Motion In One Dimension

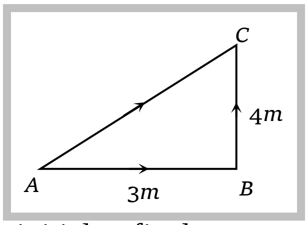
Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	*e.g.* Motion of car on a circular turn. Motion of billiards ball.	*e.g.*. Motion of flying kite. Motion of flying insect.

2.5 Distance and Displacement

(1) **Distance :** It is the actual path length covered by a moving particle in a given interval of time.

(i) If a particle starts from A and reach to C through point B as shown in the figure.



Then distance travelled by particle $= AB + BC = 7\ m$

(ii) Distance is a scalar quantity.

(iii) Dimension : $[M^0L^1T^0]$

(iv) Unit : *metre* (S.I.)

(2) **Displacement :** Displacement is the change in position vector *i.e.*, A vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension : $[M^0L^1T^0]$

(iii) Unit : *metre* (S.I.)

(iv) In the above figure the displacement of the particle $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$® \; |AC| = \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^o} = 5m$$

(v) If $\vec{S}_1, \vec{S}_2, \vec{S}_3 \, ... \, \vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

(3) **Comparison between distance and displacement :**

(i) The magnitude of displacement is equal to minimum possible distance between two positions.

So distance $\geq$|Displacement|.

(ii) For a moving particle distance can never be negative or zero while displacement can be.

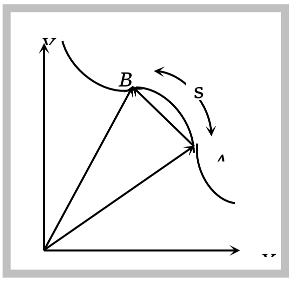
(zero displacement means that body after motion has came back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

(iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.

(iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.



(vi) If $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of particle initially and finally.

Then displacement of the particle

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

and s is the distance travelled if the particle has gone through the path *APB*.

Sample problems based on distance and displacement

Problem **1.** A man goes 12 *m* towards North, then 5 *m* towards east then displacement is

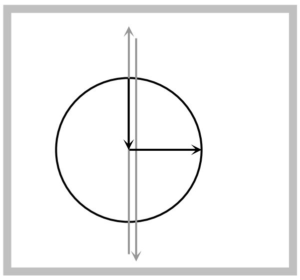
(a) 17 *m* (b) 14 *m* (c) 13 *m* (d) 12 *m*

Solution : (a) If we take east as $x-$axis and north as $y-$axis, then displacement $= 12\ \hat{i} + 5\hat{j}$

So, magnitude of displacement $= \sqrt{12^2 + 5^2} = 13$ *m*.

Problem **2.** A body moves over one fourth of a circular arc in a circle of radius *r*. The magnitude of distance travelled and displacement will be respectively

(a) $\frac{\pi r}{2}, r\sqrt{2}$ (b) $\frac{\pi r}{4}, r$ (c) $\pi r, \frac{r}{\sqrt{2}}$ (d) $\pi r, r$



Solution : (a) Let particle start from *A*, its position vector $\vec{r}_{OA} = r\hat{i}$

After one quarter position vector $\vec{r}_{OB} = r\,\hat{j}$.

So displacement $= r\hat{j} - r\hat{i}$

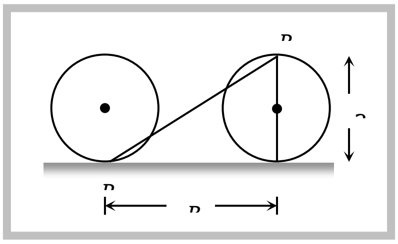
Magnitude of displacement $= r\sqrt{2}$.

and distance = one fourth of circumference $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

Problem **3.** The displacement of the point of the wheel initially in contact with the ground, when the wheel roles forward half a revolution will be (radius of the wheel is R)

(a) $\dfrac{R}{\sqrt{\pi^2+4}}$ (b) $R\sqrt{\pi^2+4}$ (c) $2\pi R$ (d) πR

Solution : (b) Horizontal distance covered by the wheel in half revolution = πR



So the displacement of the point which was initially in contact with a ground = $\sqrt{(\pi R)^2+(2R)^2}$

$= R\sqrt{\pi^2+4}$.

2.6 Speed and Velocity

(1) **Speed :** Rate of distance covered with time is called speed.

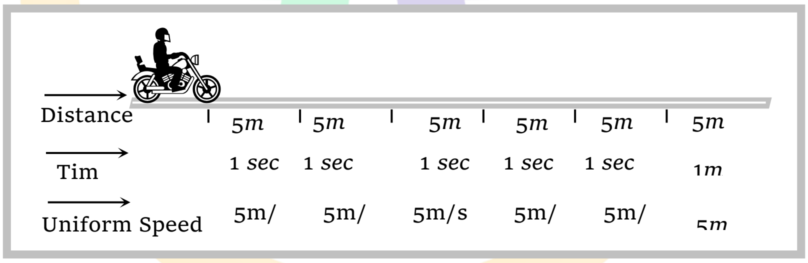
(i) It is a scalar quantity having symbol υ .

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

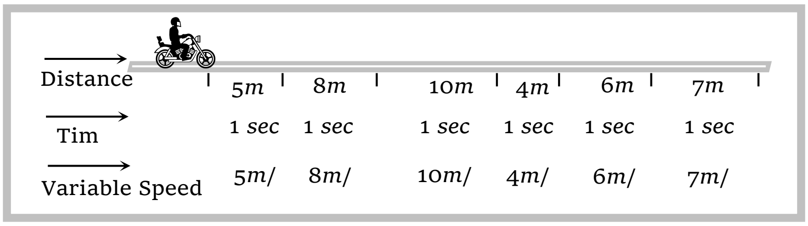
(iv) Types of speed :

(a) **Uniform speed :** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5m) in each second. So we can say that particle is moving with uniform speed of 5 m/s.



(b) **Non-uniform (variable) speed :** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5m in 1st second, 8m in 2nd second, 10m in 3rd second, 4m in 4th second *etc*.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



(c) **Average speed :** The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time taken}}$; $v_{av} = \dfrac{\Delta s}{\Delta t}$

✈ *Time average speed* : When particle moves with different uniform speed υ_1, υ_2, υ_3 ... *etc* in different time intervals t_1, t_2, t_3, ... *etc* respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Special case : When particle moves with speed v_1 upto half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \dfrac{v_1 + v_2}{2}$

✈ *Distance averaged speed* : When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds $v_1, v_2, v_3 ......$ respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{d_1 + d_2 + d_3 + ...}{\dfrac{d_1}{\upsilon_1} + \dfrac{d_2}{\upsilon_2} + \dfrac{d_3}{\upsilon_3} + ...}$$

✈ When particle moves the first half of a distance at a speed of v_1 and second half of the distance at speed v_2 then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

✈ When particle covers one-third distance at speed v_1, next one third at speed v_2 and last one third at speed v_3, then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

(d) **Instantaneous speed :** It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \to 0$). Thus

Instantaneous speed $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

(2) **Velocity :** Rate of change of position *i.e.* rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol v .

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

(iv) Types

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity :** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

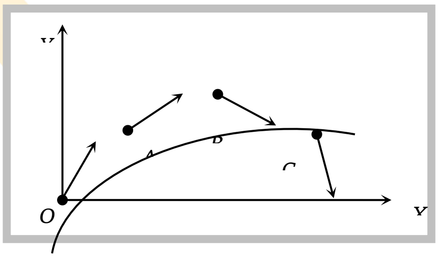
$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}\ ; \quad \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

(d) **Instantaneous velocity :** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Instantaneous velocity $\vec{v} = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(v) **Comparison between instantaneous speed and instantaneous velocity**

(a) instantaneous velocity is always tangential to the path followed by the particle.



When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is v_1, at point A the instantaneous velocity of stone is v_2, similarly at point B and C are v_3 and v_4 respectively.

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$$\vec{v} = -A_1 + 2A_2 t$$

For the given value of t, we can find out the instantaneous velocity.

e.g. for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as $(\Delta s > 0)$.

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be *i.e.* $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.

Sample problems based on speed and velocity

Problem 4. A train has a speed of 60 *km/h* for the first one hour and 40 *km/h* for the next half hour. Its average speed in *km/h* is

(a) 50 (b) 53.33 (c) 48 (d) 70

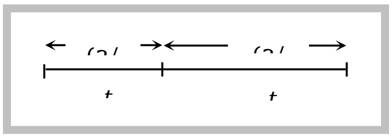
Solution : (b) Total distance travelled = $60 \times 1 + 40 \times \frac{1}{2} = 80\,km$ and Total time taken = $1\,hr + \frac{1}{2}hr = \frac{3}{2}hr$

4 Average speed $= \frac{80}{3/2} = 53.33$ *km/h*

Problem 5. If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is

(a) $\frac{1}{2}\sqrt{v_1 v_2}$ (b) $\frac{v_1 + v_2}{2}$ (c) $\frac{2v_1 v_2}{v_1 + v_2}$ (d) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

Solution : (d) Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{x}{t_1 + t_2}$



$$= \frac{x}{\frac{(2/5)x}{v_1} + \frac{(3/5)x}{v_2}} = \frac{5v_1 v_2}{2v_2 + 3v_1}$$

Problem 6. A car accelerated from initial position and then returned at initial point, then

(a) Velocity is zero but speed increases (b) Speed is zero but velocity increases

(c) Both speed and velocity increase (d) Both speed and velocity decrease

Solution : (a) As the net displacement = 0

Hence velocity = 0 ; but speed increases.

Note : ✈ $\frac{|\text{Average velocity}|}{|\text{Average speed}|} \le 1 \Rightarrow |\text{Av.speed}| \ge |\text{Av.velocity}|$

Problem 7. The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in *metres* and *t* in sec. The displacement, when velocity is zero, is **[CPMT 2000]**

(a) 24 *metres* (b) 12 *metres* (c) 5 *metres* (d) Zero

Solution : (d) $3t = \sqrt{3x} + 6$ ® $\sqrt{3x} = (3t-6)$ ® $3x = (3t-6)^2$ ® $x = 3t^2 - 12t + 12$

4 $v = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 12t + 12) = 6t - 12$

If velocity = 0 then, $6t - 12 = 0 \Rightarrow t = 2 sec$

Hence at $t = 2$, $x = 3(2)^2 - 12(2) + 12 = 0$ *metres*.

Problem 8. The motion of a particle is described by the equation $x = a + bt^2$ where $a = 15\,cm$ and $b = 3\,cm$. Its instantaneous velocity at time 3 *sec* will be

(a) 36 *cm/sec* (b) 18 *cm/sec* (c) 16 *cm/sec* (d) 32 *cm/sec*

Solution : (b) $x = a + bt^2$ 4 $v = \frac{dx}{dt} = 0 + 2bt$

At $t = 3sec$, $v = 2 \times 3 \times 3 = 18\,cm/sec$ (As $b = 3cm$)

Problem 9. A person completes half of its his journey with speed υ_1 and rest half with speed υ_2. The average speed of the person is

(a) $\upsilon = \frac{\upsilon_1 + \upsilon_2}{2}$ (b) $\upsilon = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$ (c) $\upsilon = \frac{\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$ (d) $\upsilon = \sqrt{\upsilon_1\upsilon_2}$

Solution : (b) In this problem total distance is divided into two equal parts. So

$$\upsilon_{av} = \frac{d_1 + d_2}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2}} = \frac{\frac{d}{2} + \frac{d}{2}}{\frac{d/2}{\upsilon_1} + \frac{d/2}{\upsilon_2}} \text{ ® } \upsilon_{av} = \frac{2}{\frac{1}{\upsilon_1} + \frac{1}{\upsilon_2}} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$$

Problem 10. A car moving on a straight road covers one third of the distance with 20 *km/hr* and the rest with 60 *km/hr*. The average speed is **[MP PMT 1999; CPMT 2002]**

(a) 40 *km/hr* (b) 80 *km/hr* (c) $46\frac{2}{3}\,km/hr$ (d) 36 *km/hr*

Solution : (d) Let total distance travelled = x and total time taken $t_1 + t_2 = \frac{x/3}{20} + \frac{2x/3}{60}$

4 Average speed $= \frac{x}{\frac{(1/3)x}{20} + \frac{(2/3)x}{60}} = \frac{1}{\frac{1}{60} + \frac{2}{180}} = 36\,km/hr$

2.7 Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes

Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	*e.g.* Motion under gravity	*e.g.* Projectile motion

(3) Dimension : $[M^0L^1T^{-2}]$

(4) Unit : *metre/second*^{2} (S.I.); *cm/second*^{2} (C.G.S.)

(5) Types of acceleration :

(i) **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : ✈ If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. *e.g.* Projectile motion.

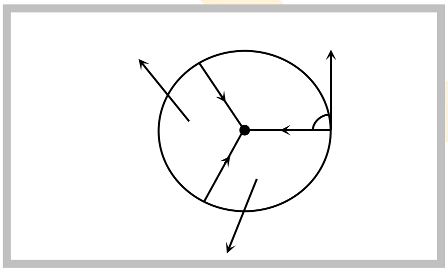
(ii) **Non-uniform acceleration :** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

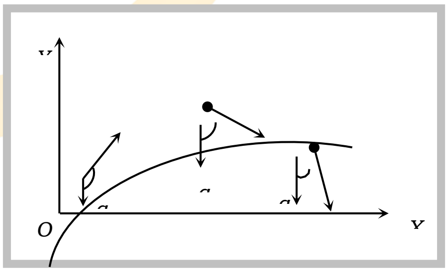
(iii) **Average acceleration :** $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.





e.g. (a) In uniform circular motion (= 90º always

(b) In a projectile motion (is variable for every point of trajectory.

(vi) If a force $\vec{F}$ acts on a particle of mass *m*, by Newton's 2nd law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

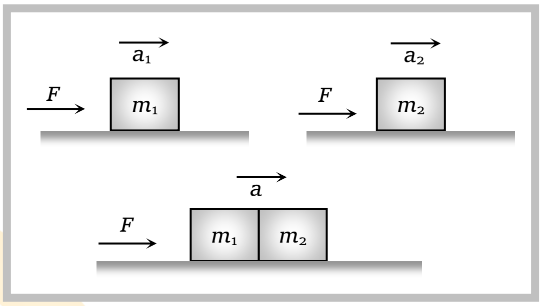
(vii) By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}\left[\text{As } \vec{v} = \frac{d\vec{x}}{dt}\right]$

i.e., if *x* is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{d\upsilon}{dx}\left[\text{as } v = \frac{dx}{dt}\right]$

(ix) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is $a_{a\upsilon} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$

(x) If same force is applied on two bodies of different masses m_1 and m_2 separately then it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then



$F = (m_1 + m_2)a$ ® $\frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$

So, $\frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2}$ ® $a = \frac{a_1 a_2}{a_1 + a_2}$

(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(xii) For motion of a body under gravity, acceleration will be equal to "g", where g is the acceleration due to gravity. Its normal value is 9.8 m/s^2 or 980 cm/s^2 or 32 feet/s^2.

Sample problems based on acceleration

Problem **11.** The displacement of a particle, moving in a straight line, is given by $s = 2t^2 + 2t + 4$ where s is in *metres* and t in seconds. The acceleration of the particle is

(a) 2 m/s^2 (b) 4 m/s^2 (c) 6 m/s^2 (d) 8 m/s^2

Solution : (b) Given $s = 2t^2 + 2t + 4$ 4 velocity (v) $= \frac{ds}{dt} = 4t + 2$ and acceleration (a) $= \frac{dv}{dt} = 4(1) + 0 = 4m/s^2$

Problem **12.** The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to

(a) $\frac{a}{b}$ (b) $\frac{2a}{3b}$ (c) $\frac{a}{3b}$ (d) Zero

Solution : (c) Given $x = at^2 - bt^3$ 4 velocity $(v) = \frac{dx}{dt} = 2at - 3bt^2$ and acceleration (a) = $\frac{dv}{dt} = 2a - 6bt$.

When acceleration = 0 ® $2a - 6bt = 0$ ® $t = \frac{2a}{6b} = \frac{a}{3b}$.

Problem **13.** The displacement of the particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively

(a) $b, -4d$ (b) $-b, 2c$ (c) $b, 2c$ (d) $2c, -4d$

Solution : (c) Given $y = a + bt + ct^2 - dt^4$ 4 v = $\frac{dy}{dt} = 0 + b + 2ct - 4dt^3$

Putting $t = 0$, $v_{\text{initial}} = b$

So initial velocity = b

Now, acceleration (a) $= \frac{dv}{dt} = 0 + 2c - 12\,d\,t^2$

Putting $t = 0$, $a_{\text{initial}} = 2c$

Problem **14.** The relation between time t and distance x is $t = \alpha x^2 + \beta x$, where 〈 and ® are constants. The retardation is (v is the velocity)

(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$

Solution : (a) differentiating time with respect to distance $\frac{dt}{dx} = 2\alpha x + \beta$ ® $v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$

So, acceleration (a) = $\frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx} = \frac{-v.2\alpha}{(2\alpha x + \beta)^2} = -2\alpha.v.v^2 = -2\alpha v^3$

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Problem **15.** A particle is moving eastwards with velocity of 5 *m/s*. In 10 *sec* the velocity changes to 5 *m/s* northwards. The average acceleration in this time is

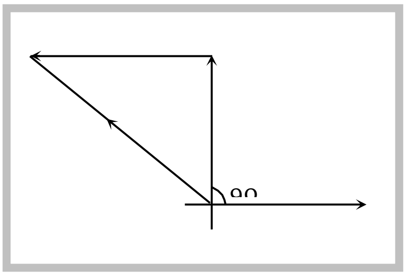
(a) Zero (b) $\frac{1}{\sqrt{2}}$ m/s^2 toward north-west

(c) $\frac{1}{\sqrt{2}}$ m/s^2 toward north-east (d) $\frac{1}{2}$ m/s^2 toward north-west

Solution : (b) $\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1$

$\Delta\upsilon = \sqrt{\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2 \cos 90^o} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$\Delta\upsilon = 5\sqrt{2}$



Average acceleration $= \frac{\Delta\upsilon}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 toward north-west (As clear from the figure)

Problem **16.** A body starts from the origin and moves along the x-axis such that velocity at any instant is given by $(4t^3 - 2t)$, where t is in second and velocity is in *m/s*. What is the acceleration of the particle, when it is 2*m* from the origin?

(a) $28\ m/s^2$ (b) $22\ m/s^2$ (c) $12\ m/s^2$ (d) $10\ m/s^2$

Solution : (b) Given that $\upsilon = 4t^3 - 2t$

$x = \int \upsilon\, dt$, $x = t^4 - t^2 + C$, at $t = 0, x = 0 \Rightarrow C = 0$

When particle is 2*m* away from the origin

$2 = t^4 - t^2$ ® $t^4 - t^2 - 2 = 0$ ® $(t^2 - 2)(t^2 + 1) = 0$ ® $t = \sqrt{2}\ sec$

$a = \frac{d\upsilon}{dt} = \frac{d}{dt}\left(4t^3 - 2t\right) = 12t^2 - 2$ ® $a = 12t^2 - 2$

for $t = \sqrt{2}\ sec$ ® $a = 12 \times \left(\sqrt{2}\right)^2 - 2$ ® $a = 22\ m/s^2$

Problem **17.** A body of mass 10 *kg* is moving with a constant velocity of 10 *m/s*. When a constant force acts for 4 *sec* on it, it moves with a velocity 2 *m/sec* in the opposite direction. The acceleration produced in it is

(a) $3\ m/s^2$ (b) $-3\ m/s^2$ (c) $0.3\ m/s^2$ (d) $-0.3\ m/s^2$

Solution : (b) Let particle moves towards east and by the application of constant force it moves towards west $\vec{v}_1 = +10\ m/s$ and $\vec{v}_2 = -2\ m/s$. Acceleration $= \dfrac{\text{Change in velocity}}{\text{Time}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t}$

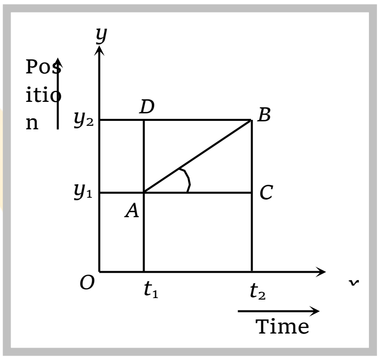
® $a = \dfrac{(-2)-(10)}{4} = \dfrac{-12}{4} = -3\ m/s^2$

2.8 Position Time Graph

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

Let AB is a position-time graph for any moving particle



As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

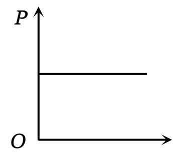
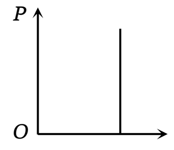
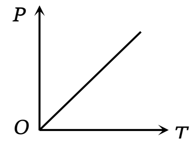
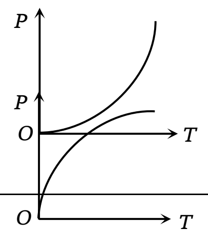
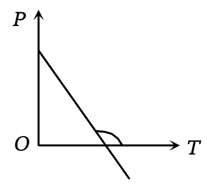
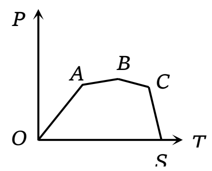
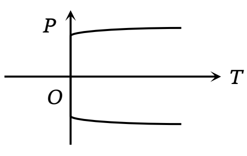
From triangle ABC $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

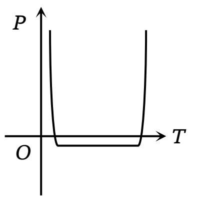
By comparing (i) and (ii) Velocity = tan (

$v = \tan($

It is clear that slope of position-time graph represents the velocity of the particle.

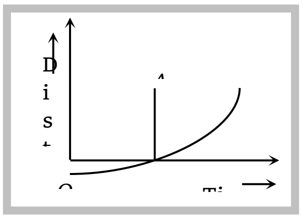
Various position – time graphs and their interpretation

	(= 0^o so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
	(= 90^o so v = □ *i.e.*, line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible.
	(= constant so v = constant, $a = 0$ *i.e.*, line with constant slope represents uniform velocity of the particle.
	(is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	(is decreasing so v is decreasing, a is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	(constant but > 90^o so v will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.



The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note : ✈ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.



✈ For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{\upsilon_1}{\upsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$

Sample problems based on position-time graph

<u>Problem</u> **18.** The position of a particle moving along the x-axis at certain times is given below :

t (s)	0	1	2	3
x (m)	– 2	0	6	16

Which of the following describes the motion correctly **[AMU (Engg.) 2001]**

(a) Uniform, accelerated (b) Uniform, decelerated

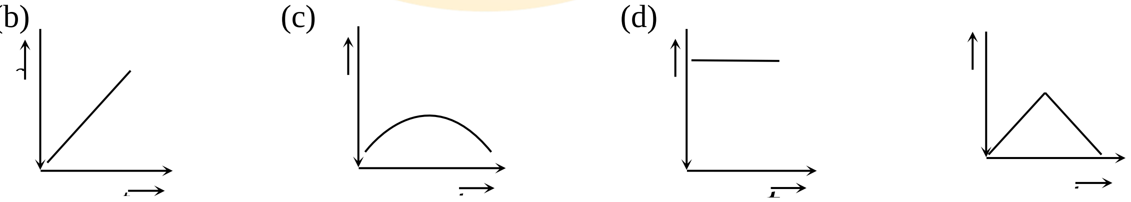
(c) Non-uniform, accelerated (d) There is not enough data for generalisation

Solution : (a) Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$, By using the data from the table

$v_1 = \frac{0-(-2)}{1} = 2m/s$, $v_2 = \frac{6-0}{1} = 6m/s$ and $v_3 = \frac{16-6}{1} = 10\ m/s$ *i.e.* the speed is increasing at a constant rate so motion is uniformly accelerated.

<u>Problem</u> **19.** Which of the following graph represents uniform motion **[DCE 1999]**

(a) (b) (c) (d)



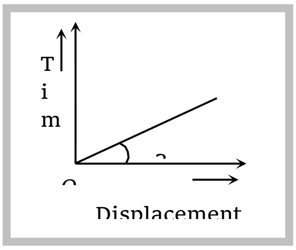
Solution : (a) When distance time graph is a straight line with constant slope than motion is uniform.

<u>Problem</u> **20.** The displacement-time graph for two particles A and B are straight lines inclined at angles of 30^o and 60^o with the time axis. The ratio of velocities of $v_A : v_B$ is

(a) 1 : 2 (b) $1:\sqrt{3}$ (c) $\sqrt{3}:1$ (d) 1 : 3

Solution : (d) $v = \tan\theta$ from displacement graph. So $\frac{v_A}{v_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$

<u>Problem</u> **21.** From the following displacement time graph find out the velocity of a moving body



(a) $\frac{1}{\sqrt{3}}$ m/s

(b) 3 m/s

(c) $\sqrt{3}$ m/s

(d) $\frac{1}{3}$

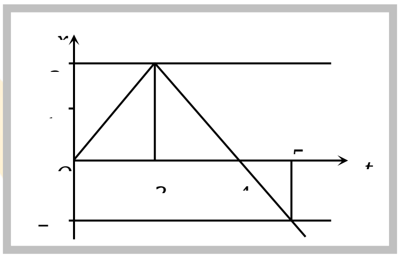
Solution : (c) In first instant you will apply $\upsilon = \tan\theta$ and say, $\upsilon = \tan 30^o = \frac{1}{\sqrt{3}}$ m/s.

But it is wrong because formula $\upsilon = \tan\theta$ is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis $= 90^o - 30^o = 60^o$.

Now $\upsilon = \tan 60^o = \sqrt{3}$

Problem **22** The diagram shows the displacement-time graph for a particle moving in a straight line. The average velocity for the interval $t = 0$, $t = 5$ is



(a) 0

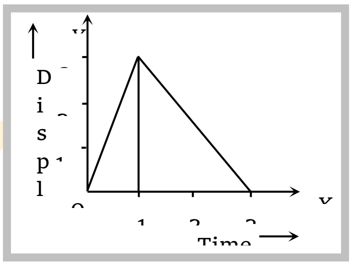
(b) 6 ms^{-1}

(c) – 2 ms^{-1}

(d) 2 ms^{-1}

Solution : (c) Average velocity = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{(20)+(-20)+(-10)}{5} = -2\ m/s$

Problem **23.** Figure shows the displacement time graph of a body. What is the ratio of the speed in the first second and that in the next two seconds



(a) 1 : 2

(b) 1 : 3

(c) 3 : 1

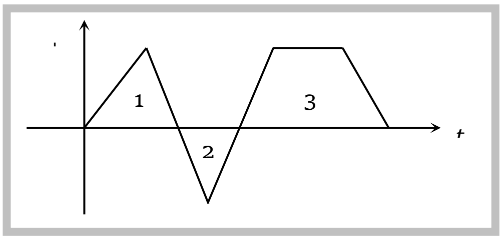
(d) 2 : 1

Solution: (d) Speed in first second = 30 and Speed in next two seconds = 15. So that ratio 2 : 1

2.9 Velocity Time Graph

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.



Then Total distance $= |A_1| + |A_2| + |A_3|$

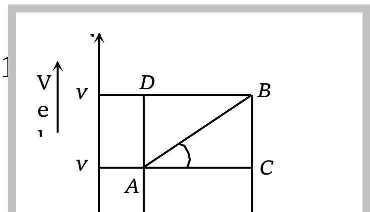
= Addition of modulus of different area. *i.e.* $s = \int |\upsilon|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. *i.e.* $r = \int \upsilon\, dt$

here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration : Let AB is a velocity-time graph for any moving particle



As Acceleration = $\frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$ …(i)

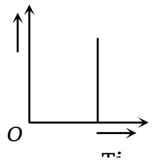
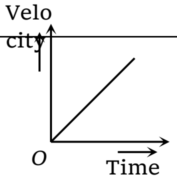
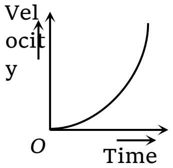
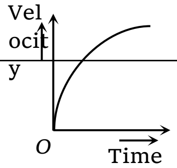
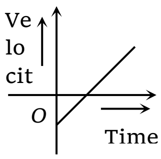
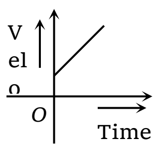
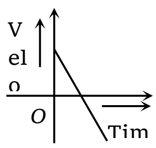
From triangle *ABC*, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1}$ ….(ii)

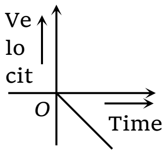
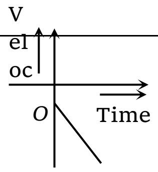
By comparing (i) and (ii)

Acceleration (a) = $\tan\theta$

It is clear that slope of velocity-time graph represents the acceleration of the particle.

Various velocity – time graphs and their interpretation

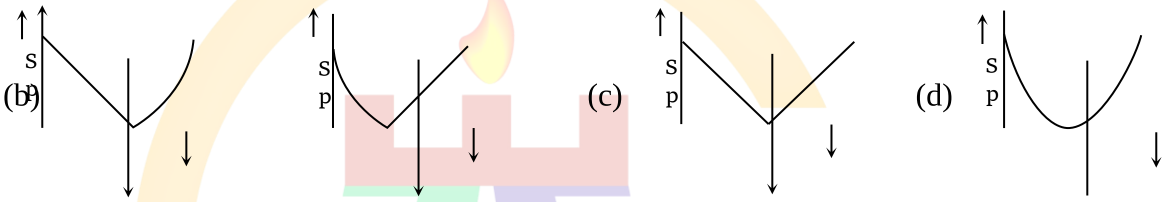
Velo city	(= 0, a = 0, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
	(= 90°, a = □, v = increasing *i.e.*, line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	(=constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	(increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.
	(decreasing so acceleration decreasing *i.e.* line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because (is constant and < 90° but initial velocity of the particle is negative.
	Positive constant acceleration because (is constant and < 90° but initial velocity of particle is positive.
	Negative constant acceleration because (is constant and > 90° but initial velocity of the particle is positive.

	Negative constant acceleration because (is constant and > 90° but initial velocity of the particle is zero.
	Negative constant acceleration because (is constant and > 90° but initial velocity of the particle is negative.

Sample problems based on velocity-time graph

Problem **24.** A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored **[AIIMS 2003]**

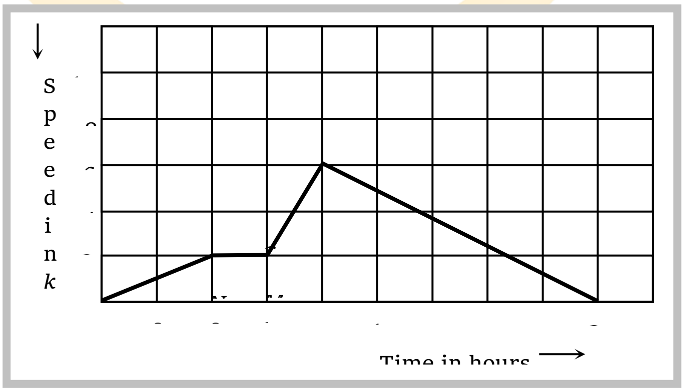
(a) (b) (c) (d)



Solution : (c) In first half of motion the acceleration is uniform & velocity gradually decreases, so slope will be negative but for next half acceleration is positive. So slope will be positive. Thus graph '*C*' is correct.

Not ignoring air resistance means upward motion will have acceleration ($a + g$) and the downward motion will have $(g - a)$.

Problem **25.** A train moves from one station to another in 2 hours time. Its speed-time graph during this motion is shown in the figure. The maximum acceleration during the journey is **[Kerala (Engg.) 2002]**

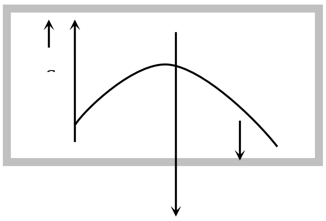


(a) 140 $km\ h^{-2}$ (b) 160 $km\ h^{-2}$ (c) 100 $km\ h^{-2}$ (d) 120 $km\ h^{-2}$

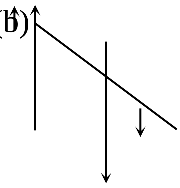
Solution : (b) Maximum acceleration means maximum slope in speed – time graph.

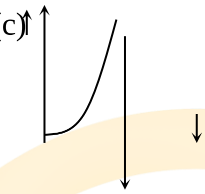
that slope is for line *CD*. So, $a_{\max}$ = slope of *CD* = $\dfrac{60-20}{1.25-1.00} = \dfrac{40}{0.25} = 160\,km\,h^{-2}$.

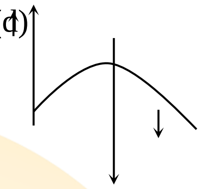
***Problem* 26.** The graph of displacement *v/s* time is

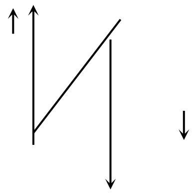


Its corresponding velocity-time graph will be

(a) 

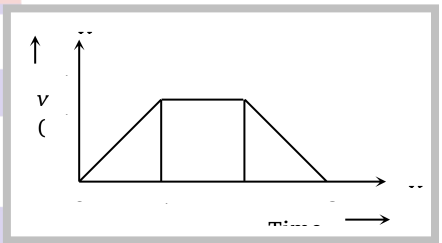
(b) 

(c) 

(d) 

Solution : (a) We know that the velocity of body is given by the slope of displacement – time graph. So it is clear that initially slope of the graph is positive and after some time it becomes zero (corresponding to the peak of the graph) and then it will be negative.

***Problem* 27.** In the following graph, distance travelled by the body in *metres* is



(a) 200

(b) 250

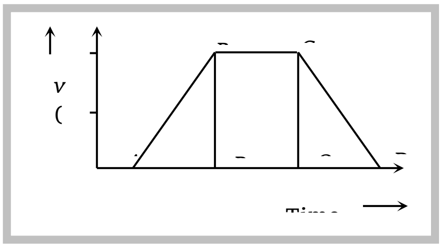
(c) 300

(d) 400

Solution : (a) Distance = The area under $v-t$ graph

$S = \frac{1}{2}(30+10)\times 10 = 200$ metre

***Problem* 28.** For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds



(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

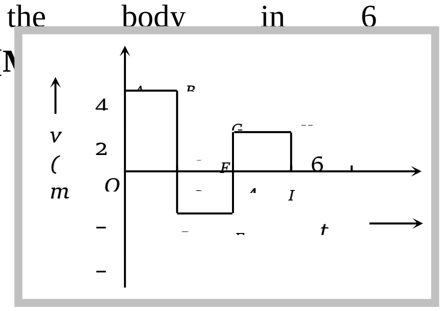
(d) $\frac{2}{3}$

Solution : (b) Distance covered in total 7 seconds = Area of trapezium *ABCD* $= \frac{1}{2}(2+6)\times 10 = 40\ m$

Distance covered in last 2 second = area of triangle *CDQ* $= \frac{1}{2}\times 2\times 10 = 10\ m$

So required fraction $= \frac{10}{40} = \frac{1}{4}$

Problem 29. The velocity time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 *sec* are respectively



(a) $8\,m, 16\,m$

(b) $16\,m, 8\,m$

(c) $16\,m, 16\,m$

(d) $8\,m, 8\,m$

Solution : (a) Area of rectangle *ABCO* = 4 . 2 = 8 *m*

Area of rectangle *CDEF* = 2 × (– 2) = – 4 *m*

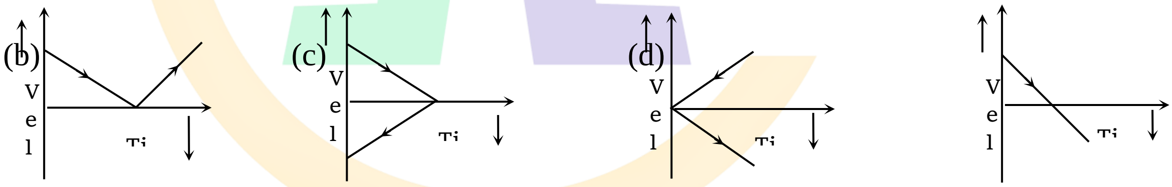
Area of rectangle *FGHI* = 2×2 = 4 *m*

Displacement = sum of area with their sign = 8 + (– 4) + 4 = 8 *m*

Distance = sum of area with out sign = 8 + 4 + 4 = 16 *m*

Problem 30. A ball is thrown vertically upward which of the following graph represents velocity time graph of the ball during its flight (air resistance is neglected)

(a) (b) (c) (d)



Solution : (d) In the positive region the velocity decreases linearly (during rise) and in negative region velocity increase linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

Problem 31. A ball is dropped vertically from a height *d* above the ground. It hits the ground and bounces up vertically to a height $\frac{d}{2}$. Neglecting subsequent motion and air resistance, its velocity υ varies with the height *h* above the ground as.

(a) (b) (c) (d)

Solution : (a) When ball is dropped from height d its velocity will be zero.

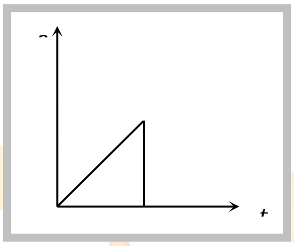
As ball comes downward h decreases and υ increases just before the rebound from the earth

$h = 0$ and v = maximum and just after rebound velocity reduces to half and direction becomes opposite.

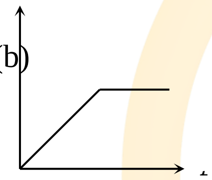
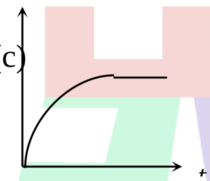
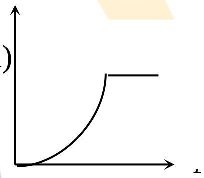
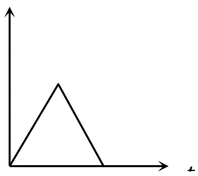
As soon as the height increases its velocity decreases and becomes zero at $h = \frac{d}{2}$.

This interpretation is clearly shown by graph (a).

***Problem* 32.** The acceleration-time graph of a body is shown below –



The most probable velocity-time graph of the body is

(a)  (b)  (c)  (d) 

Solution : (c) From given $a - t$ graph acceleration is increasing at constant rate

$\therefore \frac{da}{dt} = k$ (constant) ® $a = kt$ (by integration)

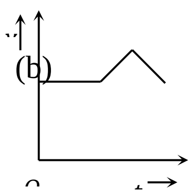
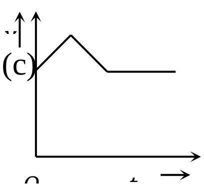
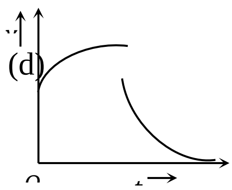
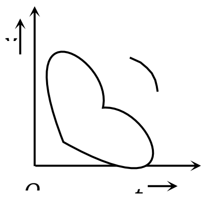
® $\frac{dv}{dt} = kt$ ® $dv = ktdt$ ® $\int dv = k\int tdt$ ® $v = \frac{kt^2}{2}$

i.e., v is dependent on time parabolically and parabola is symmetric about v-axis.

and suddenly acceleration becomes zero. i.e. velocity becomes constant.

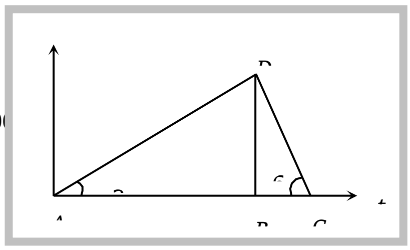
Hence (c) is most probable graph.

***Problem* 33.** Which of the following velocity time graphs is not possible

(a)  (b)  (c)  (d) 

Solution : (d) Particle can not possess two velocities at a single instant so graph (d) is not possible.

***Problem* 34.** For a certain body, the velocity-time graph is shown in the figure. The ratio of applied forces for intervals AB and BC is



(a) $+\frac{1}{2}$

(b) $-\frac{1}{2}$

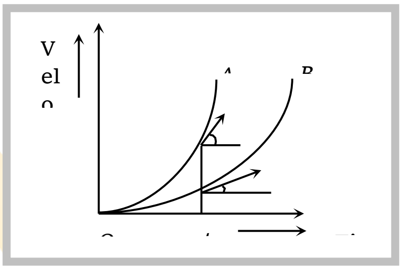
(c) $+\frac{1}{3}$

(d) $-\frac{1}{3}$

Solution : (d) Ratio of applied force = Ratio of acceleration

$$= \frac{a_{AB}}{a_{BC}} = \frac{\tan 30}{\tan(120)} = \frac{1/\sqrt{3}}{-\sqrt{3}} = -1/3$$

Problem **35.** Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that



(a) Initial velocity of *A* is greater than the initial velocity of *B*

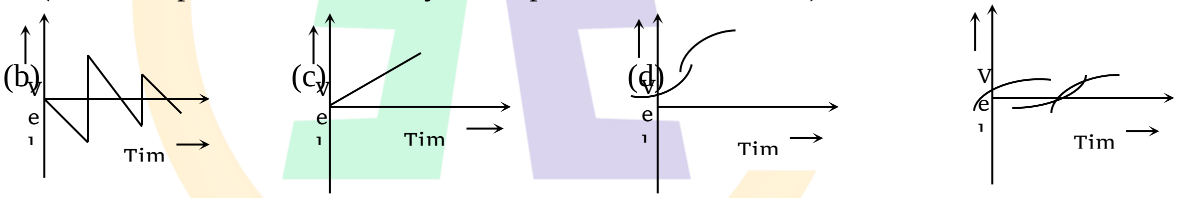
(b) Acceleration in *A* is increasing at lesser rate than in *B*

(c) Acceleration in *A* is greater than in *B*

(d) Acceleration in *B* is greater than in *A*

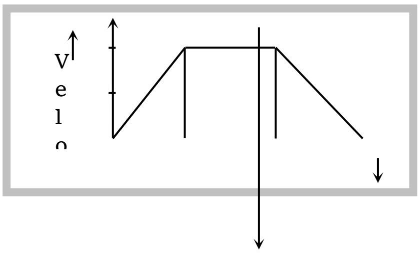
Solution : (c) At a certain instant *t* slope of *A* is greater than *B* ($\theta_A > \theta_B$), so acceleration in *A* is greater than *B*

Problem **36.** Which one of the following graphs represent the velocity of a steel ball which fall from a height on to a marble floor? (Here υ represents the velocity of the particle and *t* the time)

(a) 

Solution : (a) Initially when ball falls from a height its velocity is zero and goes on increasing when it comes down. Just after rebound from the earth its velocity decreases in magnitude and its direction gets reversed. This process is repeated untill ball comes to at rest. This interpretation is well explained in graph (a).

Problem **37.** The adjoining curve represents the velocity-time graph of a particle, its acceleration values along *OA*, *AB* and *BC* in *metre/sec*^{2} are respectively



(a) 1, 0, – 0.5

(b) 1, 0, 0.5

(c) 1, 1, 0.5

(d) 1, 0.5, 0

Solution : (a) Acceleration along *OA* $= \frac{v_2 - v_1}{t} = \frac{10-0}{10} = 1 m/s^2$

Acceleration along *OB* $= \frac{0}{10} = 0$

Acceleration along *BC* $= \frac{0-10}{20} = -0.5\ m/s^2$

2.10 Equations of Kinematics

These are the various relations between *u*, *v*, *a*, *t* and *s* for the moving particle where the notations are used as :

u = Initial velocity of the particle at time $t = 0$ *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n^{th} *sec*

(1) **When particle moves with zero acceleration**

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As $a = 0$]

(2) **When particle moves with constant acceleration**

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar from

$$\upsilon = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$\upsilon^2 = u^2 + 2as$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Equation of motion in vector from

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$$

$$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$$

$$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

(3) **Important points for uniformly accelerated motion**

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t *sec* is proportional to t^2 (*i.e.* $s \propto t^2$).

So we can say that the ratio of distance covered in 1 *sec*, 2 *sec* and 3 *sec* is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in *n*th sec is proportional to $(2n-1)$ (*i.e.* $s_n \propto (2n-1)$)

So we can say that the ratio of distance covered in I *sec*, II *sec* and III *sec* is 1 : 3 : 5.

(iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $\upsilon^2 = u^2 - 2as$ ® $0 = u^2 - 2as$ ® $s = \frac{u^2}{2a}$, $s \propto u^2$ [since a is constant]

So we can say that if u becomes n times then s becomes n^2 times that of previous value.

(iv) A particle moving with uniform acceleration from A to B along a straight line has velocities υ_1 and υ_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$\upsilon = \sqrt{\frac{\upsilon_1^2 + \upsilon_2^2}{2}}$$

Sample problems based on uniform acceleration

<u>*Problem*</u> 3.8 A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity υ. The two bodies meet after a time t. The value of t is **[MP PET 2003]**

(a) $\frac{2\upsilon}{a}$ (b) $\frac{\upsilon}{a}$ (c) $\frac{\upsilon}{2a}$ (d) $\sqrt{\frac{\upsilon}{2a}}$

Solution : (a) Let they meet after time 't' . Distance covered by body A = $\frac{1}{2}at^2$;

Distance covered by body B = vt

and $\frac{1}{2}at^2 = vt$ $\therefore$ $t = \frac{2v}{a}$.

<u>*Problem*</u> 39. A student is standing at a distance of 50metres from the bus. As soon as the bus starts its motion with an acceleration of 1ms^{-2}, the student starts running towards the bus with a uniform velocity u . Assuming the motion to be along a straight road, the minimum value of u , so that the students is able to catch the bus is **[KCET 2003]**

(a) 5 ms^{-1} (b) 8 ms^{-1} (c) 10 ms^{-1} (d) 12 ms^{-1}

Solution : (c) Let student will catch the bus after t *sec*. So it will cover distance ut.
Similarly distance travelled by the bus will be $\frac{1}{2}at^2$ for the given condition

$ut = 50 + \frac{1}{2}at^2 = 50 + \frac{t^2}{2}$ ® $u = \frac{50}{t} + \frac{t}{2}$ (As $a = 1\, m/s^2$)

To find the minimum value of u, $\frac{du}{dt} = 0$, so we get t = 10 *sec*

then u = 10 *m/s*.

<u>*Problem*</u> 40. A car, moving with a speed of 50 km/*hr*, can be stopped by brakes after at least 6*m*. If the same car is moving at a speed of 100 *km/hr*, the minimum stopping distance is **[AIEEE 2003]**

(a) 6*m* (b) 12*m* (c) 18*m* (d) 24*m*

Solution : (d) $v^2 = u^2 - 2as$ ® $0 = u^2 - 2as$ ® $s = \frac{u^2}{2a} \Rightarrow s \propto u^2$ (As a = constant)

$$\frac{s_2}{s_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{100}{50}\right)^2 \Rightarrow s_2 = 4s_1 = 4 \times 12 = 24\ m.$$

***Problem* 41.** The velocity of a bullet is reduced from 200*m/s* to 100*m/s* while travelling through a wooden block of thickness 10*cm*. The retardation, assuming it to be uniform, will be **[AIIMS 2001]**

(a) $10\times10^4\ m/s^2$ (b) $12\times10^4\ m/s^2$ (c) $13.5\times10^4\ m/s^2$ (d) $15\times10^4\ m/s^2$

Solution : (d) $u = 200\ m/s$, $v = 100\ m/s$, $s = 0.1\,m$

$$a = \frac{u^2 - v^2}{2s} = \frac{(200)^2 - (100)^2}{2\times0.1} = 15\times10^4\ m/s^2$$

***Problem* 42.** A body A starts from rest with an acceleration a_1. After 2 seconds, another body *B* starts from rest with an acceleration a_2. If they travel equal distances in the 5th second, after the start of *A*, then the ratio $a_1 : a_2$ is equal to **[AIIMS 2001]**

(a) 5 : 9 (b) 5 : 7 (c) 9 : 5 (d) 9 : 7

Solution : (a) By using $S_n = u + \frac{a}{2}(2n-1)$, Distance travelled by body *A* in 5th second = $0 + \frac{a_1}{2}(2\times5-1)$

Distance travelled by body B in 3rd second is = $0 + \frac{a_2}{2}(2\times3-1)$

According to problem : $0 + \frac{a_1}{2}(2\times5-1) = 0 + \frac{a_2}{2}(2\times3-1)$ ® $9a_1 = 5a_2 \Rightarrow \frac{a_1}{a_2} = \frac{5}{9}$

***Problem* 43.** A particle travels 10*m* in first 5 *sec* and 10*m* in next 3 *sec*. Assuming constant acceleration what is the distance travelled in next 2 *sec* **[RPET 2000]**

(a) 8.3 *m* (b) 9.3 *m* (c) 10.3 *m* (d) None of above

Solution : (a) Let initial $(t=0)$ velocity of particle = u

for first 5 sec of motion $s_5 = 10\ metre$, so by using $s = ut + \frac{1}{2}at^2$

$10 = 5u + \frac{1}{2}a(5)^2$ ® $2u + 5a = 4$ …. (i)

for first 8 sec of motion $s_8 = 20\ metre$

$20 = 8u + \frac{1}{2}a(8)^2$ ® $2u + 8a = 5$ …. (ii)

By solving (i) and (ii) $u = \frac{7}{6}m/s$ $a = \frac{1}{3}m/s^2$

Now distance travelled by particle in total 10 *sec*. $s_{10} = u\times10 + \frac{1}{2}a(10)^2$

by substituting the value of u and a we will get $s_{10} = 28.3\ m$

So the distance in last 2 *sec* = $s_{10} - s_8 = 28.3 - 20 = 8.3\ m$

***Problem* 44.** A body travels for 15 *sec* starting from rest with constant acceleration. If it travels distances S_1, S_2 and S_3 in the first five seconds, second five seconds and next five seconds respectively the relation between S_1, S_2 and S_3 is

(a) $S_1 = S_2 = S_3$ (b) $5S_1 = 3S_2 = S_3$ (c) $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$ (d) $S_1 = \frac{1}{5}S_2 = \frac{1}{3}S_3$

Solution : (c) Since the body starts from rest. Therefore $u = 0$.

$$S_1 = \frac{1}{2}a(5)^2 = \frac{25a}{2}$$

$S_1 + S_2 = \frac{1}{2}a(10)^2 = \frac{100a}{2}$ ® $S_2 = \frac{100a}{2} - S_1 = 75\frac{a}{2}$

$S_1 + S_2 + S_3 = \frac{1}{2}a(15)^2 = \frac{225\,a}{2}$ ® $S_3 = \frac{225\,a}{2} - S_2 - S_1 = \frac{125\,a}{2}$

Thus Clearly $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$

Problem **45.** If a body having initial velocity zero is moving with uniform acceleration $8\,m/\sec^2$, the distance travelled by it in fifth second will be **[MP PMT 1996; DPMT 2000]**

(a) 36 *metres* (b) 40 *metres* (c) 100 *metres* (d) Zero

Solution : (a)

$$S_n = u + \frac{1}{2}a(2n-1) = 0 + \frac{1}{2}(8)\,[2\times 5 - 1] = 36\,metres$$

Problem **46.** A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second.

(a) 7/5 (b) 5/7 (c) 7/3 (d) 3/7

Solution : (a) As $S_n \propto (2n-1)$, $\frac{S_4}{S_3} = \frac{7}{5}$

2.11 Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol *g*.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h << R$) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) **If a body dropped from some height (initial velocity zero)**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

$u = 0$ [As body starts from rest]

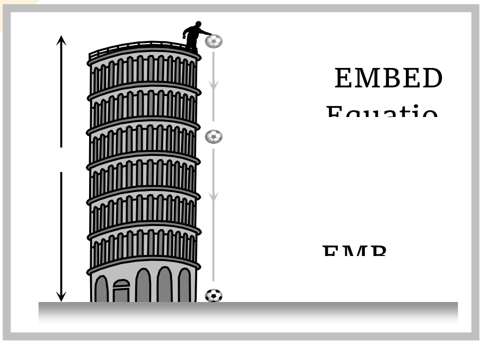
$a = +g$ [As acceleration is in the direction of motion]

$v = g\,t$ …(i)

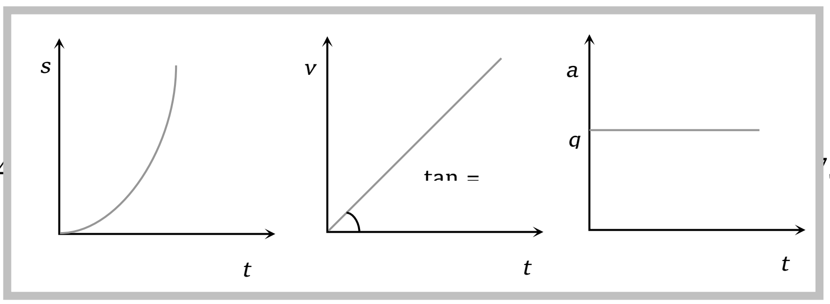
$h = \frac{1}{2}gt^2$ …(ii)

$\upsilon^2 = 2gh$ …(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)



(ii) Graph of distance velocity and acceleration with respect to time :



(iii) As $h = (1/2)gt^2$, *i.e.*, $h \propto t^2$, distance covered in time t, $2t$, $3t$, *etc*., will be in the ratio of $1^2 : 2^2 : 3^2$, *i.e.*, square of integers.

(iv) The distance covered in the *nth sec*, $h_n = \frac{1}{2} g(2n-1)$

So distance covered in I, II, III *sec*, *etc*., will be in the ratio of 1 : 3 : 5, *i.e.*, odd integers only.

(2) **If a body is projected vertically downward with some initial velocity**

Equation of motion : $\upsilon = u + gt$

$$h = ut + \frac{1}{2}gt^2$$

$$\upsilon^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

(3) **If a body is projected vertically upward**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$\upsilon = u - gt$; $h = ut - \frac{1}{2}gt^2$; $\upsilon^2 = u^2 - 2gh$; $h_n = u - \frac{g}{2}(2n-1)$

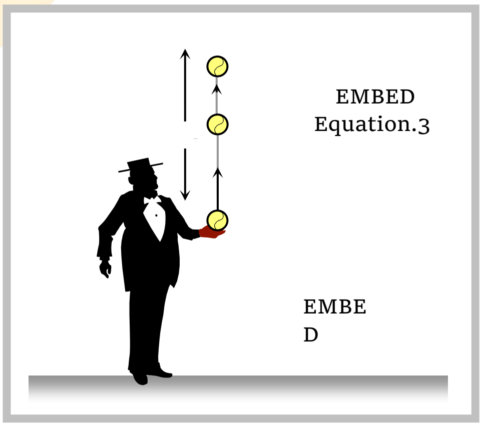
(ii) For maximum height $v = 0$

So from above equation

$u = gt$,

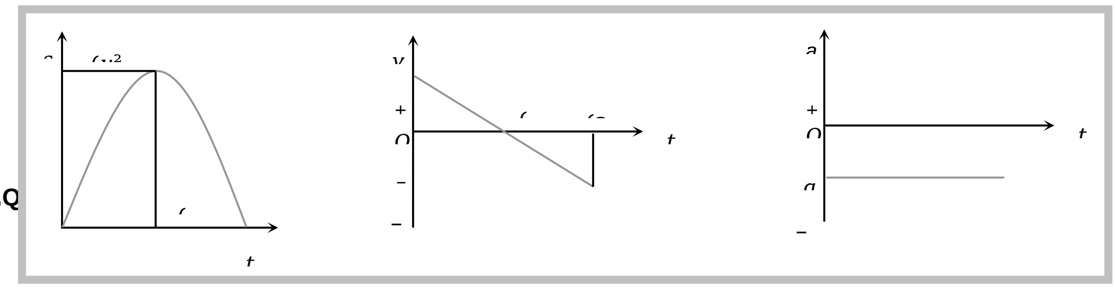
$h = \frac{1}{2}gt^2$

and $u^2 = 2gh$



(iii) Graph of distance, velocity and acceleration with respect to time (for maximum height) :

H.Q



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

(5) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

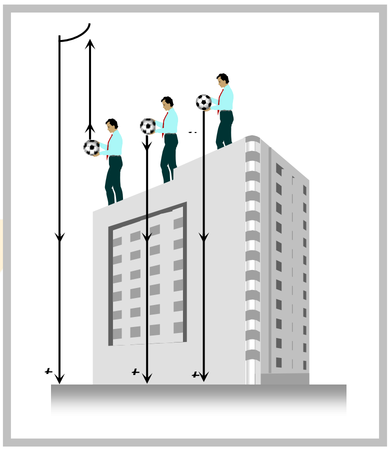
(6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_1) = time of ascent (t_2) = u/g

4 Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

(7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(8) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then



$t = \sqrt{t_1 t_2}$

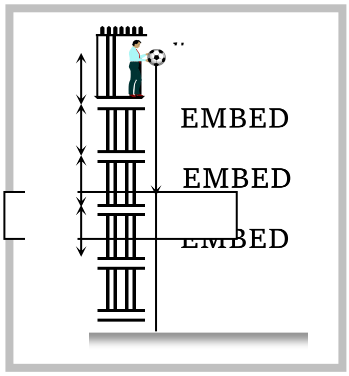
(9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \dfrac{u}{g+a}$ and $h = \dfrac{u^2}{2(g+a)}$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So $h = \frac{1}{2}(g-a)t_2^2$ ® $\frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2$ ® $t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$



Comparing t_1 and t_2 we can say that $t_2 > t_1$ since $(g + a) > (g - a)$

(10) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1*m* each will then be in the ratio of the difference in the square roots of the integers *i.e.*

$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}).......(\sqrt{4}-\sqrt{3}),........$

Sample problems based on motion under gravity

***Problem* 47.** If a body is thrown up with the velocity of 15 *m/s* then maximum height attained by the body is ($g = 10\ m/s^2$)

[MP PMT 2003]

(a) 11.25 *m* (b) 16.2 *m* (c) 24.5 *m* (d) 7.62 *m*

Solution : (a) $H_{max} = \frac{u^2}{2g} = \frac{(15)^2}{2 \times 10} = 11.25m$

***Problem* 48.** A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is $(g = 10m/s^2)$ **[MP PET 2003]**

(a) 25*m* (b) 45*m* (c) 90*m* (d) 125*m*

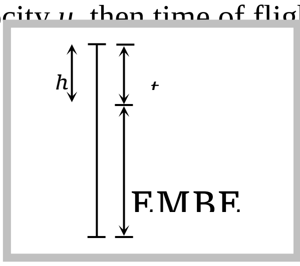
Solution : (b) $h_n = \frac{g}{2}(2n-1) \Rightarrow h_{5th} = \frac{10}{2}(2 \times 5 - 1) = 45\ m.$

***Problem* 49.** If a ball is thrown vertically upwards with speed *u*, the distance covered during the last *t* seconds of its ascent is

[CBSE 2003]

(a) $\frac{1}{2}gt^2$ (b) $ut - \frac{1}{2}gt^2$ (c) $(u-gt)t$ (d) ut

Solution : (a) If ball is thrown with velocity *u*, then time of flight $= \frac{u}{g}$



velocity after $\left(\frac{u}{g}-t\right)$ sec : $v = u - g\left(\frac{u}{g}-t\right) = gt.$

So, distance in last '*t*' *sec* : $0^2 = (gt)^2 - 2(g)h.$

® $h = \frac{1}{2}gt^2.$

***Problem* 50.** A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time (Given $g = 9.8m/s^2$) **[CBSE PMT 2003]**

(a) At least 0.8 *m/s* (b) Any speed less than 19.6 *m/s*

(c) Only with speed 19.6 *m/s* (d) More than 19.6 *m/s*

Solution : (d) Interval of ball throw = 2 *sec.*

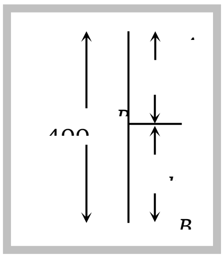
If we want that minimum three (more than two) ball remain in air then time of flight of first ball must be greater than 4 *sec*. *i.e.* $T > 4\ sec$ or $\frac{2U}{g} > 4\ sec \Rightarrow u > 19.6\ m/s$

It is clear that for $u = 19.6$ First ball will just strike the ground (in sky), second ball will be at highest point (in sky), and third ball will be at point of projection or on ground (not in sky).

Problem **51.** A man drops a ball downside from the roof of a tower of height 400 *meters*. At the same time another ball is thrown upside with a velocity 50 *meter/sec*. from the surface of the tower, then they will meet at which height from the surface of the tower

(a) 100 *meters* (b) 320 *meters* (c) 80 *meters* (d) 240 *meters*

Solution : (c) Let both balls meet at point *P* after time *t*.



The distance travelled by ball *A* $(h_1) = \frac{1}{2} gt^2$ .....(i)

The distance travelled by ball *B* $(h_2) = ut - \frac{1}{2} gt^2$ .....(ii)

By adding (i) and (ii) $h_1 + h_2 = ut$ = 400 (Given $h = h_1 + h_2 = 400$.)

$\therefore$ $t = 400/50 = 8\,sec$ and $h_1 = 320\ m,\ h_2 = 80\ m$

Problem **52.** A very large number of balls are thrown vertically upwards in quick succession in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5*m*, the number of ball thrown per minute is (take $g = 10\,ms^{-2}$)

(a) 120 (b) 80 (c) 60 (d) 40

Solution : (c) Maximum height of ball = 5*m*, So velocity of projection $\Rightarrow u = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10\ m/s$

time interval between two balls (time of ascent) = $\frac{u}{g} = 1\,sec = \frac{1}{60}\,min.$

So no. of ball thrown per min = 60

Problem **53.** A particle is thrown vertically upwards. If its velocity at half of the maximum height is 10 *m/s*, then maximum height attained by it is (Take $g = 10\ m/s^2$) **[CBSE PMT 2001]**

(a) 8 *m* (b) 10 *m* (c) 12 *m* (d) 16 *m*

Solution : (b) Let particle thrown with velocity *u* and its maximum height is H then $H = \frac{u^2}{2g}$

When particle is at a height $H/2$, then its speed is $10\ m/s$

From equation $v^2 = u^2 - 2gh$, $(10)^2 = u^2 - 2g\left(\frac{H}{2}\right) = u^2 - 2g\frac{u^2}{4g}$ ® $u^2 = 200$

$\therefore$ Maximum height $H = \frac{u^2}{2g} = \frac{200}{2 \times 10}$ = 10*m*

Problem **54.** A stone is shot straight upward with a speed of 20 *m/sec* from a tower 200 *m* high. The speed with which it strikes the ground is approximately

(a) 60 *m/sec* (b) 65 *m/sec* (c) 70 *m/sec* (d) 75 *m/sec*

Solution : (b) Speed of stone in a vertically upward direction is 20 *m/s*. So for vertical downward motion we will consider $u = -20\ m/s$

$v^2 = u^2 + 2gh = (-20)^2 + 2 \times 10 \times 200 \Rightarrow v = 65\ \ m/s$

Problem **55.** A body freely falling from the rest has a velocity '*v*' after it falls through a height '*h*'. The distance it has to fall down for its velocity to become double, is

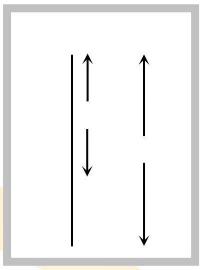
(a) $2h$ (b) $4h$ (c) $6h$ (d) $8h$

Solution : (b) Let at point A initial velocity of body is equal to zero

For path *AB* : $v^2 = 0 + 2gh$ … (i)

For path *AC* : $(2v)^2 = 0 + 2gx$ ® $4v^2 = 2gx$ … (ii)

Solving (i) and (ii) $x = 4h$



Problem **56.** A body sliding on a smooth inclined plane requires 4 *seconds* to reach the bottom starting from rest at the top. How much time does it take to cover one-fourth distance starting from rest at the top **[BHU 1998]**

(a) 1 *s* (b) 2 *s* (c) 4 *s* (d) 16 *s*

Solution : (b) $S = \frac{1}{2}at^2$ ® $t \propto \sqrt{s}$ (As a = constant)

$$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2} \Rightarrow t_2 = \frac{t_1}{2} = \frac{4}{2} = 2\,s$$

Problem **57.** A stone dropped from a building of height *h* and it reaches after *t* seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity *u* and they reach the earth surface after t_1 and t_2 seconds respectively, then

(a) $t = t_1 - t_2$ (b) $t = \frac{t_1 + t_2}{2}$ (c) $t = \sqrt{t_1 t_2}$ (d) $t = t_1^2\, t_2^2$

Solution : (c) For first case of dropping $h = \frac{1}{2}gt^2$.

For second case of downward throwing $h = -ut_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt^2$

® $-ut_1 = \frac{1}{2}g(t^2 - t_1^2)$ ......(i)

For third case of upward throwing $h = ut_2 + \frac{1}{2}gt_2^2 = \frac{1}{2}gt^2$

® $ut_2 = \frac{1}{2}g(t^2 - t_2^2)$ ....(ii)

on solving these two equations : $-\frac{t_1}{t_2} = \frac{t^2 - t_1^2}{t^2 - t_2^2}$ ® $t = \sqrt{t_1 t_2}$.

Problem **58.** By which velocity a ball be projected vertically downward so that the distance covered by it in 5th second is twice the distance it covers in its 6th second ($g = 10\ m/s^2$) **[CPMT 1997; MH CET 2000]**

(a) $58.8\ m/s$ (b) $49\ m/s$ (c) $65\ m/s$ (d) $19.6\ m/s$

Solution : (c) By formula $h_n = u + \frac{1}{2}g(2n-1)$ ®

$u - \frac{10}{2}[2\times 5 - 1] = 2\{u - \frac{10}{2}[2\times 6 - 1]\}$

® $u - 45 = 2\times(u - 55)$ ® $u = 65\,m/s.$

Problem **59.** Water drops fall at regular intervals from a tap which is 5 *m* above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant

(a) 2.50 *m* (b) 3.75 *m* (c) 4.00 *m* (d) 1.25 *m*

Solution : (b) Let the interval be *t* then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ .....(i) For second drop $x = \frac{1}{2}gt^2$ .....(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and hence required height $h = 5 - \frac{5}{4} = 3.75\,m.$

Problem **60.** A balloon is at a height of 81 *m* and is ascending upwards with a velocity of $12\,m/s$. A body of 2 *kg* weight is dropped from it. If $g = 10\,m/s^2$, the body will reach the surface of the earth in

(a) 1.5 *s* (b) 4.025 *s* (c) 5.4 *s* (d) 6.75 *s*

Solution : (c) As the balloon is going up we will take initial velocity of falling body $= -12\,m/s,\ h = 81\,m,\ g = +10\,m/s^2$

By applying $h = ut + \frac{1}{2}gt^2$; $81 = -12t + \frac{1}{2}(10)t^2$ ® $5t^2 - 12t - 81 = 0$

® $t = \frac{12 \pm \sqrt{144 + 1620}}{10} = \frac{12 \pm \sqrt{1764}}{10} \approx 5.4$ sec.

Problem **61.** A particle is dropped under gravity from rest from a height $h (g = 9.8\,m/s^2)$ and it travels a distance 9*h*/25 in the last second, the height *h* is

(a) 100 *m* (b) 122.5 *m* (c) 145 *m* (d) 167.5 *m*

Solution : (b) Distance travelled in n sec = $\frac{1}{2}gn^2 = h$ .....(i)

Distance travelled in n^{th} sec $= \frac{g}{2}(2n-1) = \frac{9h}{25}$ .....(ii)

Solving (i) and (ii) we get. $h = 122.5\,m$.

Problem **62.** A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is **[CPMT 1983; Kerala PMT 2002]**

(a) 4.9 *m* (b) 9.8 *m* (c) 19.6 *m* (d) 24.5 *m*

Solution : (d) The separation between two bodies, two second after the release of second body is given by :

$s = \frac{1}{2}g(t_1^2 - t_2^2) = \frac{1}{2}\times 9.8\times(3^2 - 2^2) = 24.5\ m.$

2.12 Motion With Variable Acceleration

(i) If acceleration is a function of time

$a = f(t)$ then $v = u + \int_0^t f(t)dt$ and $s = ut + \int\left(\int f(t)dt\right)dt$

(ii) If acceleration is a function of distance

$a = f(x)$ then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)dx$

(iii) If acceleration is a function of velocity

$a = f(v)$ then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{vdv}{f(v)}$

Sample problems based on variable acceleration

Problem 63. An electron starting from rest has a velocity that increases linearly with the time that is $v = kt$, where $k = 2m/\sec^2$. The distance travelled in the first 3 *seconds* will be

(a) 9 *m* (b) 16 *m* (c) 27 *m* (d) 36 *m*

Solution : (a) $x = \int_{t_1}^{t_2} v\,dt = \int_0^3 2t\,dt = 2\left[\frac{t^2}{2}\right]_0^3 =$ 9 *m.*

Problem 64. The acceleration of a particle is increasing linearly with time *t* as *bt*. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time *t* will be

(a) $v_0 t + \frac{1}{3}bt^2$ (b) $v_0 t + \frac{1}{3}bt^3$ (c) $v_0 t + \frac{1}{6}bt^3$ (d) $v_0 t + \frac{1}{2}bt^2$

Solution : (c) $\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt = \int_{t_1}^{t_2} (bt)dt$

$\Rightarrow v_2 - v_1 = \left(\frac{bt^2}{2}\right)_{t_1}^{t_2}$

® $v_2 = v_1 + \left(\frac{bt^2}{2}\right)_0^t = v_0 + \frac{bt^2}{2}$

® $S = \int v_0 dt + \int \frac{bt^2}{2} dt = v_0 t + \frac{1}{6}bt^3$

Problem 65. The motion of a particle is described by the equation $u = at$. The distance travelled by the particle in the first 4 seconds

(a) $4a$ (b) $12a$ (c) $6a$ (d) $8a$

Solution : (d) $u = at$ ® $\frac{ds}{dt} = at$ ® $s = \int_0^4 at\,dt = a\left[\frac{t^2}{2}\right]_0^4 =$ $8a$.