Medical Image Processing for Interventional Applications

Vesselness Filter

Online Course – Unit 7

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Topics

Vessel Segmentation

- **Problem 1:** Vessels have different diameters.
  $\rightarrow$ We need to model different scales.
- **Problem 2:** Edges are only a weak model of vessels.
  $\rightarrow$ Structure tensor is insufficient for vessel modeling.

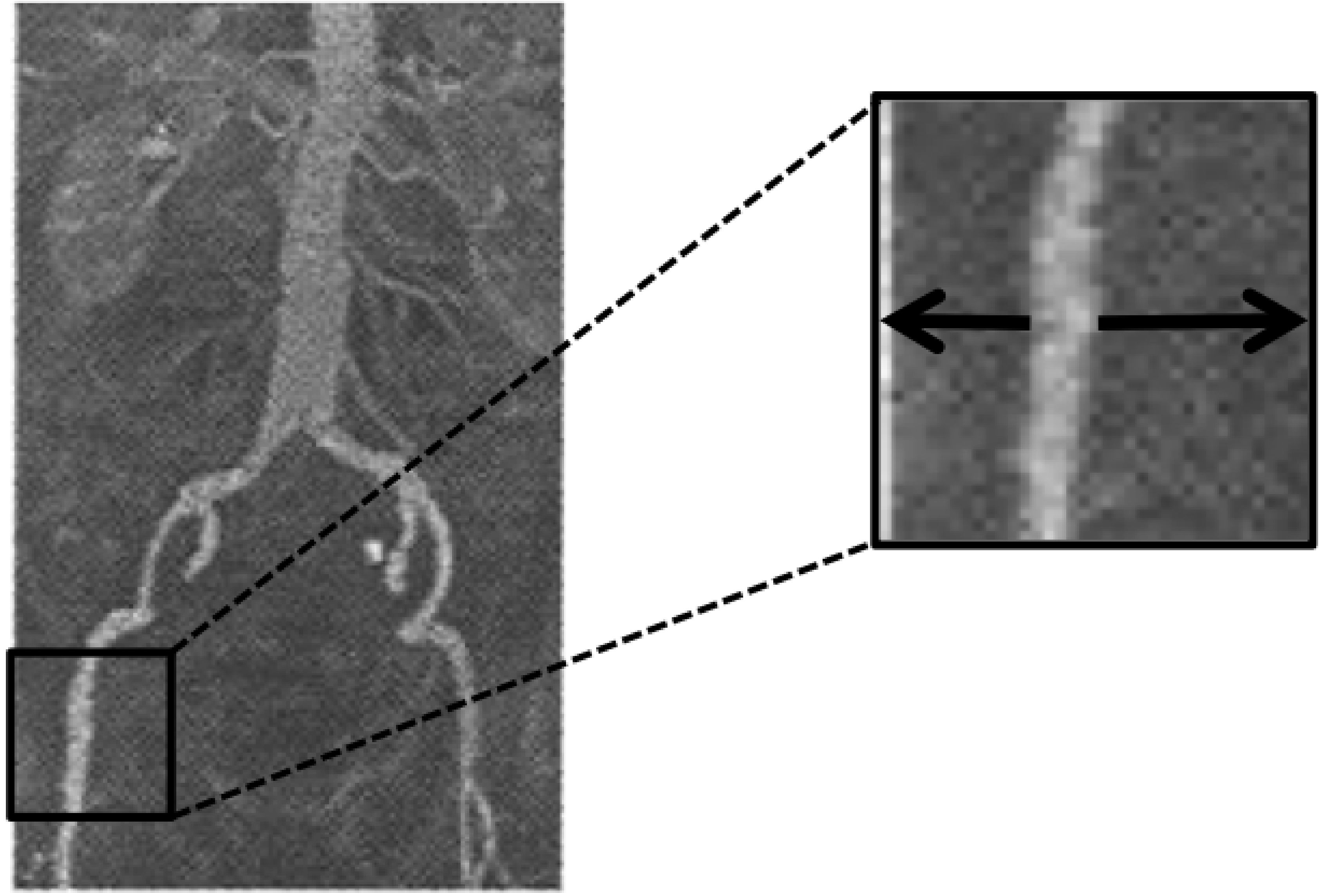


Figure 1: Example for a medical image containing vessels (left), detail on an area showing the edge property well (right)

Problem 1: Scale Modelling

Question: How big must the window/volume be?

- Solution using scale space with parameter s:

$$I(x,s) = I(x) * G(x,s), \quad G(x,s) = \frac{1}{\sqrt{2\pi s}} \exp\left(-\frac{\|x\|^2}{2s^2}\right)$$

- Derivative of Gaussians (in 1-D):

$$\frac{\partial}{\partial x} I(x,s) = I(x) * \frac{\partial}{\partial x} G(x,s)$$

Problem 2: Vessel Model

From the properties of the structure tensor we can determine:

- rank 0 $\rightarrow$ flat area,
- rank 1 $\rightarrow$ edge,
- rank 2 $\rightarrow$ corner.

But: Edges are not a good model of a vessel.

$\rightarrow$ Modeling of curvature seems better suited to tackle this problem.

Problem 2: Vessel Model

Approach: compute Hessian (here 2-D)

$$H_s = \begin{pmatrix} \frac{\partial^2}{\partial x^2} I(\boldsymbol{x}, s) & \frac{\partial^2}{\partial x \partial y} I(\boldsymbol{x}, s) \\ \frac{\partial^2}{\partial y \partial x} I(\boldsymbol{x}, s) & \frac{\partial^2}{\partial y^2} I(\boldsymbol{x}, s) \end{pmatrix}$$

This matrix looks very similar to the structure tensor, but the analysis of its eigenvalues yields:

- rank 0 $\rightarrow$ no curvature, i. e., **flat** or **linear** behaviour,
- rank 1 $\rightarrow$ curvature in one direction, i. e., a **tubular** structure,
- rank 2 $\rightarrow$ curvature in two directions, i. e., a **blob-like** structure.

Good Vessels in 2-D

Indication of a vessel (we assume sorting $|\lambda_1| \geq |\lambda_2|$):

$$|\lambda_1| > |\lambda_2| \qquad \wedge \qquad |\lambda_2| \approx 0$$

Vesselness measures:

$$R_B = \frac{\lambda_2}{\lambda_1} \qquad \text{(close to 0 if vessel)}$$

$$S = \sqrt{\lambda_1^2 + \lambda_2^2} \qquad \text{(high contrast if vessel)}$$

Probability map for vessels (β, c are control parameters, e. g., $\beta = 0.5$, and c depends on scaling):

$$V(\boldsymbol{x}, s) = \begin{cases} 0, & \lambda_1 > 0 \\ \exp\left(-\frac{R_B^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right), & \text{otherwise} \end{cases}$$

Maximum over all scales:

$$V(\boldsymbol{x}) = \max_{s_{\min} < s < s_{\max}} V(\boldsymbol{x}, s)$$

3-D Case

Good vessels (we assume sorting $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$):

$$|\lambda_3| \approx 0 \qquad \wedge \qquad |\lambda_2| \gg \lambda_3 \qquad \wedge \qquad \lambda_1 \approx \lambda_2$$

3-D vesselness:

$$R_B = \frac{|\lambda_3|}{\sqrt{|\lambda_1 \lambda_2|}} \qquad \text{(close to 0 if vessel)}$$

$$R_A = \frac{|\lambda_2|}{|\lambda_1|} \qquad \text{(close to 0 if surface point)}$$

$$S = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \qquad \text{(high if vessel } \rightarrow \text{ contrast)}$$

3-D Case

Vesselness on scale s (probability map):

$$V(\boldsymbol{x}, s) = \begin{cases} 0, & \lambda_1 > 0 \text{ or } \lambda_2 > 0 \\ \left(1 - \exp\left(-\frac{R_A^2}{2\alpha^2}\right)\right) \exp\left(-\frac{R_B^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right), & \text{otherwise} \end{cases}$$

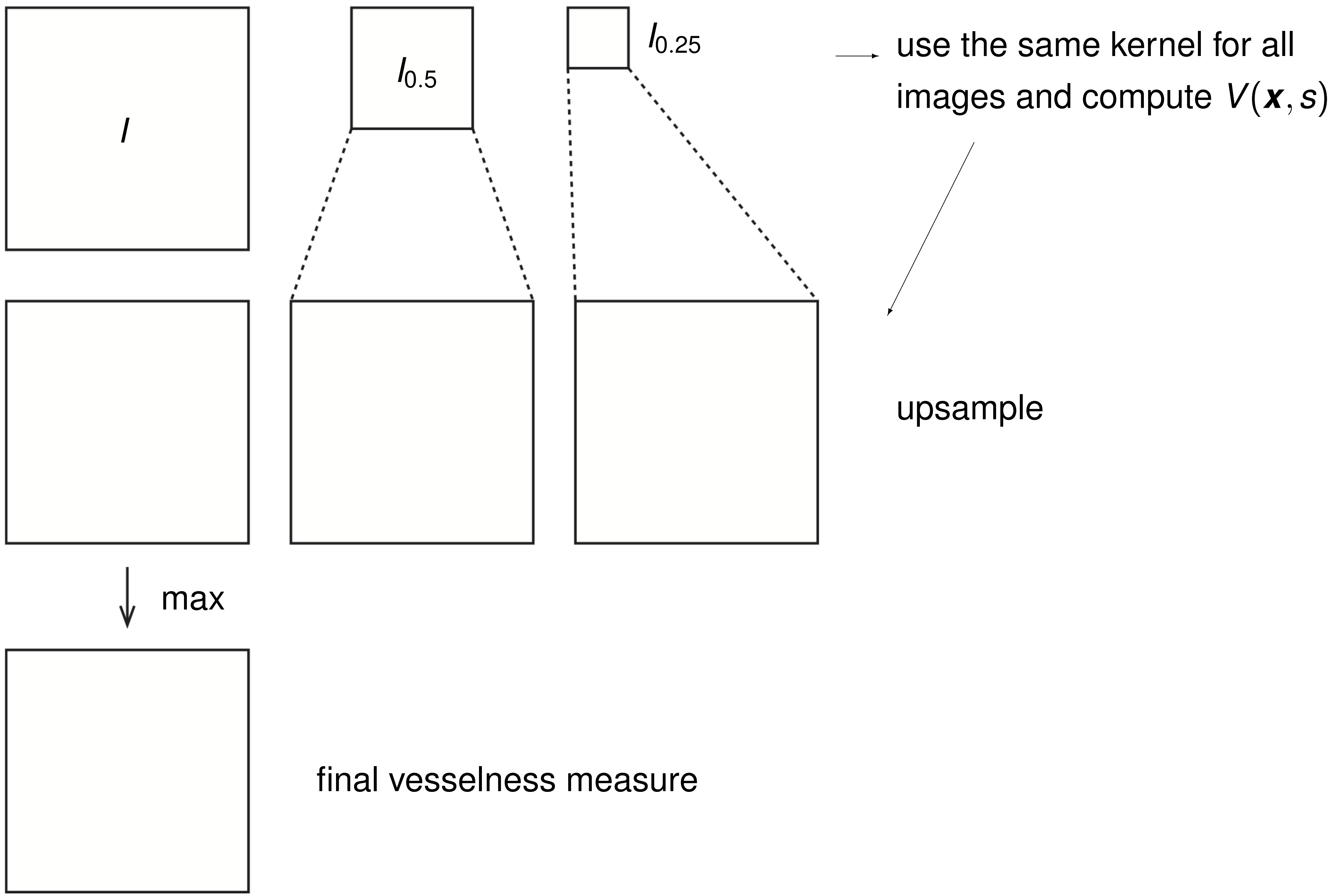
α, β, c are parameters to be selected by the user ($\alpha = \beta = 0.5$, c depends on contrast)

Maximum over all scales:

$$V(\boldsymbol{x}) = \max_s V(\boldsymbol{x}, s)$$

Faster Implementation

Instead of increasing s, downsample the image I to images I_d accordingly:
$\rightarrow$ less eigenvalue analyses
$\rightarrow$ faster computation for high resolution images



Topics

Take Home Messages

- The Hessian and its eigenvalue analysis can be used to define a measure of vesselness.
- It is often more reliably than the structure tensor.
- The vesselness filter is well-known in medical imaging and can be considered a requisite.
- When implementing methods using scale-space, downsampling instead of computing several kernels can speed up your code.

Further Readings

- Alejandro F. Frangi et al. “Multiscale Vessel Enhancement Filtering”. In: *Medical Image Computing and Computer-Assisted Intervention – MICCAI’98*. Ed. by William M. Wells, Alan Colchester, and Scott Delp. Vol. 1496. Lecture Notes in Computer Science. Springer Berlin Heidelberg, 1998, pp. 130–137. DOI: 10.1007/BFb0056195
- A. Budai et al. “Robust Vessel Segmentation in Fundus Images”. In: *International Journal of Biomedical Imaging* 2013.154860 (Sept. 2013), pp. 1–11. DOI: 10.1155/2013/154860