Medical Image Processing for Interventional Applications

Epipolar Geometry – Part 2

Online Course – Unit 35

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TECHNISCHE FAKULTÄT

Topics

Radon Transform (Refresher)

Epipolar Geometry

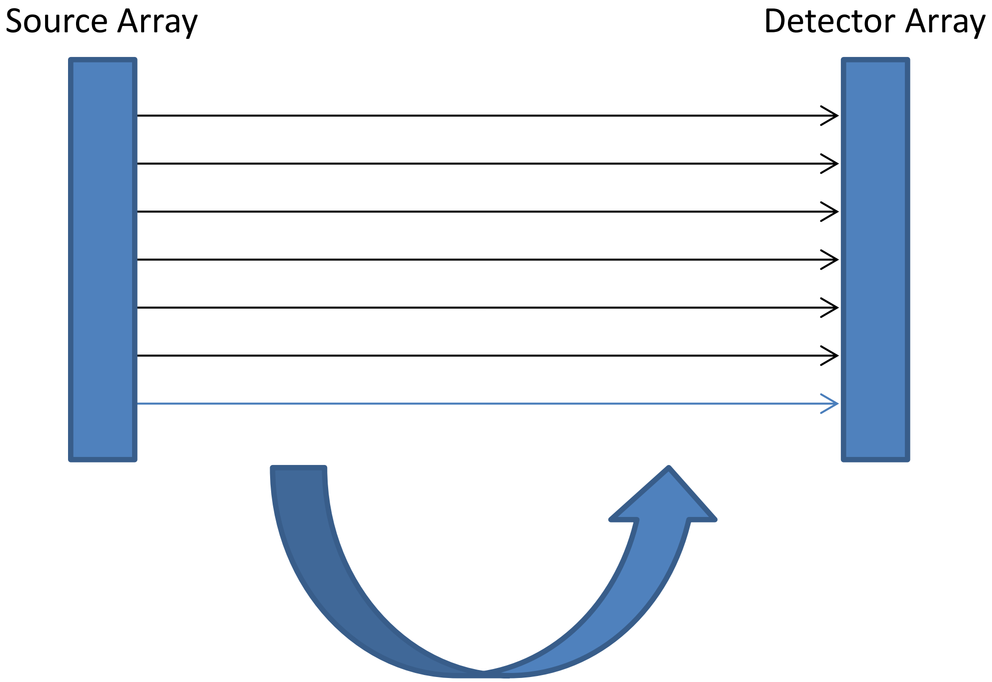
- In Diagrams
- Redundancies on Epipolar Lines
- Grangeat's Theorem

Applied Example

Summary

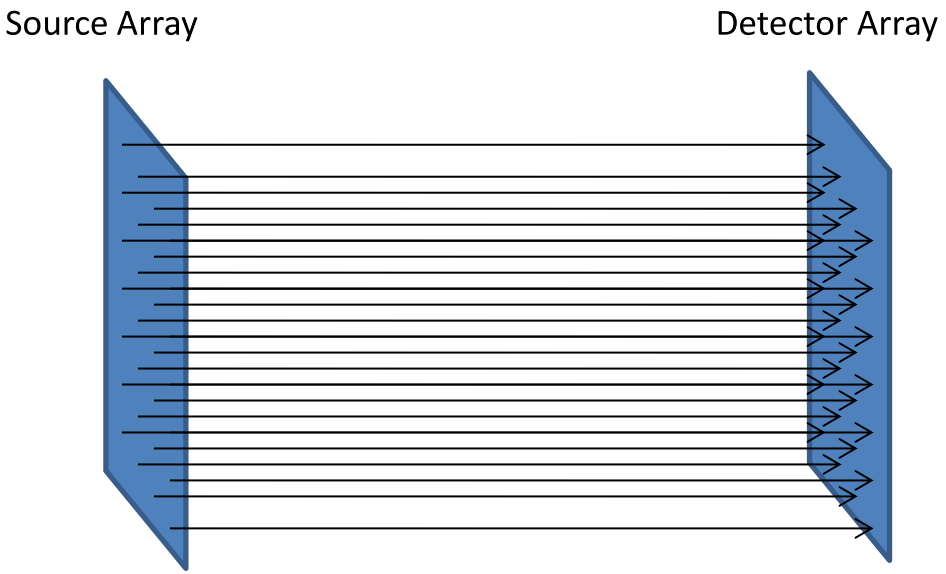
- Take Home Messages
- Further Readings

Radon Transform: 2-D Case



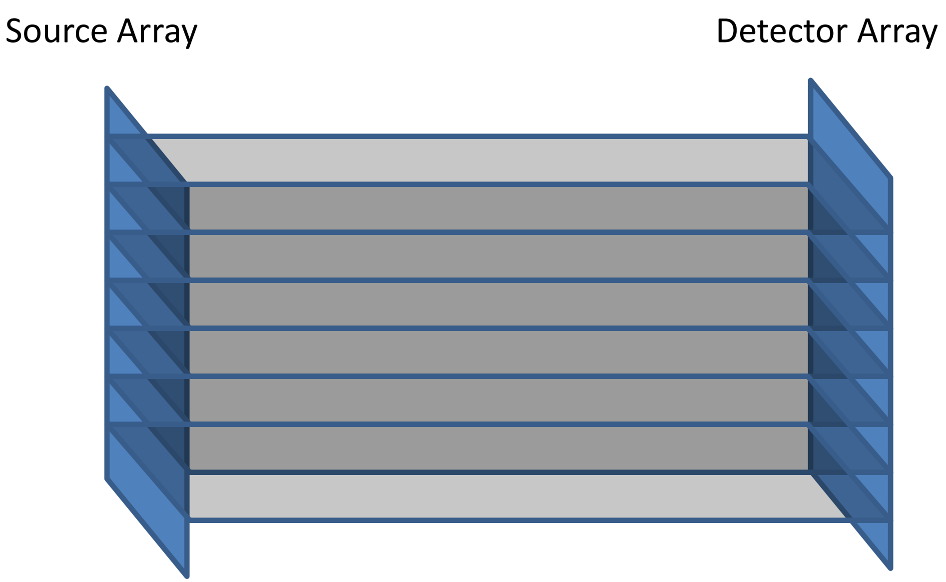
2-D Radon Transform | Sinogram | X-Ray Transform

Radon Transform: 3-D Case



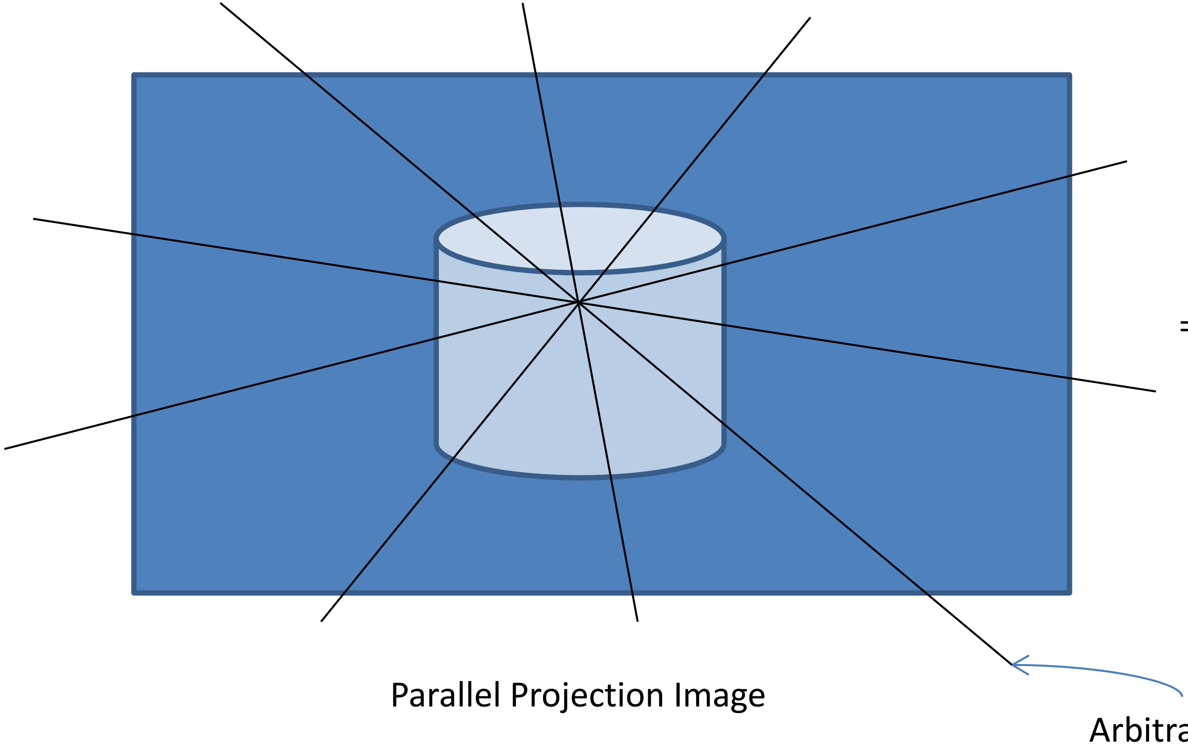
3-D X-Ray Transform !

Radon Transform: 3-D Case



3-D X-Ray Transform !

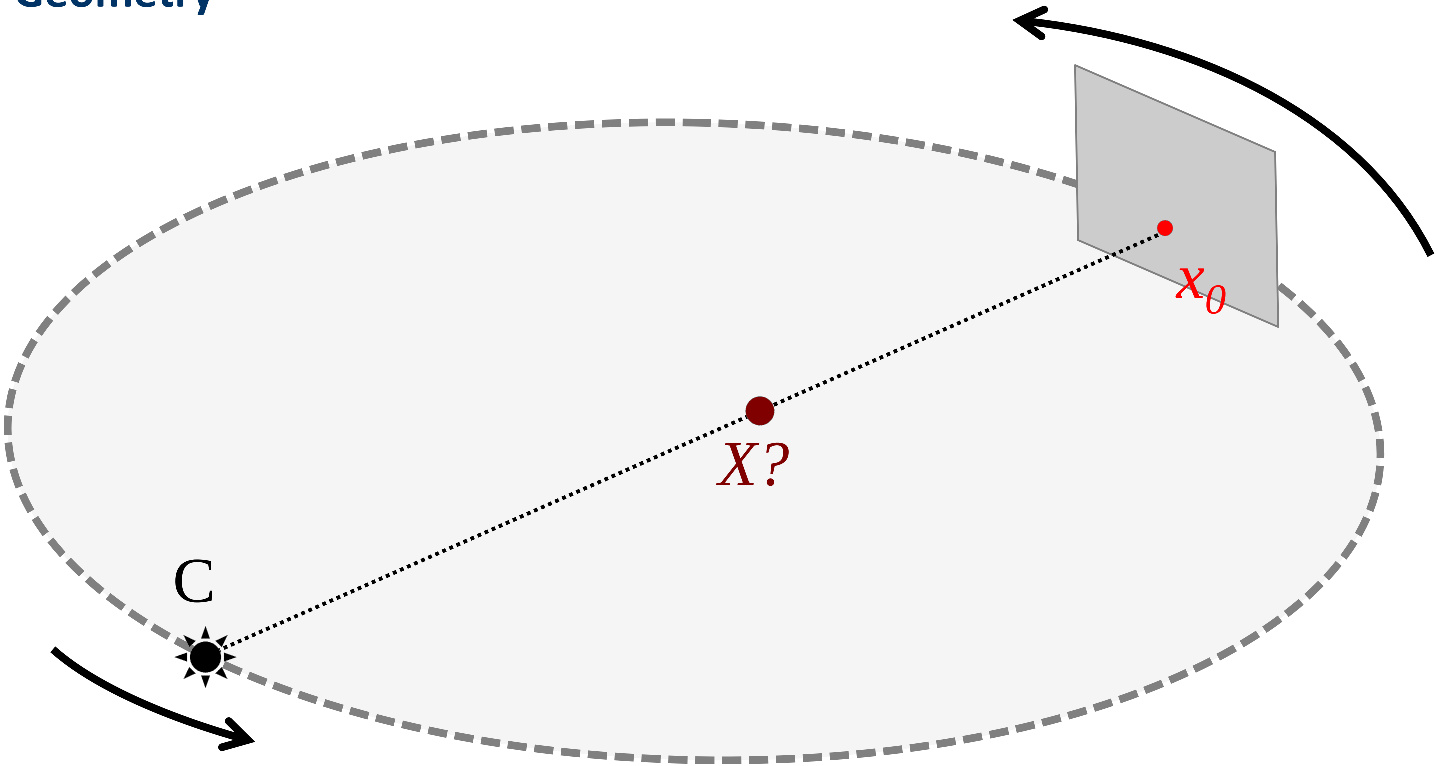
X-Ray Transform and Radon Transform



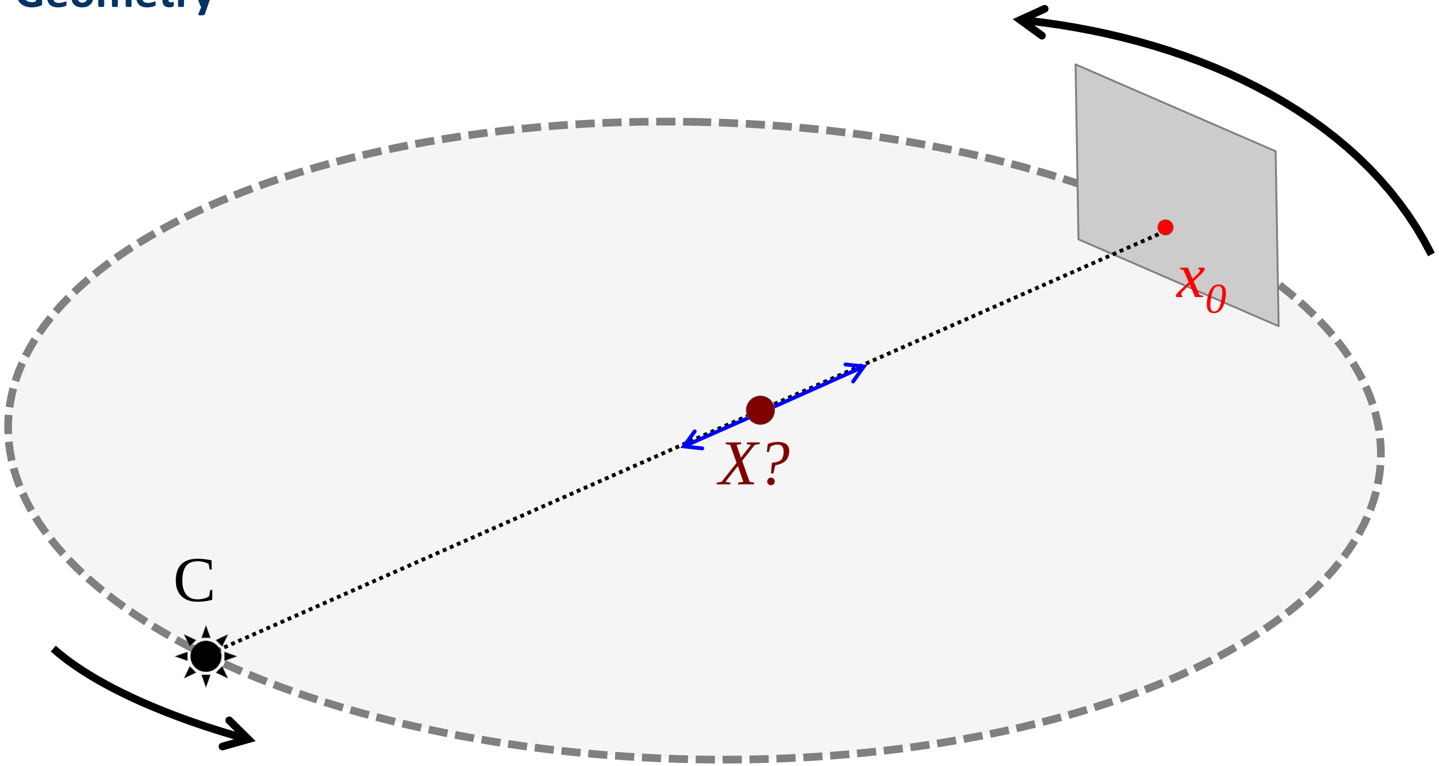
⇒ 2-D Radon Transfrom

Topics

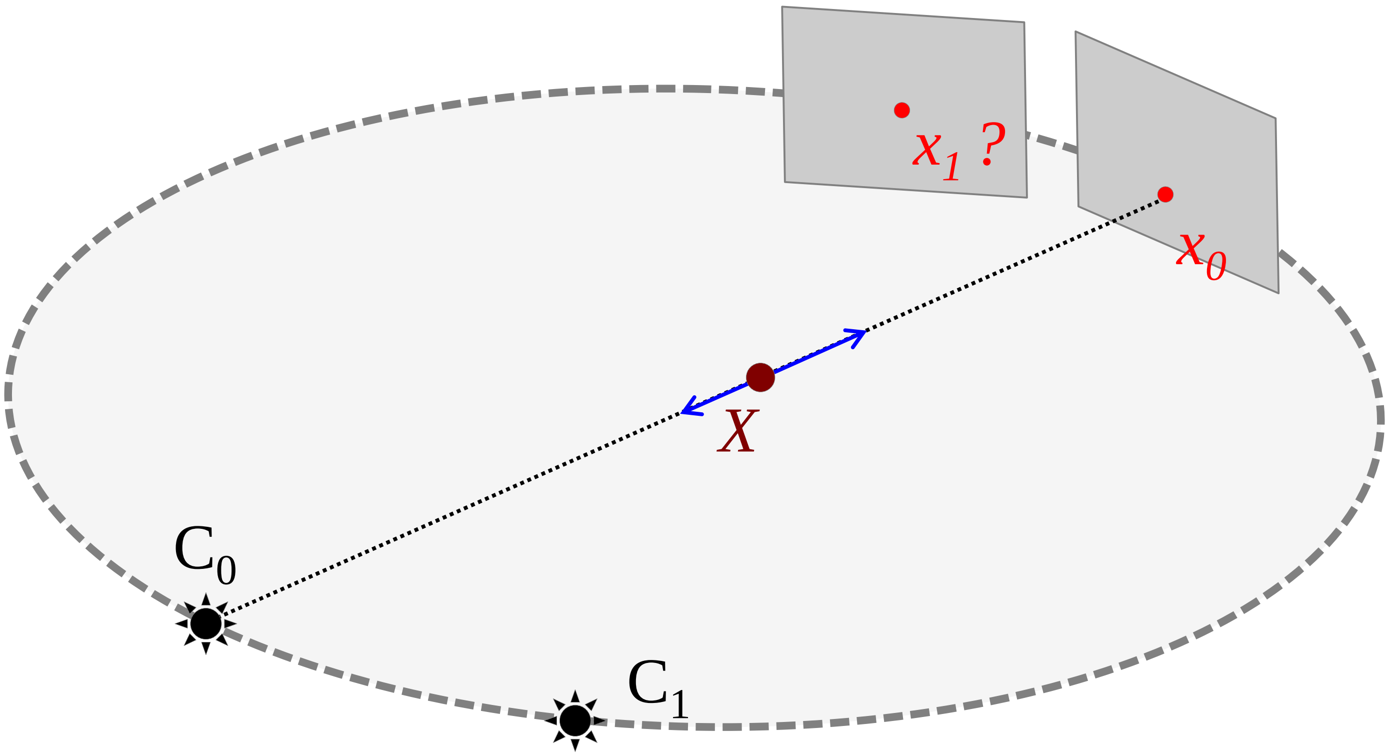
Epipolar Geometry



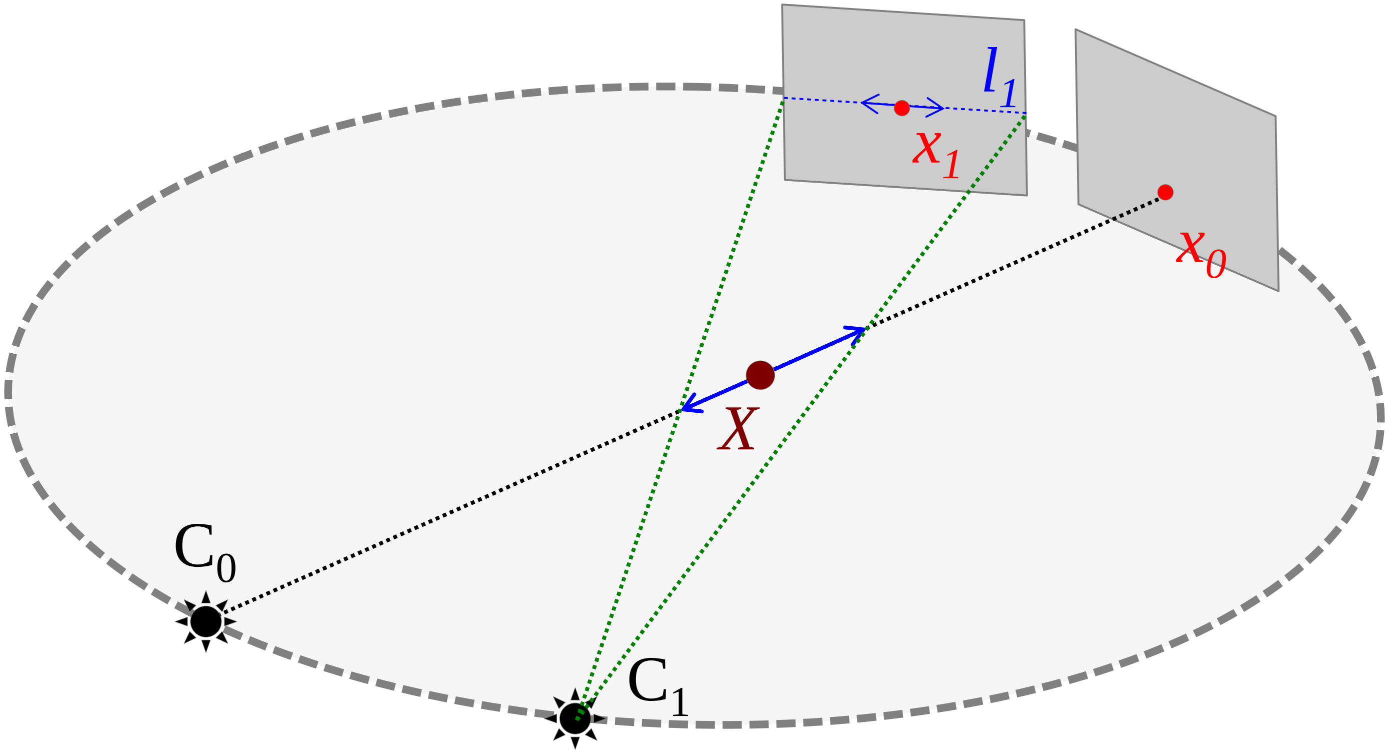
Epipolar Geometry



Epipolar Geometry

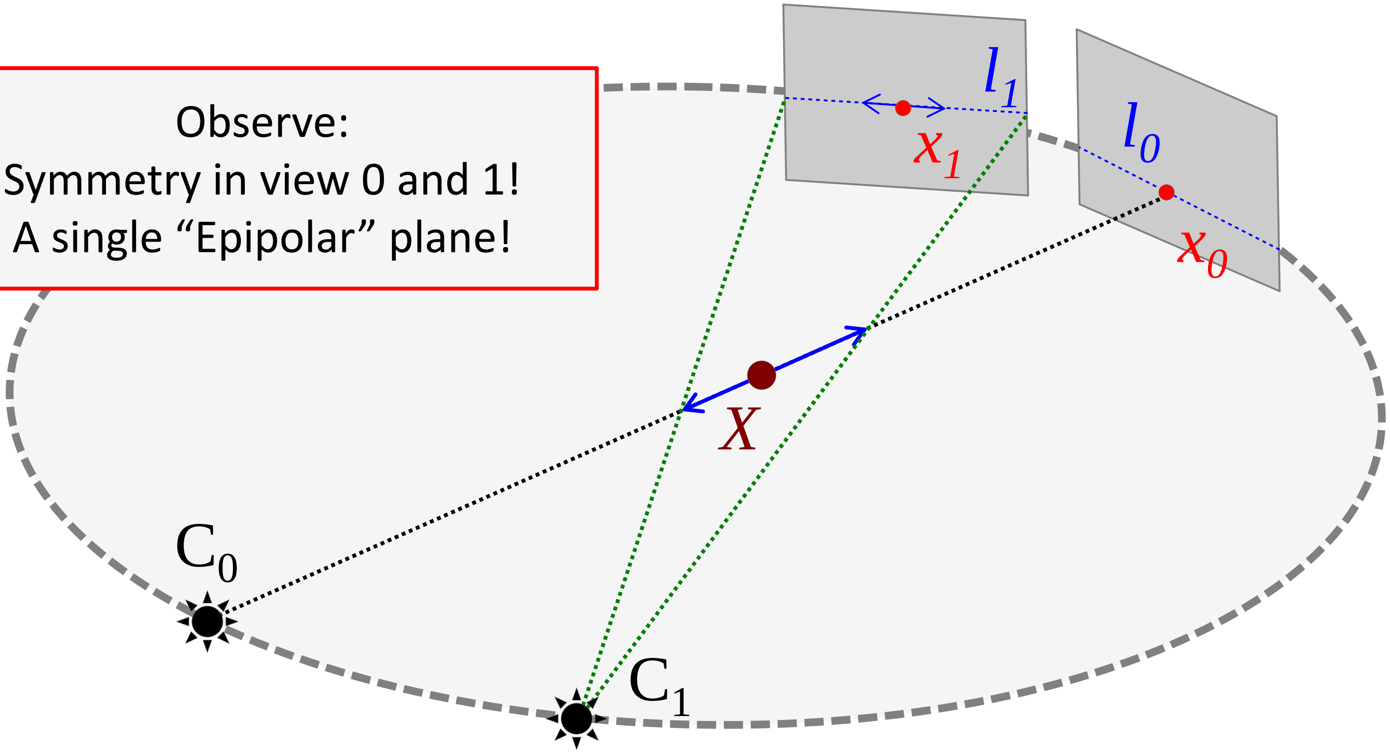


Epipolar Geometry

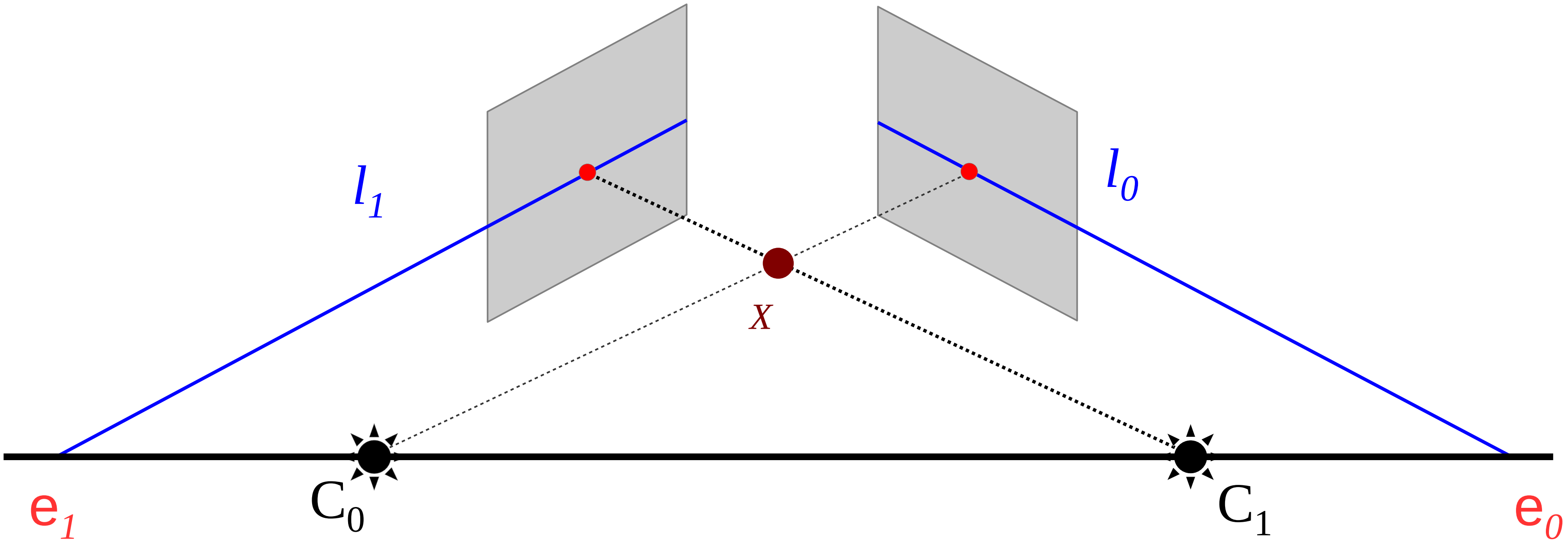


Epipolar Geometry

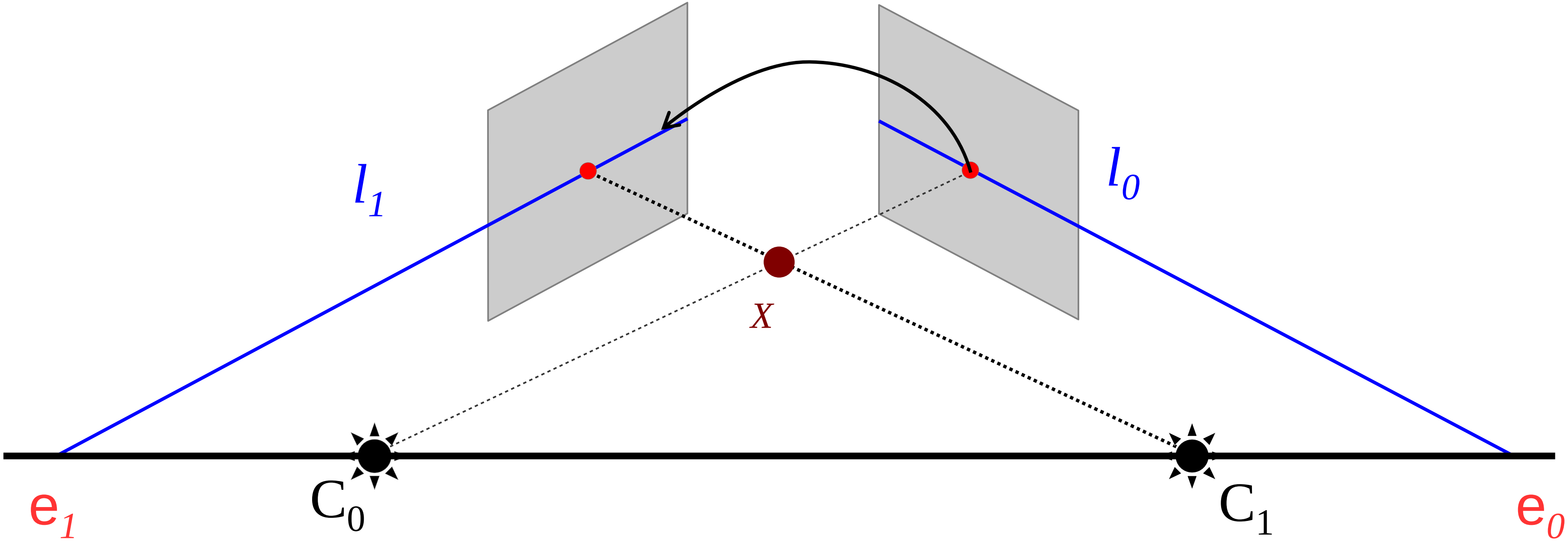
Observe:
Symmetry in view 0 and 1!
A single “Epipolar” plane!



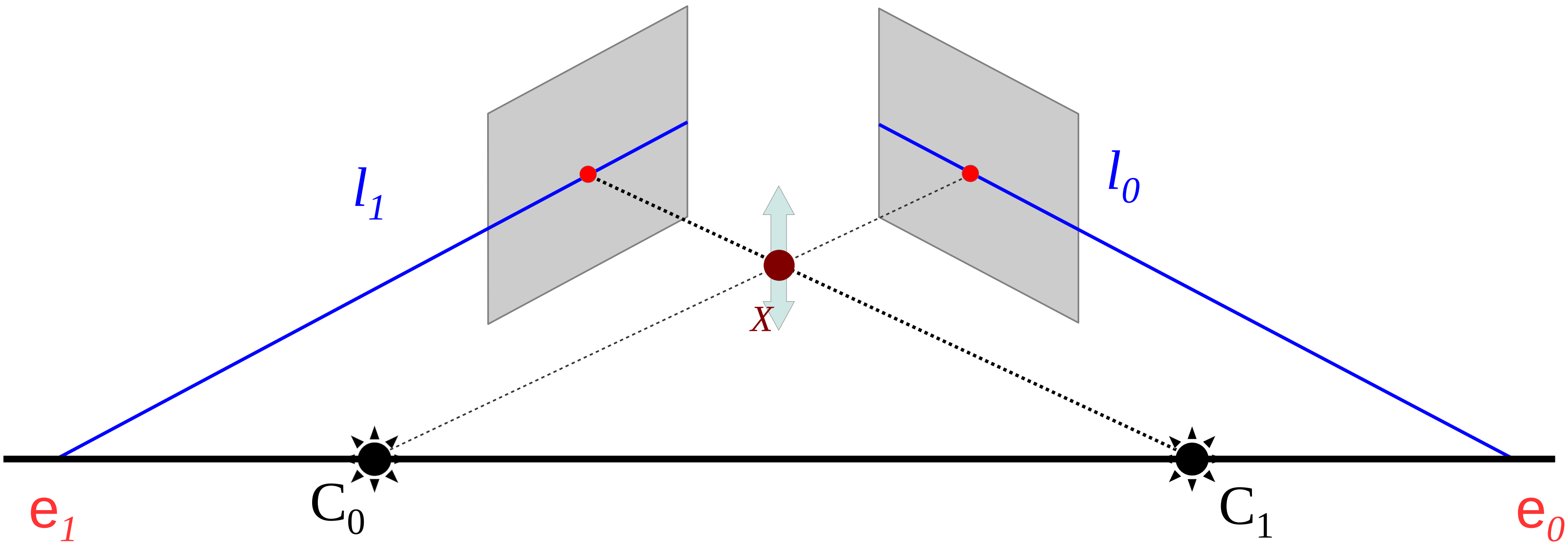
Epipolar Geometry



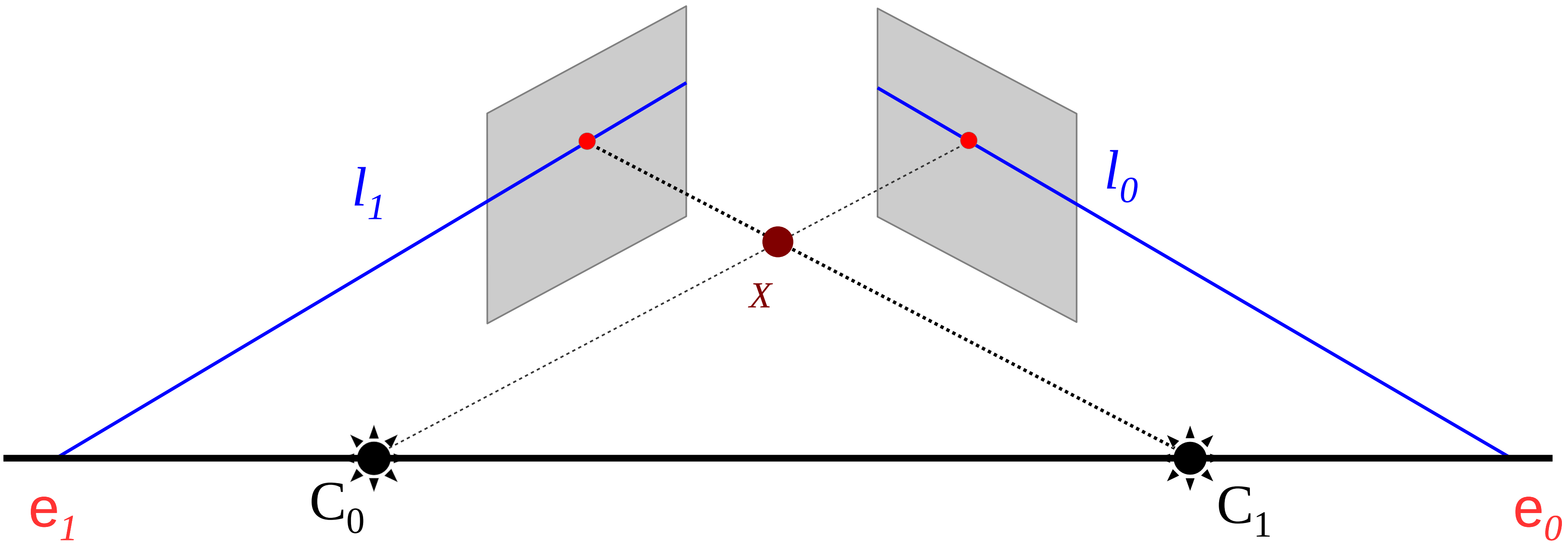
Epipolar Geometry



Epipolar Geometry



Epipolar Geometry



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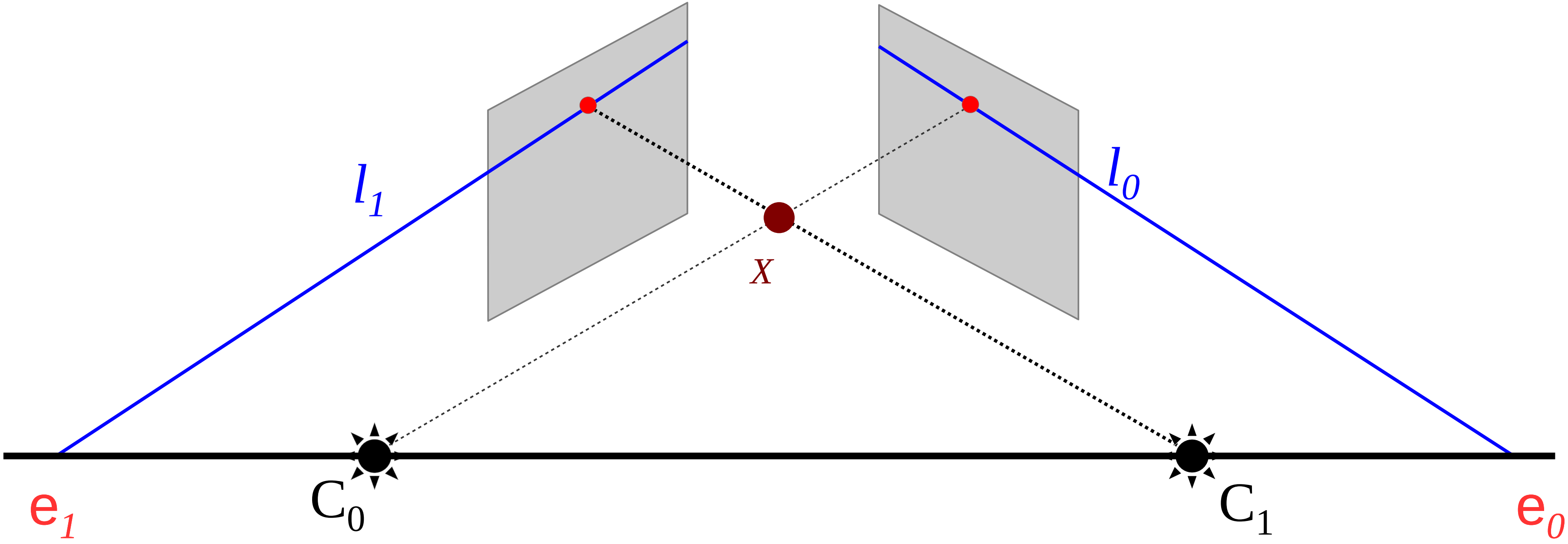

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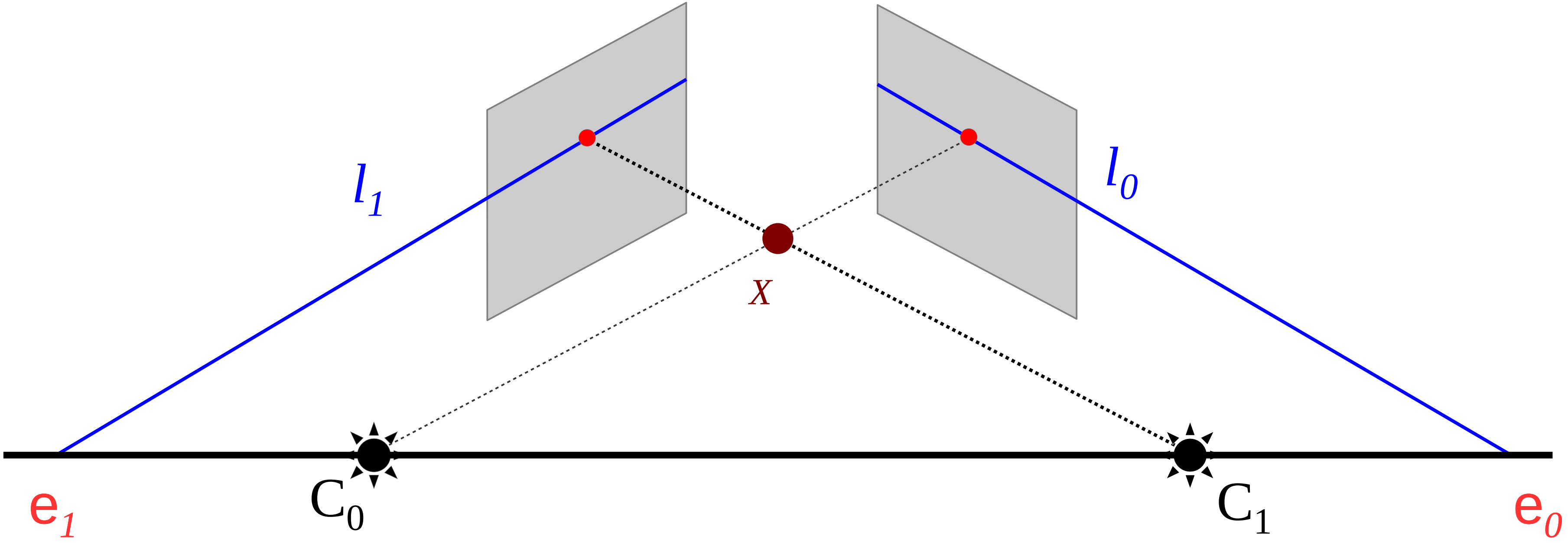
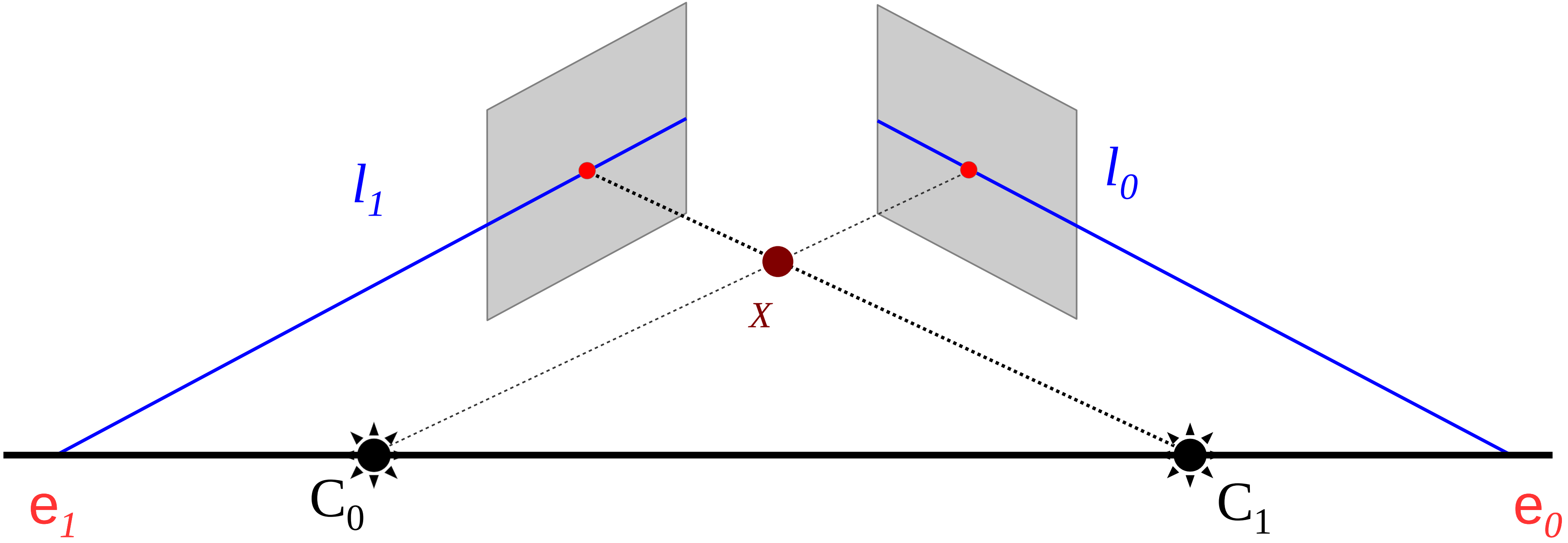
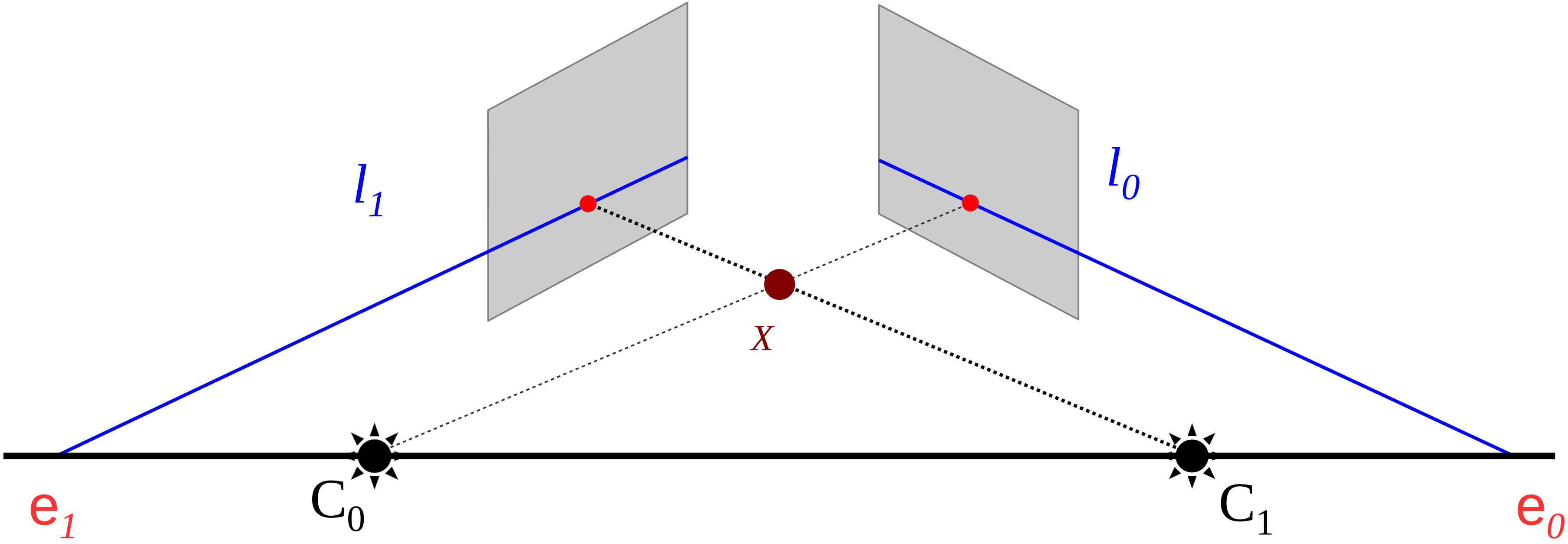


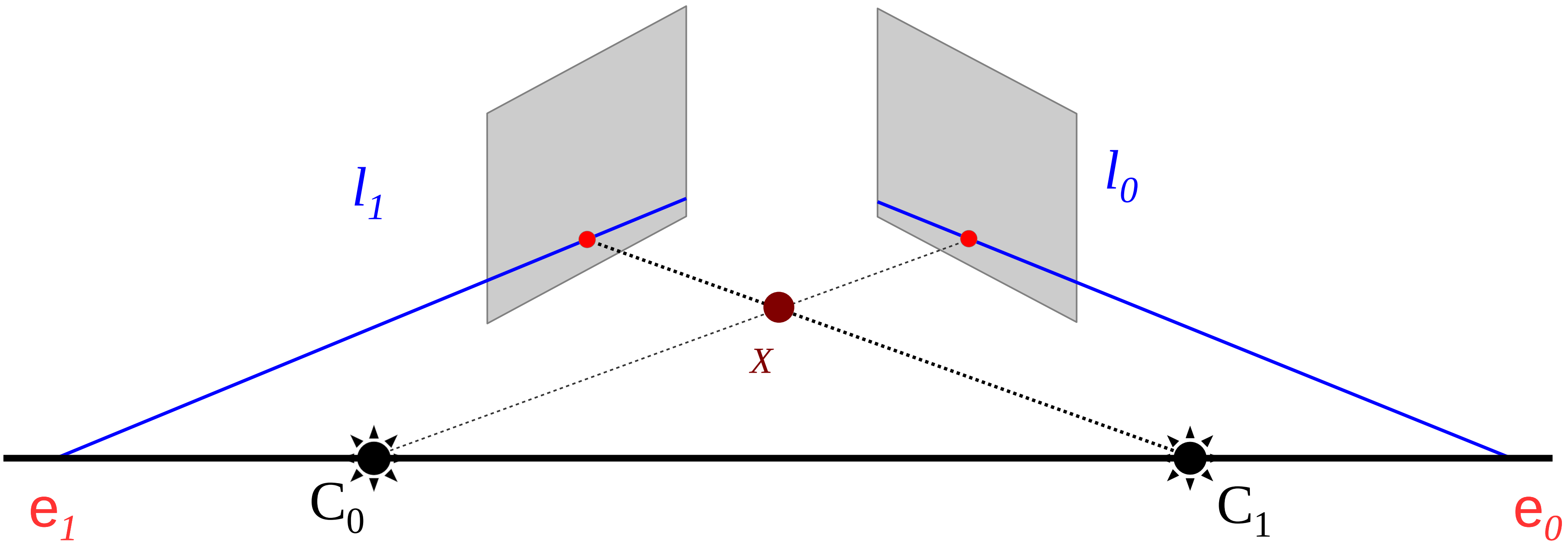
Epipolar Geometry



Epipolar Geometry



Epipolar Geometry



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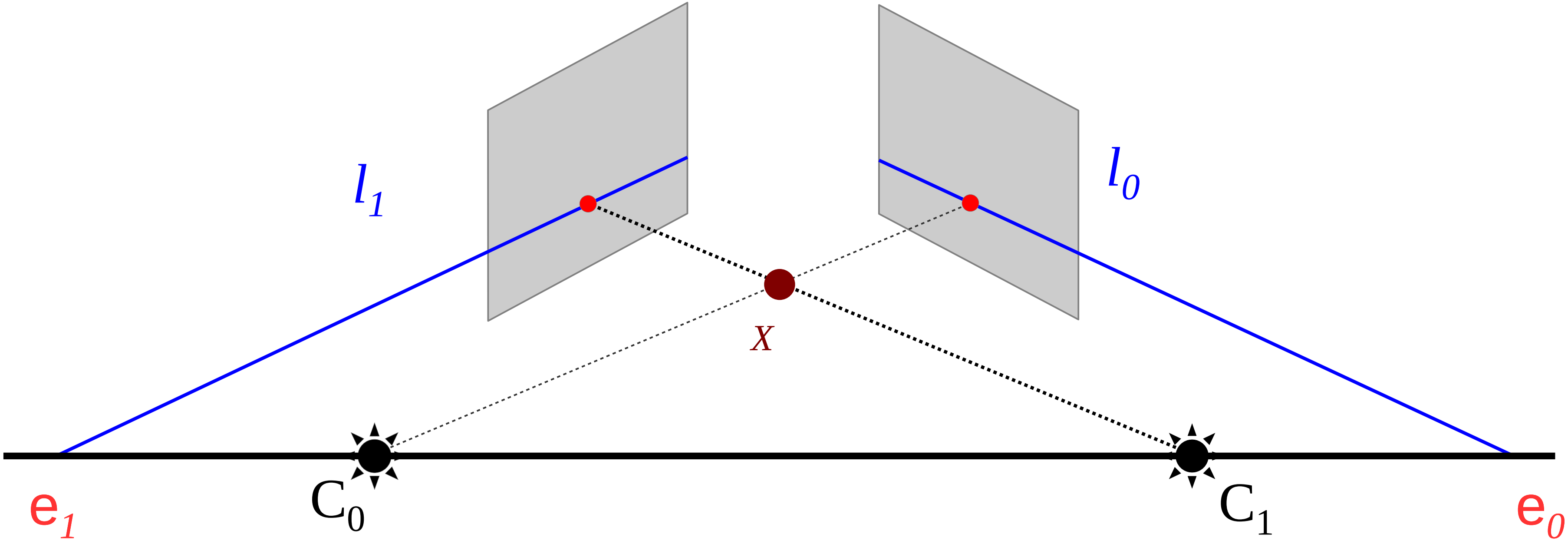

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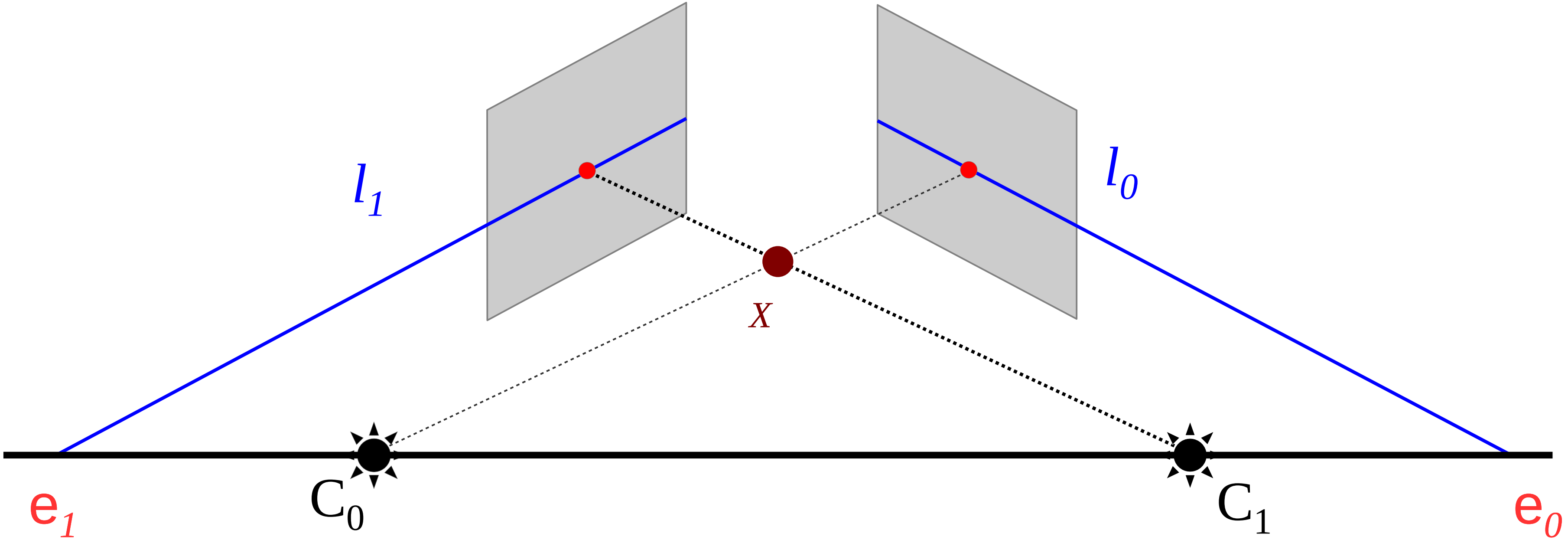
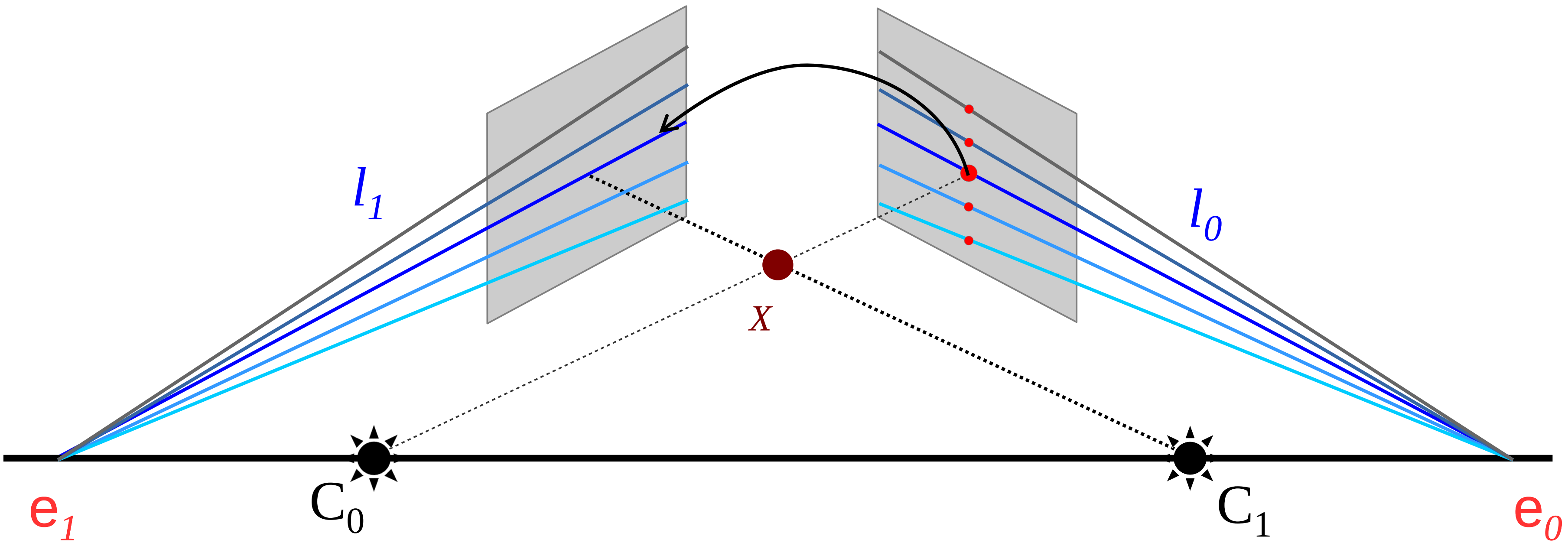
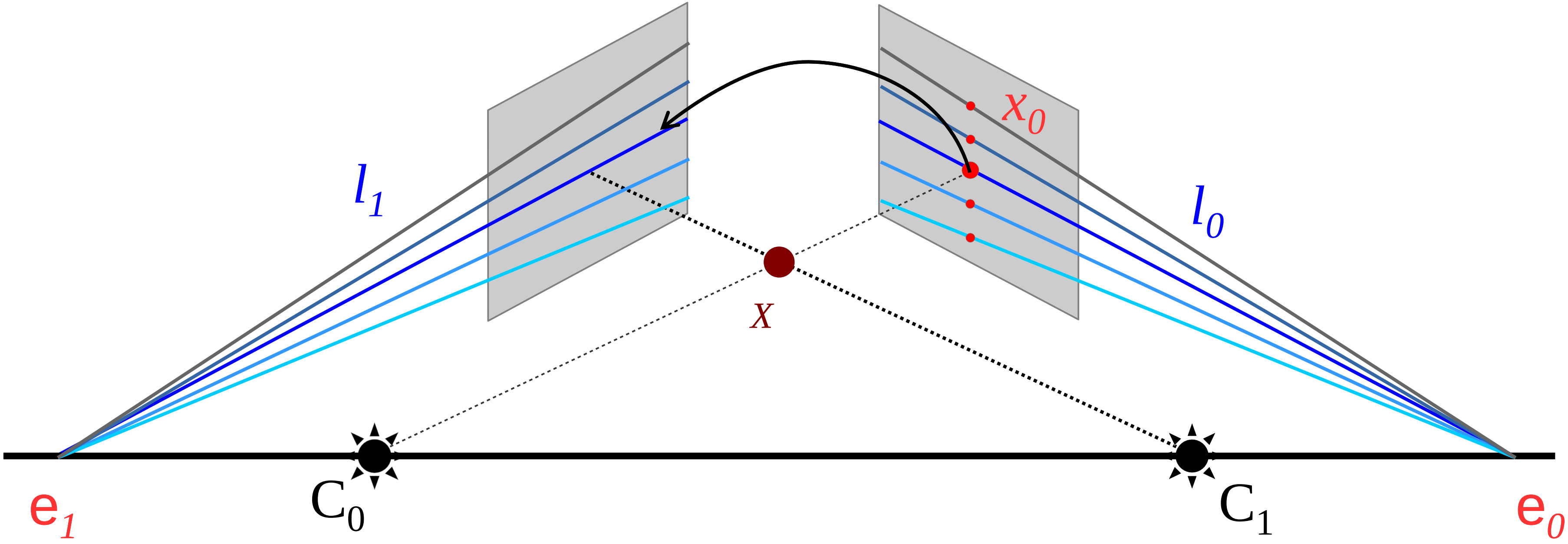


Epipolar Geometry



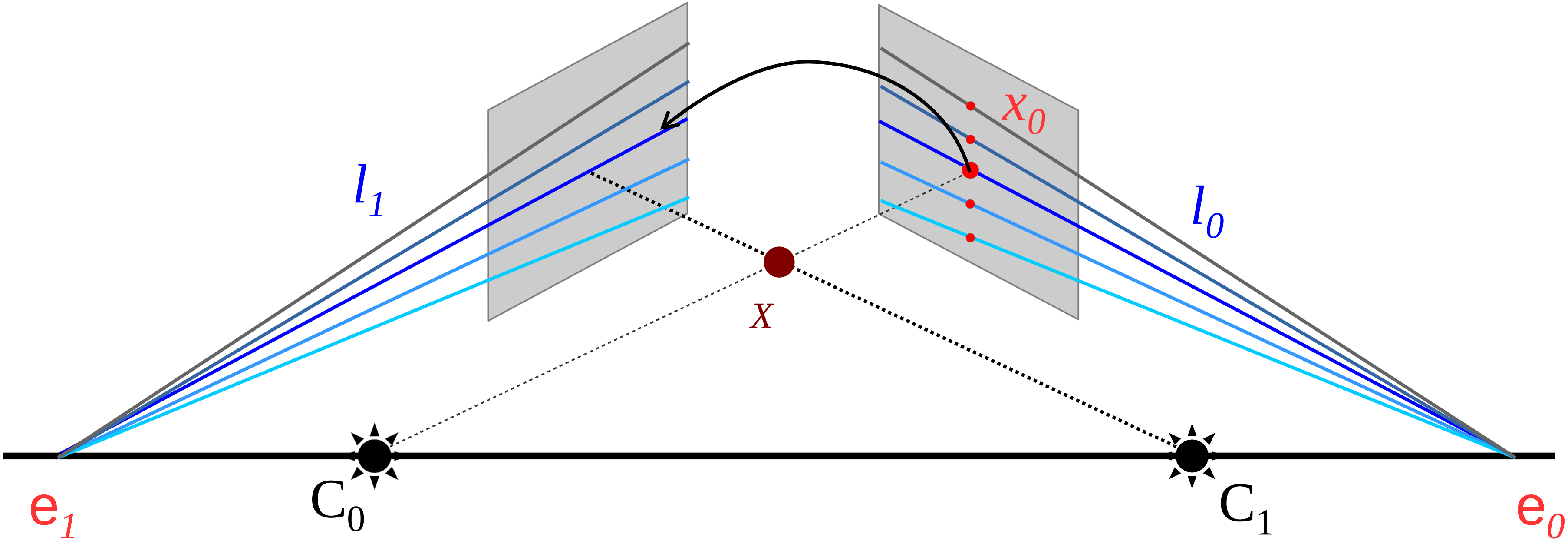
Two line bundles with 1-1 correspondences!

Epipolar Geometry



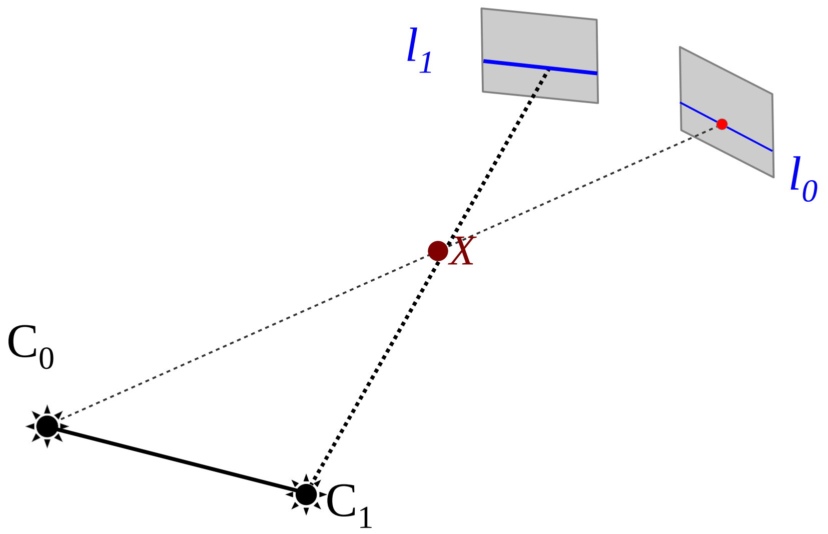
A single 3x3 matrix ***F*** encodes the relative geometry!

Epipolar Geometry

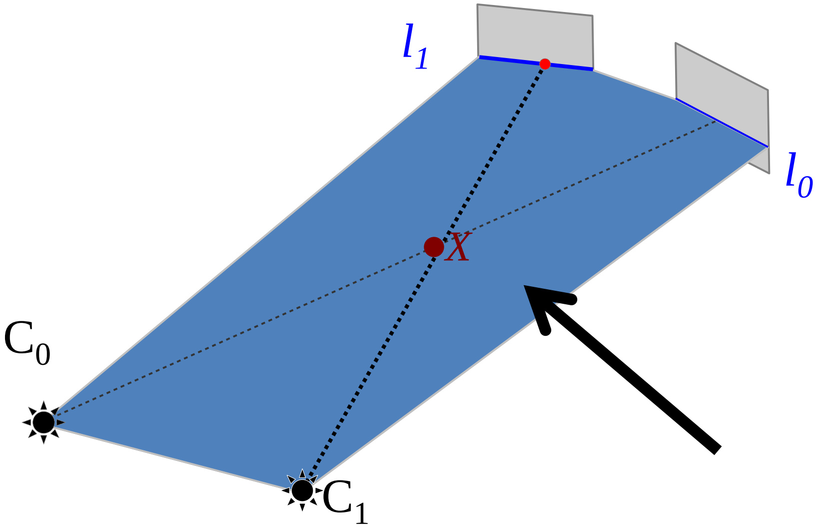


All epipolar lines intersect in the epipole!

Redundancies on Epipolar Lines

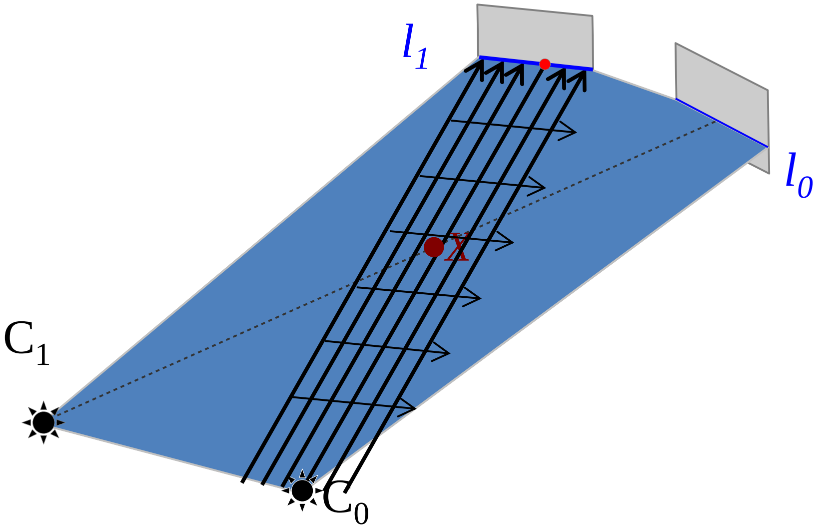


Redundancies on Epipolar Lines

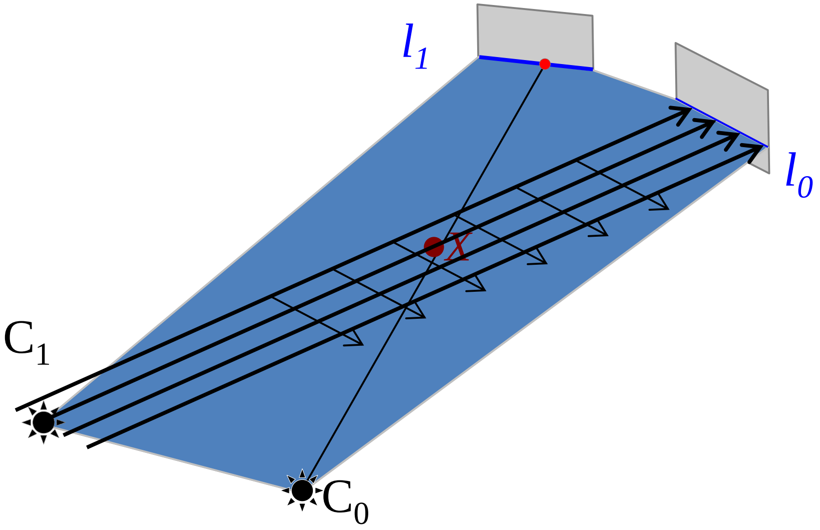


Epipolar Plane

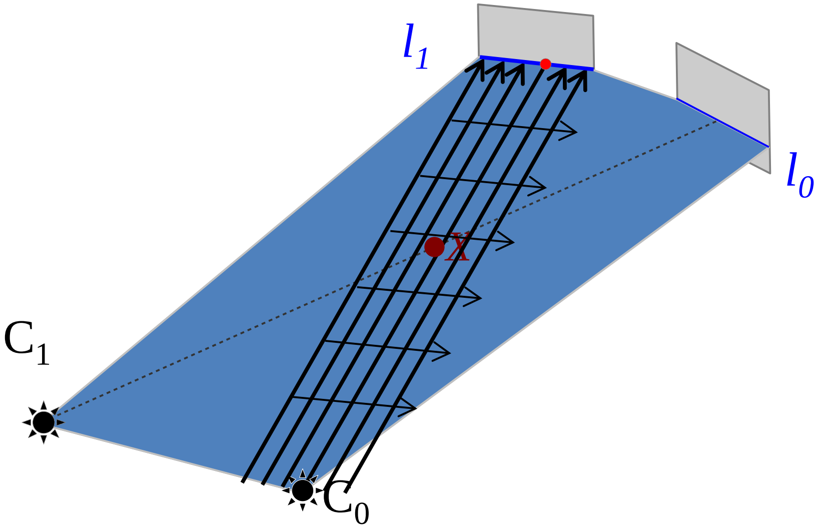
Redundancies on Epipolar Lines



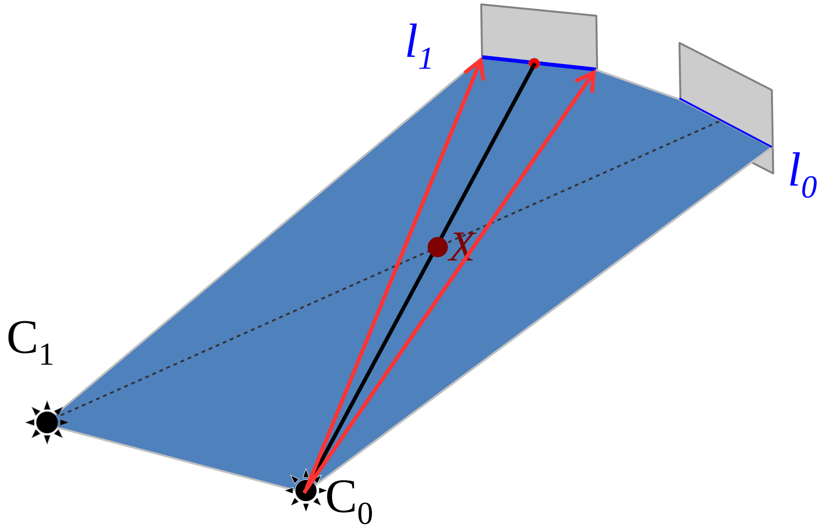
Redundancies on Epipolar Lines



Redundancies on Epipolar Lines



Redundancies on Epipolar Lines



Not true for fan-beam geometry

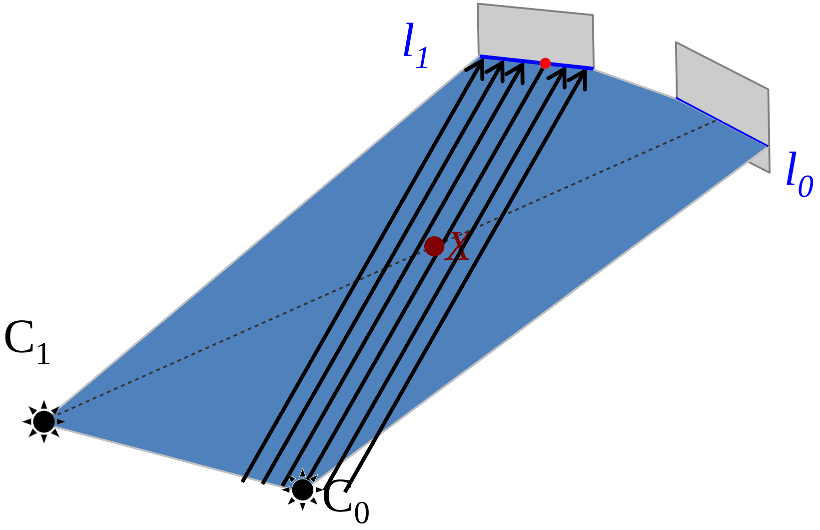
Suppose C were far away.

-> Rays would be parallel.

-> Then: **plane integral = line integral!**

Grangeat's Theorem

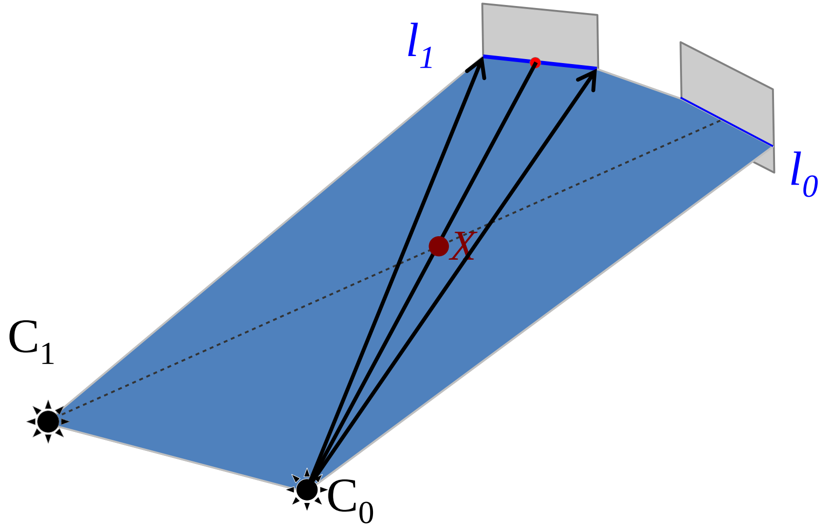
Grangeat's Theorem



Integral of Epipolar Plane

$$\rho_f(\mathbf{E}) = \iint f(x, y, 0)\,dx\,dy$$

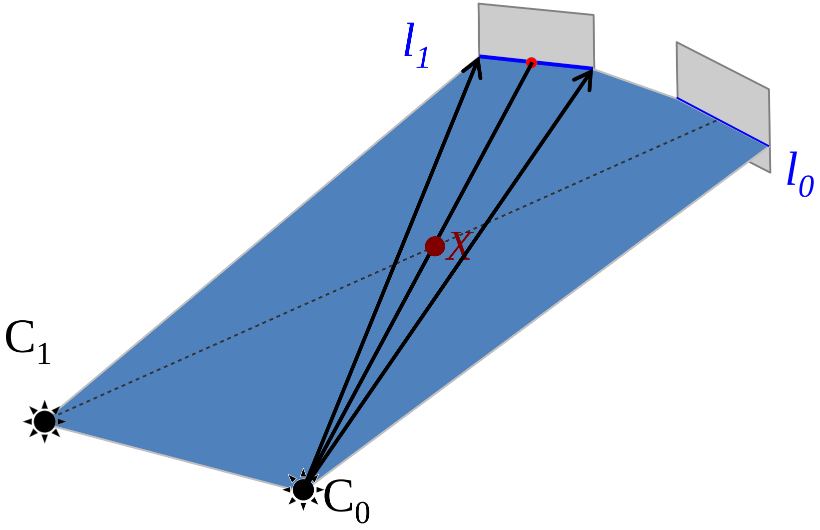
Grangeat's Theorem



Integral of Epipolar Plane

$$\rho_f(\mathbf{E}) = \iint f(x,y,0)dxdy$$

Grangeat's Theorem



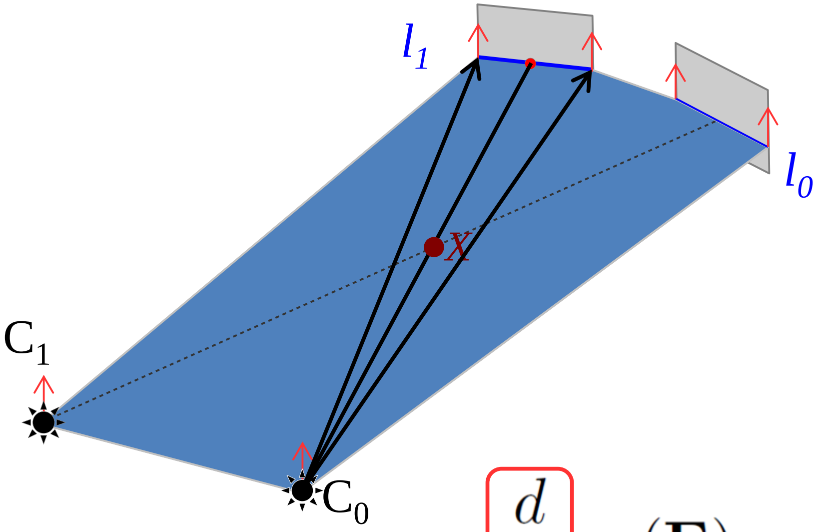
Integral of Epipolar Plane

$$\rho_f(\mathbf{E}) = \iint f(x,y,0)\,dx\,dy$$

$$= \iint f(\Phi(\varphi,r))\boxed{\det(J_\Phi)}\,dr\,d\varphi$$

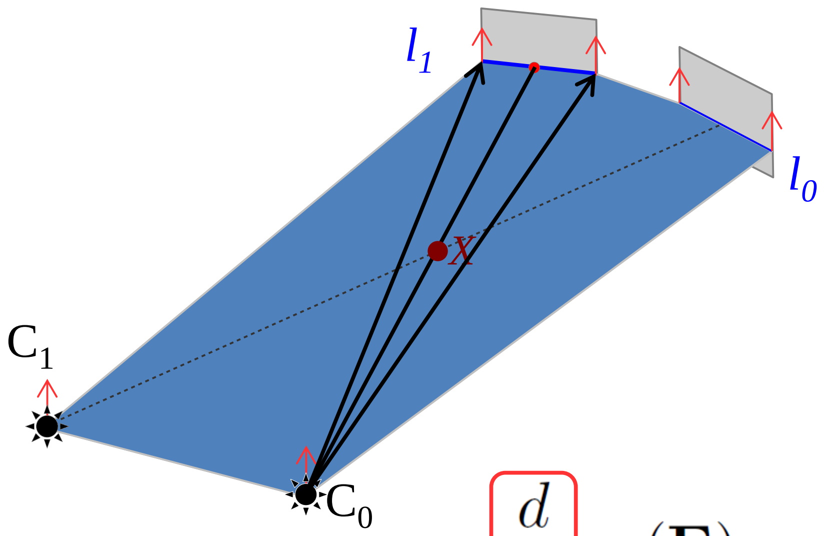
Weighted line integral on detector

Grangeat's Theorem



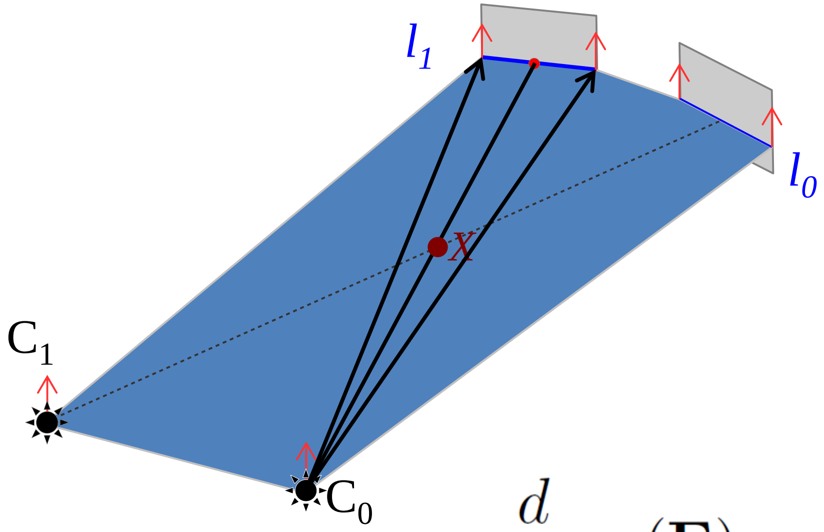
$$\boxed{\frac{d}{dn}}\rho_f(\mathbf{E})$$

Grangeat's Theorem



$$\boxed{\frac{d}{dn}}\rho_f(\mathbf{E}) \approx \frac{d}{d\kappa}\iint f(\Phi(\varphi, r))\,dr\,d\varphi \approx \boxed{\frac{d}{dt}}\rho_I(\mathbf{l})$$

Grangeat's Theorem



The derivative of the 3-D Radon transform in normal direction is approximately the derivative of the 2-D Radon transform in intercept direction.

$$\frac{d}{dn}\rho_f(\mathbf{E}) \approx \frac{d}{d\kappa}\iint f(\Phi(\varphi, r))drd\varphi \approx \frac{d}{dt}\rho_I(\mathbf{l})$$

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Applied Example

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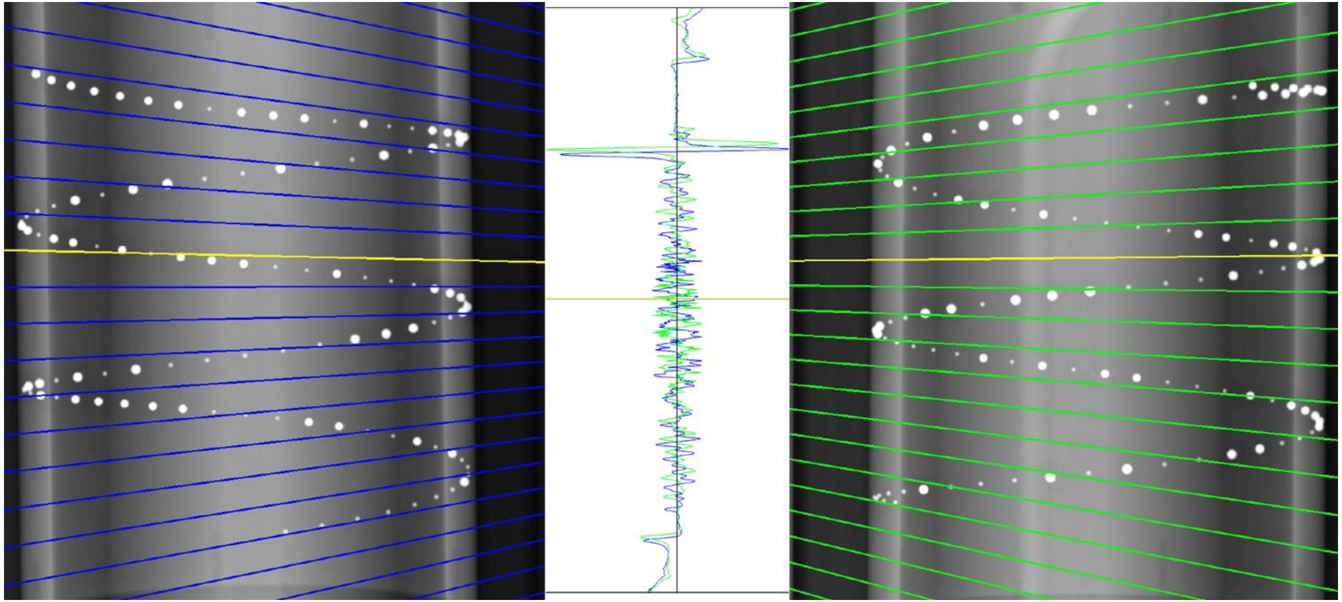
Metric for Geometric Consistency

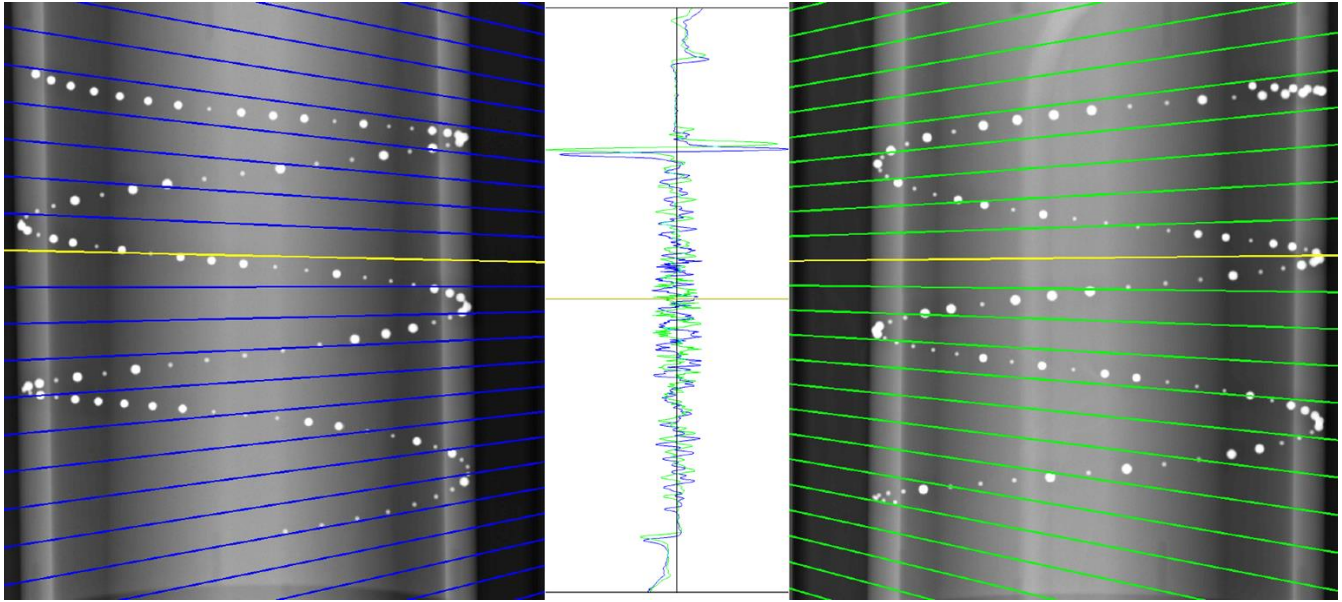
Derivative of line integrals



Plane angle (around baseline)

Metric for Geometric Consistency

Derivative of line integrals

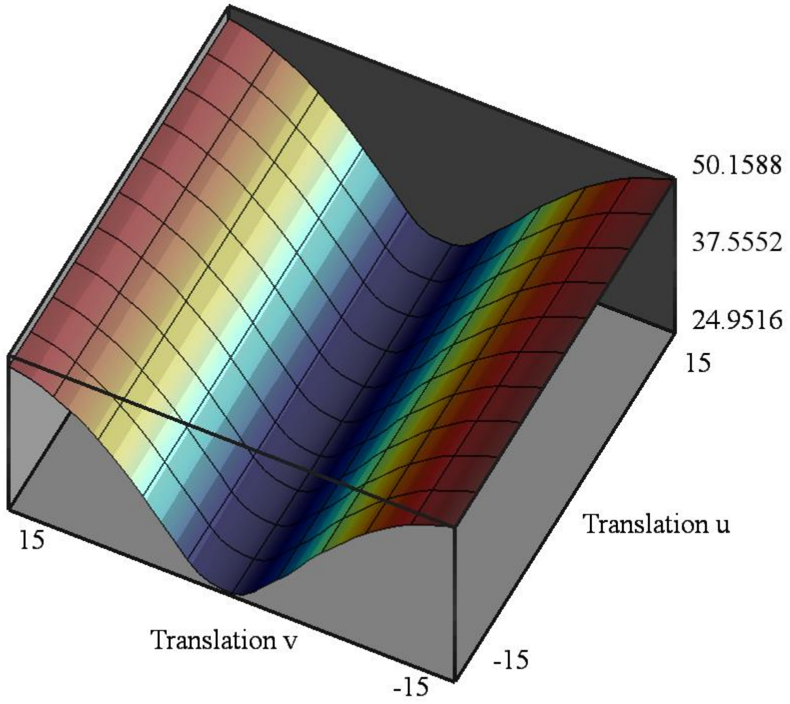


Plane angle (around baseline)

Metric defined as difference between blue and green curves!

A Plot for Detector Shifts and Just Two Views

- For a “close” image pair
- Range: 15 pixels
- Epipolar lines almost parallel to u-axis



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Take Home Messages

- In this unit you got visual insight into the intricacies of epipolar geometry.
- We also connected line integrals on the detector with integrals of the epipolar plane.
- In the next unit we will learn how to use this to develop an epipolar consistency metric.

Further Readings

André Aichert et al. "Epipolar Consistency in Transmission Imaging". In: *IEEE Transactions on Medical Imaging* 34.11 (Nov. 2015), pp. 2205–2219. DOI: 10.1109/TMI.2015.2426417

Acknowledgements:

MEDICAL VALLEY
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SIEMENS

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