Medical Image Processing for Interventional Applications

Camera Rotation and Translation

Online Course – Unit 33
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Topics

Camera Rotation and Translation

Summary

Take Home Messages

Further Readings

Motivation

What needs to be computed:

- Translation $\rightarrow$ null space of the essential matrix
- Rotation $\rightarrow$ linear estimator based on quaternions
- Coordinates of 3-D points $\rightarrow$ triangulation

Computation of Camera Rotation and Translation

For computing rotation and translation, compute $\boldsymbol{E}$ from $\boldsymbol{F}$:

$$\boldsymbol{F} = \left(\boldsymbol{K}^{-1}\right)^{\mathsf{T}} \cdot \boldsymbol{E} \cdot \boldsymbol{K}^{-1}.$$

Since intrinsic parameters are known

$$\boldsymbol{E} = \boldsymbol{K}^{\mathsf{T}} \cdot \boldsymbol{F} \cdot \boldsymbol{K}.$$

$\rightarrow$ $\boldsymbol{E}$ depends on the extrinsic camera parameters only.

Computation of Camera Rotation and Translation

Known:

- Essential matrix $\boldsymbol{E} = \boldsymbol{R} \cdot [t]_\times$
- Translation vector $\boldsymbol{t}$ (spans nullspace of essential matrix)

Computation of rotation matrix $\boldsymbol{R}$

Compute by solving the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & \|\boldsymbol{E} - \boldsymbol{R} \cdot [\boldsymbol{t}]_\times\|_2^2, \\ \text{subject to} \quad & \det(\boldsymbol{R}) = 1, \\ & \boldsymbol{R} \text{ is an orthogonal matrix, i. e., } \boldsymbol{R}^{-1} = \boldsymbol{R}^\mathsf{T}. \end{aligned}$$

This is a **nonlinear** and **difficult** optimization problem.

But it can be converted into a **linear** problem by using quaternions.

Essential Matrix and Quaternions

First we rewrite the objective function:

$$\|\boldsymbol{E} - \boldsymbol{R} \cdot [\boldsymbol{t}]_\times\|_2^2 = \|\boldsymbol{R}^\mathsf{T}\boldsymbol{E} - [\boldsymbol{t}]_\times\|_2^2,$$

where $\det \boldsymbol{R} = 1$.

Writing $\boldsymbol{E} = (\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$, where $\boldsymbol{e}_i \in \mathbb{R}^3$, we therefore minimize:

$$\min_{\boldsymbol{R}} \|\boldsymbol{R}^\mathsf{T}\boldsymbol{E} - [\boldsymbol{t}]_\times\|_2^2 = \min_{\boldsymbol{R}} \sum_{i=1}^{3} \|\boldsymbol{R}^\mathsf{T}\boldsymbol{e}_i - \left([\boldsymbol{t}]_\times\right)_i\|_2^2.$$

Essential Matrix and Quaternions

In quaternion notation:

- $\boldsymbol{R}^{\mathrm{T}}$ defines a quaternion $\boldsymbol{r}$,
- $\boldsymbol{p}_i = (0, \boldsymbol{e}_i^{\mathrm{T}})$,
- $\boldsymbol{p}'_i = (0, ([\boldsymbol{t}]_\times)_i^{\mathrm{T}})$.

Thus we get

$$\min_{\boldsymbol{r}} \sum_{i=1}^{3} \|\boldsymbol{r} \cdot \boldsymbol{p}_i \cdot \bar{\boldsymbol{r}} - \boldsymbol{p}'_i\|^2,$$

where $\|\boldsymbol{r}\| = 1$.

Essential Matrix and Quaternions

Multiplication by $\boldsymbol{r}$ from the right results in the objective function:

$$\widehat{\boldsymbol{r}} = \min_{\boldsymbol{r}} \sum_{i=1}^{3} \|\boldsymbol{r} \cdot \boldsymbol{p}_i - \boldsymbol{p}_i' \cdot \boldsymbol{r}\|^2.$$

Since this expression is linear in $\boldsymbol{r}$, there exist matrices $\boldsymbol{M}_i$ with:

$$\widehat{\boldsymbol{r}} = \min_{\boldsymbol{r}} \sum_{i=1}^{3} \left\|\boldsymbol{M}_i \cdot \boldsymbol{r}^{\mathsf{T}}\right\|^2.$$

Topics

Camera Rotation and Translation

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Both camera translation and rotation can be computed using the essential matrix.
- If we use quaternions, we can formulate the optimization problem for the rotation linearly.

Further Readings

Epipolar geometry is nicely introduced in:

Emanuele Trucco and Alessandro Verri. *Introductory Techniques for 3-D Computer Vision*. Upper Saddle River, NJ, USA: Prentice Hall, 1998

All the math regarding epipolar geometry can be found in:

Richard Hartley and Andrew Zisserman. *Multiple View Geometry in Computer Vision*. 2nd ed. Cambridge: Cambridge University Press, 2004. DOI: 10.1017/CBO9780511811685

Magnetic navigation:

Michelle P. Armacost et al. "Accurate and Reproducible Target Navigation with the Stereotaxis Niobe® Magnetic Navigation System". In: *Journal of Cardiovascular Electrophysiology* 18 (Jan. 2007), S26–S31. DOI: 10.1111/j.1540-8167.2007.00708.x