LESSON-17

PROBABILITY

1. INTRODUCTION

The word probability and the word chance are synonymous and may be taken, in this context, to be indistinguishable.

Probability, in the conventional sense, is a ratio between what may be called as the 'number of favourable' to the 'total number'; and, as such this ratio is a positive fraction varying in value between zero (when the events is certain not to happen) to one (when an event is certain to happen).

2. CONCEPT OF PROBABILITY IN SET THEORETIC LANGUAGE

1. RANDOM EXPERIMENT

It is an operation which can result in any one of its well defined outcomes and the outcome cannot be predicted with certainity.

2. SAMPLE SPACE AND SAMPLE POINTS

The set of all possible outcomes of a random experiment is called sample space and is denoted by *S*. Every possible outcome i.e. every element of this set is a sample point.

3. TRIAL

When an experiment is repeated under similar conditions and it does not give the same result each time but may result in any one of the several possible outcomes, the experiment is called a trial and the outcomes are called cases. The number of times the experiment is repeated is called the number of trials.

4. EVENT

A subset of sample space, i.e. a set of some of possible outcomes of a random experiment is called as event.

Simple event

Each sample point in the sample space is called an elementary event or simple event. For example occurrence of head in throw of a coin is simple event.

Sure event

The set containing all sample points is a sure event as in the of a throw die the occurrence of natural number less than 7, is a sure event.

Null event

The set which does not contain any sample point.

Mixed/compound event

A subset of sample space S containing more than one element is called a mixed event or a compound event.

Compliment of an event

Let S be the sample space and E be an event then E^c or $\overline{E}$ represents complement of event E which is a subset containing all sample points in S which are not in E. It refers to the non occurrence of event E.

5. ALGEBRA OF EVENTS

In connection with basic probability laws we shall need the following concepts and facts about events (subsets) A, B, C, …… of a given sample space S.

The union $A \cup B$ of A and B consists of all points in A or B or both.

The intersection $A \cap B$ of A and B consists of all points that are in both A and B.

If A and B have no points in common, we write

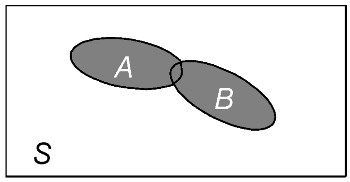
$$A \cap B = \varphi$$

where φ is the empty set (set with no elements) and we call A and B mutually exclusive (or disjoint) because the occurrence of A excludes that of B (and conversely) if your die turns up an odd number, it cannot turn up an even number in the same trial. Similarly, a coin cannot turn up Head and Tail at the same time.

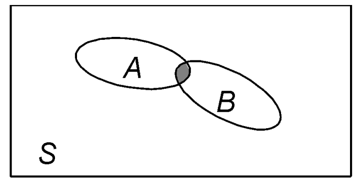
The complement A^c of A consists of all the points of S not in A. Thus,

$$A \cap A^c = \varphi, \quad A \cup A^c = S$$

Working with events can be illustrated and facilitated by Venn diagrams for showing union, intersections, and complements, as shown in the figure.



Union $A \cup B$



Intersection $A \cap B$

Venn diagrams showing two events A and B in a sample space S

and their union A ∪ B (coloured) and intersection A ∩ B (coloured)

6. EQUALLY LIKELY EVENTS

The events are said to be equally likely if none of them is expected to occur in preference to the other one. For example

In throw of a fair coin occurrence of a head or a tail have equal chances. Hence event that a head appears and event that a tail appears are equally likely events.

7. MUTUALLY EXCLUSIVE EVENTS

A set of events is said to be mutually exclusive if occurrence of one of them precludes the occurrence of any other. For example:

- In throw of a die, the event of occurrence of an even number and the event of occurrence of an odd number are mutually exclusive.
- In throw of a fair coin, occurrence of a head or a tail are mutually exclusive.

8. EXHAUSTIVE EVENTS

A set of events is exhaustive if the performance of the experiment results in occurrence of atleast one of them. For example:

- In throw of a die, the event of occurrence of an even numbers and the event of occurrence of an odd number are exhaustive.

3. DEFINITION OF PROBABILITY WITH DISCRETE SAMPLE SPACE

If the sample space S of an experiment consists of finitely many outcomes (points) that are equally likely, then the probability of occurrence of an event A is

$$P(A) = \frac{\text{Number of sample points in A}}{\text{Number of sample points in S}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

In particular $P(S) = 1$ and $0 \le P(A) \le 1$.

Example: **Ten items out of a set of 100 are defective. What is the probability that 3 out of any four chosen are defective?**

Solution: Probability $= \dfrac{{}^{90}C_1 \cdot {}^{10}C_3}{{}^{100}C_4} = \dfrac{144}{52283}$

Example: **Seven persons are to be seated on one side of a straight table. What is the probability that two particular persons will be seated next to each other ?**

Solution: Total number of ways of 7 persons being seated is ${}^7P_7 = \lfloor 7$ ways

If two are to be seated next to each other, treat them as one unit – and this one unit with the remaining 5 can be seated in $\lfloor 6$ ways – and in each one of these $\lfloor 6$ ways the two persons can be interchanged in 2 ways.

$\therefore$ probability $= \dfrac{2\lfloor 6}{\lfloor 7} = \dfrac{2}{7}$

4. AXIOMATIC DEFINITION OF PROBABILITY

Given a sample space S, with each event A of S, there is associated a number P(A), called the probability of A, such that the following axioms of probability are satisfied.

- For every A in S, $0 \le P(A) \le 1$
- The entire sample space has the probability $P(S) = 1$
- For mutually exclusive events A and B ($A \cap B = \varphi$), $P(A \cup B) = P(A) + P(B)$.

5. BASIC THEORIES OF PROBABILITY

1. For an event A and its compliment A^c in sample space S

$\boldsymbol{P(A^c) = 1 - P(A)}$ as $A \cap A^c = \varphi$ and $A \cup A^c = S$

and $P(A \cup A^c) = P(A) + P(A^c)$

$\Rightarrow$ $P(S) = P(A) + P(A^c)$

$\Rightarrow$ $1 = P(A) + P(A^c)$

Example: **A cricket club has 15 members, among whom only 5 can bowl. What is the probability of forming a team of 11 to consist at least 3 bowlers?**

Solution: Total number of ways of forming the team $= {}^{15}C_{11} = {}^{15}C_4$

Of these, number of ways of formation of the team

(i) with one bowler $= {}^5C_1 \cdot {}^{10}C_{10} = 5$

(ii) with two bowlers $= {}^5C_2 \cdot {}^{10}C_9 = 100$

Probability that at least 3 bowlers are in the team $= 1 - \dfrac{105}{{}^{15}C_4} = \dfrac{12}{13}$

Example: **Five coins are tossed simultaneously. Find the probability of event that atleast one head turns up. (Assume that wins are fair)**

Solution: Let A be the event that 'at least one head turns up' since each coin turns up on either a head or a tail hence, the sample space consists of $2^5 = 32$ outcomes. Each outcome having a probability of occurrence as $\dfrac{1}{32}$. Then A^c is the event that 'No head turns up'. Thus A^c consists of only one outcome,

Hence $P(A^c) = \dfrac{1}{32}$

$$P(A) = 1 - \frac{1}{32} = \frac{31}{32}$$

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2. ADDITION RULE OF PROBABILITY

For events A and B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example: **In tossing a fair die, what is the probability of getting an odd number or a number less than 4?**

Solution: $S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event that odd number occurs then $P(A) = \dfrac{3}{6} = \dfrac{1}{2}$ as $A = \{1, 3, 5\}$.

Let B be the event that a number less than 4 occurs then $B = \{1, 2, 3\}$ and $P(B) = \dfrac{3}{6} = \dfrac{1}{2}$
then $A \cap B = \{1, 3\}$ (odd number less than 4)

$\Rightarrow \quad P(A \cap B) = \dfrac{2}{6} = \dfrac{1}{3}$

$$P(A \cup B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = \frac{2}{3}$$

Example: **If the probability that on any workday a garage will get 10−20, 21−30, 31−40 over 40 cars to service is 0.20, 0.35, 0.25, 0.12 respectively. What is the probability that on a given workday the garage gets atleast 21 cars to service?**

Solution: Since these are mutually exclusive events. Hence required probability is

$0.35 + 0.25 + 0.12 = 0.72.$