outside. Let the outside pressure be P_o and inside pressure be P_i.

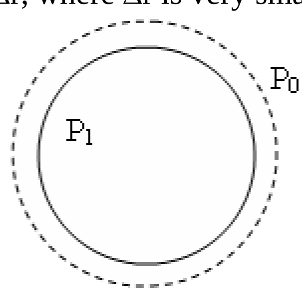
Let the radius of the drop increase from r to r + Δr, where Δr is very small.

Initial surface area = $4\pi r^2$.

Final surface = $4\pi(r^2 + \Delta r)^2$

$= 4\pi(r^2 + 2r\Delta r + \Delta r^2)$

$= 4\pi r^2 + 8\pi r\Delta r$.



Δr^2 is neglected as it is small.

Increase in surface area = $4\pi r^2 + 8\pi r\Delta r - 4\pi r^2 = 8\pi r\Delta r$

From the definition of surface tension, work done to increase the surface area by $8\pi r\Delta r$ is $dW = 8\pi r\Delta r \times T$.

This work done is also equal to excess force times the distance moved.

Excess force = excess pressure *x* area

$= (P_i – P_o)\, 4\pi r^2$

The distance moved is Δr.

So work done $dW = (P_i – P_o)\, 4\pi r^2 \times \Delta r$

$(P_i – P_o)\, 4\pi r^2 \times \Delta r = 8\pi r\Delta rT$

$(P_i - P_o) = \frac{8\pi r T}{4\pi r^2} = \frac{2T}{r}$

This is called Laplace's Law for a spherical membrance.

In the case of a bubble there are two free surfaces in contact . Hence the total increase in area is 2 x 8πrΔr x T

Work done, dW = 16πrΔr x T

Work done due to excess pressure = $(P_i - P_o)\ 4\pi r^2 \times \Delta r$

From above two equations.

$(P_i - P_o)\ 4\pi r^2 \Delta r = 16\pi r \Delta r \times T$

$P_i - P_o = \frac{4T}{r}$

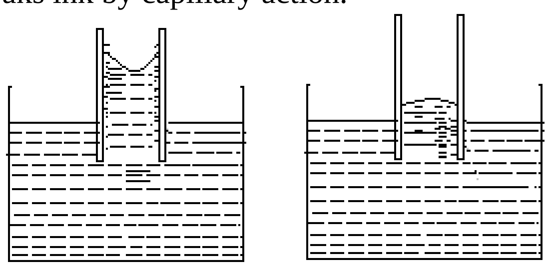
The excess pressure inside a spherical bubble is $\frac{4T}{r}$.

Capillarity

A tube having very small diameter is called a capillary tube. If a capillary tube in a liquid which wets the glass, the liquid rises up above the level outside. This is called capillary rise. If the tube is dipped in a liquid like mercury which does not wet the glass the liquid is depressed below the level outside. **This is called capillary depression.**

For 1. Kerosene oil rise in lamp wick due to capillarity.

2. Blotting paper soaks ink by capillary action.



(a) (b)

Expression for Capillary Rise

Consider a capillary tube of radius r dipped vertically in a liquid of density ρ and surface tension T acts. The angle of contact is θ which is acute angle,so the liquid wets the glass tube. The surface tension T acts along the tangent at the point of contact to the liquid meniscus whose radius is R. The pressure Pi at A is greater than the pressure P_o at B.

So the liquid will rise in the tube and reach a height h, above the horizontal surface outside the tube until the difference in pressure balances the hydrostatic pressure due to height h of the liquid column.

At equilibrium $(P_i - P_o) = h\rho g$ … (a)

But $(P_i - P_o) = \frac{2T}{R}$

$\cos = \frac{r}{R}, i.e., \frac{\cos\theta}{r} = \frac{1}{R}$

$i.e., (P_i - P_o) = \frac{2T\cos\theta}{r}$

$$\frac{2T\cos\theta}{r} = h\rho g$$

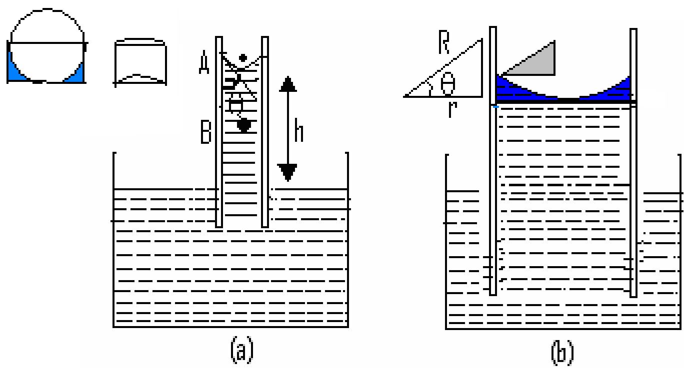
$$T = \frac{r\,h\rho\,g}{2\cos\,\theta}$$

The capillary rise h $= \dfrac{2T\cos\theta}{r\rho g}$

If the effect of liquid contained in the meniscus is taken into account then,
Mass of liquid in meniscus is

$$(\pi r^2 r - \frac{2\pi r^3}{3})\rho g = \frac{\pi r^3}{3}\rho g$$

So, $T = \dfrac{r(h + r/3)\rho g}{2\cos\theta}$ **This relation is called capillary ascent formula**



Risk of Liquid in a Tube of Insuffcient Length
The height to which liquid rise in capillary tube is

$$h = \frac{2T\cos\theta}{r\rho g}$$

The radius of the capillary tube r and the radius of the meniscus R are related by
r = R cosθ

$$h = \frac{2T\cos\theta}{R\cos\theta\rho g} = \frac{2T}{R\rho g}$$

T,ρ and g are constant

$$\therefore\ hR = \frac{2T}{\rho g} = a\ constant \quad \therefore h \propto \frac{1}{R}.$$

Jurin's Law. The smaller the radius of the capillary tube greater is the rise or fall of liquid in it. This means if h is small R should be large so that the product is a constant. If the length of the tube is less than h, then the radius of curvature increases to R′, so that hR = h′R′ where h′ is length of the tube less than h. Now the meniscus will not hemispherical .
Thus in a tube of insufficient length the liquid rises and will spread out to a new radius of curvature R′ = hR/h′. The liquid will not overflow.