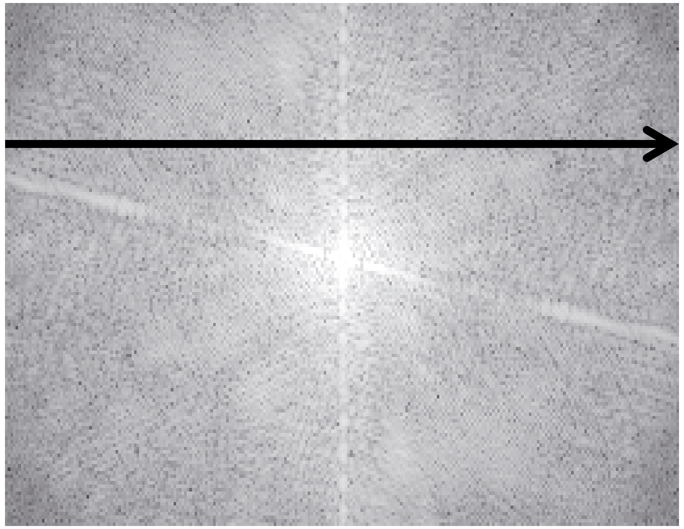
Repetition

Learning objectives

Understand important effects of finite
k-space velocity on Cartesian MRI



Chemical shift, field inhomogeneities, T_2 relaxation

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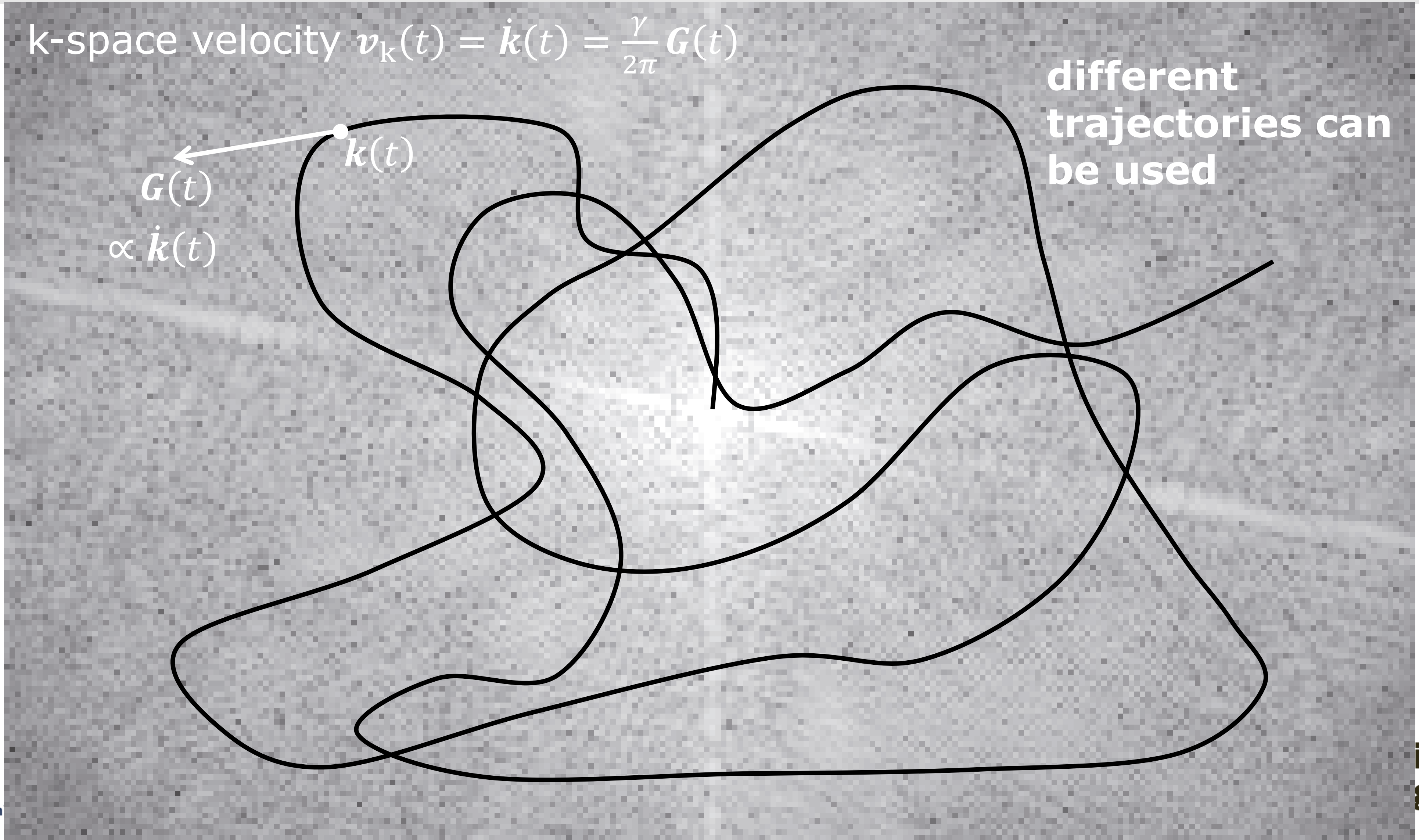
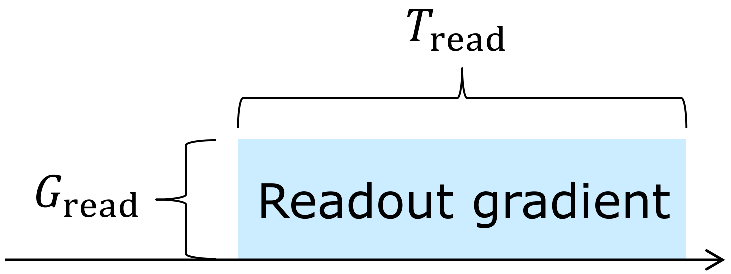
We can sample arbitrary trajectories

$$\boldsymbol{k}(t) = \frac{\gamma}{2\pi}\int_{t_0}^{t} \boldsymbol{G}(t')dt'$$



Summary

$$\boldsymbol{k}(t) = \frac{\gamma}{2\pi}\int_0^t \boldsymbol{G}(t')dt'$$



$$k_{\text{read}}(t) = \frac{\gamma}{2\pi} G_{\text{read}} t - k_{\max}$$

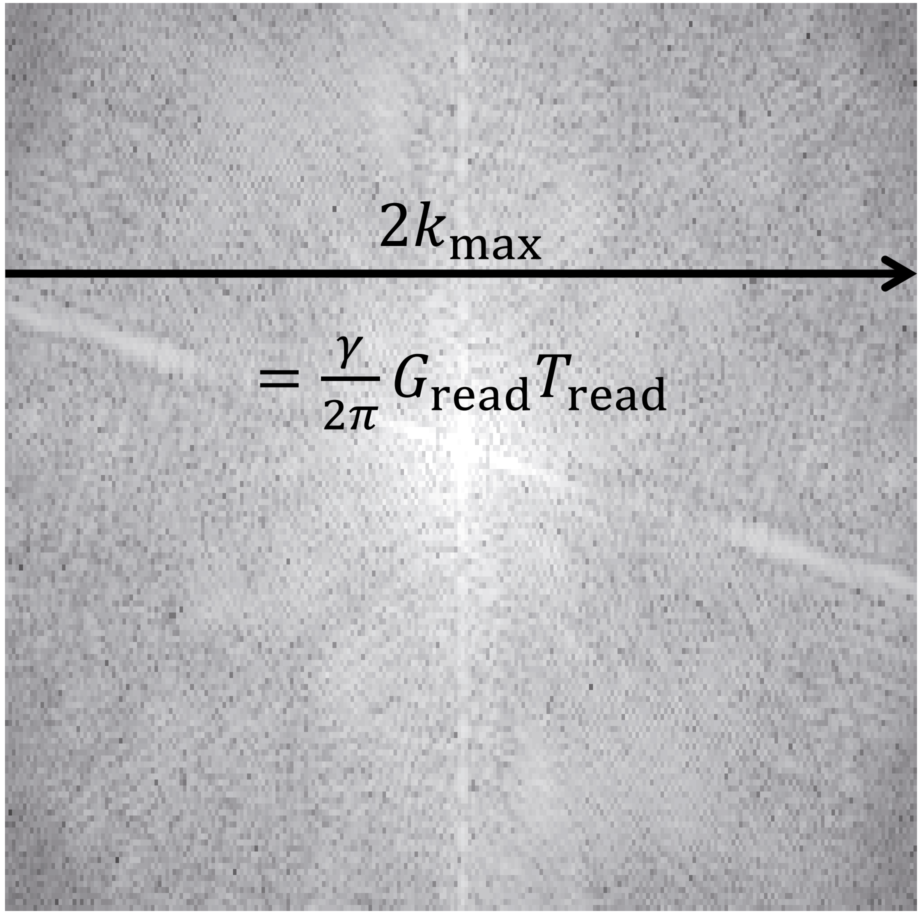
k-space velocity $v_{\text{k,read}} = \dot{k}_{\text{read}}(t) = \frac{\gamma}{2\pi} G_{\text{read}}$

Alternatively: $v_{\text{k,read}} = \dfrac{k_{\text{read}}(T_{\text{read}}) - k_{\text{read}}(0)}{T_{\text{read}}}$

$$= \frac{2k_{\max}}{T_{\text{read}}} = \frac{\gamma}{2\pi} G_{\text{read}}$$

indirect

Three measures of speed in k-space



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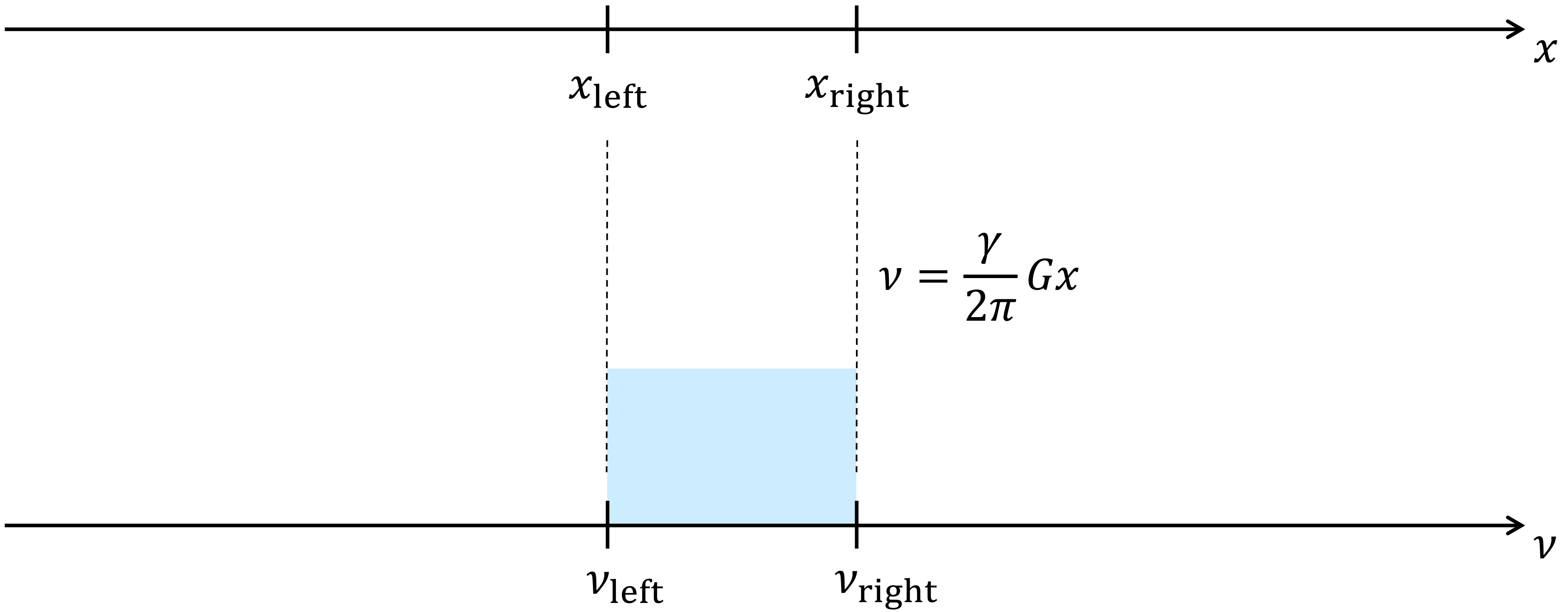
Bandwidth

$x_{\text{right}} - x_{\text{left}}$ = edge length of voxel Δx



$$\nu = \frac{\gamma}{2\pi} G x$$

Acquisition bandwidth $BW = \nu_{\text{right}} - \nu_{\text{left}} = \frac{1}{T_{\text{read}}}$

indirect

indirect

Fourth measure of speed in k-space

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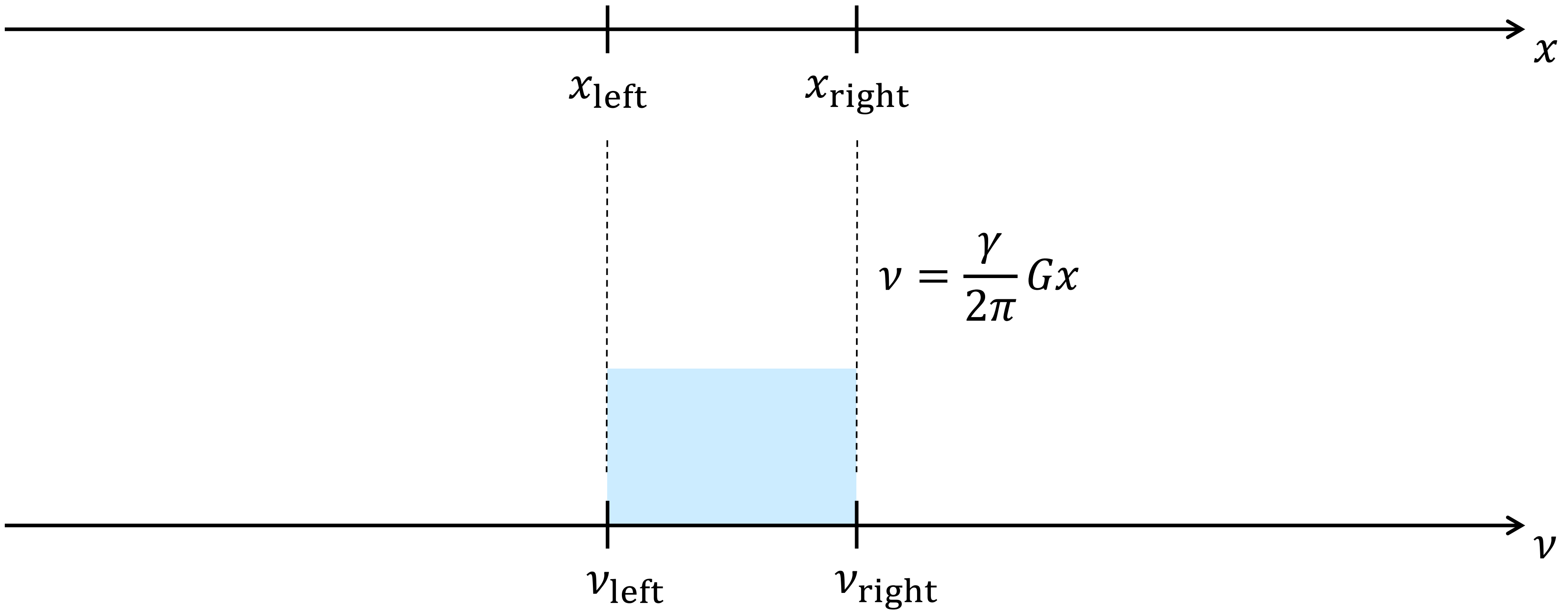
Bandwidth
2nd definition, not used in this course

$x_{\text{right}} - x_{\text{left}} =$ field of view FOV



Fourth measure of speed in k-space

Acquisition bandwidth $BW = \nu_{\text{right}} - \nu_{\text{left}} = \frac{N_{\text{voxel}}}{T_{\text{read}}}$

indirect

indirect

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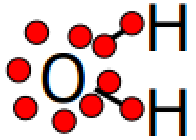
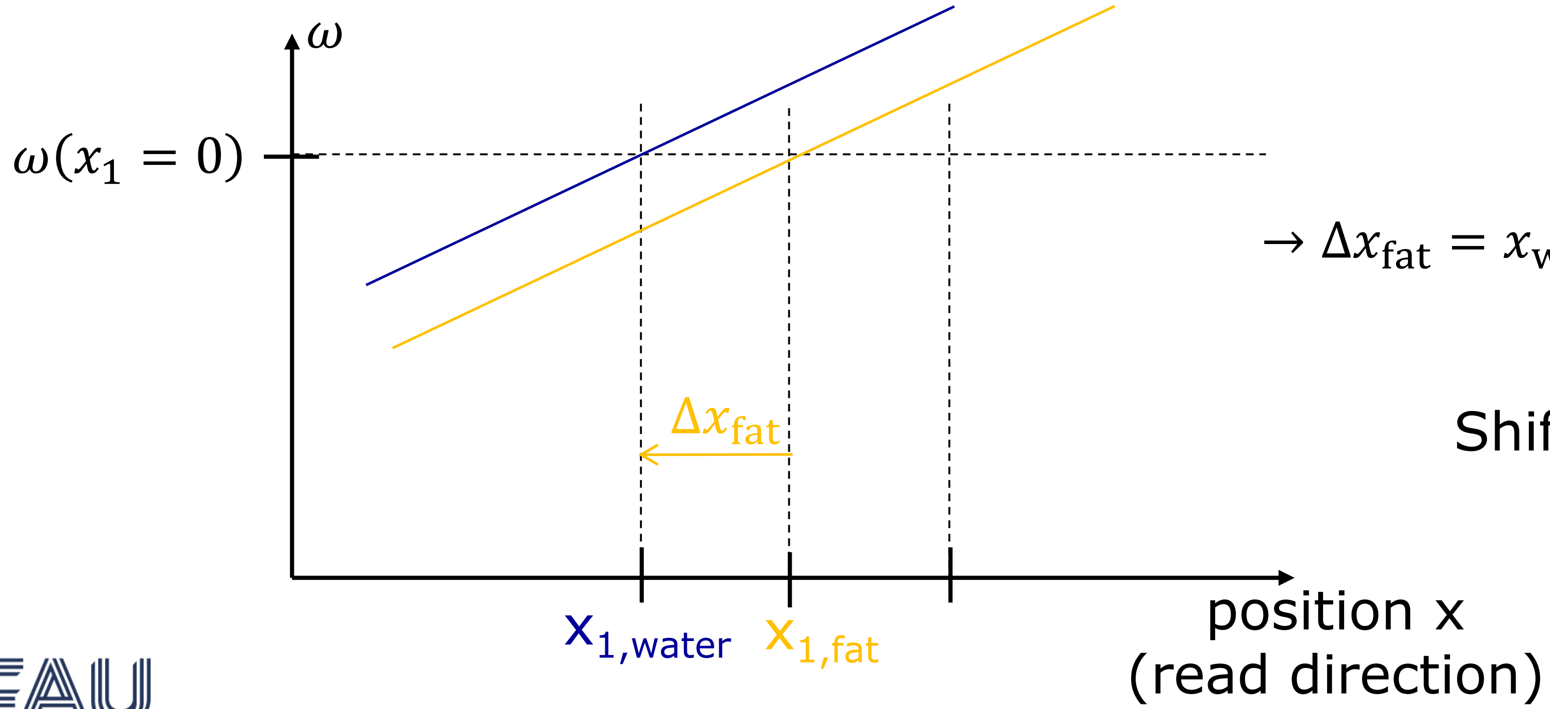
Chemical shift

$$\nu_{0,\text{of a certain chemical group}} = \frac{\gamma}{2\pi} B_0 \cdot (1 + \delta)$$

δ is the chemical shift

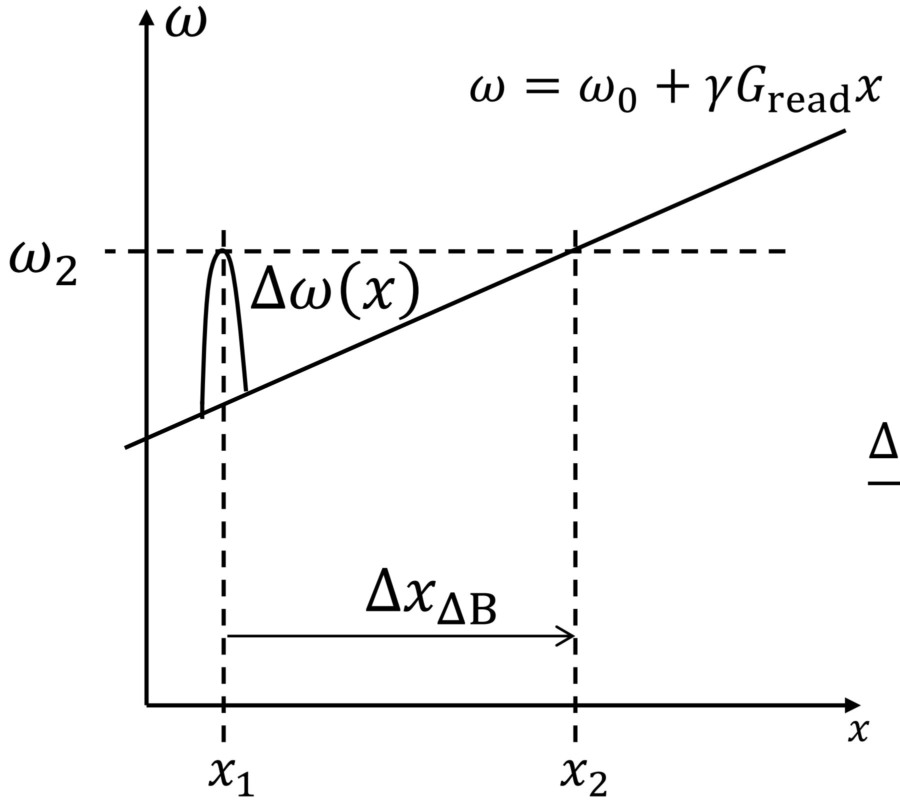




$$\rightarrow \Delta x_{\text{fat}} = x_{\text{water}} - x_{\text{fat}} = \frac{B_0 \cdot (\delta_{\text{fat}} - \delta_{\text{water}})}{G_{\text{read}}}$$

Shift in #voxel $= \frac{\Delta x_{\text{fat}}}{\Delta x} = \frac{\Delta \nu_{\text{fat}}}{BW}$

Field inhomogenity



$$\frac{\Delta x_{\Delta \text{B}}(\boldsymbol{r})}{\Delta x} = \frac{\Delta \nu(\boldsymbol{r})}{BW}$$

In the image:
point x_1 is shifted to x_2

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Summary

Physical Effect
Chemical shift: $\omega \to \gamma B_0 \cdot (1 + \delta)$
Field inhomogenity: $\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \Delta \boldsymbol{B}(\boldsymbol{r})$

Summary

Physical Effect	**Effect on Cartesian MRI**	$\propto B_0$	$\propto BW^{-1}$	$\propto G_{\text{read}}^{-1}$	$\propto v_{\text{k}}^{-1}$
Chemical shift: $\omega \to \gamma B_0 \cdot (1+\delta)$	Shift of fat signal by Δx_{fat}	✓	✓	✓	✓
Field inhomogenity: $\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \Delta \boldsymbol{B}(\boldsymbol{r})$	Shift of signal by $\Delta x_{\Delta\text{B}}(\boldsymbol{r})$	✓	✓	✓	✓

$$\boxed{\text{SNR} \propto T_{\text{read}}^{1/2} \propto BW^{-1/2}}$$

→ Reduction of these artifacts by a factor of 2 is paid with a factor $\sqrt{2}$ of SNR

1.5. Transversal relaxation

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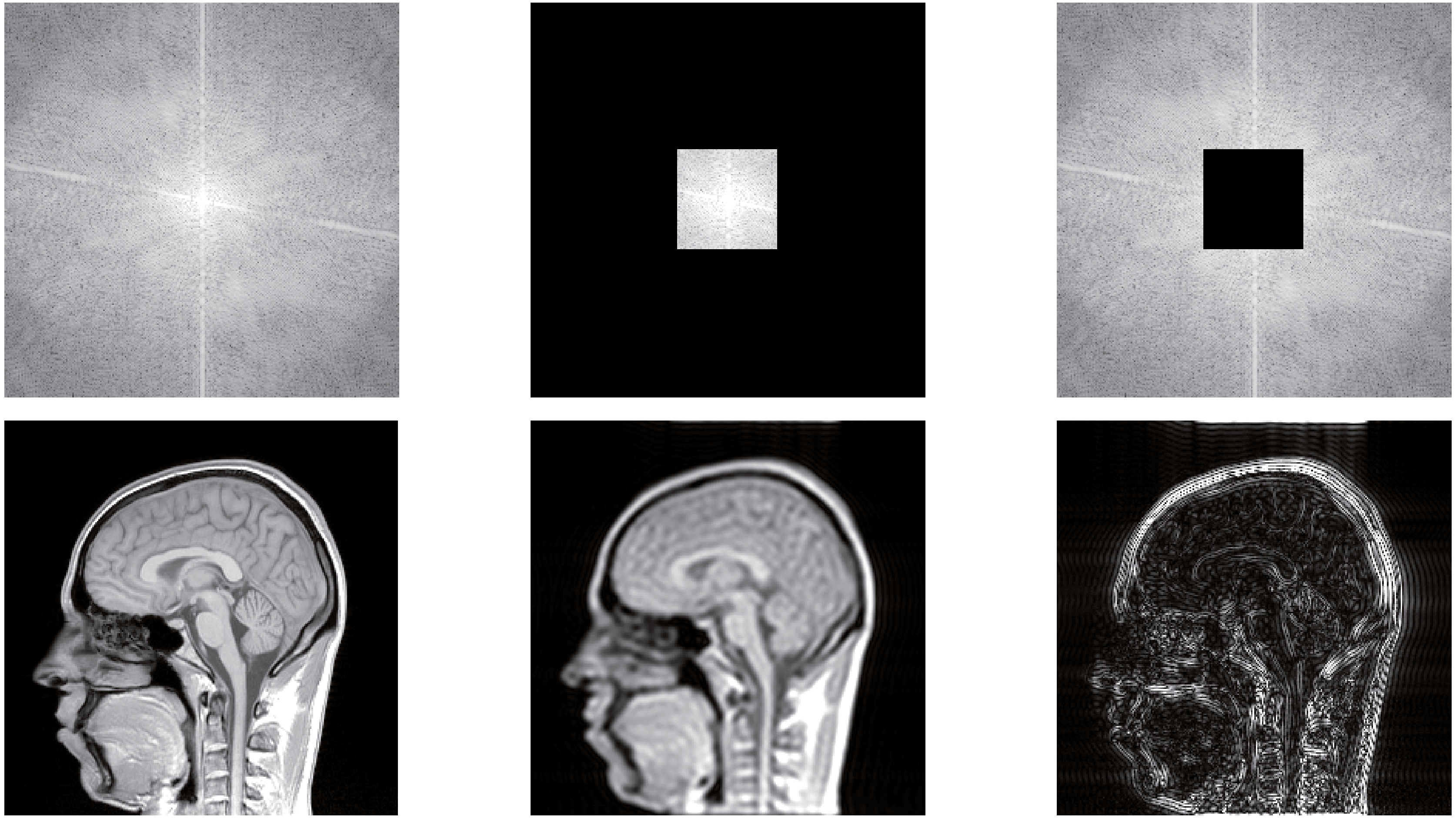
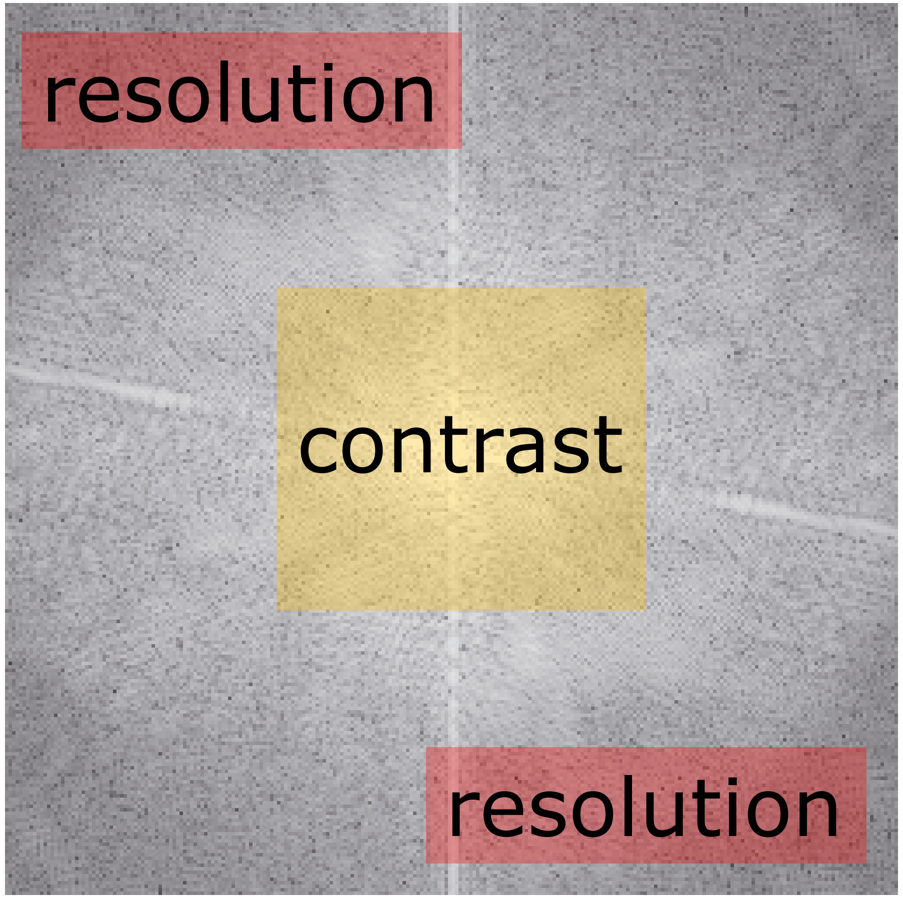
Reminder:
k-Space & image space



Reminder:
k-Space: resolution and contrast



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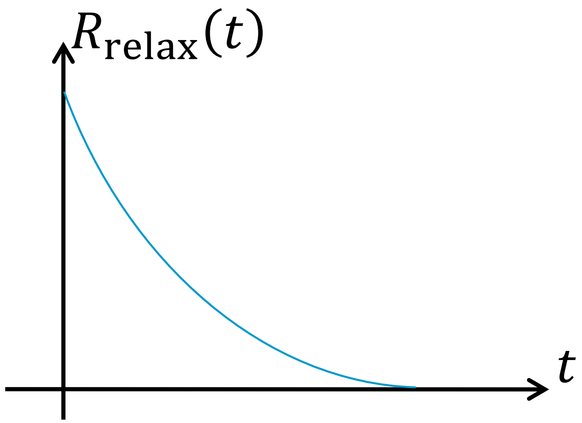
Transversal relaxation and relation to k-space position

- Due to T_2 relaxation, the transversal magnetization decays as $M_\perp(t) = M_\perp(0)e^{-t/T_2} = M_\perp(0)R_{\text{relax}}(t)$
- If $\Delta B(\boldsymbol{r}) \neq 0$ → $R_{\text{relax}}(t)$ might take a different functional form



Say that k-space is sampled with $\boldsymbol{k}(t)$.

How much time has evolved until a certain k-space position is reached?
→ find $t(\boldsymbol{k})$

The one can define
$\tilde{R}_{\text{relax,k}}(\boldsymbol{k}) \coloneqq R_{\text{relax}}\big(t(\boldsymbol{k})\big)$

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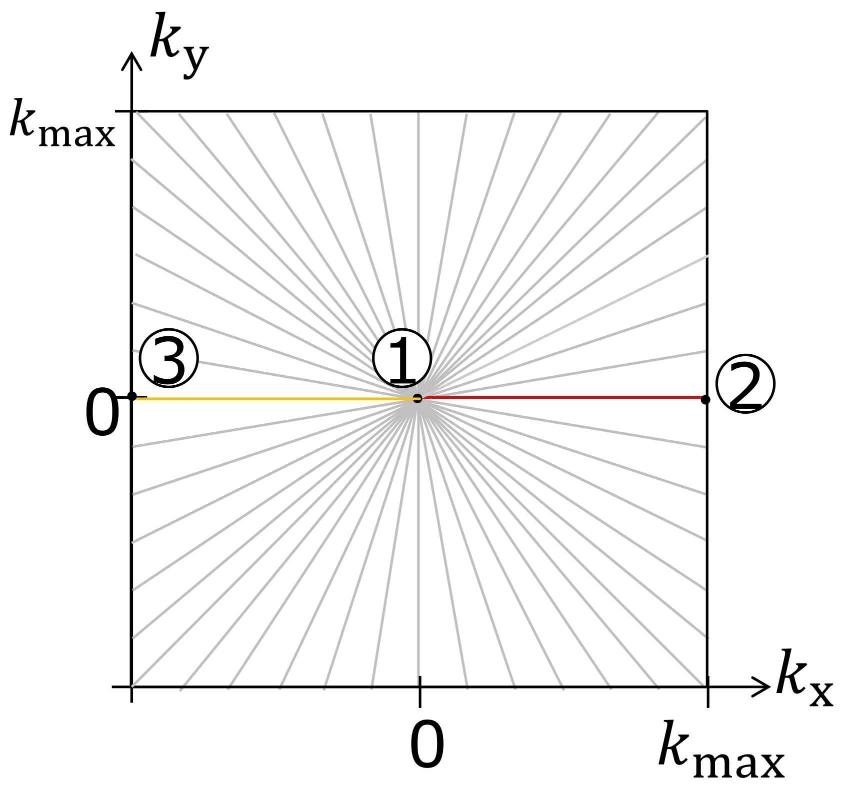
T2 blurring

depends on k-space path $\boldsymbol{k}(t)$

- In general: $\boxed{\tilde{S}_{\text{MRI, with T2}}(\boldsymbol{k}) = \tilde{S}_{\text{MRI, without T2}}(\boldsymbol{k})\tilde{R}_{\text{relax,k}}(\boldsymbol{k})}$
- Associated signal $S_{\text{MRI, with T2}}(\boldsymbol{r}) = \mathcal{F}^{-1}\{\tilde{S}_{\text{MRI, without T2}}(\boldsymbol{k})\tilde{R}_{\text{relax,k}}(\boldsymbol{k})\}$
- Convolution theorem: $\mathcal{F}^{-1}\{\tilde{f}(\boldsymbol{k}) \cdot \tilde{g}(\boldsymbol{k})\} = f(\boldsymbol{r}) * g(\boldsymbol{r})$
- $\boxed{S_{\text{MRI, with T2}}(\boldsymbol{r}) = S_{\text{MRI, without T2}}(\boldsymbol{k}) * \mathcal{F}^{-1}\{\tilde{R}_{\text{relax,k}}(\boldsymbol{k})\}}$
- I.e. a „T_2 blurring“ results
- T_2 blurring depends on position $\boldsymbol{r}$ if T_2 depends $\boldsymbol{r}$
- The larger $\mathrm{v_k} \propto G_{\text{read}} \propto \mathrm{BW}$ and the larger T_2, the smaller is the blurring

Example: Radial sampling, exponential signal decay, 1D case



1st excitation:

From ① to ② : $k(t) = \frac{1}{2\pi}\gamma G_{\text{read}} t$

2nd excitation:

From ① to ③ : $k(t) = \frac{1}{2\pi}\gamma \cdot (-G_{\text{read}})t$

$\rightarrow t(k) = \frac{2\pi|k|}{\gamma G_{\text{read}}}$

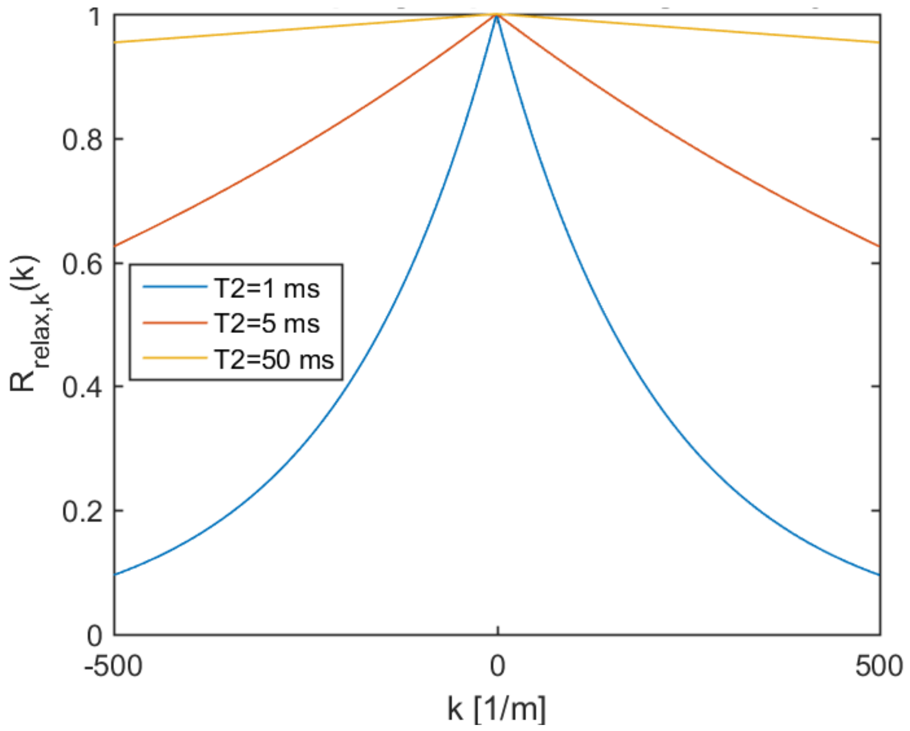
$R_{\text{relax}}(t) = \exp(-t(k)/T_2)$

$\tilde{R}_{\text{relax,k}}(k) := \exp\left(-\frac{2\pi|k|}{\gamma G_{\text{read}} T_2}\right)$

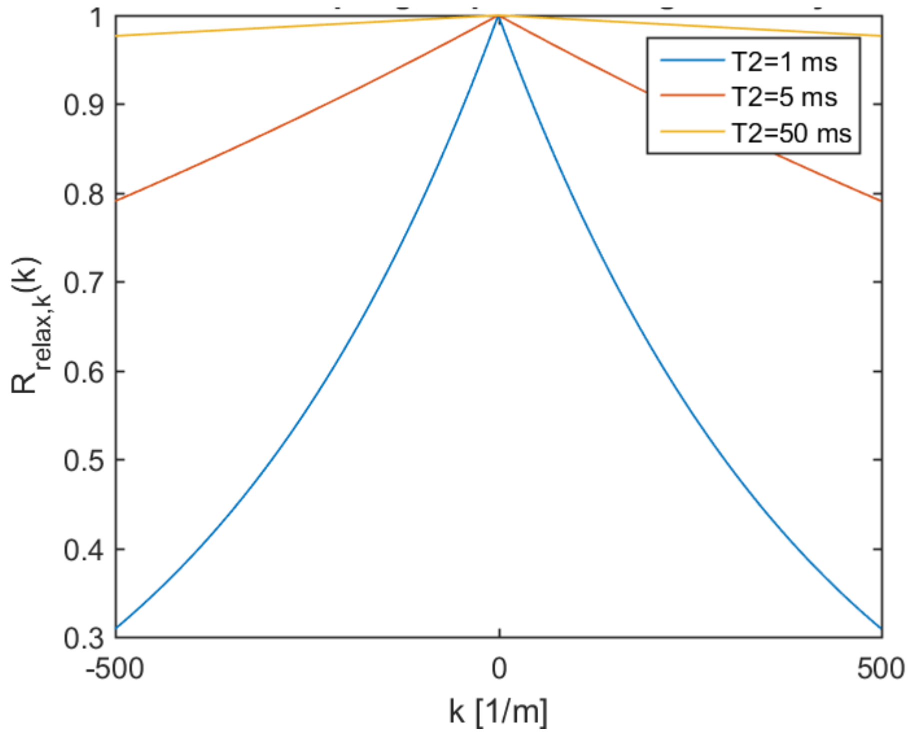
$\mathcal{F}^{-1}\{\tilde{R}_{\text{relax,k}}(k)\} = \frac{1}{\pi} \frac{\gamma G_{\text{read}} T_2}{1+\gamma^2 G_{\text{read}}^2 T_2^2 x^2}$

Example: Radial sampling, exponential signal decay, 1D case, 201 voxels

G_{read}=5 mT/m, $T_{\text{read}} \approx 2.3$ ms



G_{read}=10 mT/m, $T_{\text{read}} \approx 1.2$ ms



Example: Radial sampling, exponential signal decay, 1D case, 201 voxels

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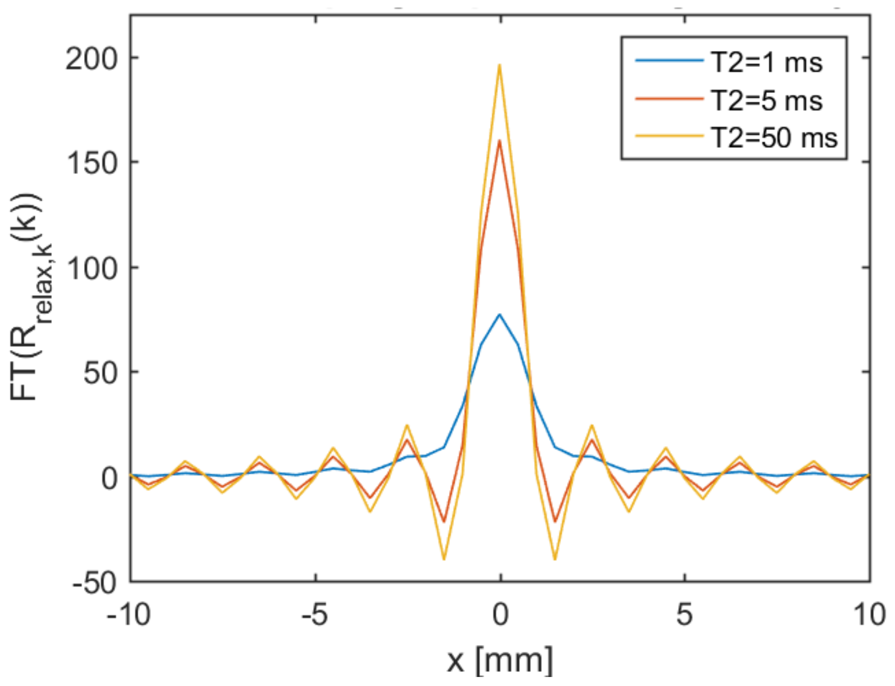

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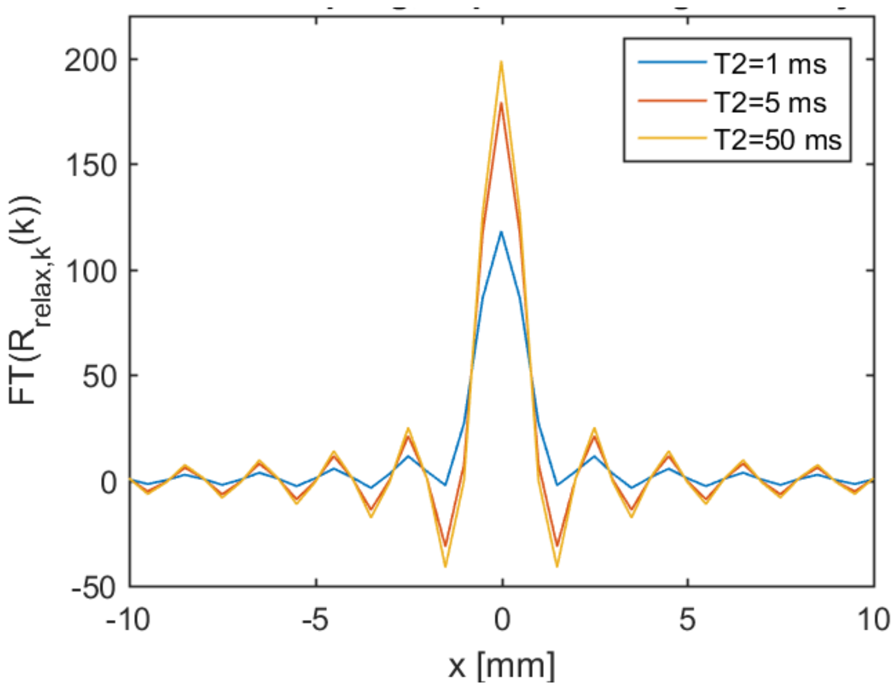

Summary

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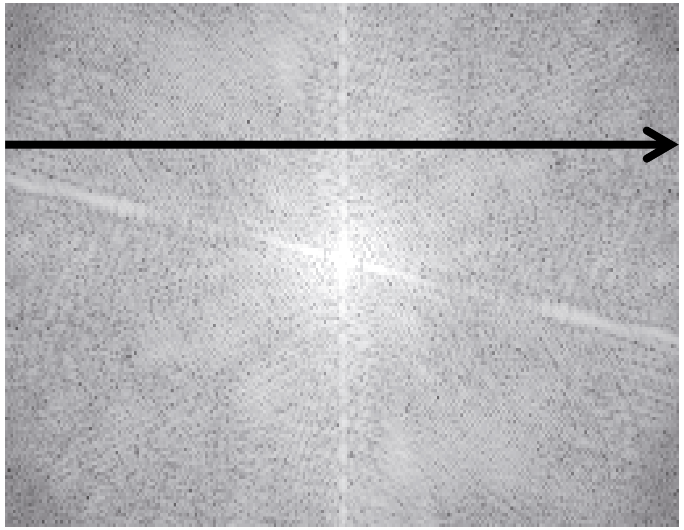
Summary

We discussed three effects
of finite k-space velocity



Summary

Physical Effect
Chemical shift: $\omega \to \gamma B_0 \cdot (1+\delta)$
Field inhomogenity: $\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \Delta\boldsymbol{B}(\boldsymbol{r})$
T_2 relaxation

Summary

Physical Effect	Effect on Cartesian MRI	$\propto B_0$	$\propto BW^{-1}$	$\propto G_{\text{read}}^{-1}$	$\propto v_{\text{k}}^{-1}$
Chemical shift: $\omega \rightarrow \gamma B_0 \cdot (1+\delta)$	Shift of fat signal by Δx_{fat}	✓	✓	✓	✓
Field inhomogenity: $\boldsymbol{B}_0 \rightarrow \boldsymbol{B}_0 + \Delta\boldsymbol{B}(\boldsymbol{r})$	Shift of signal by $\Delta x_{\Delta\text{B}}(\boldsymbol{r})$	✓	✓	✓	✓
T_2 relaxation	T_2 blurring		✓	✓	✓

$$\boxed{\text{SNR} \propto T_{\text{read}}^{1/2} \propto BW^{-1/2}}$$

→ Reduction of these artifacts by a factor of 2 is paid with a factor $\sqrt{2}$ of SNR

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Echo planar imaging (EPI):

The principle

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Which vehicle would like to drive?

Porsche Diesel
v_{max} = 27.9 km/h

Porsche 911
v_{max} > 300 km/h

Images: de.wikipedia.org

Best vehicle depends on the intended purpose

https://de.wiktionary.org/wiki/Acker

https://de.wikipedia.org/wiki/Nordschleife

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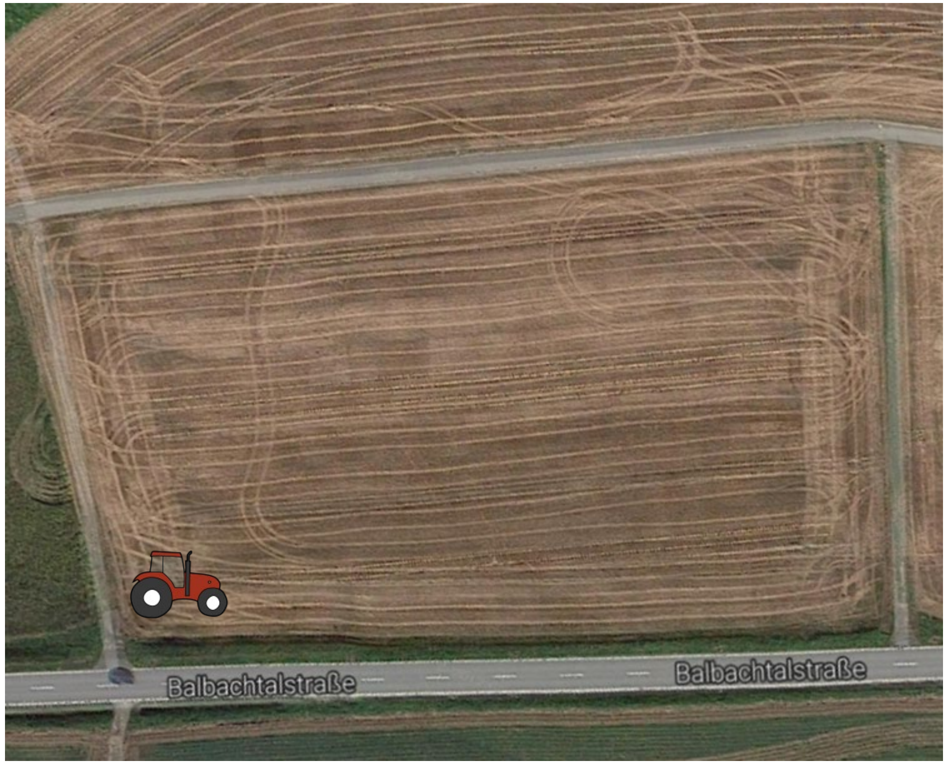
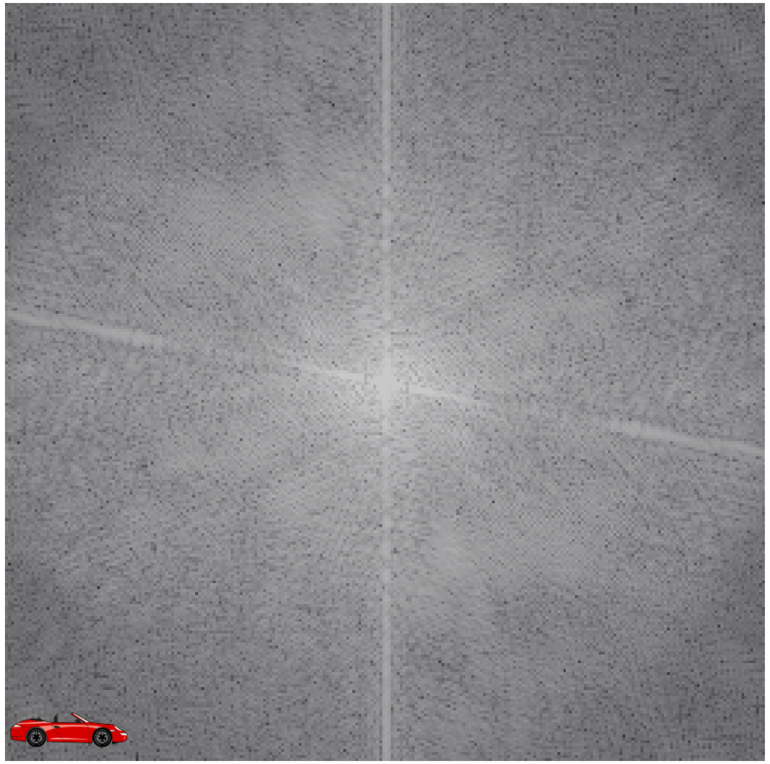
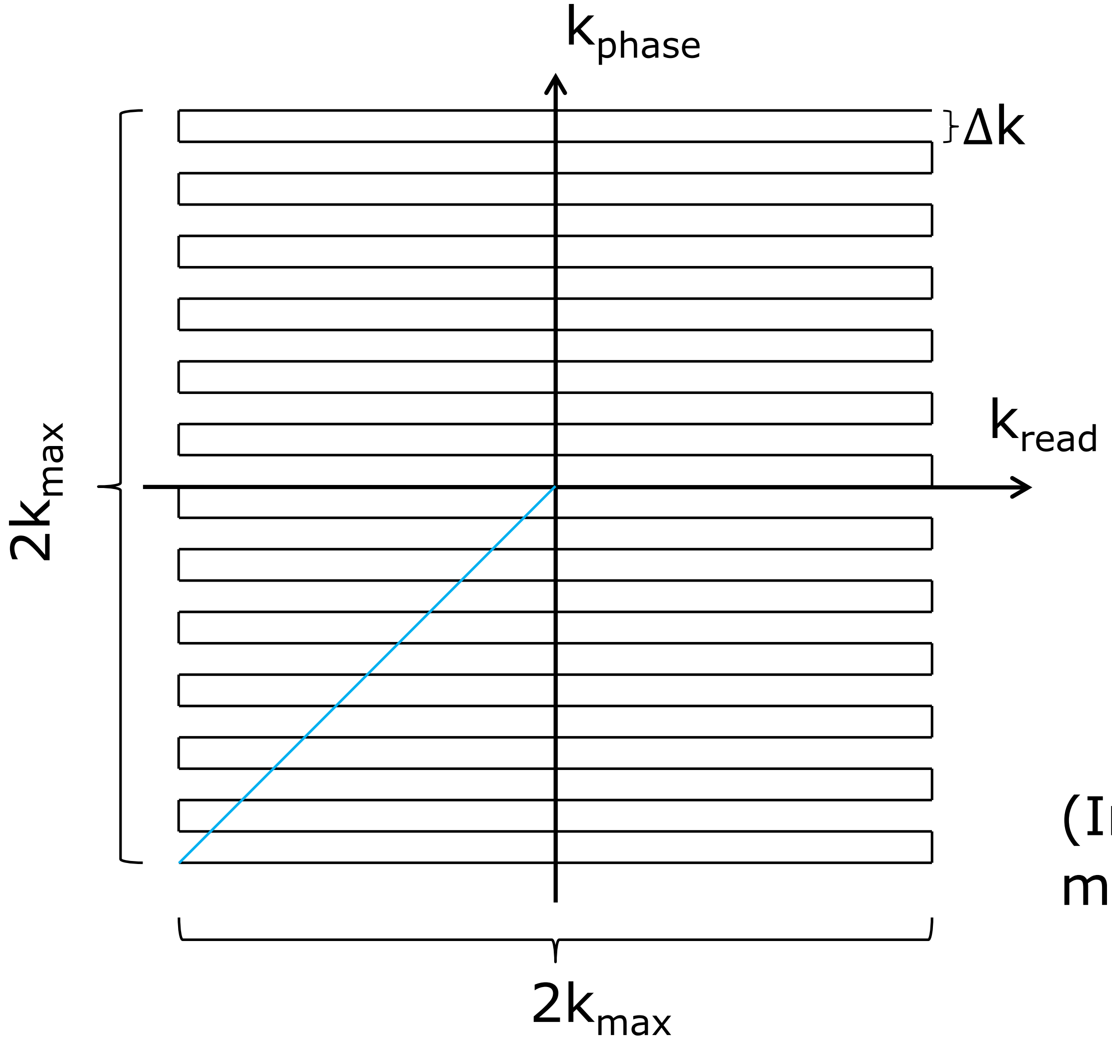
Definition EPI



EPI: A snail through k-space



EPI trajectory



(In general, $k_{\text{max,phase}}$ may be $\neq k_{\text{max,read}}$)

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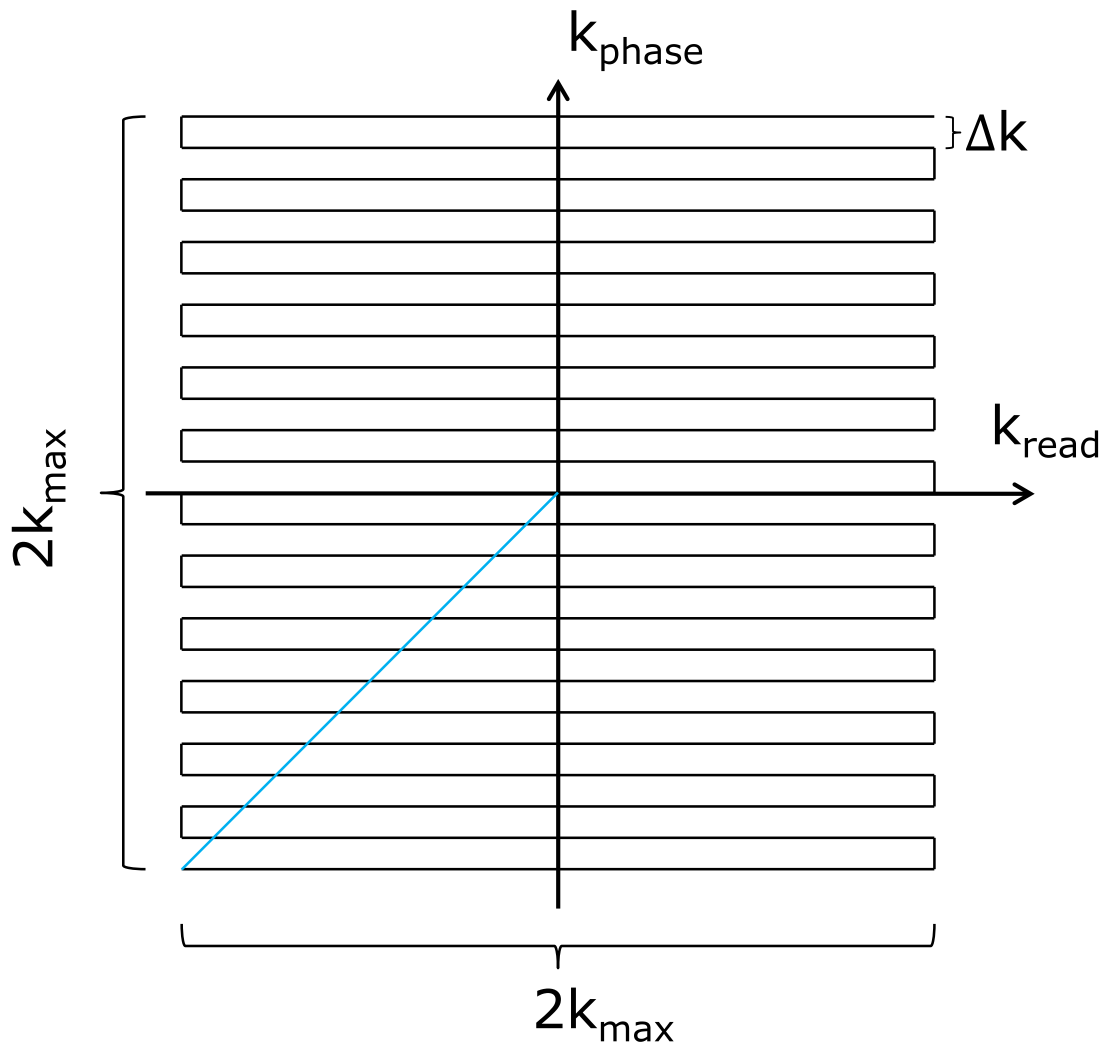
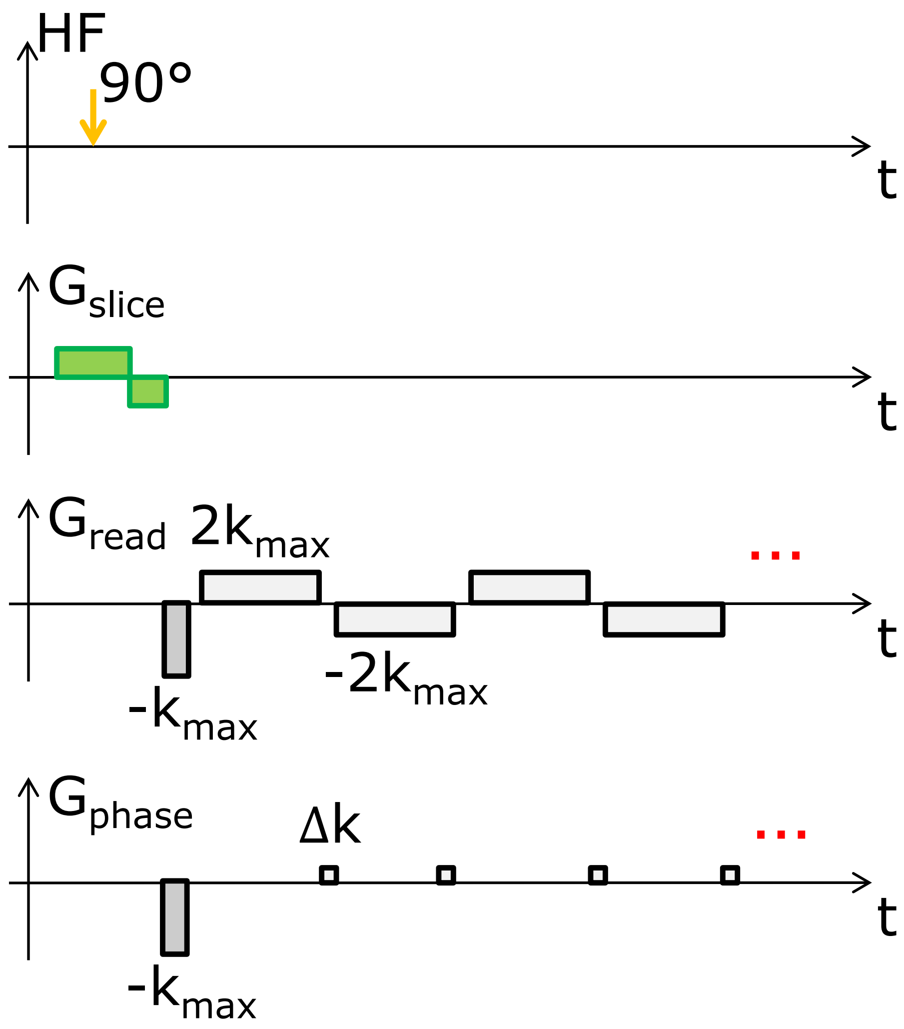
EPI timing table

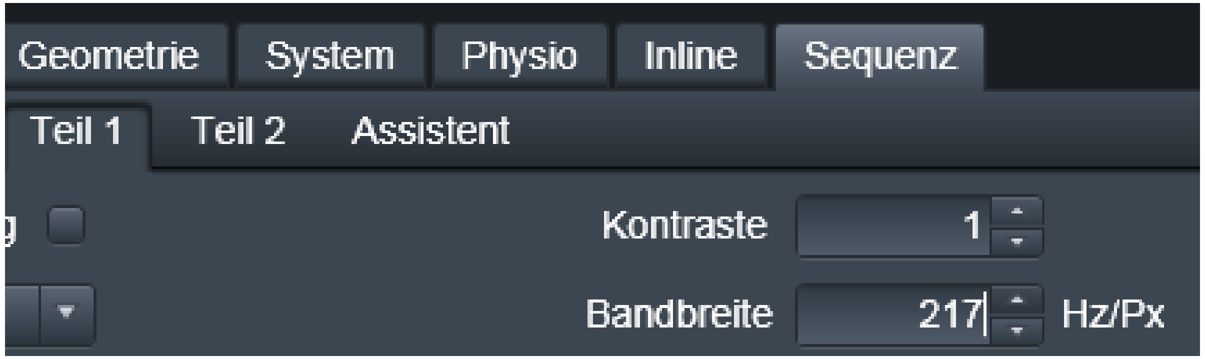
$$\boldsymbol{k}(t) = \frac{\gamma}{2\pi}\int_0^t \boldsymbol{G}(t')\,dt' \cdot \boldsymbol{r}$$



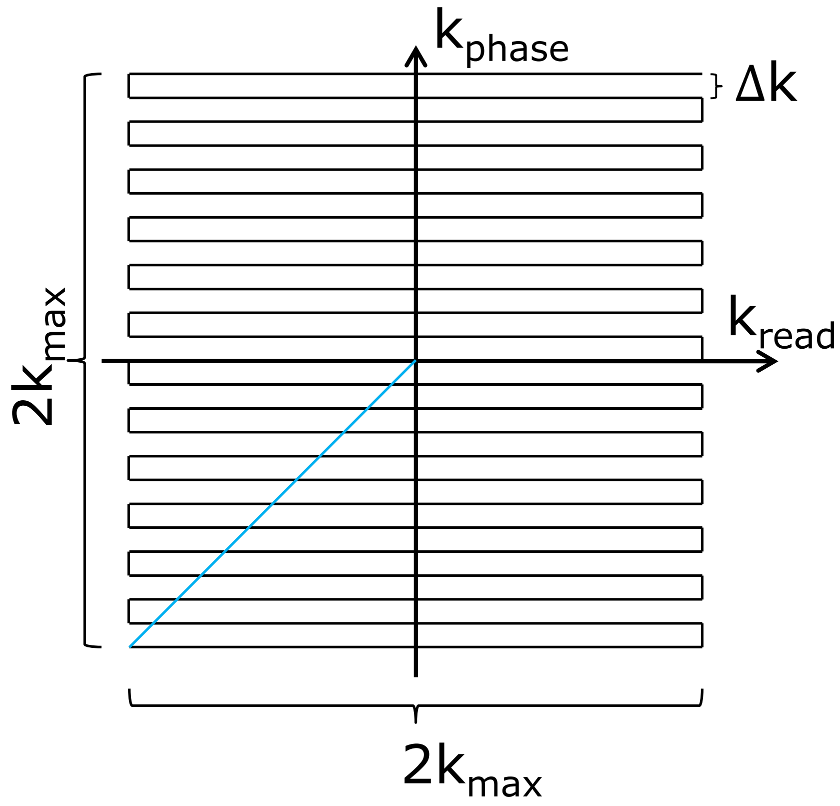


Bandwidth in EPI

Bandwidth is defined as for a single line sequence



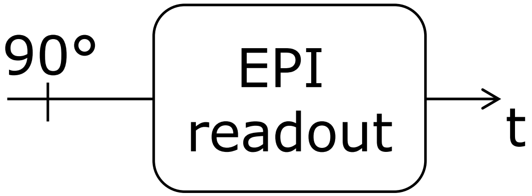
$$\mathrm{BW} = \frac{1}{T_{\mathrm{read}}}$$



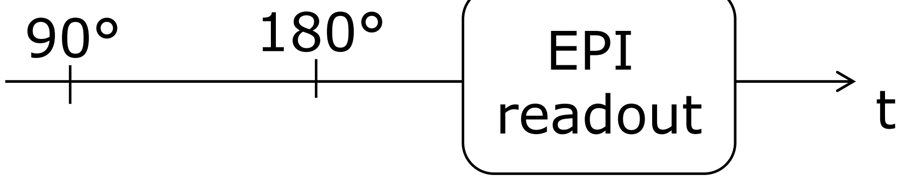
Properties of EPI

- Echo time TE is defined as the time when the k-space center is reached
- Excitation angle <90° possible
- Typical acquisition time for one image: 0.1 s → fast technique
- It is a “single-shot technique”
- Can be run as spin echo or gradient echo sequence

gradient echo:



spin echo:



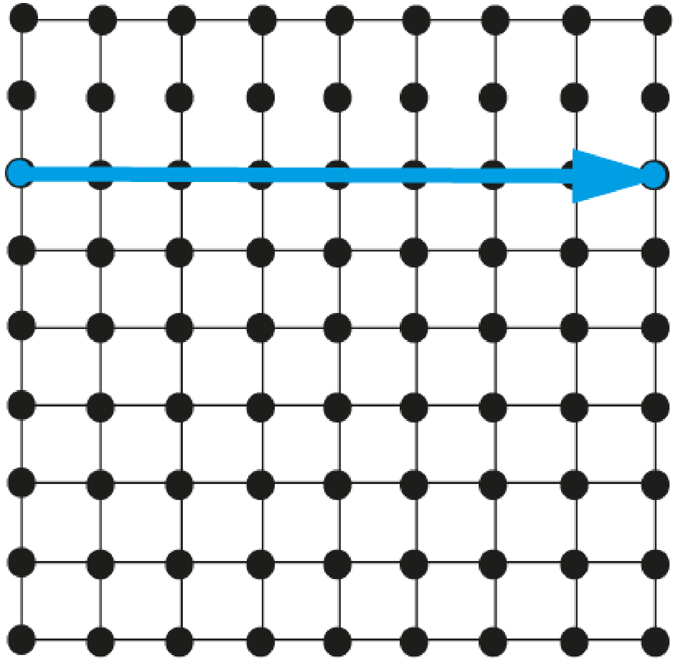
Advantages and Disadvantages of EPI

- Advantages:
  - Very fast
  - SNR-efficient
  - Very good for some special applications (diffusion imaging, fMRI, …)
- Disadvantages:
  - Long echo train makes EPI prone to artifacts
  - not well-suited for high-resolution imaging

EPI and chemical shift

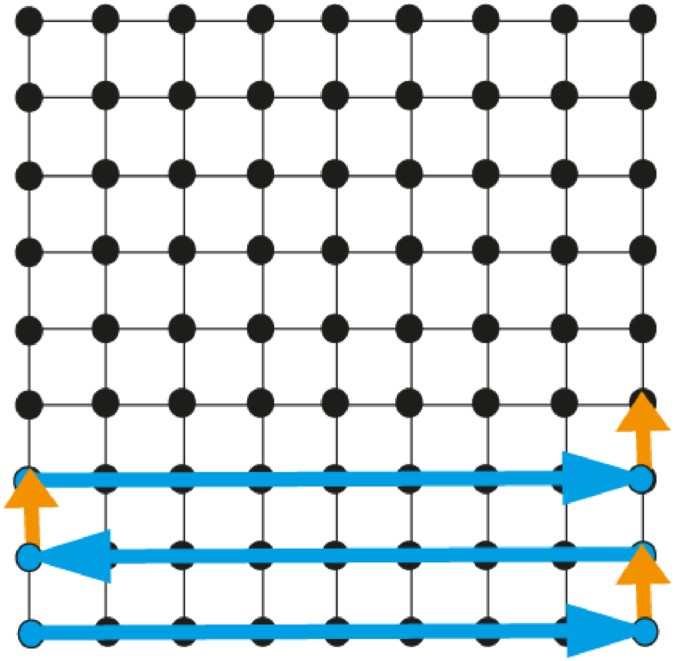
Fat - water shift in EPI

single line readout



$$\Delta \boldsymbol{x}_{\text{fat}} = \frac{\gamma B_0 \cdot (\sigma_{\text{fat}} - \sigma_{\text{water}})}{2\pi v_{\text{k,read}}} \boldsymbol{e}_{\text{read}}$$

EPI



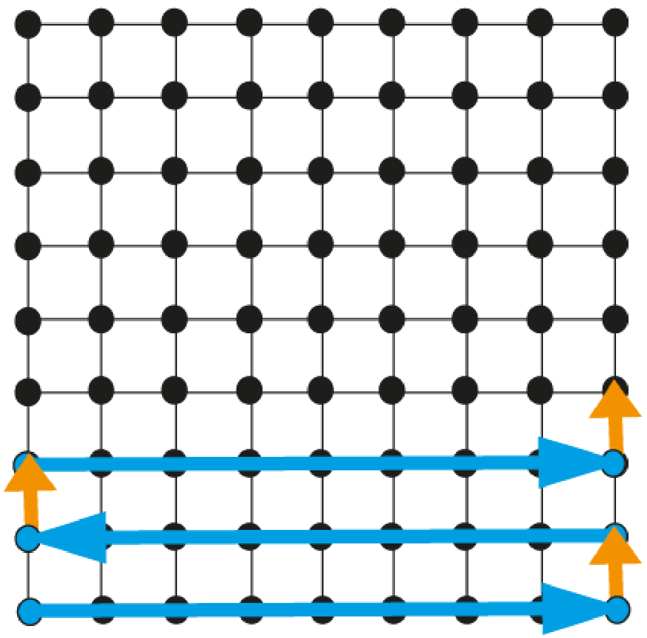
$$\Delta \boldsymbol{x}_{\text{fat}} = ?$$

Fat - water shift in EPI

- read direction:
  - time needed for one line is T_{read}
  - travelled distance is $2k_{\text{max,read}}$
  - the k-space velocity is $v_{\text{k,read}} = \frac{2k_{\text{max,read}}}{T_{\text{read}}}$
- phase direction:
  - N lines must be acquired
  - needed time is $N \cdot T_{\text{read}}$
  - travelled distance is $2k_{\text{max,phase}}$
  - the k-space velocity is $v_{\text{k,phase}} = \frac{2k_{\text{max,phase}}}{N \cdot T_{\text{read}}}$
- $v_{\text{k,phase}} = v_{\text{k,read}}/N$

→ The chemical shift artifact is N times larger for EPI

EPI



$\Delta \boldsymbol{x}_{\text{fat}} = ?$

$\boxed{\Delta x_{\text{fat}} \propto v_{\text{k}}^{-1}}$

Fat - water shift in EPI

$$\frac{\Delta x_{\text{fat}}}{\Delta x} = \frac{\Delta \nu_{\text{fat}}}{BW}$$

- Single line encoding:

$$\Delta \boldsymbol{x}_{\text{fat,1line}} = \frac{\Delta x \cdot \Delta \nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{read}}$$

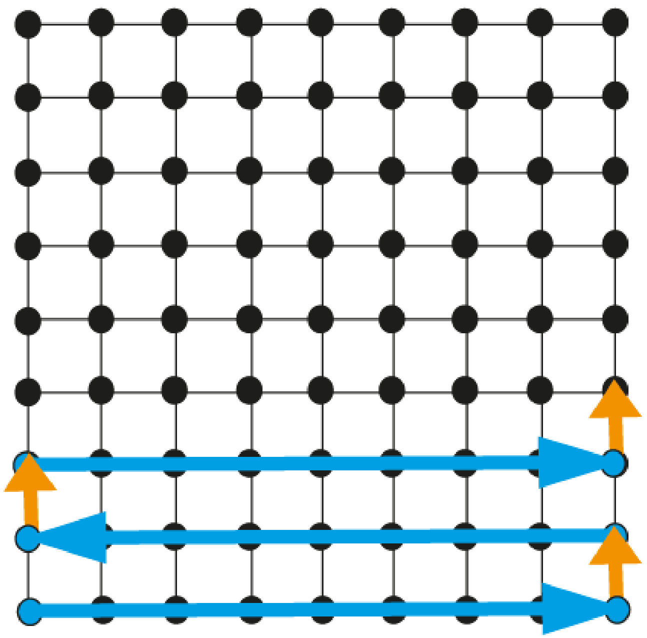
- EPI: just adapt this formula

$$\Delta \boldsymbol{x}_{\text{fat,EPI}} = \frac{N \cdot \Delta x \cdot \Delta \nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{phase}}$$

$$= \frac{FOV_{\text{phase}} \cdot \Delta \nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{phase}}$$

- The shift in read direction is much smaller and somewhat more involved because of travelling back and forth. We neglect it.

EPI



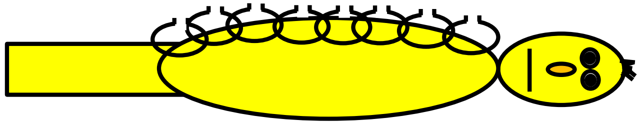
Options to minimize Δx_{fat} in EPI

$$\Delta \boldsymbol{x}_{\text{fat,EPI}} = \frac{FOV_{\text{phase}} \cdot \Delta\nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{phase}}$$

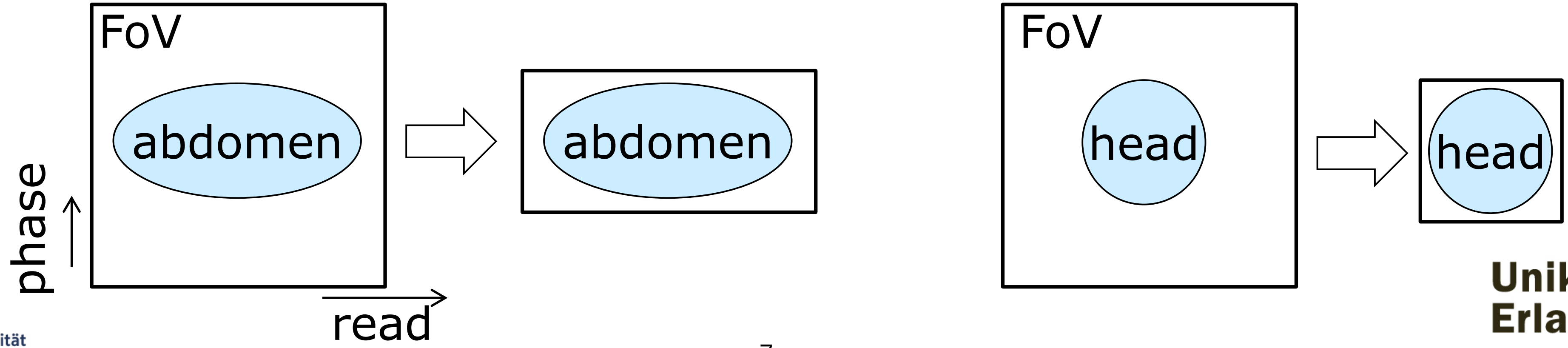
increase the BW

Typical BW ≈ 2000 Hz/Px

use parallel imaging
(discussed later in the course)



reduce the field of view along phase direction



No free lunch

$$\Delta \boldsymbol{x}_{\text{fat,EPI}} = \frac{FOV_{\text{phase}} \cdot \Delta\nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{phase}}$$

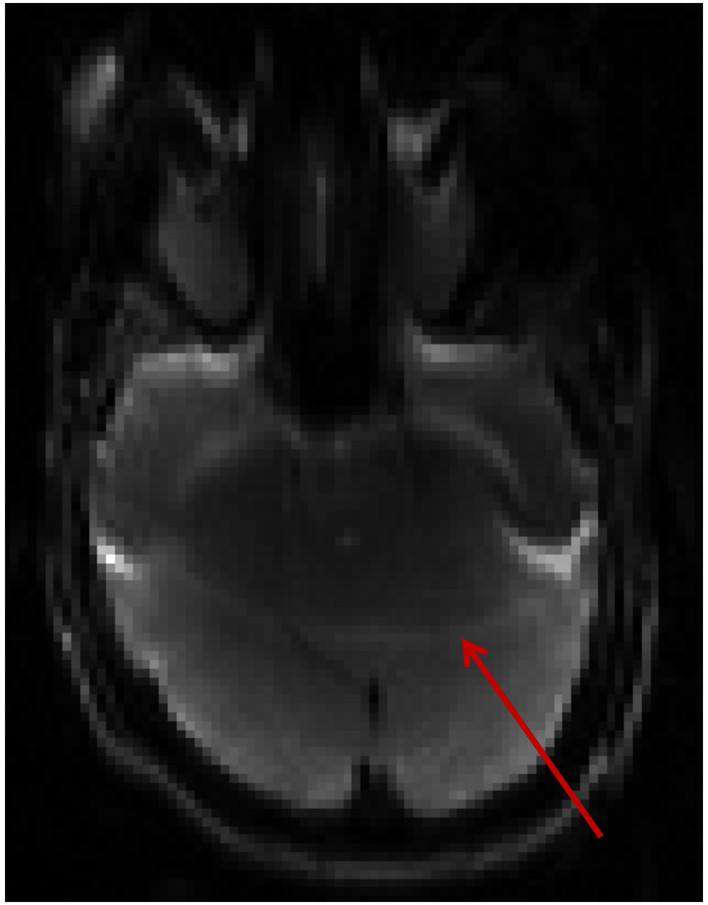
- $\Delta x_{\text{fat}} \propto BW^{-1}$ $\propto T_{\text{data acquisition}}$
- $\Delta x_{\text{fat}} \propto FoV_{\text{phase}}$ $\propto T_{\text{data acquisition}}$
- $\Delta x_{\text{fat}} \propto (\text{parallel imaging acceleration factor})^{-1}$ $\propto T_{\text{data acquisition}}$
- All these factors have in common: they reduce the time spend on acquiring data, i.e. $T_{\text{data acquisition}}$

- The price one has to pay:

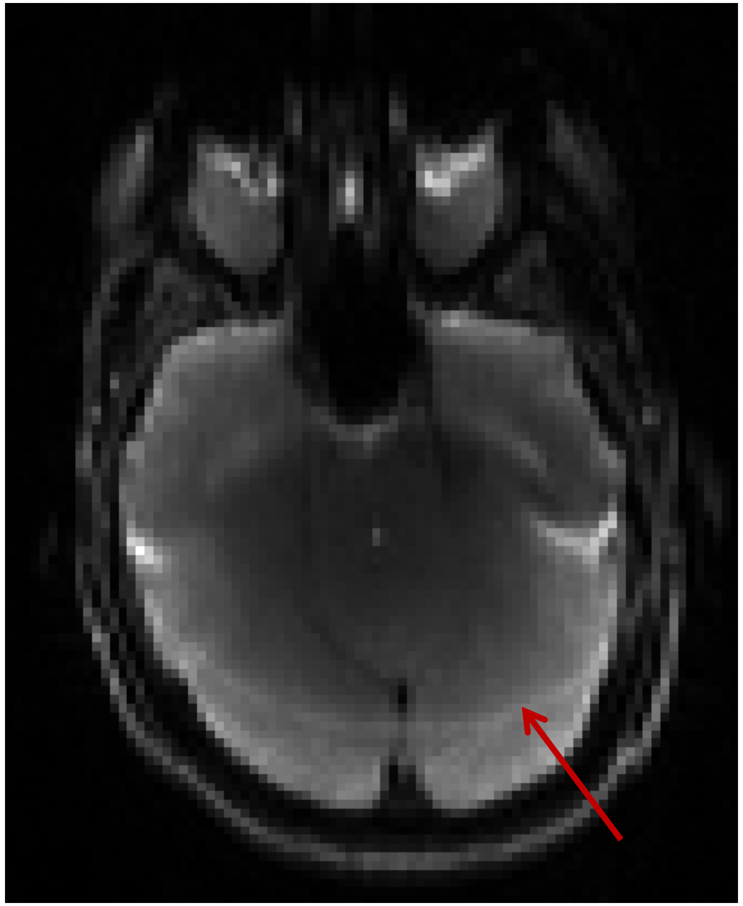
  signal-to-noise ratio $\text{SNR} \propto \sqrt{T_{\text{data acquisition}}}$

Fat-water shift in the brain

BW = 1000 Hz/Px



BW = 2000 Hz/Px
(only half the shift)



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Dank an A. Riexinger

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EPI and magnetic field inhomogeneities

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$\Delta x_{\Delta \mathrm{B}}$ formulas for EPI (just replace $B_0 \cdot (\sigma_{\mathrm{fat}} - \sigma_{\mathrm{water}})$ with $\Delta B_{\mathrm{z}}(\boldsymbol{r})$)

$$\Delta \boldsymbol{x}_{\mathrm{fat,EPI}} = \frac{\gamma B_0 \cdot (\sigma_{\mathrm{fat}} - \sigma_{\mathrm{water}}) \cdot FoV_{\mathrm{phase}}}{2\pi BW} \boldsymbol{e}_{\mathrm{phase}}$$

$$\Downarrow$$

$$\Delta \boldsymbol{x}_{\Delta \mathrm{B,EPI}} = \frac{\gamma \Delta B_{\mathrm{z}}(\boldsymbol{r}) \cdot FoV_{\mathrm{phase}}}{2\pi BW} \boldsymbol{e}_{\mathrm{phase}}$$

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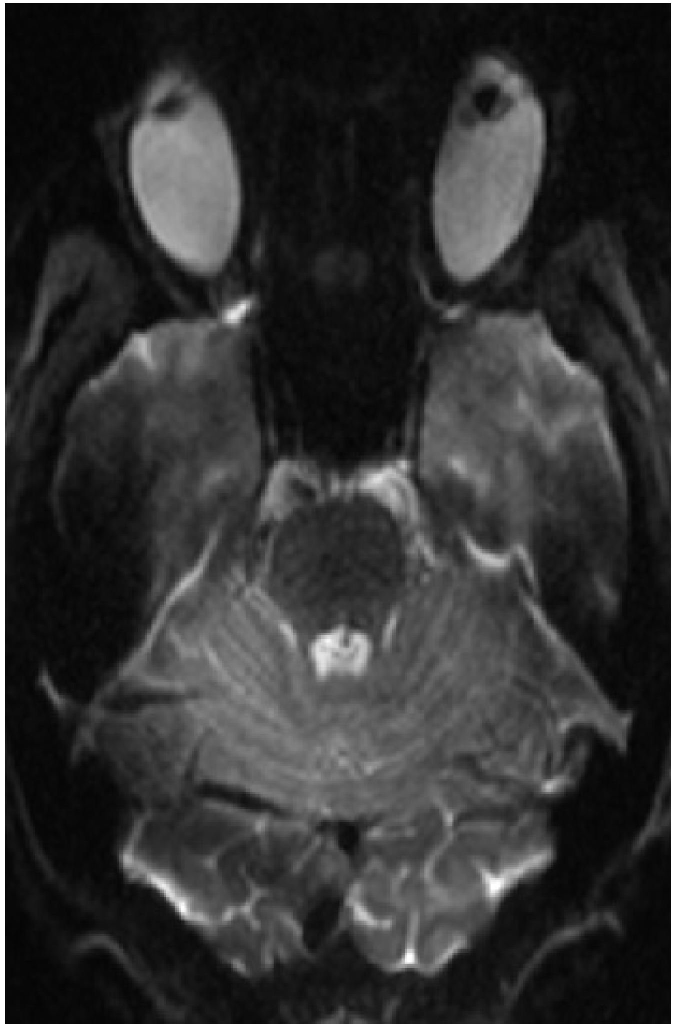
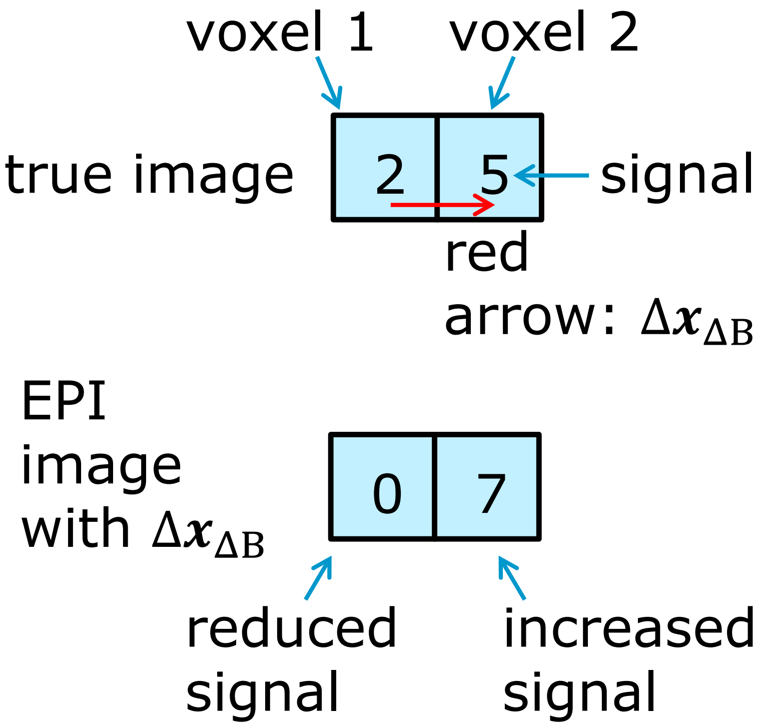
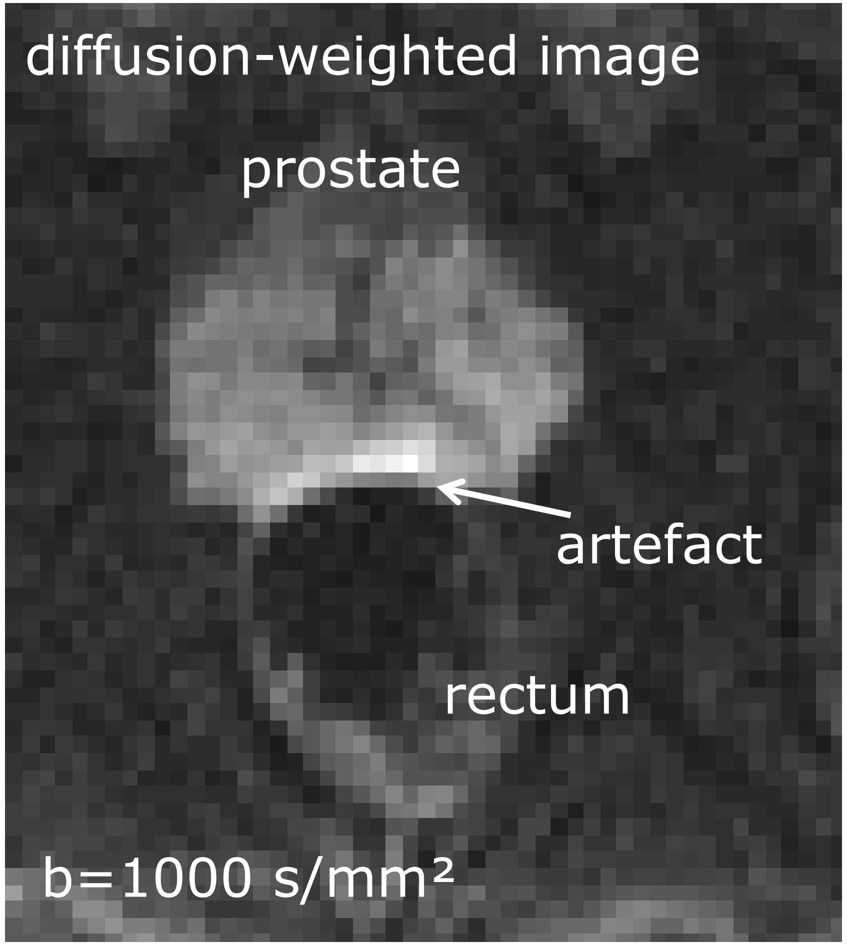
Effect 1: image shift



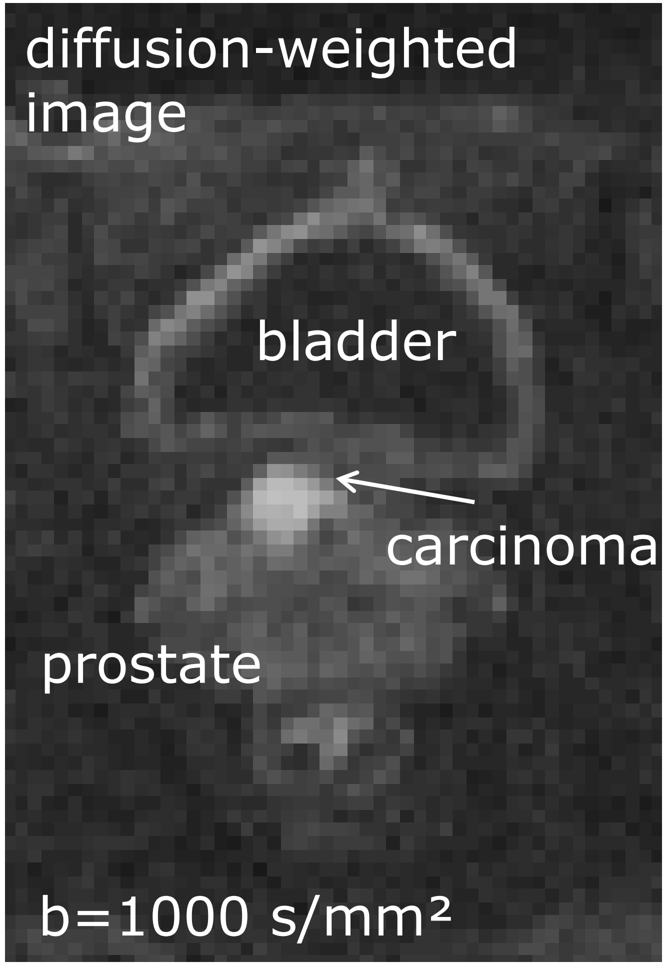
squeezed eye balls

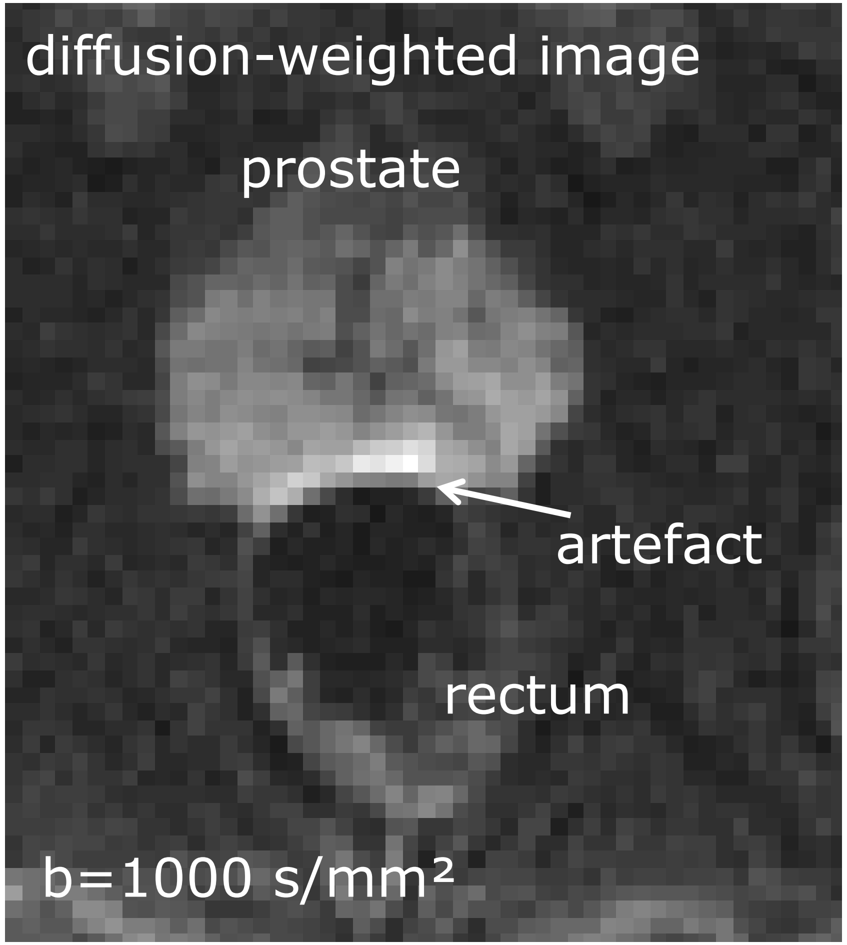
Effect 2: Intensity variation





Effect 2: Intensity variation





Transversal relaxation

T2 blurring

- $S_{\text{MRI, with T2}}(\boldsymbol{r}) = S_{\text{MRI, without T2}}(\boldsymbol{r}) * \mathcal{F}^{-1}\{\tilde{R}_{\text{relax,k}}(\boldsymbol{k})\}$
- Again a „T_2 blurring“ occurs
- Unlike for single-line encoding: along phase encoding direction for EPI
- For EPI, the T_2 blurring is $N \cdot \frac{k_{\text{max,read}}}{k_{\text{max,phase}}}$ times worse
- In EPI, actually T_2^* blurring

Echo spacing

Echo spacing

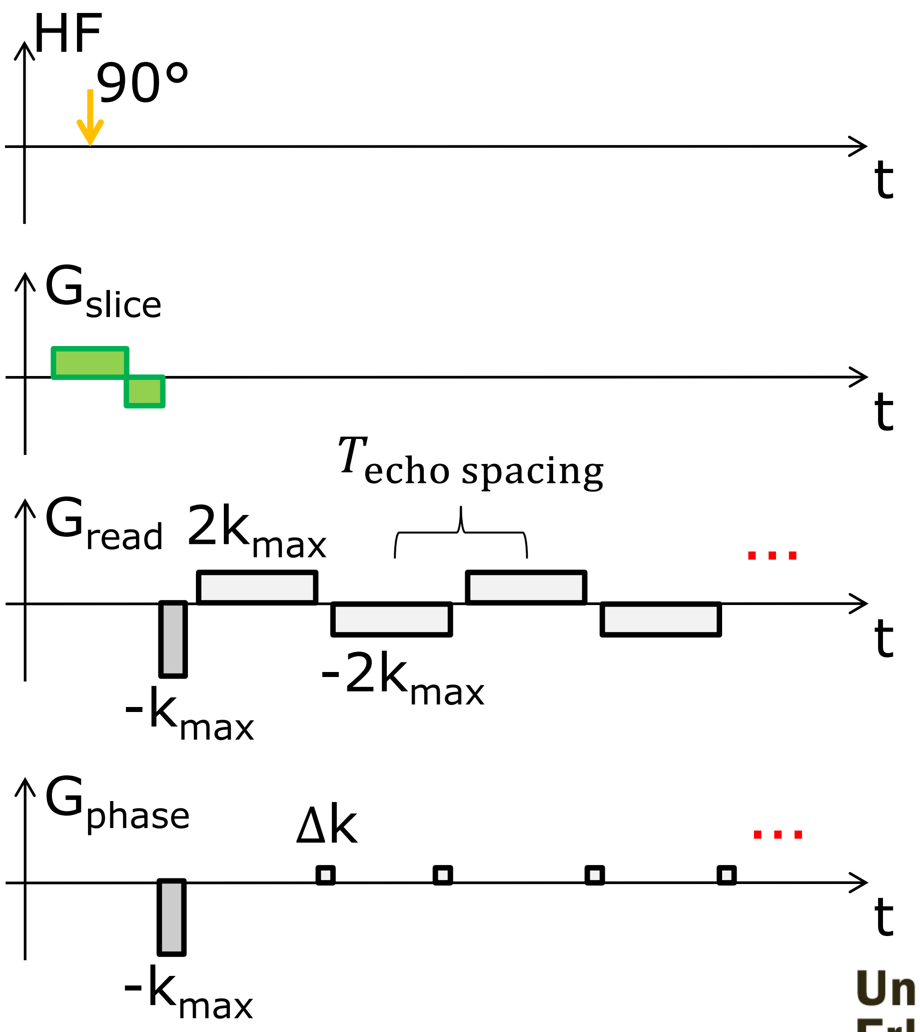
$$\Delta \boldsymbol{x}_{\text{fat,EPI}} = \frac{FOV_{\text{phase}} \cdot \Delta\nu_{\text{fat}}}{BW} \boldsymbol{e}_{\text{phase}}$$

$$BW = T_{\text{read}}^{-1}$$

A bit more correct:
use echo spacing

$$\Delta \boldsymbol{x}_{\text{fat,EPI}}$$

$$= FOV_{\text{phase}} \Delta\nu_{\text{fat}} T_{\text{echo spacing}} \boldsymbol{e}_{\text{phase}}$$



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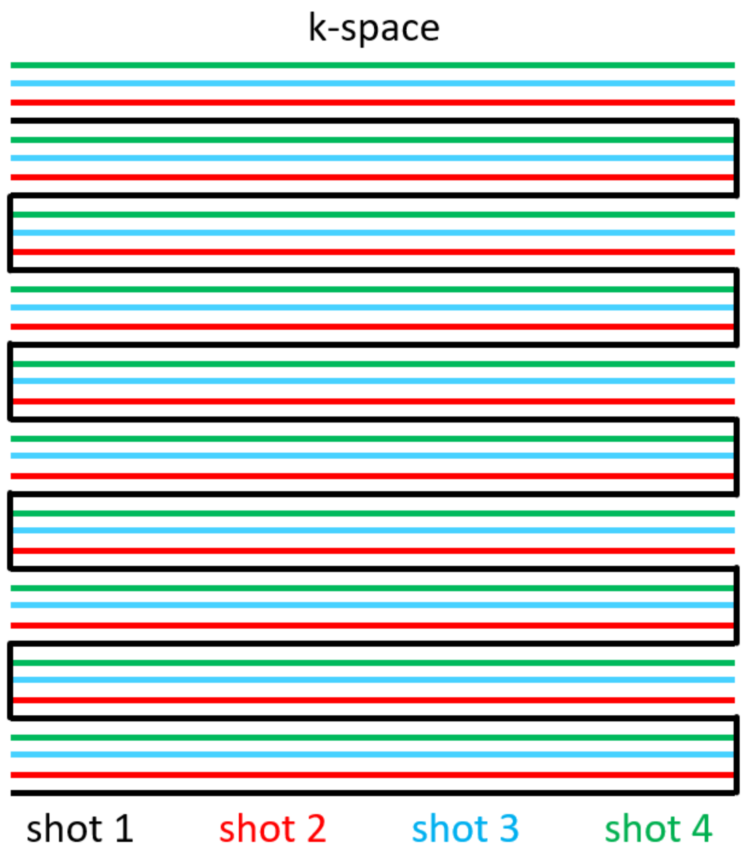
Variations of EPI

Interleaved EPI

Interleaved EPI

- Advantage:
  - The discussed artifacts go down by a factor N_{shot}
- Disadvantage:
  - More prone to patient motion
  - Acquisition time goes up



Readout-segmented EPI

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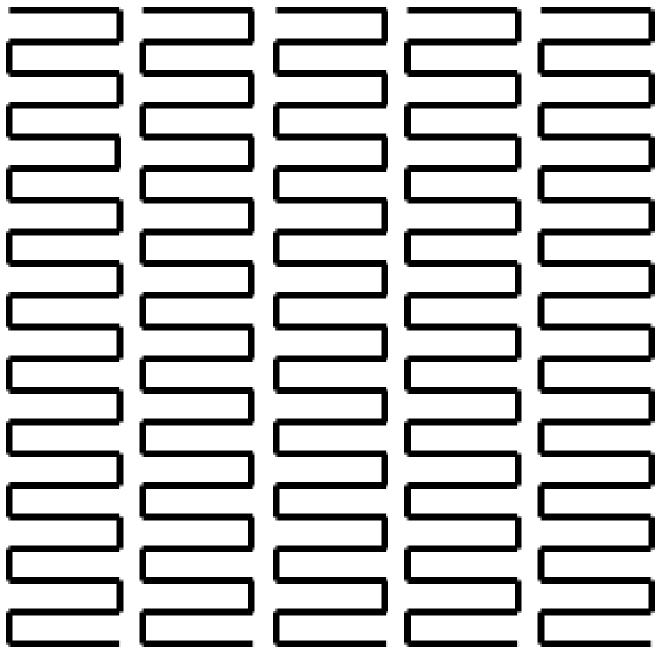
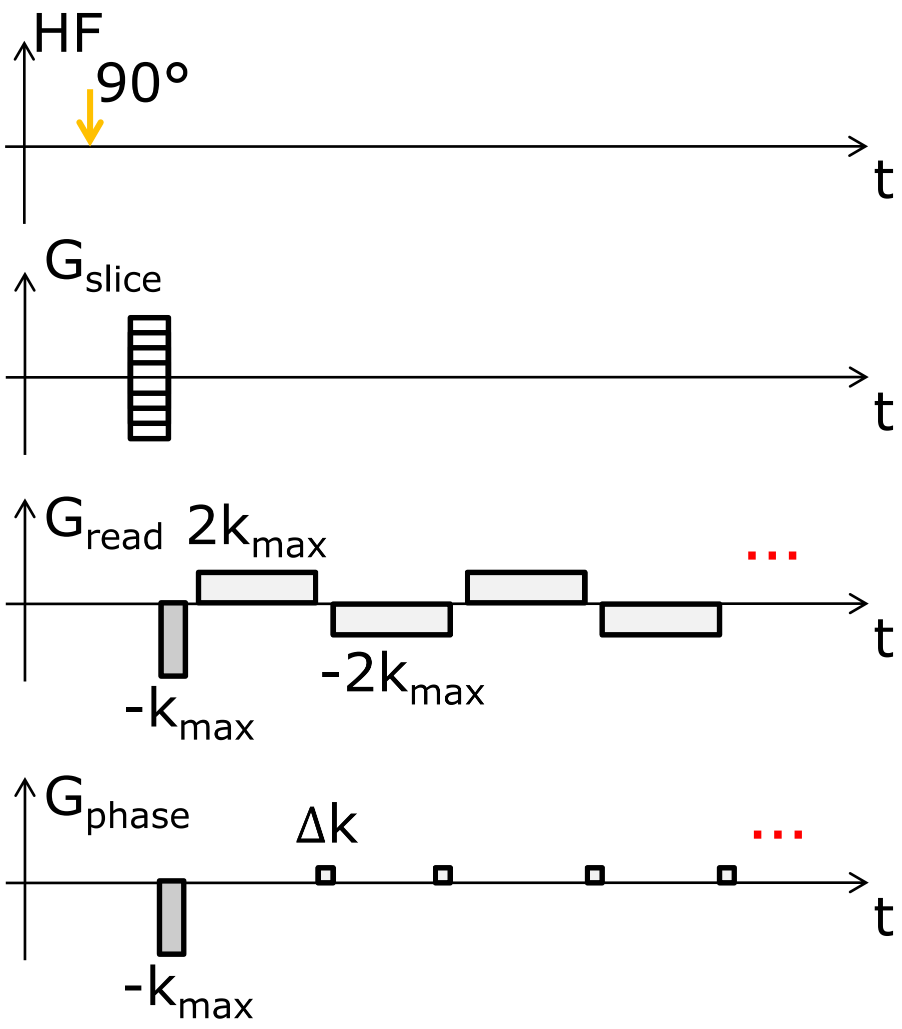
Readout-segmented EPI

- Advantage:
  - The discussed artifacts go down by a factor N_{shot}
- Disadvantage:
  - More prone to patient motion
  - Acquisition time goes up



3D EPI

3D EPI



Summary

Summary

- EPI is fast (acquisition time)
- EPI is slow (long readout train)
  - $\Delta x_{\text{fat}}, \Delta x_{\Delta \text{B}}$, T_2 blurring worse by factor N
  - These artifacts: along phase direction
- N/2 ghost may arise