Medical Image Processing for Interventional Applications

Feature Descriptors – SIFT (Part 1)

Online Course – Unit 11

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Pattern Recognition Lab (CS 5)

Topics

Feature Descriptors

SIFT – Feature Detection

- Scale Space
- Laplace of Gaussians (LoG)
- Difference of Gaussians (DoG)

Summary

- Take Home Messages
- Further Readings

Feature Descriptors

Basic principle: Describe the neighborhood of a key point in an invariant manner → feature vector.

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Popular: analysis of local gradient distribution (SIFT, SURF, HOG, GLOH, RIFF …)

SIFT – Scale Invariant Feature Transform

1. Scale-space extrema detection → feature detection
2. Key point localization and filtering → feature selection
3. Orientation assignment → local coordinate system
4. Computation of key point descriptor → encode local gradient distribution

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SIFT – Scale-space Extrema Detection (cf. Lowe, 2004)

→ Feature detector

Challenge: scale invariance

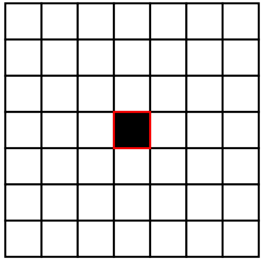

Figure 1: Feature detector fits object size

SIFT – Scale-space Extrema Detection (cf. Lowe, 2004)

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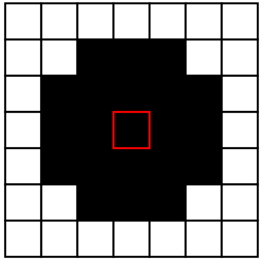


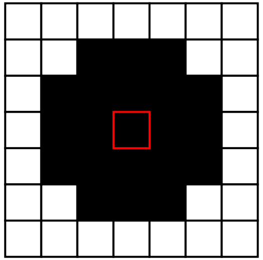
Figure 2: Different detector/object scales

SIFT – Scale-space Extrema Detection (cf. Lowe, 2004)

→ Feature detector

Challenge: scale invariance

What about Harris corner detector?



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Objects have characteristic scale where they 'make sense'.

→ Search over all scales and image locations!

Scale-space Representation

- Represent an image as one-parameter family of Gaussian-smoothed images
- Scale as a third image dimension (x, y, σ)

$$L(x,y,\sigma) = G(x,y,\sigma) * I(x,y), \qquad G(x,y,\sigma) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

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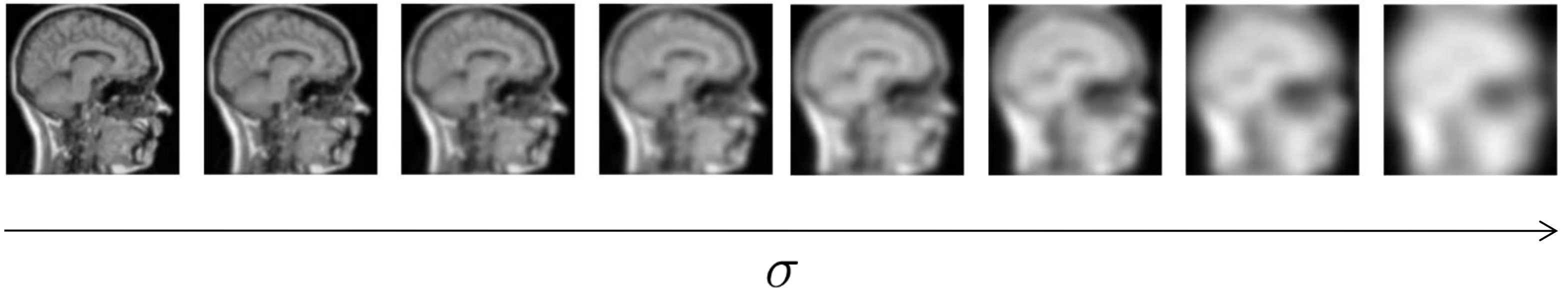

Laplacian of Gaussian (cf. Marr and Hildreth, 1980)

Laplacian of Gaussian (LoG):

$$\nabla^2\big(G(x,y,\sigma) * I(x,y)\big), \qquad G(x,y,\sigma) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

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LoG/Marr-Hildreth operator:

$$\nabla^2 G(x,y,\sigma) = \frac{\partial^2 G(x,y,\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y,\sigma)}{\partial y^2}$$

$$= -\frac{1}{\pi\sigma^4}\left(1 - \frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

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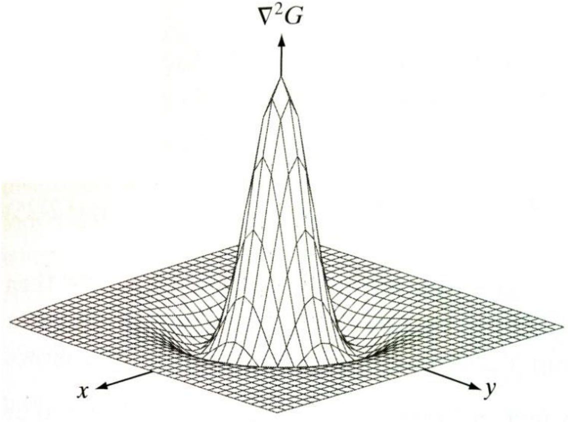


Figure 4: LoG mesh plot

LoG: Examples

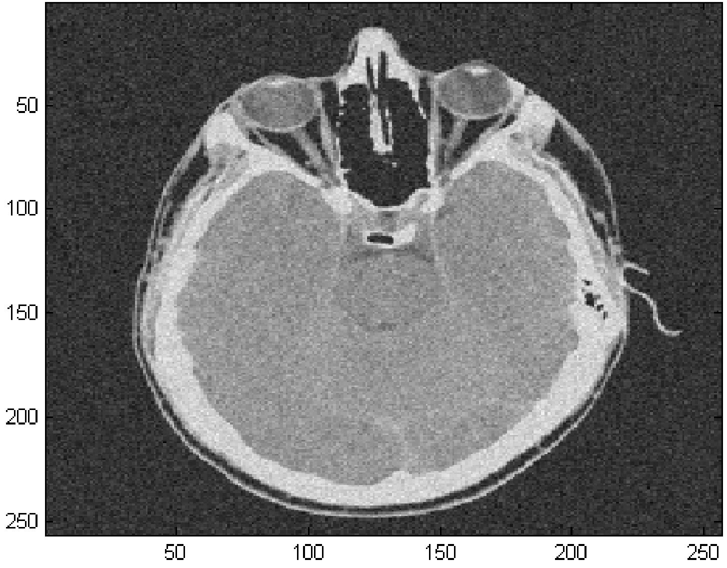


Figure 5: Noisy input

Figure 6: Laplace operator

LoG: Examples

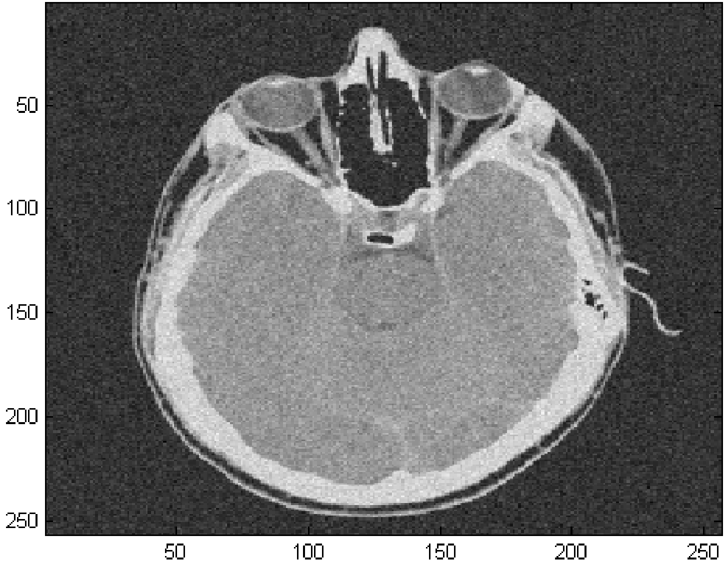


Figure 5: Noisy input

$\nabla^2 G(x, y, \sigma)$

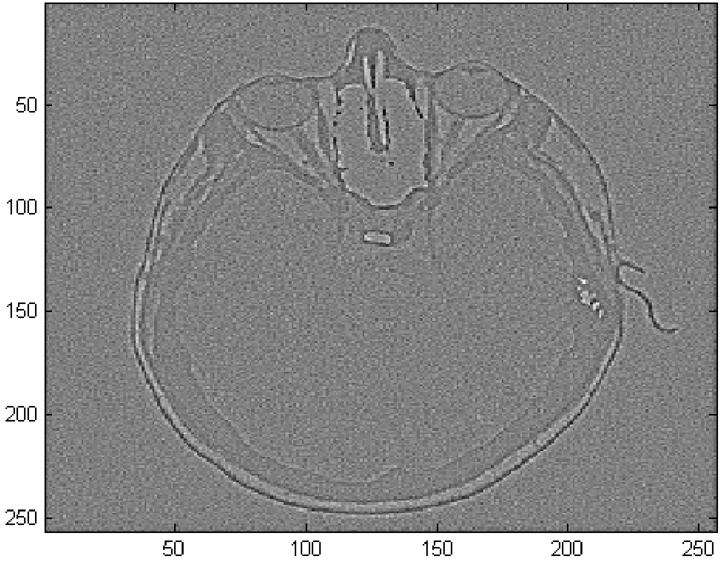
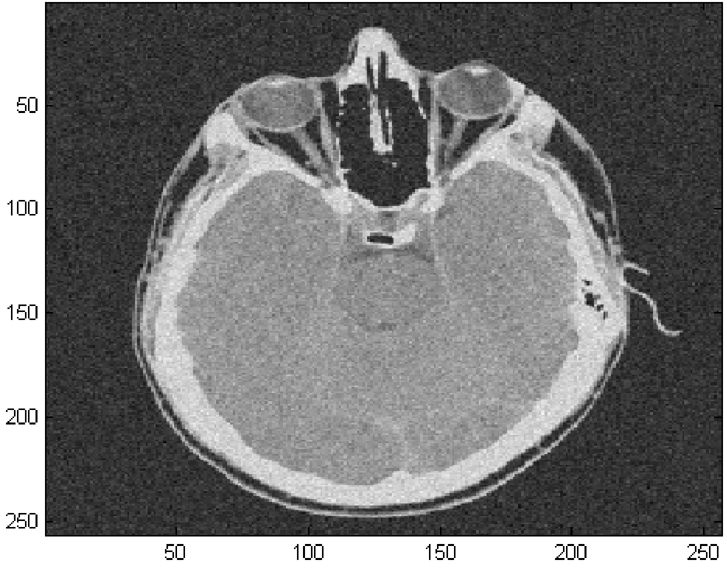


Figure 7: LoG operator, $\sigma = 0.5$

LoG: Examples



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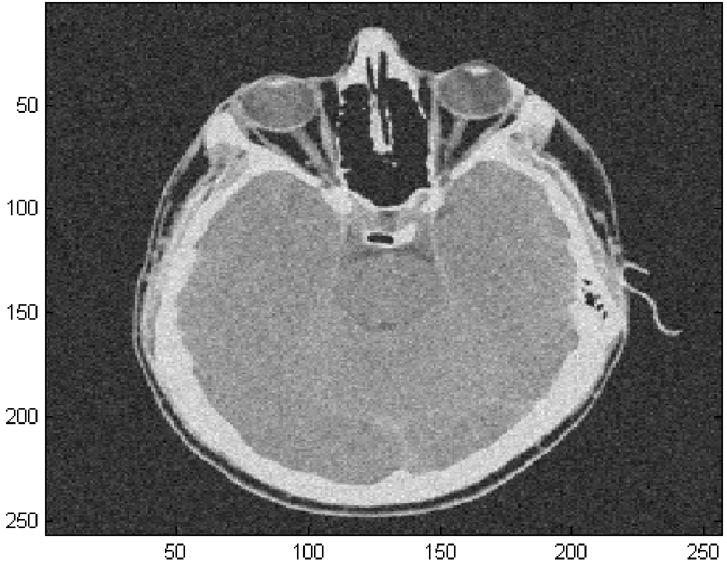
$\nabla^2 G(x, y, \sigma)$

Figure 8: LoG operator, $\sigma = 1$

LoG: Examples

$$\nabla^2 G(x, y, \sigma)$$



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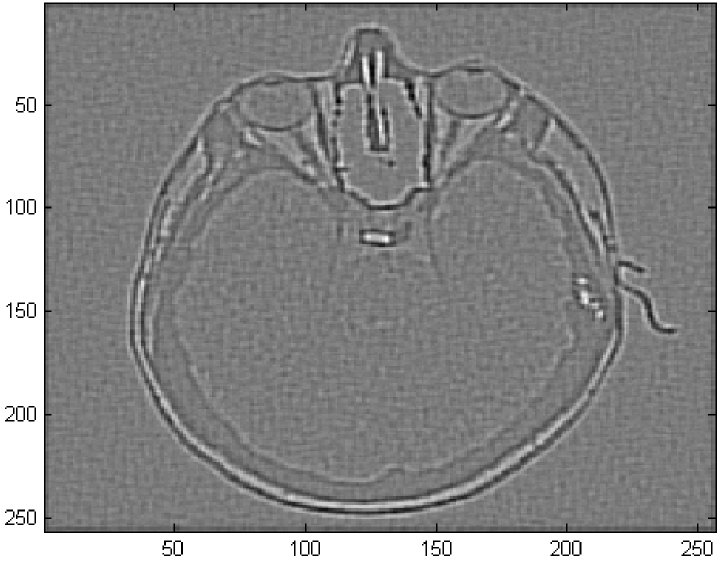

Figure 9: LoG operator, $\sigma = 2$

Difference of Gaussians (cf. Lowe, 2004)

Difference of Gaussians (DoG):

→ Approximation of scale-normalized LoG operator $\sigma^2 \nabla^2 G(x, y)$

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y)$$

$$D_k(x, y, \sigma) = \big(G(x, y, k\sigma) - G(x, y, \sigma)\big) * I(x, y)$$

$$= L(x, y, k\sigma) - L(x, y, \sigma), \qquad 0 < k < +\infty$$

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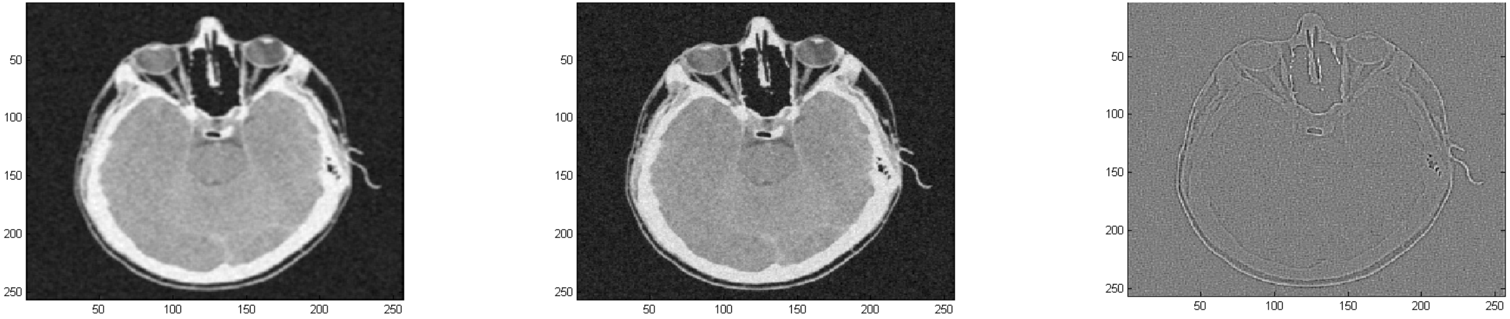

Figure 10: $L(x, y, k\sigma)$ on the left, $L(x, y, \sigma)$ in the middle, and $D_k(x, y, \sigma)$ on the right

Difference of Gaussians (cf. Lowe, 2004)

Difference of Gaussians (DoG):

$\rightarrow$ Approximation of scale-normalized LoG operator $\sigma^2 \nabla^2 G(x, y)$

$$
\begin{aligned}
L(x, y, \sigma) &= G(x, y, \sigma) * I(x, y) \\
D_k(x, y, \sigma) &= \big(G(x, y, k\sigma) - G(x, y, \sigma)\big) * I(x, y) \\
&= L(x, y, k\sigma) - L(x, y, \sigma), \qquad 0 < k < +\infty
\end{aligned}
$$

Interpretation in the frequency domain?

DoG Scale-space (cf. Lowe, 2004)

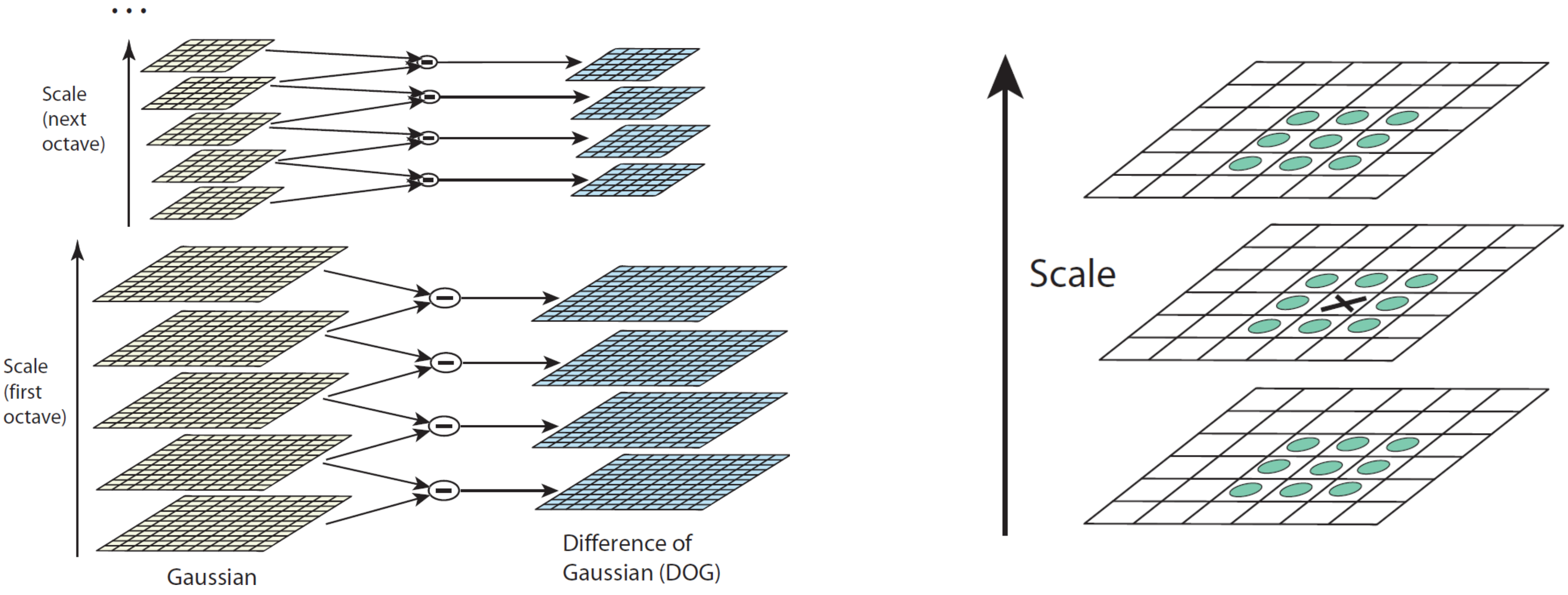

Figure 11: Detect local extrema **across scale and space** → characteristic scale σ

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- Many feature descriptors are based on an analysis of the derivatives of an image.
- In order to make the feature detector scale invariant, this analysis is usually performed on different scales.
- The **s**cale **i**nvariant **f**eature **t**ransform (SIFT) utilizes differences of Gaussians (DoG) to detect extrema in scale-space.

Credits:
We acknowledge the contributions of F.F. Li, E. Angelopoulou, D. Lowe, and A. Berg for their material in units 9-14 (on feature detectors/descriptors).

Further Readings

- David G. Lowe. “Distinctive Image Features from Scale-Invariant Keypoints”. In: *International Journal of Computer Vision* 60.2 (Nov. 2004), pp. 91–110. DOI: 10.1023/B:VISI.0000029664.99615.94
- D. Marr and E. Hildreth. “Theory of Edge Detection”. In: *Proceedings of the Royal Society of London B: Biological Sciences* 207.1167 (Feb. 1980), pp. 187–217. DOI: 10.1098/rspb.1980.0020
- Chris Harris and Mike Stephens. “A Combined Corner and Edge Detector”. In: *Proceedings of Fourth Alvey Vision Conference*. 1988, pp. 147–152