Medical Image Processing for Interventional Applications

Super-Resolution: Introduction

Online Course – Unit 20

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Topics

What is Image Super-Resolution?

Cameras and Sampling

- The Sampling Theorem
- Sampling of Real Cameras
- Quantization and Image Noise

Summary

- Take Home Messages
- Further Readings

What is Image Super-Resolution?

Digital imaging systems perform a non-ideal mapping of a scene to the image plane of a camera:

- **(Down-)sampling**: continuous real world scene $\leftrightarrow$ discrete representation with finite resolution
- **Blur/diffraction**: non-ideal mapping of points and edges
- **Noise**: induced by camera sensors

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Definition

Super-resolution is the process of obtaining high-resolution images from observed low-resolution images.

What is Image Super-Resolution?

Basic approaches to image super-resolution:

- Instrumental super-resolution (hardware-based):
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  - Widely used in microscopy (STED, RESOLFT)

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- *In this course*: computational super-resolution (software-based):
  - Retrospective approach to image super-resolution (reconstruction algorithms)
  - Aims at overcoming limitations related to digital sampling and/or diffraction
  - No modifications of the underlying camera hardware (sensor and optics) $\rightarrow$ low-cost solution

Super-Resolution Applications

Various applications for image super-resolution algorithms:

- Consumer electronics
- Surveillance cameras
- Remote sensing
- Medical imaging:
  - Ophthalmic imaging
  - Image-guided surgery
  - Radiology
- and more . . .

Figure 1: Super-resolving car license plates

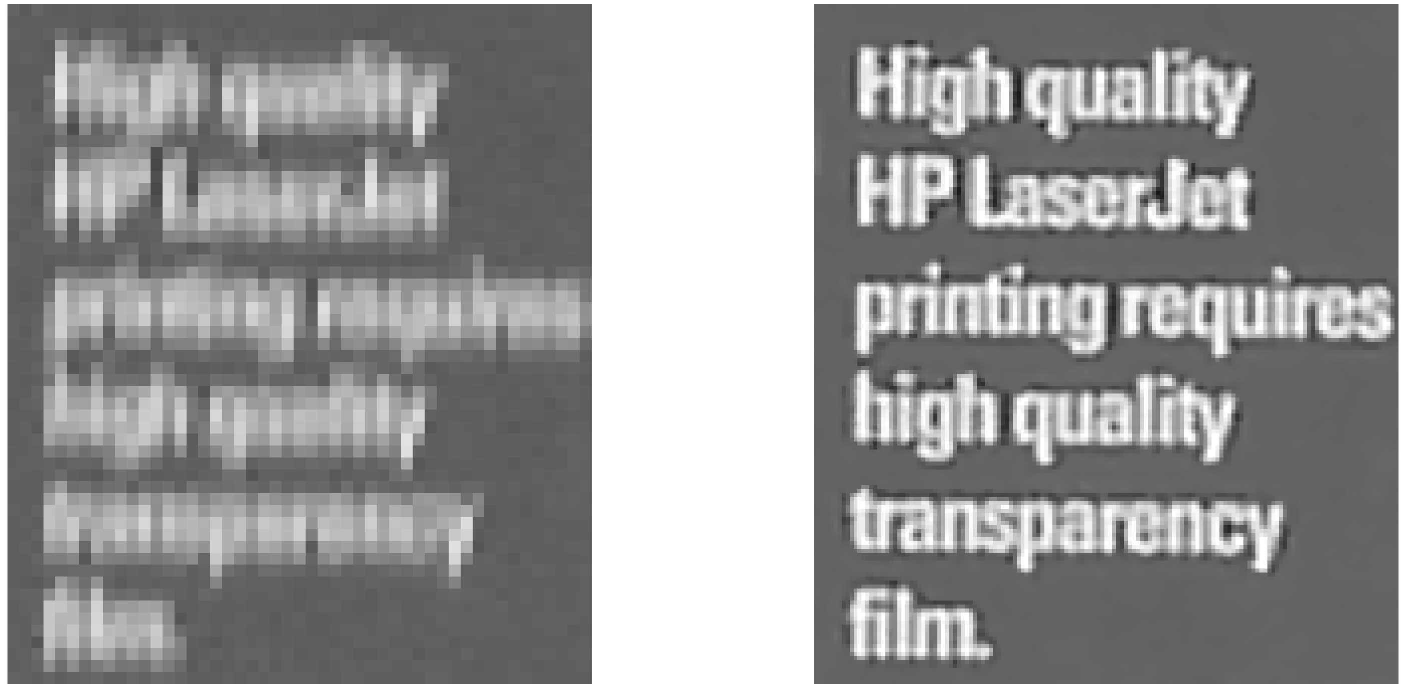

Figure 2: Super-resolving text document images

Topics

Sampling in 2-D

Definitions:

- $f(x, y)$ is a continuous, real-valued signal.
- If $f(x, y)$ is sampled in x- and y-direction, it can be represented by discrete values $f_{m,n}$ where:
  - $f_{m,n} = f(m \cdot \Delta x, n \cdot \Delta y) \in \mathbb{R}$,
  - Δx and Δy denote the sample spacing (sample pitch) on a regular grid,
  - for a finite regular grid $f(x, y)$ is limited to a range $x_0 \leq x \leq x_1$ and $y_0 \leq y \leq y_1$.

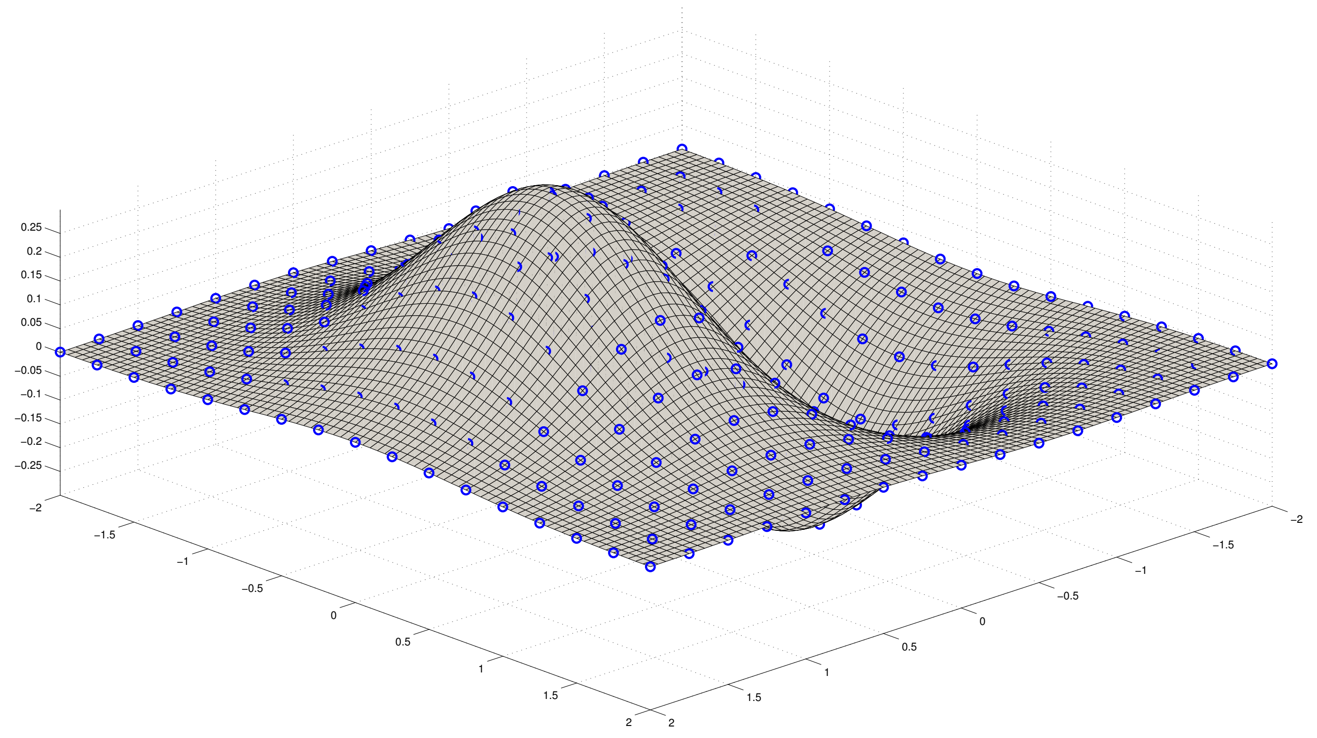

Figure 3: Example of a function sampling

Is it possible to reconstruct $f(x, y)$ from samples $f_{m,n}$ without loss of information? $\rightarrow$ Sampling theorem

Sampling in 2-D

Mathematical modeling of the sampling process:

- **Ideal sampling** is modeled by a sequence of Dirac delta functions:

$$\Delta(x,y) = \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - m\Delta x, y - n\Delta y),$$

  where a single Dirac delta is given by

$$\delta(x,y) = \begin{cases} 1, & \text{if } x = 0 \text{ and } y = 0, \\ 0, & \text{otherwise.} \end{cases}$$

- The discrete samples are determined by

$$f_{m,n} = \Delta(x,y) f(x,y).$$

Sampling Theorem

Band-limited signals:

- Let $F(u,v)$ be the Fourier transform of the continuous signal $f(x,y)$:

$$F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-2\pi i(ux+vy)}\,\mathrm{d}x\,\mathrm{d}y$$

- For the formulation of the sampling theorem we consider **band-limited** signals $f(x,y)$:

$$F(u,v) = 0 \qquad \text{for } |u| > u_0 \text{ or } |v| > v_0.$$

Sampling Theorem

Sampling theorem according to Shannon and Nyquist (for low-pass signals):

The continuous signal $f(x,y)$ is completely determined by its discrete samples :

$$f_{m,n} = f(m\cdot\Delta x, n\cdot\Delta y) \in \mathbb{R} \qquad \text{for } m,n = 0,1,2,\ldots$$

without loss of information **if and only if**

$$\Delta x \leq \frac{1}{2u_0} \quad \text{and} \quad \Delta y \leq \frac{1}{2v_0}.$$

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In other words: We can reconstruct $f(x,y)$ from $f_{m,n}$ if the sampling rates $1/\Delta x$ and $1/\Delta y$ are high enough.

Aliasing

Violation of the sampling theorem:

- If the sampling theorem is not fulfilled, ***aliasing*** is induced.
- Aliasing: High frequencies in the original signal are mapped to low frequencies in the sampled signal.

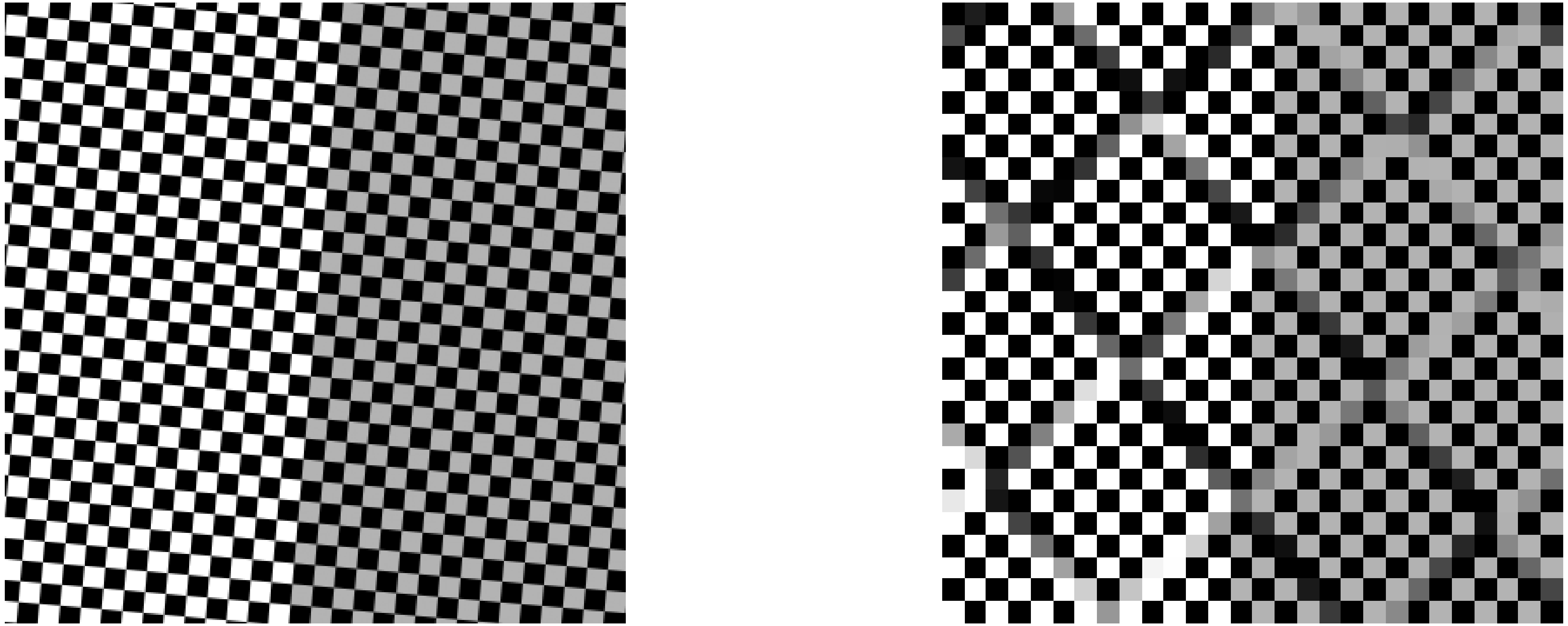


Figure 4: Original checkerboard pattern (left), resampled pattern with aliasing artifacts (right)

Sampling of Real Cameras

Generalization of the sampling process:

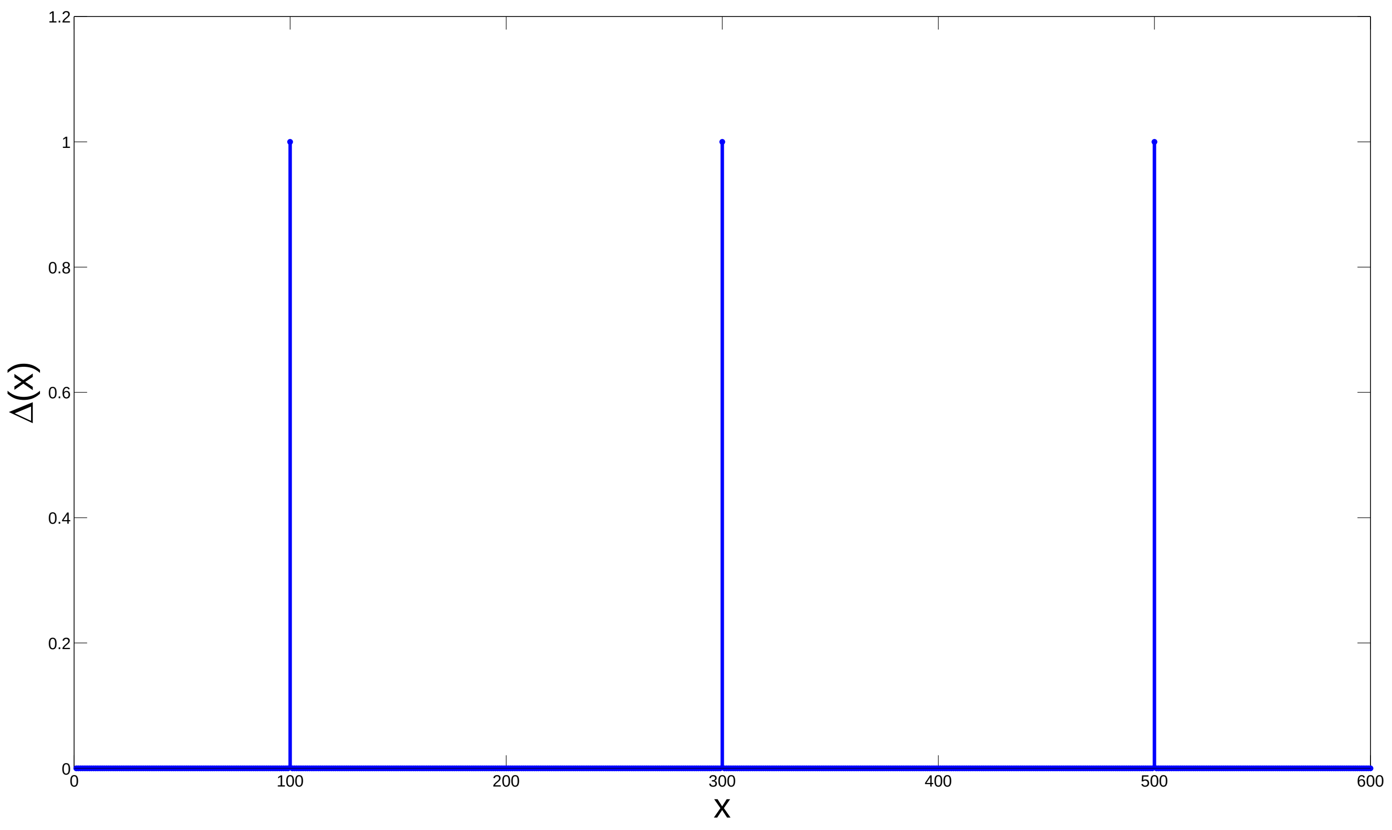
- A real camera cannot sample with ideal Dirac functions since the sensor array consists of pixels of finite size.
- The signal has to pass the ***point spread function*** (PSF) $h(x,y)$.
- For a space-invariant PSF, one discrete sample $f_{m,n}$ is given by

$$f_{m,n} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)h(x-m\Delta x, y-n\Delta y)\,\mathrm{d}x\,\mathrm{d}y,$$

i. e., $f_{m,n}$ is a weighted sum of the surrounding intensities $f(x,y)$ collected at the sensor array (convolution of $f(x,y)$ with the PSF).

Sampling of Real Cameras

Ideal sampling: If we could sample with an ideal Dirac sequence, ideal edges from the real world would be mapped onto ideal edges in an image.



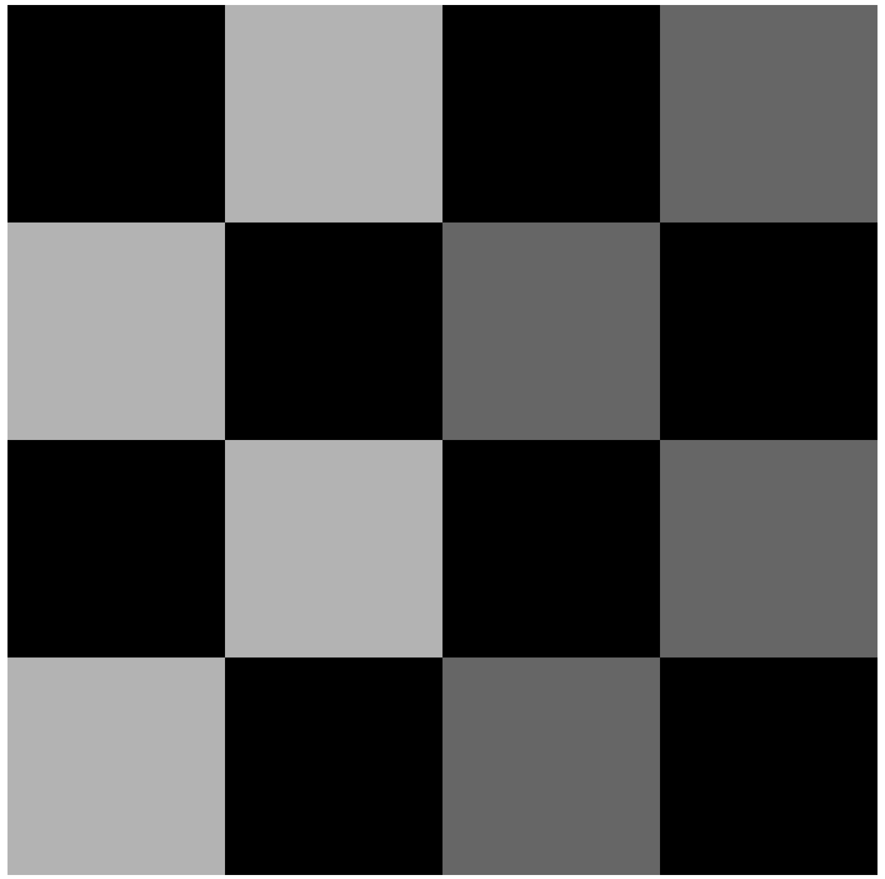


Figure 5: Ideal Dirac sequence in 1-D (left), ideal sampling (right)

Sampling of Real Cameras

Sampling under real-world conditions: If we sample with a real camera, ideal edges get blurred in the observed images.

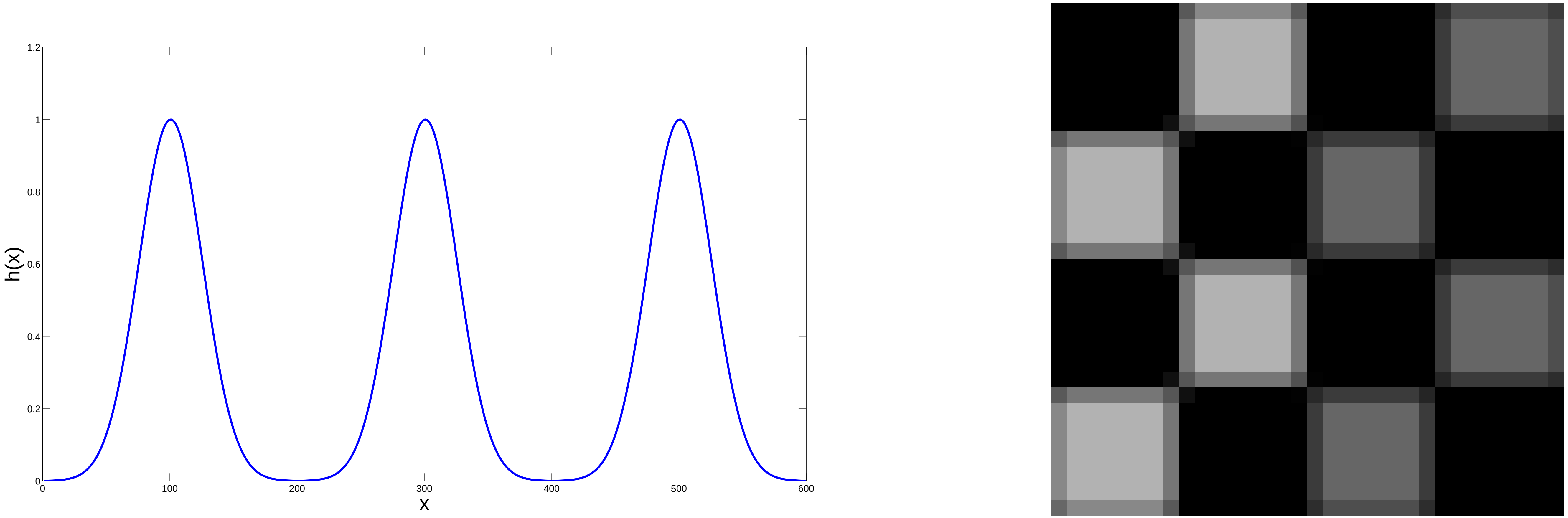


Figure 6: Gaussian PSF kernel in 1-D (left), real sampling (right)

Quantization and Image Noise

Consider noise in the image formation process:

- Discrete samples cannot be obtained and stored with infinite accuracy:
  - 8-bit quantization for grayscale images,
  - 24-bit quantization for RGB color images.
- Furthermore, noise is induced in the sensor array.

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Total observation model: The sampled signal is disturbed by additive noise $\varepsilon_{m,n}$:

$$f_{m,n} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)h(x-m\Delta x, y-n\Delta y)dxdy + \varepsilon_{m,n}.$$

We assume $\varepsilon_{m,n}$ to be the interference of different noise sources and therefore to be spatially invariant Gaussian noise ($\rightarrow$ central limit theorem).

Topics

Take Home Messages

- Super-resolution algorithms enhance the resolution of an image which makes them highly interesting not only for medical applications.
- The sampling theorem allows us to determine a discrete sampling pattern which perfectly samples a given signal.
- A real camera has a limited sampling capability and we have to deal with noise as well.

Further Readings

Theory of image super-resolution (books and review articles):

- Hayit Greenspan. "Super-Resolution in Medical Imaging". In: *The Computer Journal* 52.1 (Feb. 2008), pp. 43–63. DOI: 10.1093/comjnl/bxm075
- Peyman Milanfar, ed. *Super-Resolution Imaging*. Digital Imaging and Computer Vision. CRC Press, 2011
- Sina Farsiu et al. "Advances and Challenges in Super-Resolution". In: *International Journal of Imaging Systems and Technology* 14.2 (Aug. 2004), pp. 47–57. DOI: 10.1002/ima.20007
- Sung Cheol Park, Min Kyu Park, and Moon Gi Kang. "Super-Resolution Image Reconstruction: A Technical Overview". In: *IEEE Signal Processing Magazine* 20.3 (May 2003), pp. 21–36. DOI: 10.1109/MSP.2003.1203207