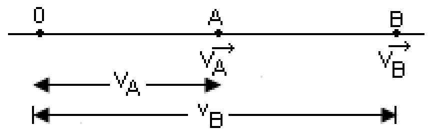
RELATIVE VELOCITY

Motion Along The Straight Line:-Let two particles A and B moves along the same straight line and at time t. Their displacement measured from some fixed origin O on the line be X_A and X_B respectably. The velocity of A and B are:-



$$V_A = \frac{dX_A}{dt}, V_B = \frac{dX_B}{dt}$$

The displacement of b relative to A = displacement if B as measured from A.

= $(X_B – X_A)$

The rate of charge of this displacement is called the velocity of B relative to A.

$$\frac{d}{dt}(X_B - X_A) = \frac{dX_B}{dt} - \frac{dX_A}{dt}$$

$$= \vec{V_B} - \vec{V_A}$$

$$\overrightarrow{V_B A} = \vec{V_B}.\vec{V_A}$$

= Relative velocity of B with respect to A.

This is the velocity which B appears to have, when seen from A.

Here observer is on A. To measure the relative velocity observer should be dept art rest.

Let a long train is moving to the right a long a straight level track. A driving car driver driving to the right along the train.

V_{TE} = velocity of train relative to the earth E.

V_{CT} = velocity of car relative to the train.

V_{CE} = velocity of car with respect to the earth.

= It is equal to the seen of the relative velocities $\vec{V}_{CT}$ and $\vec{V}_{TE}$

$V_{CE} = V_{CT} + V_{TE}$ $\left[\frac{l}{E} = \frac{C}{T} x \frac{T}{E} \right.$

Q1. A train is traveling relative to the earth at $15ms^{-1}$ and car is traveling relative to the train is $20ms^{-1}$, find the velocity of car with respect to earth. If the car were traveling to lost with velocity of $20ms^{-1}$ with respect to train. Find V_{CE}?

Ans:- V_{CE} = ?

$V_{TE} = 15ms^{-1}$

$V_{CT} = 20ms^{-1}$

$V_{CE} = V_{CT} + V_{TE} = (20 + 15)ms^{-1} = 35ms^{-1}$

$V_{CE} = -V_{CT} + V_{TE}$

$= -20 + 15 -5m/s$

Thus it would be traveling to the left with respect to earth, with 5m/s.

Q2. An automobile driver A traveling relative to the earth at 65kmhr^{-1} on a straight level road is a head of motorcycle office B traveling on the velocity of B relation to A?

Ans:- $V_{BA} = ?$

$V_{AE} = 65km/h$ $\therefore V_{AE} = -65km/h$

$V_{BE} = 80km/h$

$V_{BA} = \vec{V}_{BE} + \vec{V}_{EA}$ $\left[\frac{B}{A} = \frac{B}{E} x \frac{E}{A}\right]$

$= V_{BE} - V_{AE}$

$= (80 - 65) km/h$

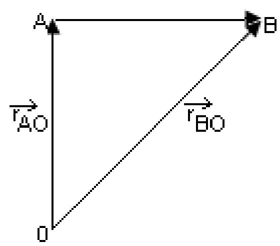
$= 15$ km / h

Hence the officer is overtaking the driver at 15km/hr.

RELATIVE VELOCITY WHEN MOTION IS IN TWO DIMENTION

Here

, $\vec{r}_{AO}, \vec{r}_{BO}$ be position vector at time t, of two moving particle with respect to a Fixed origin O .



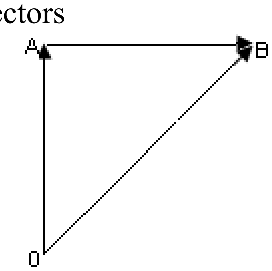
$\vec{r}_{AO}$ = position vector of w.r.t O

$\vec{r}_{BO}$ = position vector of w.r.t O

$\vec{v}_{AO} = \frac{d\vec{r}_{AO}}{dt}$ = Velocity A w.r.t O

$\vec{v}_{BO} = \frac{d\vec{r}_{AO}}{dt}$ = Velocity B w.r.t O

By the triangle law of vectors



$\vec{OA} + \vec{AB} = \vec{OB}$

$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= \vec{r}_{BO} - \vec{r}_{AO} = \vec{R}_{BA}$$

$\vec{AB}$ is the displacement of B relative to A

$\vec{R}_{BA}$ is the position vector of B relative to A

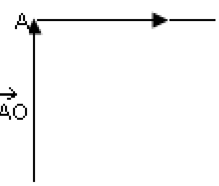
The velocity of B relative of A is $\vec{v}_{BA} = \dfrac{d\vec{R}_{BA}}{dt}$

$$= \frac{d}{dt}\left(\vec{r}_{BO} - \vec{r}_{AO}\right) = \frac{d\vec{r}_{BO}}{dt} - \frac{d\vec{r}_{AO}}{dt}$$
$$\therefore \vec{v}_{BA} = \vec{v}_{BO} - \vec{v}_{AO}$$

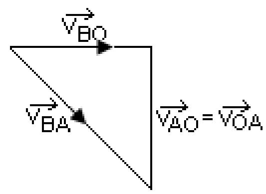
The shows that relative velocity of two moving particle is the vector difference of their velocities with respect to the same origin

Similarly , the velocity of A relative to B us

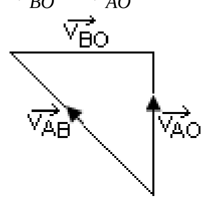
$$\vec{v}_{AB} = \vec{v}_{AO} - \vec{v}_{BO}$$



$$\vec{v}_{BA} = \vec{v}_{BO} - \vec{v}_{AO}$$



$$\vec{v}_{BA} = \vec{v}_{BO} + \left(-\vec{v}_{AO}\right) = \vec{v}_{BO} - \vec{v}_{AO}$$

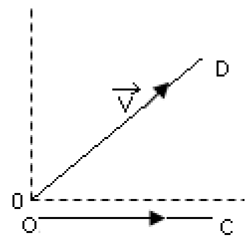


$$\vec{v}_{AB} = \vec{v}_{AO} + \left(-\vec{v}_{BO}\right) = \vec{v}_{AO} - \vec{v}_{BO}$$

If we have to find the velocity A w.r.t B , then the velocity of B should be stopped
By taking equal and opposite direction of velocity of B add there vector

ANALYTICAL RESULTS

Let two particle A and B be moving with velocities $\vec{u}$ and $\vec{v}$ along OC and OD inclined at an angle θ



The velocity of B relative to A , Parallel to OC

$= v \cos\theta - u$

Since A has no velocity perpendicular to OC

The velocity of B relative to A, perpendicular to OC

$$= (v \sin\theta - 0) = v \sin\theta$$

$$v_{BA}^2 = (v\cos\theta - u)^2 + (v\sin\theta)^2$$

$$v_{BA}^2 = v^2 + u^2 - 2uv\cos\theta$$

$$v_{BA} = \sqrt{v^2 + u^2 - 2uv\cos\theta} \qquad \text{.....(i)}$$

Let,

$\overrightarrow{v_{BA}}$ be the resultant of these two components at an angles α the OC

$$\tan\alpha = \frac{Component\ perpendicular\ to\ OC}{Component\ parallel\ to\ OC} = \frac{v\sin\theta}{v\cos\theta - u} \qquad \text{.....(ii)}$$

Eg (i) & (ii) give respectively the magnitude v_{BA} and direction α of relative velocity of B with respect the A.

RELATIVE MOTION

Question based on relative motion are usually of following five types:-

(i)River-boat problem.

(ii) Minimum distance between two bodies in motion.

(iii)Air craft-wind problem.

(iv)Rain problem

(v) Relative motion is projected.

(a) **River boat-problems**

Let,

v_r = absolute velocity or river = v_{rG}

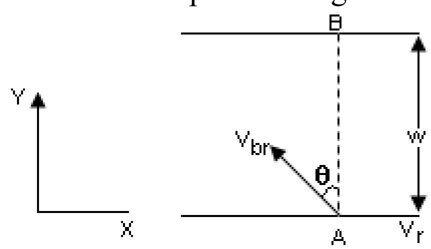
v_{br} = velocity of boatman with respect to river.

= velocity of boatman is still water

= velocity of boatman with which he stears

v_b = absolute velocity of boatman = v_{bG}

= actual velocity of boatman with respect the to ground.



A boatman starts from point A on bank of river with the velocity v_{br} an the distraction as shown. River is flowing along positive x-axis direction with velocity v_r width of river is w.

$v_b = v_{br} + v_r$

along x-axis $v_{bx} = v_{brx} + v_{rx} \quad = -\, v_{br}\sin\theta + v_r$

$= -\, v_r + v_{br}\sin\theta$

$v_{by} = v_{bry} + v_{ry} \quad = -\, v_{br}\cos\theta + 0 = v_{br}\cos\theta$

Now, time taken by boatman to cross the river is

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br}\cos\theta}$$

$$\boxed{t = \frac{w}{v_{br}\cos\theta}} \qquad \text{....(i)}$$

Displacement along x-axis when be reaches on the other bank. It is also called drift

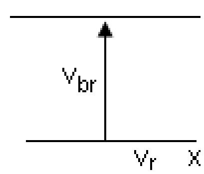
$$x = v_{bx} x t = v_{bx}\frac{w}{v_{br}\cos\theta}$$

$$x = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \text{....(ii)}$$

Two spiral cases are:-

(i)Condition when the boatman crosses the river in shortest internal

From equation (i) when t will be minimum when θ = 0 i.e. the boatman stears his boat perpendicular to the river current.



$$t_{\min} = \frac{w}{v_{or}}$$

$\cos\theta = 1$

(ii)Condition when the boatman to reach point B. i.e. at a point just opposite from where he started.

In this case, the draft (x) should be zero (x = 0)

$$(v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} = 0$$

$$v_r = v_{br}\sin\theta$$

$$\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$$

Hence to reach point B the boatman should row at one an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB

RAIN UMBRELLA AROBLEMS

The problem is asked that in which direction a cyclist should hold his umbrella so that he does not wet.

Let,

v_R = velocity of rain

v_w = velocity pf wind

v_c = velocity of cylist

The cyclist should hold his umbrella in the direction of $\overrightarrow{v_R} + \overrightarrow{v_W} - \overrightarrow{v_C}$

If $v_w = 0$ i.e. no wind is blowing, then he should hold his umbrella in the direction of $v_R - v_C = v_{RC}$ = velocity of each with respect to cylist.

The man and the rain problem

The aim is to determine the angle at which the man should hold the umbrella to prevent himself wetting.

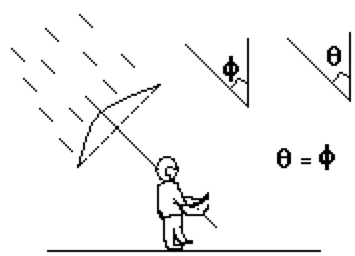
Here, V_r = velocity of rain with respect to ground

V_m = velocity of man with respect to ground

V_{rm} = velocity of rain with respect to man

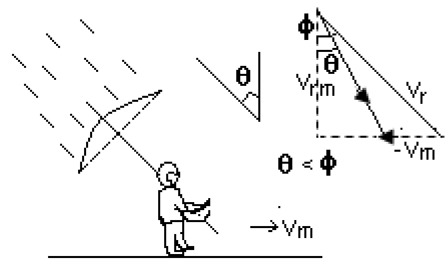
The answer to the problem is that he should hold the umbrella in the direction where from the rain appears to be falling.

Case I : Let man is stationary and the rain is falling at his back at an angle Φ with the vertical.



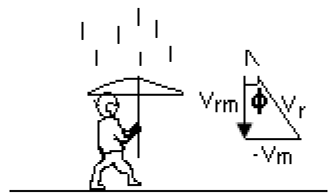
$$V_{rm} = V_r - V_m = V_r$$

Case II: The man starts moving forward. The relative velocity of rain with respect to man shifts towards vertical direction.



$$V_{rm} = V_r - V_m$$

Case III: As the man further increase his increase then at a particular value the rain appears to be falling vertically.



Case IV: if the man increases his speed further more the rain appears to be falling for the forward direction.