

MATHEMATICS
Guess question -07
CLASS-XII

SECTION A

1. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$.
2. Two dice are thrown. Find the odds in favour of getting the sum (i) 4 (ii) 5.
3. A coin is tossed. If head comes up, a die is thrown but if tail comes up, the coin is tossed again. Find the probability of obtaining
(i) Head and a number 6 (ii) head and an even number.
4. Form the differential equation of the family of curves represented by the equation $(x - a)^2 + 2y^2 = a^2$, a being the parameter.
5. Evaluate: $\int e^{3 \log x} (x^4 + 1)^{-1} dx$.
6. Find a two-parameter family of solutions of each of the following differential equation: $y'' = 1 + 2x + 3x^2 + \dots + (n + 1)x^n$, $x \in \mathbb{R}$.
7. Find $\int \frac{dx}{x^2 (x^4 + 1)^{3/4}}$
8. Examine whether the system of equations $2x - y = 5$ and $4x - 2y = 10$ is consistent or inconsistent.
9. Differentiate $\log \sin x$ from first principle
10. Evaluate by using the property, $\int_{-\pi/4}^{\pi/4} \cos^2 x dx$.
11. Find the values of a and b so that the function f given by $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \end{cases}$ is continuous both at $x = 3$ and $x = 5$.
12. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.
13. Evaluate: $\int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$.
14. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?
15. Examine the validity of the following argument:
 $S_1 : p \vee (q \vee r)$, $S_2 : \sim r$, $S : p \vee q$.
16. Using integration find the area of the triangle ABC, coordinates of whose vertices are A(2,0), B(4,5) and C(6,3).
17. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.
18. Solve the following system of equations:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2.$$

SECTION B

19. Find the area of the triangle whose adjacent sides are made by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.
20. A particle possesses two velocities 15 m/s and 20 m/s, simultaneously inclined to each other at an angle 120° . Find the magnitude and direction of the resultant velocity.
21. Find the shortest distance between the lines whose vector equations are $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.
22. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 12z + 20 = 0$, then find the coordinates of the other end of the diameter.
23. \vec{P} and \vec{Q} are two unlike parallel forces. When the magnitude of \vec{P} is doubled, it is found that the line of action of \vec{Q} is mid-way between the lines of action of the new and original resultant. Find the ratio P and Q.
24. State and prove Lami's Theorem.
25. A plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of the triangle ABC. Then, show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$.
26. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. Prove that angle of projection is $\tan^{-1} \frac{bc}{a(c-a)}$ and the velocity of projection V is given by $\frac{2V^2}{g} = \frac{a^2(c-a)^2 + b^2c^2}{ab(c-a)}$.

SECTION C

19. A bill for Rs 73000 drawn on Jan 18, 2000 for 7 months was discounted at the bank on Mar 23, 2000, the rate of interest being 6% p.a. how much did the holder receive?
20. The banker's gain and the true discount on a bill due after a certain time are respectively Rs 50 and Rs 5000. Find the face value of the bill.
21. State and prove the Recurrence formula for the binomial distribution.
22. One bag contains 4 red balls and 3 blue balls, second bag contains 3 red and 5 blue balls. One ball is drawn at random from the first bag and put unseen in the second bag. What is the probability that a ball now drawn from the second bag is blue?
23. What equivalent payment made at the beginning of each month for 8 years will pay for a house priced at Rs 500000, if money is worth 12% p.a. compounded monthly? (Use log table).
24. The marginal revenue function for a commodity is given by $MR = \frac{ab}{(x-b)^2} - C$. Find the total revenue function and the demand function.
25. A and B started a business contributing respectively Rs 100000 and Rs 120000. during the first year B took a loan of Rs 50000 from the total funds of the firm. The partnership deed provided an interest of 6% on the capital investment and allowed to charge an interest of 10% on the loan. Find the earning of A and B if the year's profit of Rs 30200 is distributed in the ratio of capital investment after paying the interest on the capital.

26. A retired person investing in two invest and amount of upto Rs 20000. His broker recommends investing in two type of bonds A and B, bond A yielding 10% return on the amount invested and bond B yielding 15% return on the amount invested. After some consideration, he decides to invest at least 5000 in bond A and no more than Rs.8000 in bond B. What should his broker suggest if he wants to maximize his return on investment? Formulate LPP and solve by I so cost method.