

MATHEMATICS
Guess question -07
CLASS -XII
SECTION A

1. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$. Show by mathematical induction that for every positive integer n , $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$.
2. A can hit a target 4 times in 5 shots, B 3 times in 4 shots and C 2 times in 3 shots. Calculate the probability that
 (i) A, B, C all may hit (ii) B, C may hit and A may lose.
3. A die is tossed once. If the random variable X is defined as

$$X = \begin{cases} 1, & \text{if the die result in an even number} \\ 0, & \text{if the die result in an odd number} \end{cases}$$
 Find the probability distribution of X .
4. Solve the following differential equation: $y' + y \cos x = e^{\sin x} \cos x$.
5. Evaluate: $\int \tan^4 x \, dx$.
6. Solve the following initial value problems.
 $\cos(x+y) \, dy = dx$
7. Evaluate: $\int_0^x 5^{5^x} \cdot 5^x \, dx$.
8. Using properties of determinants, solve for x :

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$
9. Differentiate $\tan \sqrt{x}$ w.r.t. x from first principle.
10. Evaluate $\int_{-1}^1 \sin^5 x \cdot \cos^4 x \, dx$.
11. Evaluate: $\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$.
12. If $y = \left[x + \sqrt{x^2 + a^2} \right]^n$, prove that

$$\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + y^2}}$$
13. Evaluate: $\int \frac{x^2}{x^2 + 6x + 12} \, dx$.
14. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.
15. Find a combinatorial circuit corresponding to the following Boolean expressions:
 $(x_1' + (x_2 \cdot x_3')) + x_2$.
16. Make a sketch of the region given below and find its area using integration.
 $\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3; 0 \leq x \leq 3\}$
17. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.
18. Using matrix method solve the following system of linear equation:

$$x + 2y + z = 7, \quad x + 3y = 11, \quad 2x - 3y = 1$$

SECTION B

19. In any $\triangle ABC$, prove that $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ with the help of vectors.
20. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Find the angle of projection.
21. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.
22. A particle is acted upon by three forces in one plane, equal to 2, $2\sqrt{2}$ and 1N respectively. The first is horizontal, the second acts at 45° to the horizontal and the third is vertical. Find their resultant.
23. A bead of weight W can slide on a smooth circular wire in a vertical plane. The bead is attached by a light string to the highest point of the wire, and in equilibrium, the string is taut and makes an angle θ with the vertical. Find the tension in the string and the reaction of the wire on the bead.
24. Find the vector and Cartesian forms of the equation of the plane passing through the point (1, 2, -4) and parallel to the lines $\vec{r} = \hat{i} - 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = \hat{i} - 3\hat{j} + 5\hat{k} + \mu(\hat{i} + \hat{j} - \hat{k})$.
25. Two cars start off to race with velocities u and v and travel in a straight line with uniform accelerations α and β respectively. If the race ends in a dead heat, prove that the length of the race course is $\frac{2(u-v)(u\beta - v\alpha)}{(\alpha - \beta)^2}$.
26. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the co-ordinates of the point of contact.

SECTION C

19. A banker discounts a bill for a certain amount which has 50 days to run before it matures legally at 6%. The discounted value of the bill is Rs 4525. Find the face value of the bill.
20. A bill for Rs 73200 drawn on May 1, 2002 for 6 months was discounted by a bank at 6% p.a. for Rs 72443.20. On what day the bill was discounted?
21. A bag contains 1 white and 6 red balls, and a second bag contains 4 white and 3 red balls. One of the bags is picked up at random and a ball is randomly drawn from it, and is found to be white in colour. Find the probability that the drawn ball was from the first bag.
22. A coin is tossed 4 times. Find the mean and variance of the probability distribution of the number of heads.
23. A plans to buy a new flat after 5 years, which will cost him Rs 552000. How much money should he deposited annually to accumulate this amount, if he gets 5% p.a. interest compounded annually? [Use $(1.05)^5 = 1.276$]
24. The cost function of a firm is given by $c(x) = 300x - 10x^2 + \frac{1}{3}x^3$, where x stands for the output, calculate the output at which the marginal cost is minimum.

25. A and B are partners sharing profits in the ratio of 4:3. A and B surrender $\frac{1}{4}$ of their share in favour of C, a new partner. Calculate the new profit sharing ratio and sacrificing ratio.
26. A man has Rs 1500 for purchase of rice and wheat. A bag of rice and a bag of wheat cost Rs 180 and Rs 120, respectively. He has storage capacity of 10 bags only. He earns a profit of Rs 11 and Rs 9 per bag of rice and wheat, respectively. Formulate an LPP to maximize the profit and solve it.