Medical Image Processing for Interventional Applications **Guided Filter** Online Course - Unit 16 Andreas Maier, Jakob Wasza, Frank Schebesch Pattern Recognition Lab (CS 5)













Topics

Guided Filter

Summary

Take Home Messages

Further Readings







- Proposed at the ECCV 2010 (later published as <u>He, Sun and Tang, 2013</u>)
- Applications:
 - Non-approximative edge-preserving denoising
 - HDR compression
 - Multi-modal image upsampling
 - ..
- Complexity: O(N)
- Basic idea: guide the filtering process by a dedicated image i(x).







Assumption: Output as a linear transform of the guidance i(x), i.e.:

$$f(\mathbf{x}') = a_{\mathbf{x}}i(\mathbf{x}') + b_{\mathbf{x}}, \forall \mathbf{x}' \in \omega_{\mathbf{x}}$$







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Output

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Cost function:

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$$\mathcal{J}(a_x, b_x) = \sum_{x' \in \omega_x} \left(\left(a_x i(x') + b_x - g(x') \right)^2 + \epsilon a_x^2 \right)$$

Regularization







$$\mathcal{J}(a_x, b_x) = \sum_{x' \in \omega_x} \left((a_x i(x') + b_x - g(x'))^2 + \epsilon a_x^2 \right)$$







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Partial derivatives:

$$\frac{\partial}{\partial a_x} \mathcal{J}(a_x, b_x) = 2 \sum_{x' \in \omega_x} \left((a_x i(x') + b_x - g(x')) i(x') + \epsilon a_x \right) = 0$$







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$$\frac{\partial}{\partial b_x} \mathcal{J}(a_x, b_x) = 2 \sum_{x' \in \omega_x} (a_x i(x') + b_x - g(x'))$$

$$= 2a_x \sum_{x' \in \omega_x} i(x') + 2b_x \sum_{x' \in \omega_x} 1 - 2 \sum_{x' \in \omega_x} g(x') \stackrel{!}{=} 0$$







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$$\sum_{x' \in \omega_x} 1 = |\omega_x|$$







Deriving b_x :

$$\frac{1}{2}\frac{\partial}{\partial b_x}\mathcal{J}(a_x,b_x) = a_x \sum_{x' \in \omega_x} i(x') + b_x |\omega_x| - \sum_{x' \in \omega_x} g(x') \stackrel{!}{=} 0$$







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$$b_x = \frac{1}{|\omega_x|} \sum_{x' \in \omega_x} g(x') - a_x \left(\frac{1}{|\omega_x|} \sum_{x' \in \omega_x} i(x') \right)$$







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Probabilistic formulation: Mean filtering yields the expectation value E_{ω_x} if we interprete g(x), i(x) as uniformly distributed random variables.







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$$\Rightarrow b_x = E_{\omega_x}[g(x)] - a_x E_{\omega_x}[i(x)]$$







Deriving
$$a_x$$
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$$\frac{1}{2}\frac{\partial}{\partial a_x}\mathcal{J}(a_x,b_x) = \sum_{x'\in\omega_x} \left((a_x i(x') + b_x - g(x'))i(x') + \epsilon a_x \right) = 0$$







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$$b_x = E_{\omega_x}[g(x)] - a_x E_{\omega_x}[i(x)]$$

$$a_{\boldsymbol{x}}\left(\sum_{\boldsymbol{x}'\in\omega_{\boldsymbol{x}}}i(\boldsymbol{x}')i(\boldsymbol{x}')-E_{\omega_{\boldsymbol{x}}}[i(\boldsymbol{x})]\sum_{\boldsymbol{x}'\in\omega_{\boldsymbol{x}}}i(\boldsymbol{x}')+\epsilon\sum_{\boldsymbol{x}'\in\omega_{\boldsymbol{x}}}1\right)=\sum_{\boldsymbol{x}'\in\omega_{\boldsymbol{x}}}g(\boldsymbol{x}')i(\boldsymbol{x}')-E_{\omega_{\boldsymbol{x}}}[g(\boldsymbol{x})]\sum_{\boldsymbol{x}'\in\omega_{\boldsymbol{x}}}i(\boldsymbol{x}')$$







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$$\Rightarrow a_{x} \left(\frac{1}{|\omega_{x}|} \sum_{x' \in \omega_{x}} i(x')i(x') - E_{\omega_{x}}[i(x)] \frac{1}{|\omega_{x}|} \sum_{x' \in \omega_{x}} i(x') + \epsilon \frac{1}{|\omega_{x}|} \sum_{x' \in \omega_{x}} 1 \right)$$

$$= \frac{1}{|\omega_{x}|} \sum_{x' \in \omega_{x}} g(x')i(x') - E_{\omega_{x}}[g(x)] \frac{1}{|\omega_{x}|} \sum_{x' \in \omega_{x}} i(x')$$

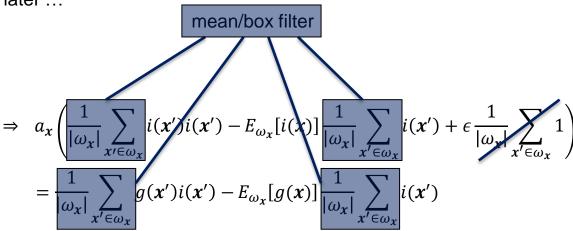






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Therefore we found:

$$a_{\mathbf{x}}\left(\mathbf{E}_{\omega_{\mathbf{x}}}[i(\mathbf{x})i(\mathbf{x})] - \mathbf{E}_{\omega_{\mathbf{x}}}[i(\mathbf{x})]\mathbf{E}_{\omega_{\mathbf{x}}}[i(\mathbf{x})] + \epsilon\right) = \mathbf{E}_{\omega_{\mathbf{x}}}[g(\mathbf{x})i(\mathbf{x})] - \mathbf{E}_{\omega_{\mathbf{x}}}[g(\mathbf{x})]\mathbf{E}_{\omega_{\mathbf{x}}}[i(\mathbf{x})].$$







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Using the computational formulas for variance and covariance:

$$Var(X) = E[X^2] - E[X]^2$$
, and $Cov[X, Y] = E[XY] - E[X]E[Y]$,

we finally obtain:

$$a_{x} = \frac{\operatorname{Cov}_{\omega_{x}}[g(x), i(x)]}{\operatorname{Var}_{\omega_{x}}[i(x)] + \epsilon}.$$







- Guided filtering, linear model: $f(x') = a_x i(x') + b_x$, $\forall x' \in \omega_x$
- Lets consider the case that i(x) = g(x):

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- Trivial case $\epsilon = 0$:
 - Flat patch: if g(x) is constant across ω_x : $a_x = 0, b_x = \mathbb{E}_{\omega_x}[g(x)]$
 - **High variance**: if g(x) changes across ω_x : $a_x = 1, b_x = 0$







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- The criterion of **flat patch** or **high variance** is given by ϵ .







- So far, only one local window was considered.
 - → Apply the model to all local windows.
 - \rightarrow Pixel x is involved in all local windows $\omega_{x'}$ that contain x.
 - \rightarrow Average all possible local coefficients $a_{x'}$, $b_{x'}$.







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→ Guided filtering can be expressed with mean filters only!







Mean or box filtering is the backbone of guided filtering:

$$\mathrm{E}_{\omega_{\mathbf{x}}}[g(\mathbf{x})] = \frac{1}{|\omega_{\mathbf{x}}|} \sum_{\mathbf{x}' \in \omega_{\mathbf{x}}} g(\mathbf{x}') = \{g * \mathcal{M}_r\}(\mathbf{x})$$







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Integral images (<u>Viola and Jones, 2001</u>):

$$\vartheta(x,y) = \sum_{\substack{x' \le x \\ y' \le y}} g(x',y')$$







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Mean filtering using integral images:

$$\{g * \mathcal{M}_r\}(x,y) = \left(\vartheta(x+r,y+r) - \vartheta(x+r,y-r-1)\right) - \left(\vartheta(x-r-1,y+r) - \vartheta(x-r-1,y-r-1)\right)$$



 $\mathcal{O}(1)$





Guided Filtering

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1	32	20	3	31	16
8	15	16	21	1	8
30	9	26	13	18	16
26	22	18	8	30	19
29	24	1	21	19	3
17	12	11	24	29	2

Figure 1: Image data

1	33	53	56	87	103
9	56	92	116	148	172
39	95	157	194	244	284
65	143	223	268	348	407
94	196	277	343	442	504
111	225	317	407	535	599

Figure 2: Integral image representation







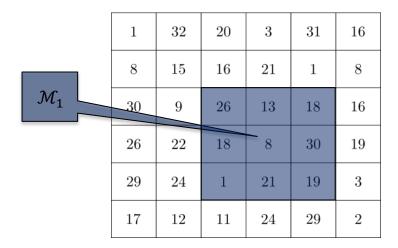


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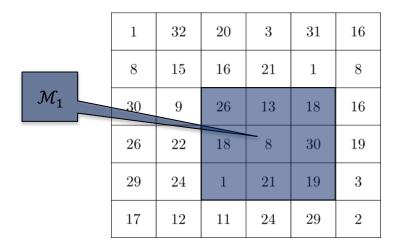


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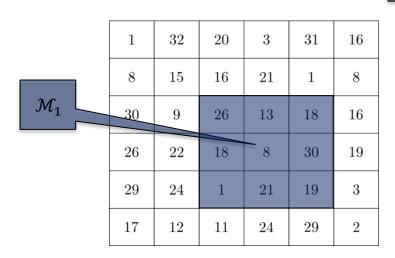


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$\vartheta(x-r-1,y-r-1)$								
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$$\vartheta(x+r,y+r)$$

26 + 13 + 18 + 18 + 8 + 30 + 1 + 21 + 19 = 154







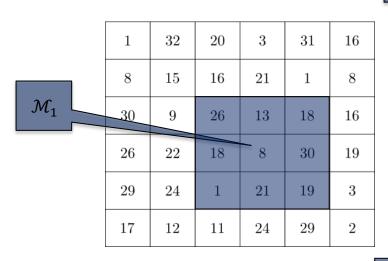
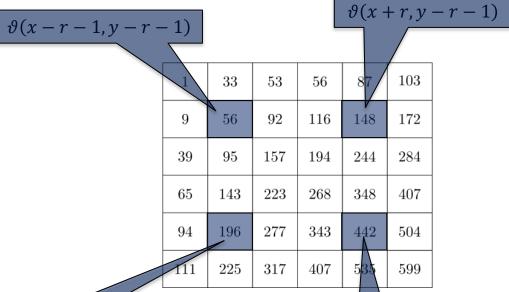


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 $\theta(x-r-1,y+r)$

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$$\theta(x+r,y+r)$$

$$26 + 13 + 18 + 18 + 8 + 30 + 1 + 21 + 19 = 154$$

$$(442 - 148) - (196 - 56) = 154$$







Topics

Guided Filter

Summary

Take Home Messages

Further Readings







Take Home Messages

- As the name expresses, a guided filter makes use of a guidance image to model, e.g., a smooth but edge-preserving filter.
- From the derivation of a guided filter by using a model that is linear in a neighborhood ω_x of x, we found a relationship of the filter with mean filtering of both guidance and target image.
- Mean or box filtering can be computed efficiently using integral images.







Further Readings

- Kaiming He, Jian Sun, and Xiaoou Tang. "Guided Image Filtering". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 35.6 (June 2013), pp. 1397–1409. DOI: 10.1109/TPAMI.2012.213
- Paul Viola and Michael Jones. "Rapid Object Detection Using a Boosted Cascade of Simple Features". In:
 Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition.
 CVPR 2001. Vol. 1. IEEE, Dec. 2001, pp. I-511—I-518. DOI: 10.1109/CVPR.2001.990517