# Medical Image Processing for Interventional Applications

## Epipolar Consistency Metric

Online Course – Unit 36

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TECHNISCHE FAKULTÄT

# Topics

## Epipolar Consistency Metric

- Idea
- Formalism

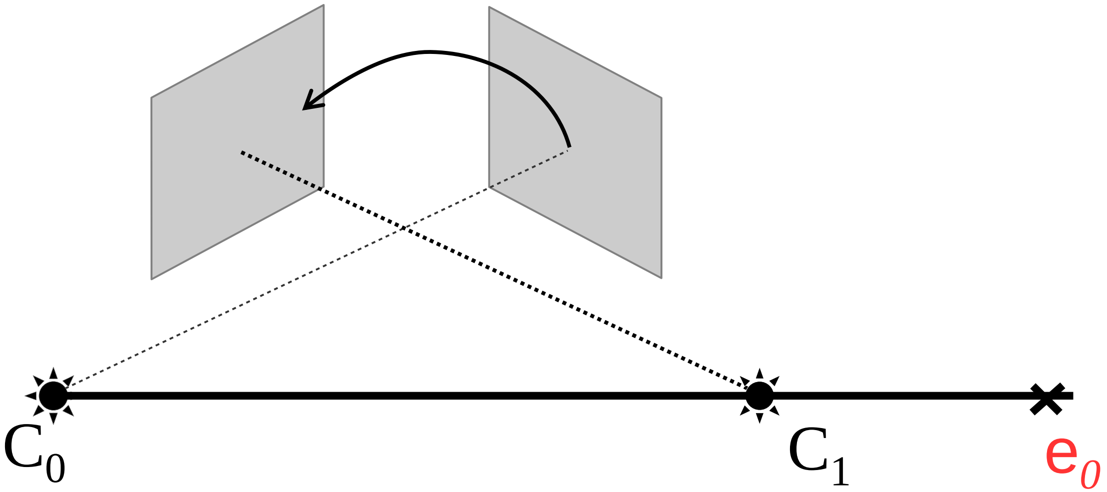
## Special Case – Circular Trajectory

## Summary

- Take Home Messages
- Further Readings

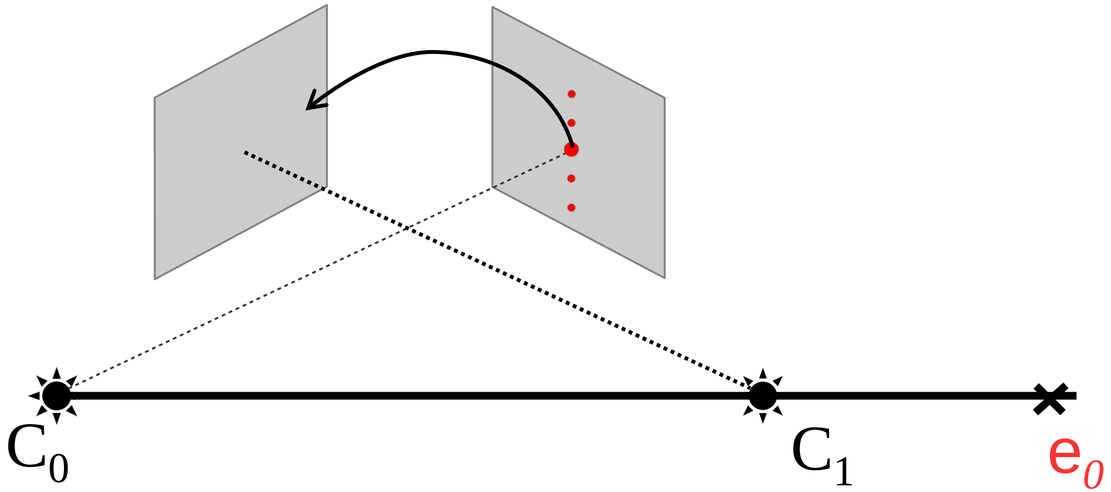
# Idea

1. Compute fundamental matrix and epipoles



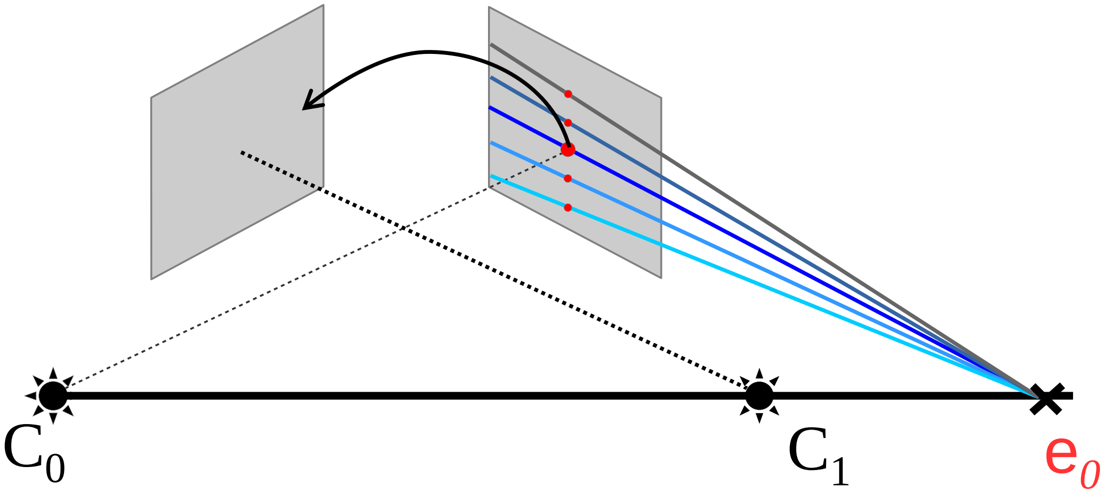
## Idea

1. Compute fundamental matrix and epipoles
2. Select points in reference image



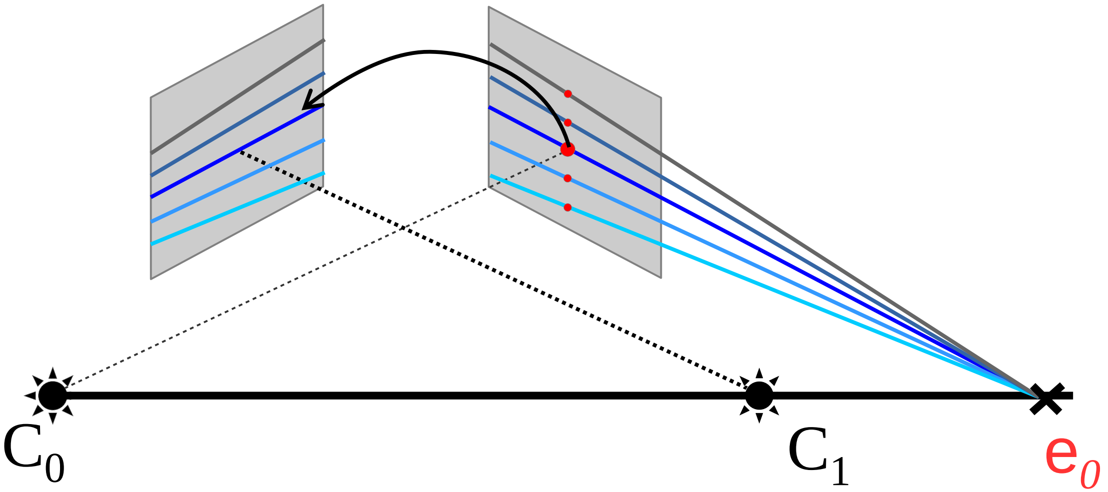
# Idea

1. Compute fundamental matrix and epipoles
2. Select points in reference image
3. Compute lines in reference image as the join with the epipole



# Idea

1. Compute fundamental matrix and epipoles
2. Select points in reference image
3. Compute lines in reference image as the join with the epipole
4. Use fundamental matrix to project to lines in the other image



## Formalism

$$M_0^i = \frac{1}{|\mathcal{X}_0^i|} \sum_{x_0 \in \mathcal{X}_0^i} \left( \frac{d}{dt} \rho_{I_0}(x_0 \times e_0) - \frac{d}{dt} \rho_{I_i}(F_0^i x_0) \right)^2$$

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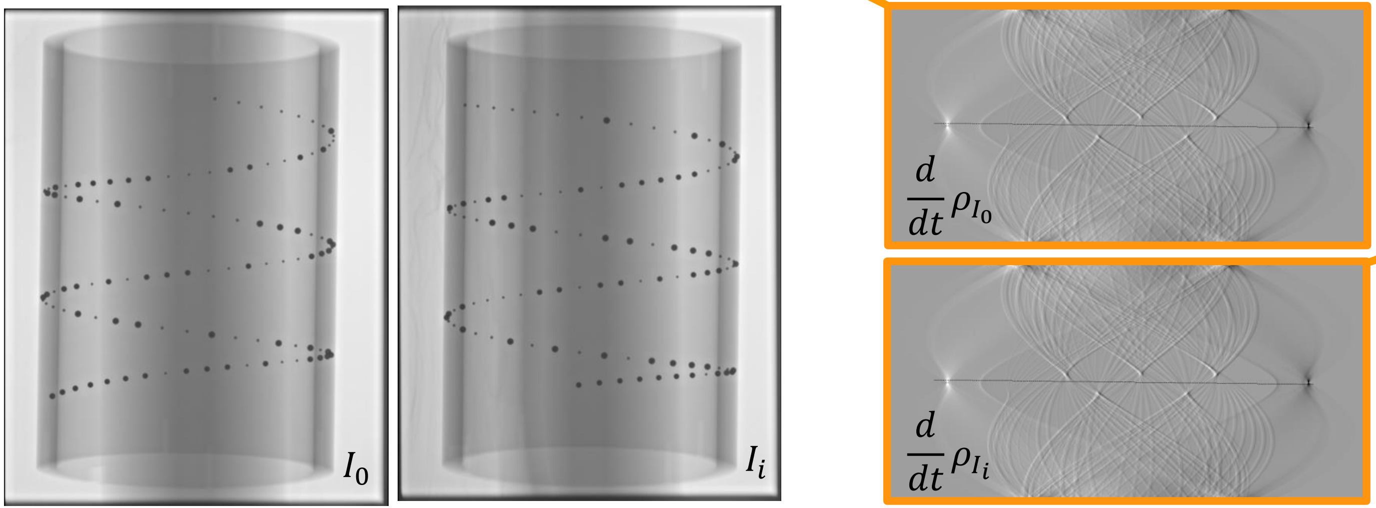

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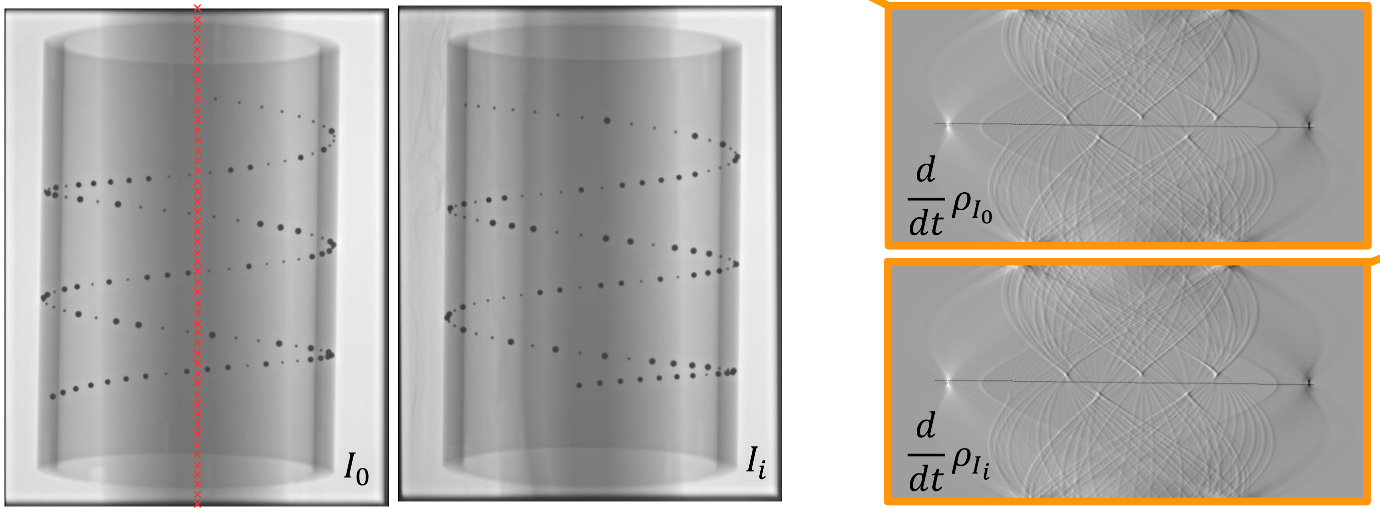

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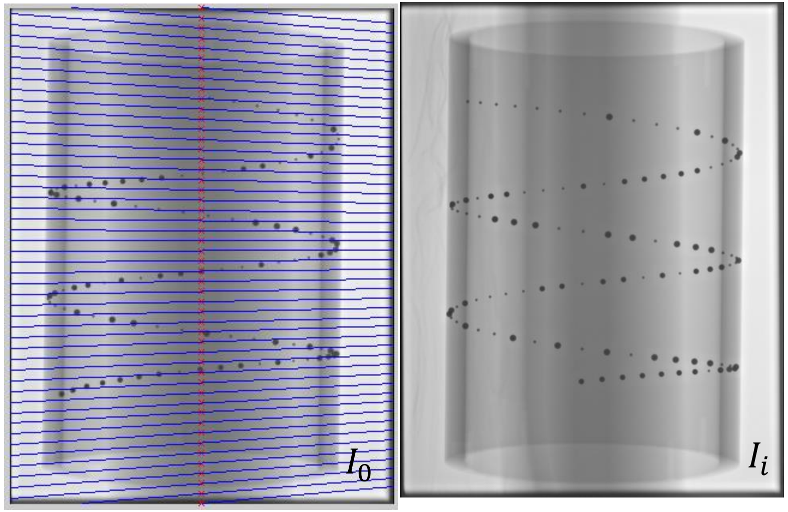


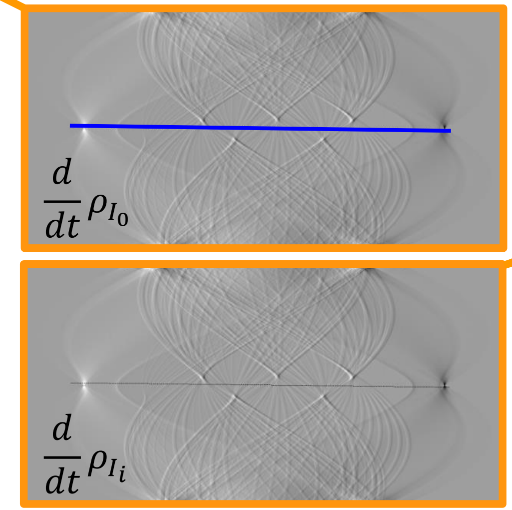

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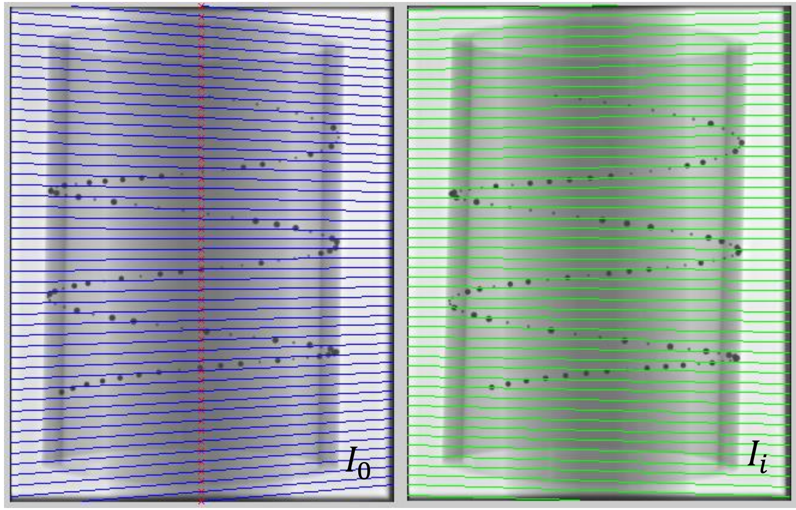


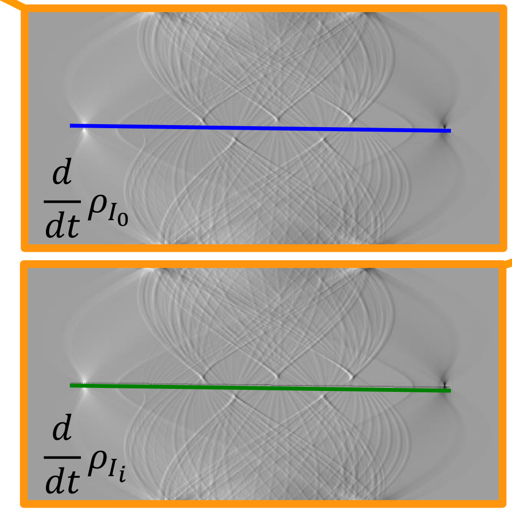

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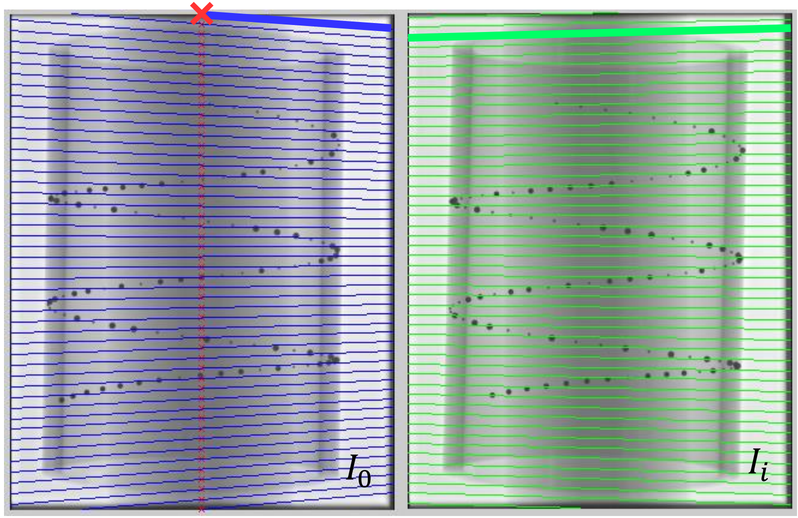


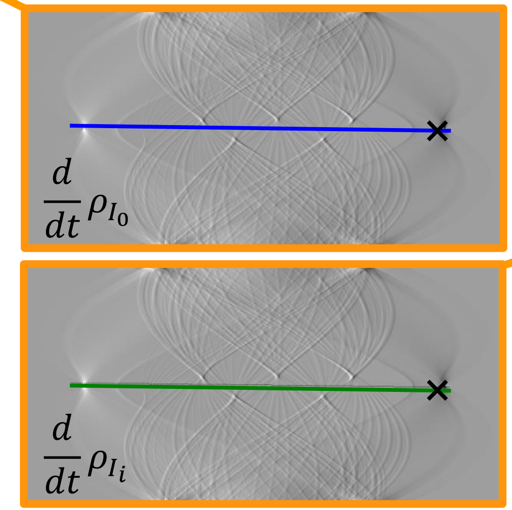

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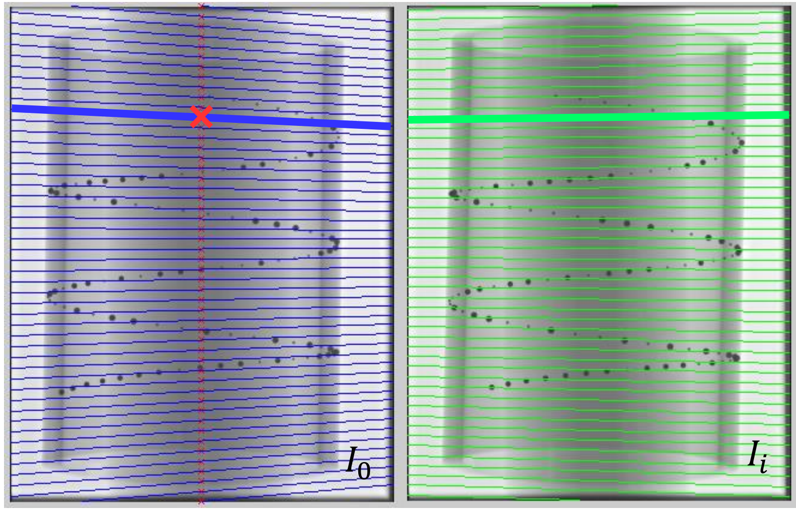


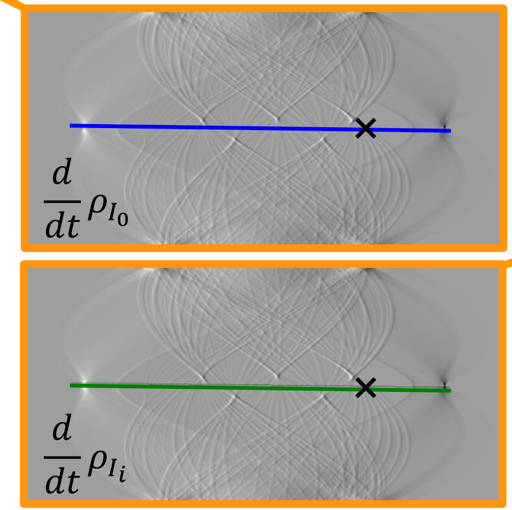

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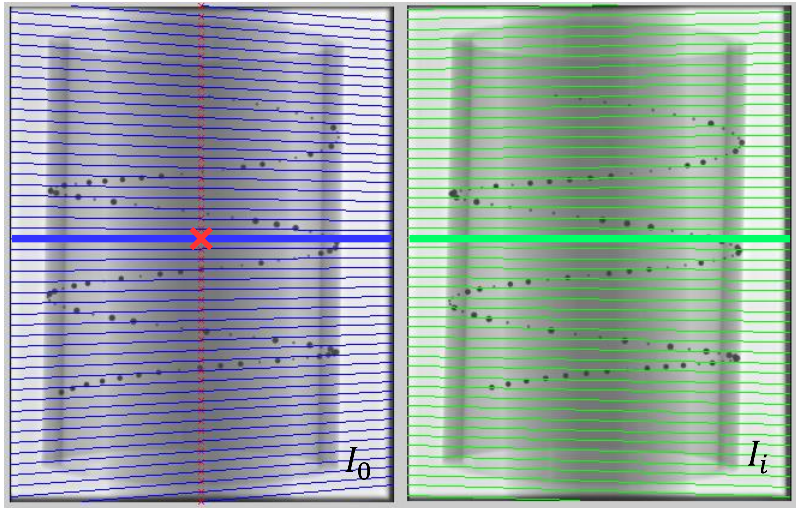


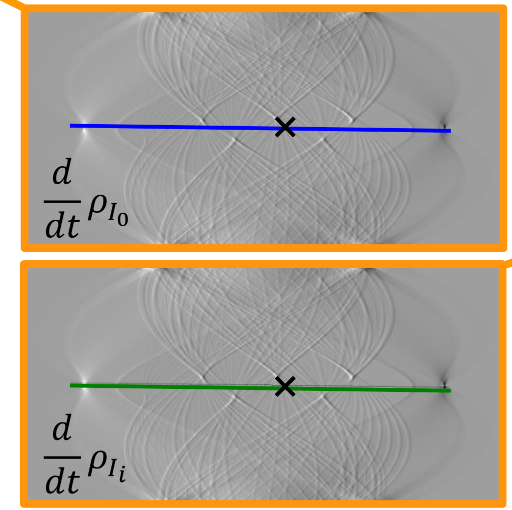

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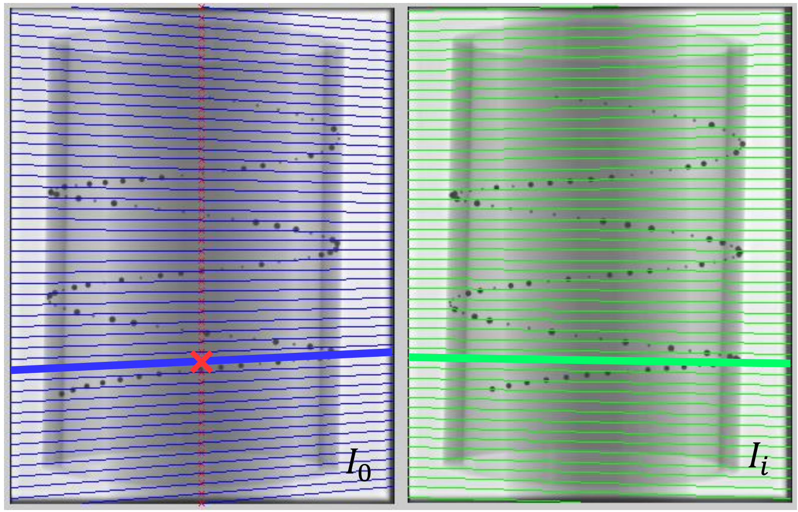


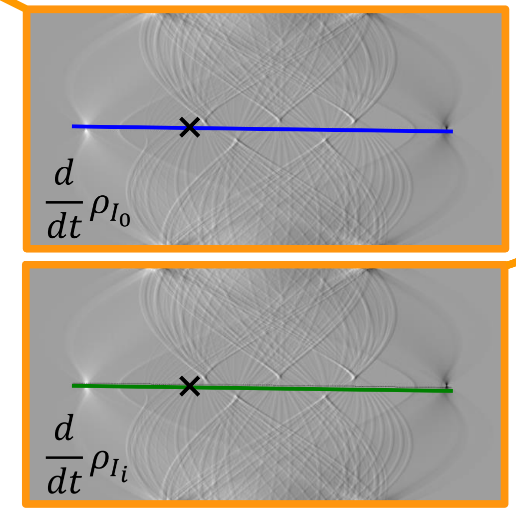

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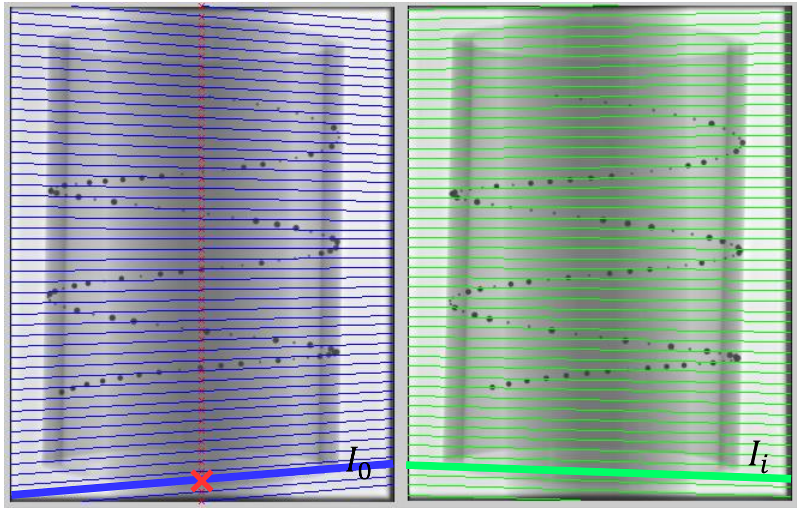


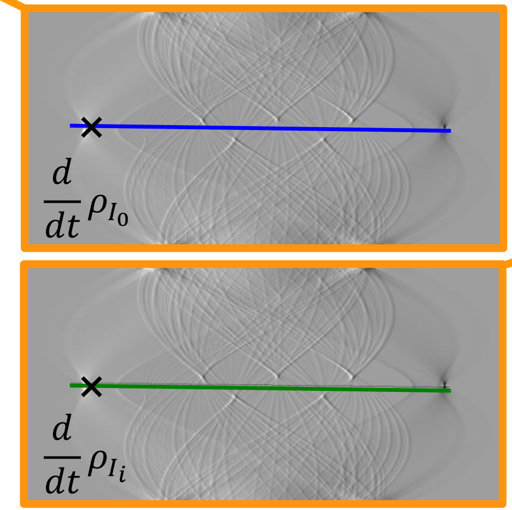

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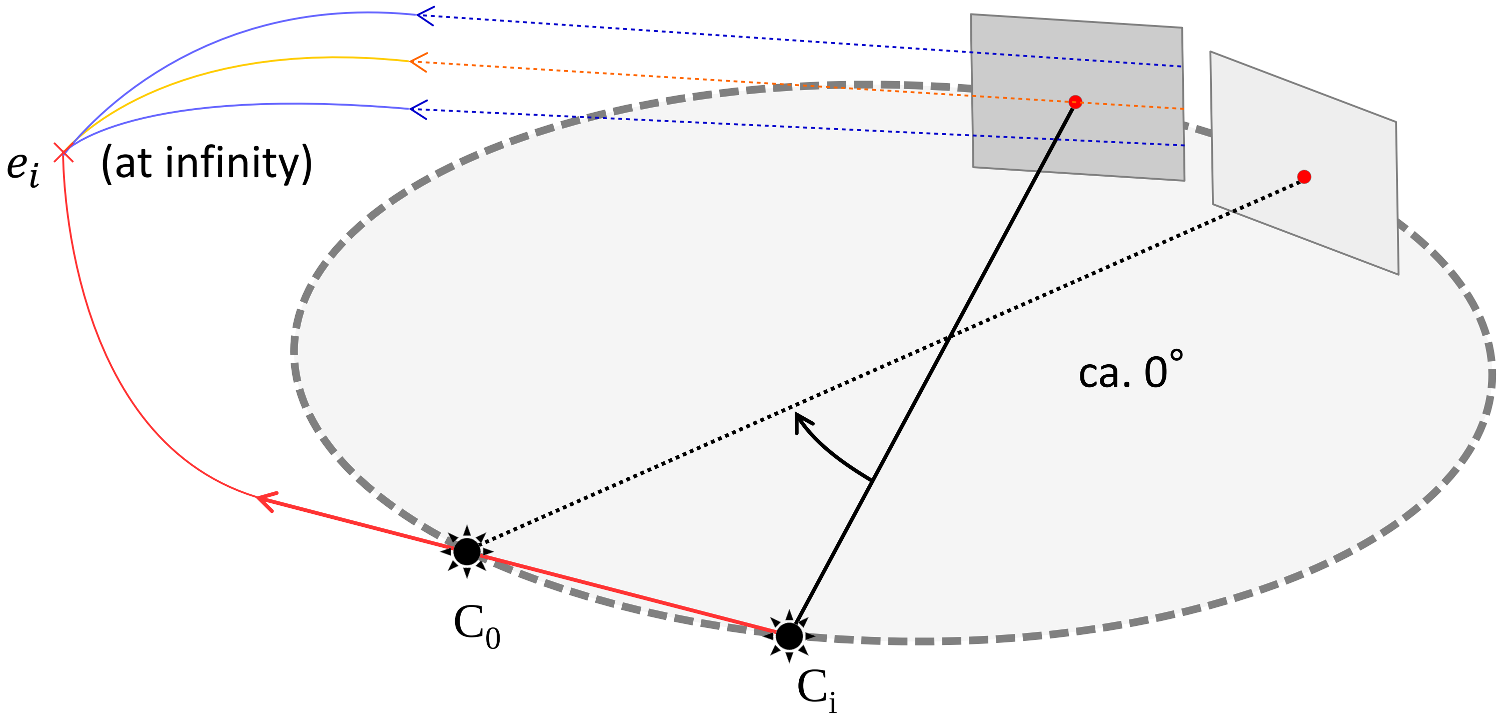

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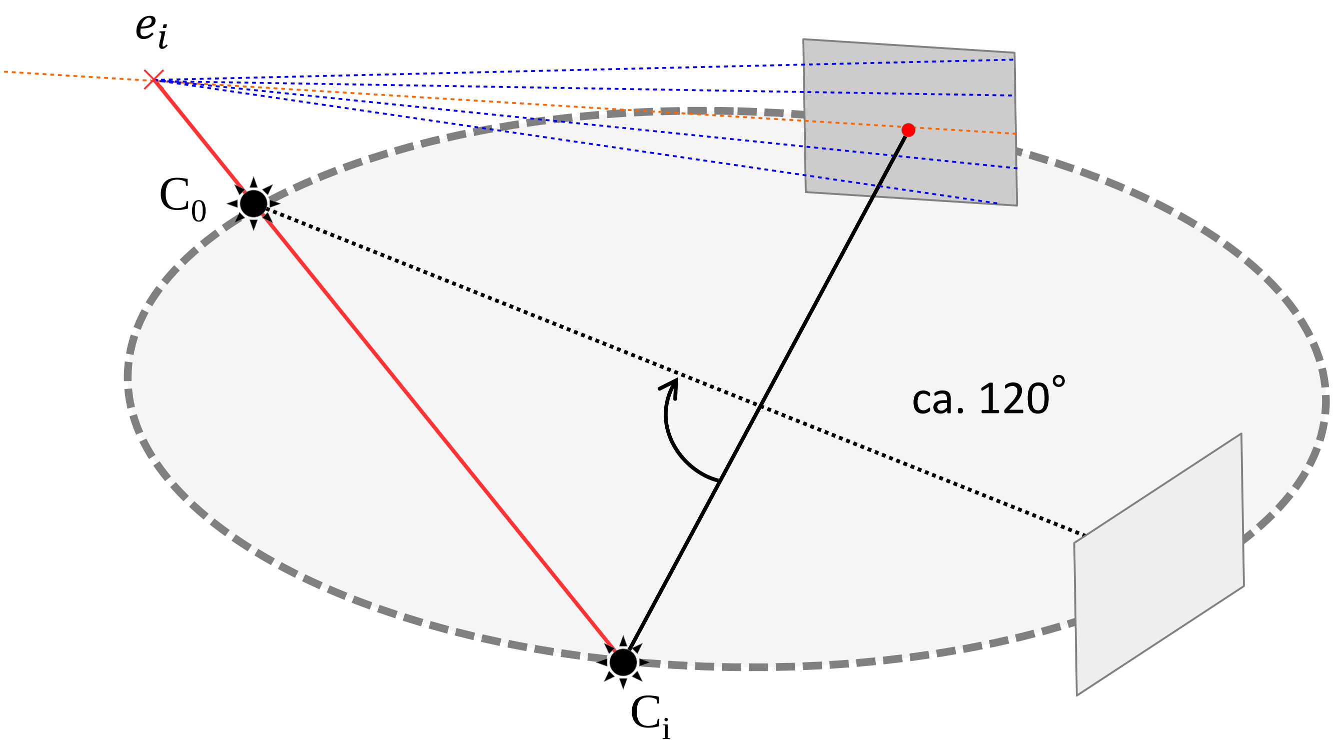

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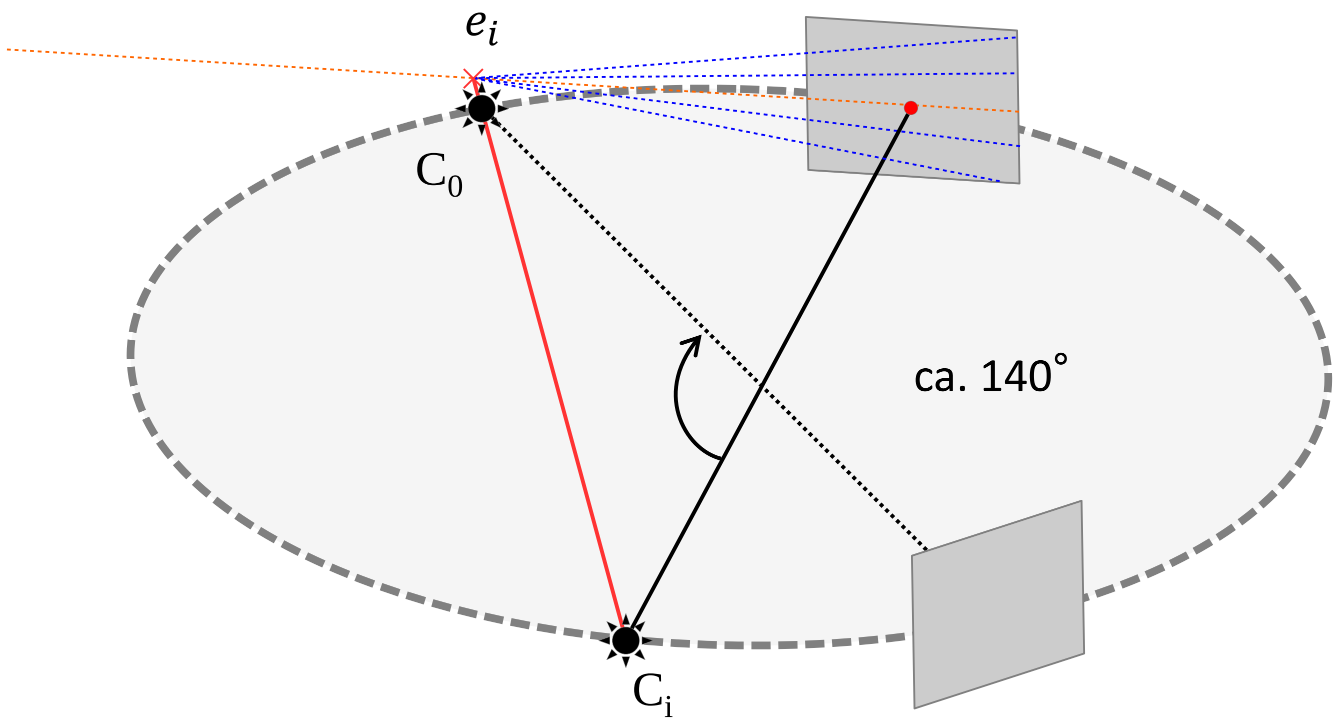

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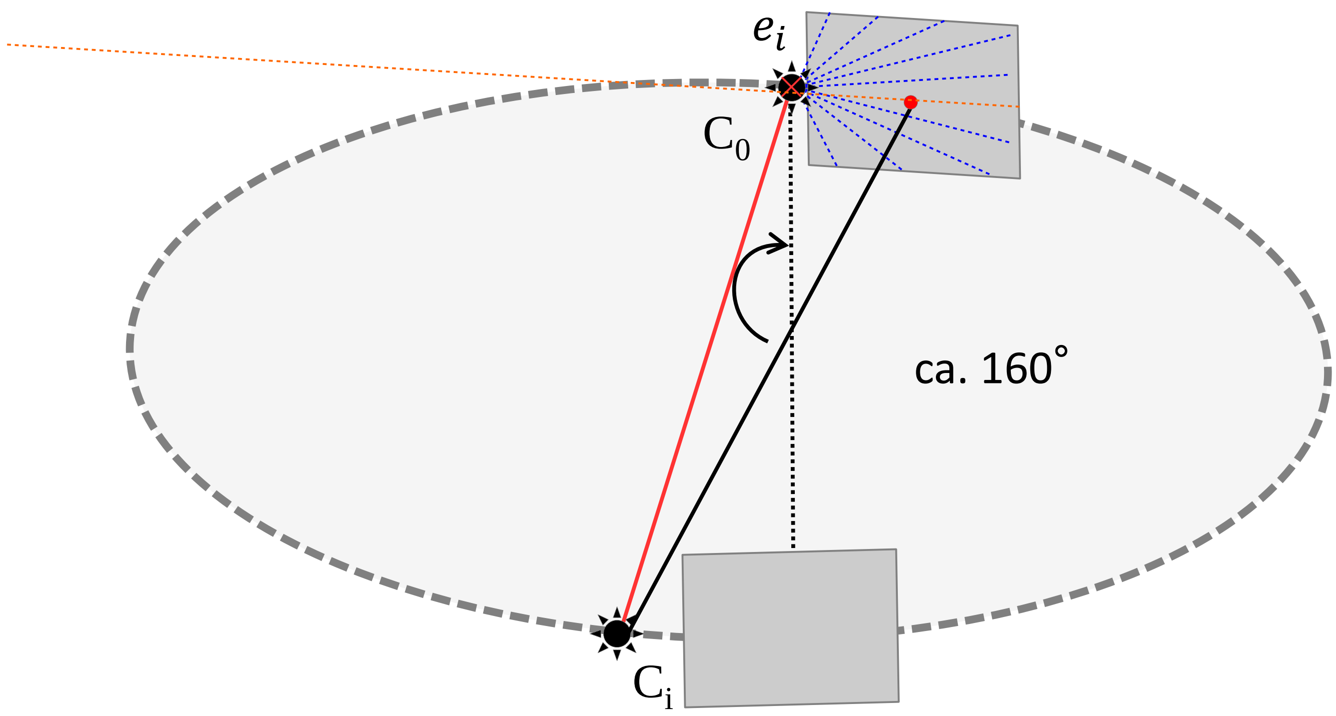

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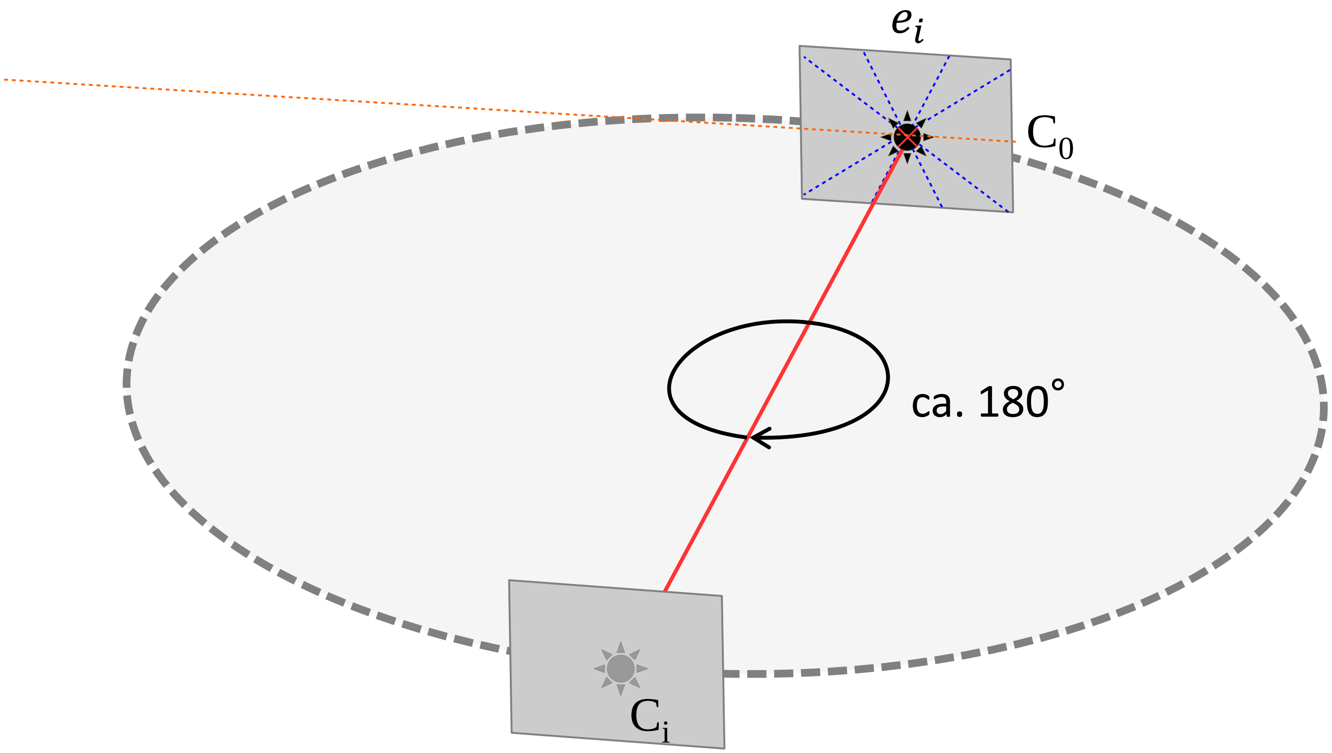
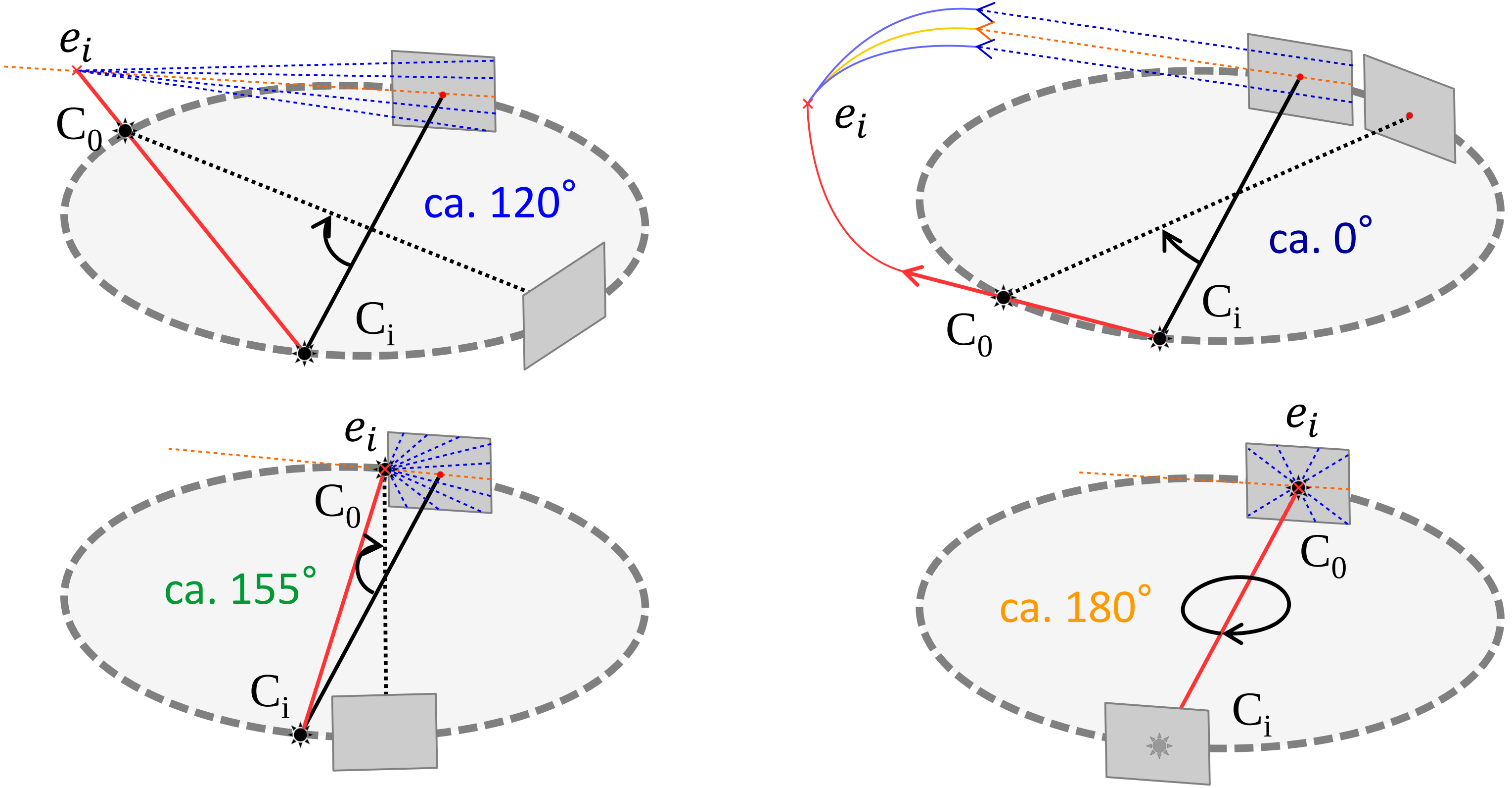


# Let's Summarize …



# Optimization in “Reliable Directions“

Ideal scenario:

- Two pairs of views with orthogonal lines
- Or when epipole is within the image (opposing views)
- Ideally random positions on a sphere around the object

→ No information in directions of parallel lines (due to summation)

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# Take Home Messages

- We described the formulation of a consistency metric which works on **any** pair of transmission images.
- In case of opposing views, the epipole is within the image, or otherwise information is given only in one direction.

# Further Readings

André Aichert et al. "Epipolar Consistency in Transmission Imaging". In: *IEEE Transactions on Medical Imaging* 34.11 (Nov. 2015), pp. 2205–2219. DOI: 10.1109/TMI.2015.2426417

**Acknowledgements:**