Medical Image Processing for Interventional Applications

Edge Detection and Structure Tensor

Online Course – Unit 6

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Topics

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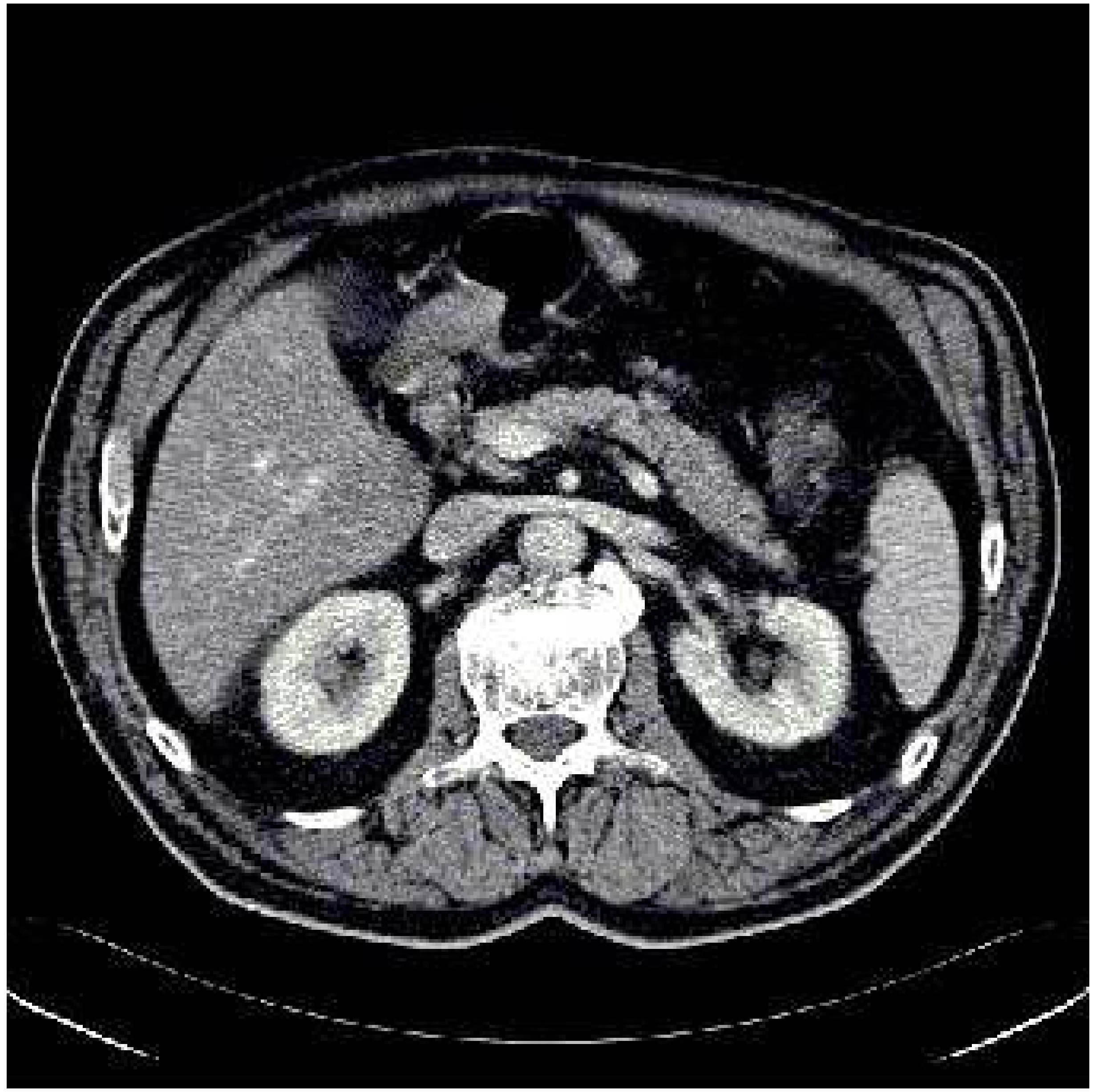
- Edges appear where we observe high differences in intensities.
- Differences in intensities can be measured by the **gradient**:

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} .$$

CT Slice and Corresponding Gradient Image



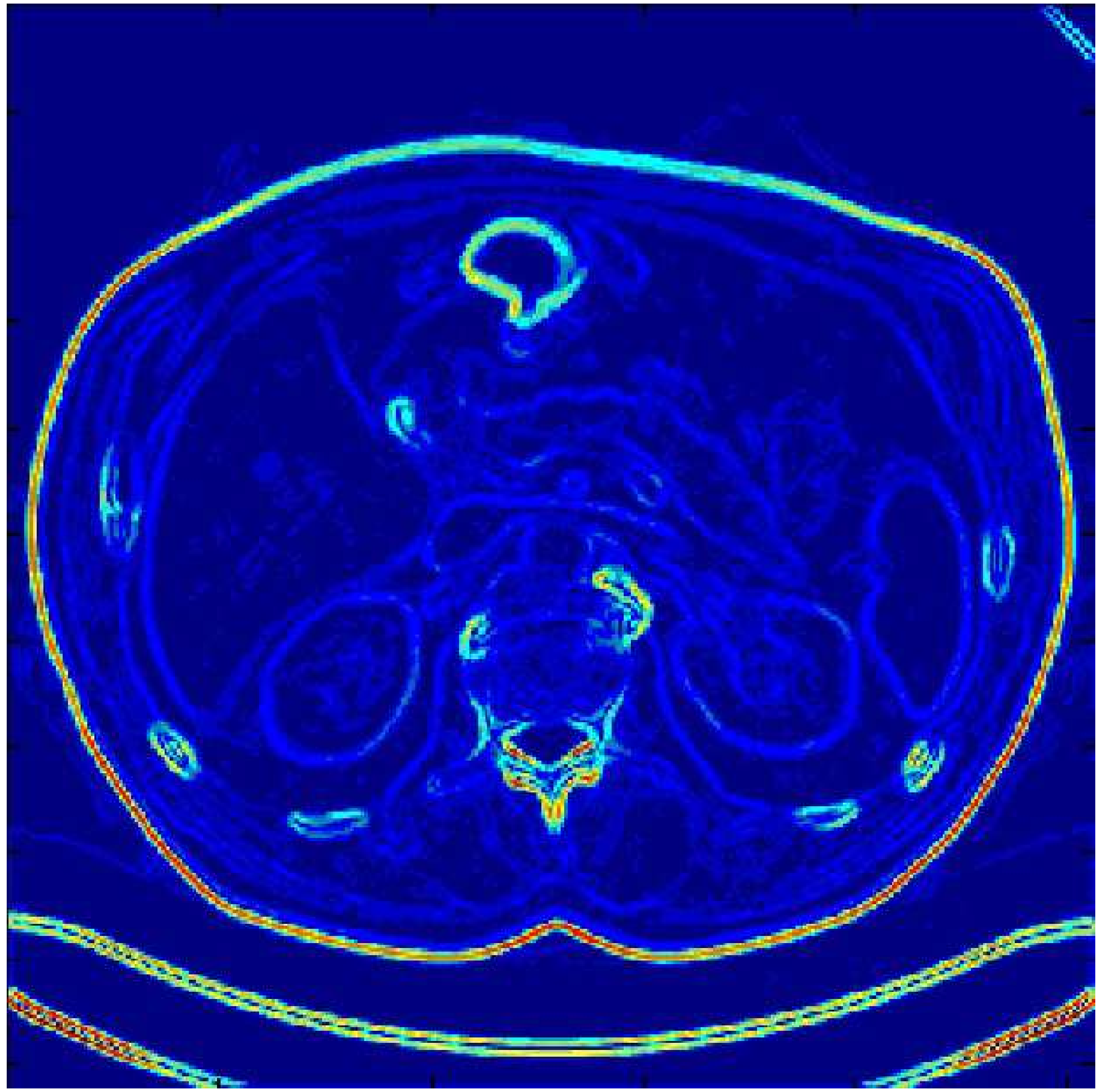


Figure 1: A CT slice (left) and its gradient image (right, gradient norm is color encoded)

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- An edge is supposed to be orthogonal to the gradient (which is often not true in practice).
- Derivatives are highly sensitive to noise (they are even ill-conditioned).
- Different discretizations exist, e.g.:
  - central differences,
  - the Sobel operator,
  - Nevatia-Babu,
  - and many more...

Computation of Discrete Derivatives

From the Taylor series expansion:

$$f(x+h) = f(x) + hf'(x) + O(h^2)$$

we get:

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h).$$

Depending on the choice of h we get:

- forward differences, e.g. $h = 1$:

$$f'(x) \approx f(x+1) - f(x),$$

- backward differences, e.g. $h = -1$:

$$f'(x) \approx f(x) - f(x-1).$$

Differentiation and Smoothing

- Differentiation is mostly combined with low pass filtering, for instance, Gaussian filtering.
- We have two choices:
  - filtering with the Gaussian kernel K_σ followed by discrete differentiation of the filtered signal, where
$$K_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right),$$
  - convolution with first derivative of filtering kernel
$$\nabla f_\sigma = \nabla\left(K_\sigma * f\right) = \left(\nabla K_\sigma\right) * f.$$

Rule of thumb:
Always prefer the computation of derivatives in continuous space to differentiation in a discrete domain.

Topics

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Structure Tensor

Summary

Take Home Messages

Further Readings

Structure Tensor

An extension of the gradient information by using the **structure tensor** was introduced by Förstner and Gülch in 1987.

Applications of the structure tensor in low-level feature analysis are:

- edge detection,
- corner detection,
- texture analysis,
- optical flow,
- tracking.

Definition of Structure Tensor

Define the tensor product of gradients (gradient tensor) by:

$$\boldsymbol{J} = \nabla f \otimes \nabla f = \nabla f (\nabla f)^{\mathsf{T}} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} (f_x, f_y) = \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}.$$

The **structure tensor** is now defined by applying spatial averaging of the components of the gradient tensor with a Gaussian K_ρ:

$$\mathbf{J}_{\rho,\sigma} = K_\rho * (\nabla f_\sigma \otimes \nabla f_\sigma) \qquad \text{(element-wise convolution)},$$

where

$$\nabla f_\sigma = (\nabla K_\sigma) * f.$$

In this context, the “standard deviations” ρ and σ act as **regularization parameters**.

Comments on the Structure Tensor

Averaging is required:

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Let λ_1, λ_2 be the eigenvalues and $\boldsymbol{v}_1, \boldsymbol{v}_2$ be the respective eigenvectors of the structure tensor. The eigenvalues describe the average integrated contrast in the eigendirections:

- **flat area**: $\lambda_1 = \lambda_2 = 0$,
- **straight edge**: $\lambda_1 \gg \lambda_2 = 0$,
- **corner**: $\lambda_1 \geq \lambda_2 \gg 0$.

Example

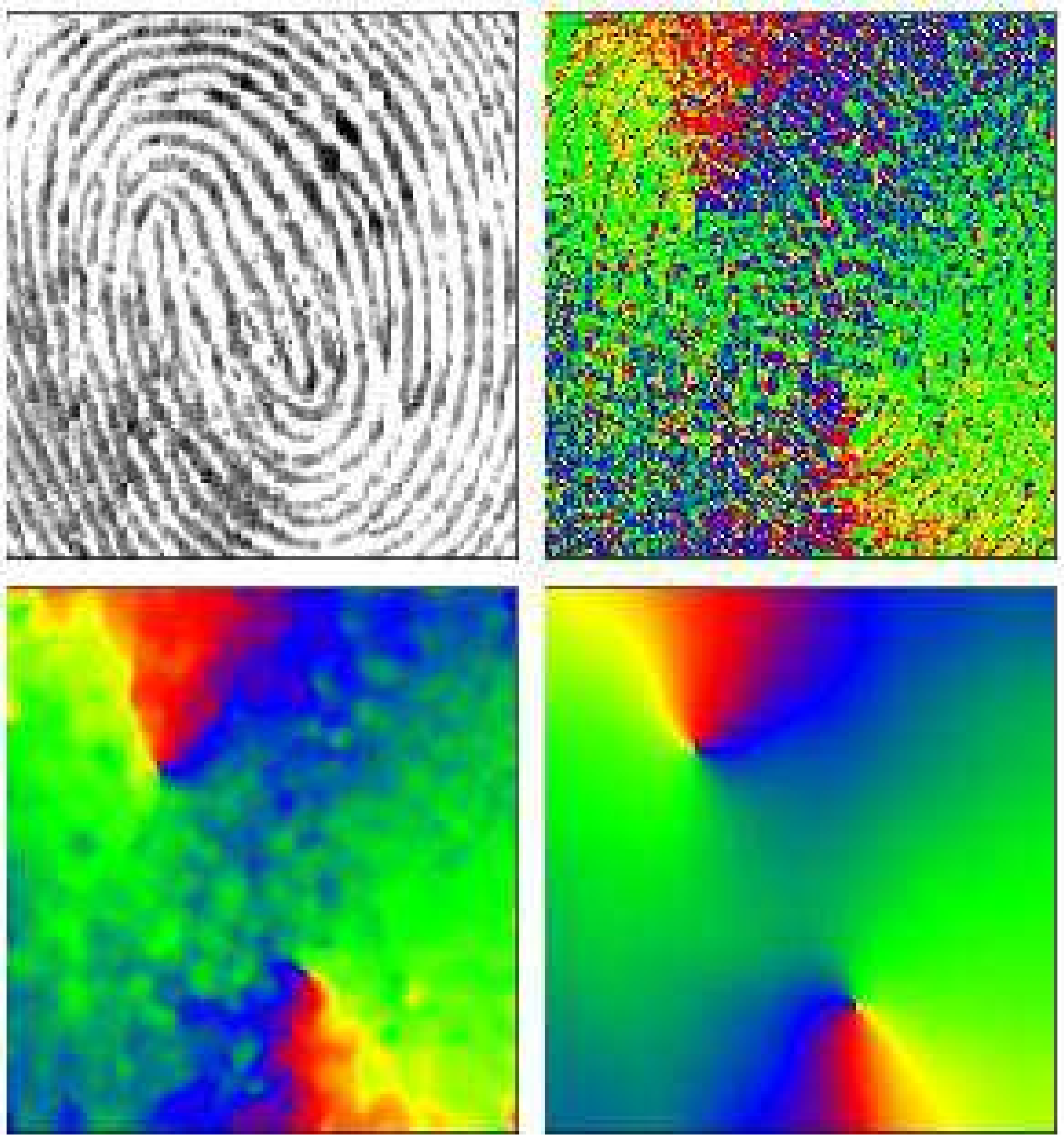

Figure 2: Original image (top left), direction of eigenvector with smaller eigenvalue for $\rho = 0$ (top right), $\rho = 4$ (bottom left), and $\rho = 26$ (bottom right) (image courtesy of Joachim Weickert)

Drawbacks of Structure Tensor

- Computation of the structure tensor violates the sampling theorem.
- Spatial averaging is done by Gaussian filtering that is not adapted to local structures.
- Corner detection has low accuracy.

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Take Home Messages

- The detection of structures like edges and corners (of objects) in images is an important task, especially for interventional imaging.
- Local gradients and smoothed versions provide a mathematical basis for edge detection.
- Although it is not perfect, the structure tensor is an important tool to estimate local image structure.

Further Readings

The fundamentals of image processing including gradient computation, structure tensor, edge and corner detection, can be found in:

Bernd Jähne. *Practical Handbook on Image Processing for Scientific and Technical Applications*. 2nd ed. CRC Press, 2004

The idea of the structure tensor was first published by:

W. Förstner and E. Gülch. “A Fast Operator for Detection and Precise Location of Distinct Points, Corners and Centres of Circular Features”. In: *Proceedings of the ISPRS Intercommission Workshop on Fast Processing of Photogrammetric Data, Interlaken, Switzerland* (June 1987), pp. 281–305

A nice introduction and improvement of the structure tensor can be found in:

Ullrich Köthe. “Edge and Junction Detection with an Improved Structure Tensor”. In: *Pattern Recognition: 25th DAGM Symposium, Magdeburg, Germany, September 10-12, 2003. Proceedings*. Ed. by Bernd Michaelis and Gerald Krell. Vol. 2781. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 2003, pp. 25–32. DOI: 10.1007/978-3-540-45243-0_4