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Study on Mathematical Model and Exact Solution of Overall Structural Response and Natural Frequency of Submerged Floating Tunnel

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ABSTRACT

Deflection of the tube of submerged floating tunnel (SFT) under environmental loading such as current, long crest wave or even sea density variation as well as its vibration characteristic are of great interest to engineering researchers. In this paper, the simplified two-dimensional SFT mathematical models are expressed by a set of partial differential equations, including four partial differential equations and four boundary conditions, totally 16 combinations. Each represents a type of SFT with a specific way by which the tube connects to the shores. They are SFT with or without cables, SFT with or without axial compression or tension, SFT with two fixed or pinned ends and SFT with only one pinned or fixed end. And the last one stands for the constructional stage. The analytical solutions to the simplified two-dimensional SFT models are obtained by using the method of variables separation. And these solutions apply to the tether type SFT in both operational stage and constructional stage, where the cable mass must be negligibly small compared to the tube mass, tube section must be constant along its length and the cable is idealized by continuous springs. Finally, the examples of application are given. The research results provide a theoretical reference for further study and construction of the SFT.

KEY WORDS: Submerged Floating Tunnel, Partial Differential Equations, Exact Solution, Structure Response, Analysis and Design

INTRODUCTION

The SFT was proposed more than 150 years ago. Norway, Turkey, Italy, Japan and China have all studied it at different times, but it has never been built [1]. The Hong Kong-Zhuhai-Macau bridge island-tunnel project has built the longest highway immersed tube tunnel in the world, overcoming a series of difficult problems such as the installation of complex sea conditions, deep burial, siltation, abnormal waves and plume. Today, immersed tube tunnel technology has been developed into the third generation, and the next breakthrough should be the realization of SFT technology [2]. After the completion of the Hong Kong-Zhuhai-Macau bridge island-tunnel project in 2018, China Communications Construction Company Limited has set up the Joint research group of suspended Tunnel Engineering Technology with 11 research directions [3], and the model of education, research and production has also been established. Participating organizations include Northwestern Polytechnical University, Dalian University of Technology, Dutch TEC company, Delft University of Technology, etc. The research team of structure and design method was established in 2019. And this paper is one of the research achievements of the project team.

Although some works have been done on the mathematical theories of SFT [1], relatively few studies have been conducted on the analytical solutions of relevant mathematical models. In [4], the similarity of beam stiffness along the continuous foundation and the intermittent support beam is studied, and the applicable range of analytical static and dynamic calculation of continuous foundation beam is proposed. Literature [5] has studied the theoretical solution of acoustic radiation when a viscoelastic foundation beam is subjected to a moving load. In [6], the authors derived the theoretical critical value of support spacing for structural instability of a SFT considering multiple factors, such as velocity and vortex propagation frequency. At present, there are relatively few studies on the analytical solutions of deformation under natural vibration frequency and common uniformly distributed loads. The analytical solutions have following advantages: 1) strengthening the intuitive understanding of the structure behavior of the SFT; 2) speeding up the calculation and judgment; 3) verifying the numerical solutions.

In this paper, the whole structure of the SFT is simplified into a two-dimensional continuous support beam problem. The partial differential equation models including different boundaries and different anchorage systems during construction and operation are systematically listed and their analytical solutions based on the method of separating variables are discussed in details. The novelty of this paper are the new combinations of mathematical models and boundary conditions when the SFT operates in design and construction stage and their analytical solutions.

MATHMATICAL MODEL

For the convenience of theoretical research, the following assumptions are made for the structure of the SFT: 1) the cross-section of the tube does not change with the length of the tube; 2) the bending stiffness of the intermediate joint (if any) is equal to that of the tube structure; 3) the natural vibration frequency ignores the mass of anchorage along the path, which is generally acceptable for anchor cable SFT. The mass of buoy cannot be ignored, so the natural vibration frequency of theoretical solution is not applicable for buoy SFT; 4) three-dimensional problems are simplified to two-dimensional problems (figure 1), which are usually simplified to vertical problems and horizontal problems according to cartesian coordinate system; 5) the tube is subjected to axial tension or pressure for various reasons, and the axial force is fixed along the tube; 6) the intermittent constraint of the anchorage system along the path is simplified to the continuous constraint, and the constraint stiffness is fixed; 7) the tube is linear.

Table 1 Partial differential equations and boundary conditions of horizontal and vertical deflections of various types of SFT

Equations	Bco1: Both ends of the tube are fixed with the shore $\begin{cases} v(0)=0 \\ v(L)=0 \\ \frac{dv(0)}{dx}=0 \\ \frac{dv(L)}{dx}=0 \end{cases}$	Bco2: Both ends of the tube are hinged with the connecting shore $\begin{cases} v(0)=0 \\ v(L)=0 \\ \frac{d^2v(0)}{dx^2}=0 \\ \frac{d^2v(L)}{dx^2}=0 \end{cases}$	Bco3: One end of the tube is fixed with the shore and the other end being unconstrained. $\begin{cases} v(0)=0 \\ \frac{dv(0)}{dx}=0 \\ \frac{d^2v(L)}{dx^2}=0 \\ \frac{d^3v(L)}{dx^3}=0 \end{cases}$	Bco4: One end of the tube is hinged with the shore and the other end is unconstrained. $\begin{cases} v(0)=0 \\ \frac{d^2v(0)}{dx^2}=0 \\ \frac{d^2v(L)}{dx^2}=0 \\ \frac{d^3v(L)}{dx^3}=0 \end{cases}$
PDE1: The tube is not anchored $EI\frac{d^4v(x)}{dx^4}=q$	Bco1+PDE1	Bco2+PDE1	Bco3+PDE1	Bco4+PDE1
PDE2: The tube is anchored along the path $EI\frac{d^4v(x)}{dx^4}+\frac{k}{h}v(x)=q$	Bco1+ PDE2	Bco2+PDE2	Bco3+PDE2	Bco4+PDE2
PDE3: The tube is not anchored but has an axial force $EI\frac{d^4v(x)}{dx^4}+N\frac{d^2v(x)}{dx^2}=q$	Bco1+PDE3	Bco2+PDE3	Bco3+PDE3	Bco4+PDE3
PDE4: The tube is anchored and has an axial force $EI\frac{d^4v(x)}{dx^4}+\frac{k}{h}v(x)+N\frac{d^2v(x)}{dx^2}=q$	Bco1+PDE4	Bco2+PDE4	Bco3+PDE4	Bco4+PDE4

Note: BCo denotes boundary conditions or Engineering implications of partial differential equations (PDE).

The deflection of the tube under vertical or horizontal uniformly distributed loads (assuming that the cross-section is uniform along the tube) and the natural vibration frequency of the anchored cable SFT (ignoring the anchorage system mass and discontinuity) can be controlled or obtained by the equations in Table 1.The first row in the table represents the "free-floating" SFT with no constraint on the tube body. The second row represents an SFT with an anchorage system along the tube body. The anchorage system can be an anchor cable or a buoy. Rows 3 and 4 consider the axial forces of the tube body on the basis of the first two columns. The first and second columns of the boundary conditions represent two extreme mechanical forms of the connection between the tube body and the shore, namely complete consolidation and complete hinge. Columns 3 and 4 of the boundary conditions indicate the construction stage of the SFT. One end of the tube has not been connected with the shore connection. Based on the equation in Table 1, the mass term is added, such as the formula 1, to calculate the natural vibration frequency.

$$m\frac{d^2v(x)}{dt^2}+EI\frac{d^4v(x)}{dx^4}+\frac{k}{h}v(x)+N\frac{d^2v(x)}{dx^2}=0 \qquad (1)$$

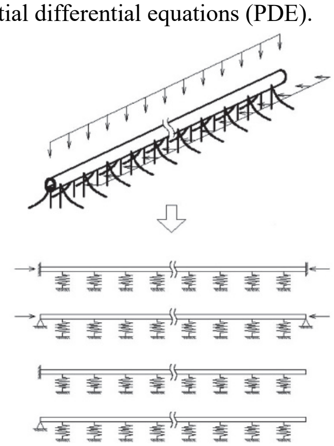


Fig. 1 Simplified mathematical model of SFT structure

SOLUTIONS OF EQUATIONS

1 The solution of PDE1: $EI\frac{d^4v(x)}{dx^4}=q$

1）Bco1

$$v(x,EI,q)=\frac{q}{24EI}x^4-\frac{qL}{12EI}x^3+\frac{qL^2}{24EI}x^2 .$$

2）Bco2

$$v(x,EI,q)=\frac{q}{24EI}x^4-\frac{qL}{12EI}x^3+\frac{qL^3}{24EI}x .$$

3）Bco3

$$v(x,EI,q)=\frac{q}{24EI}x^4-\frac{qL}{6EI}x^3+\frac{qL^2}{4EI}x^2 .$$

2 The solution of PDE2: $EI\dfrac{d^4v(x)}{dx^4}+kv(x)=q$

1）Bco1

$$v(x,EI,k,q)=e^{ax}\left(c_1\cos ax+c_2\sin ax\right)+e^{-ax}\left(c_3\cos ax+c_4\sin ax\right)+q/k$$

where $a=\dfrac{\sqrt{2}}{2}\sqrt[4]{k/EL}$,

The values of coefficient are as follows

$$\begin{cases} c_1=\dfrac{-q[e^{3aL}\left(\cos aL+\sin aL\right)-3e^{2aL}+1+e^{aL}\left(\cos aL-\sin aL\right)\left(1-2\cos aLe^{aL}\right)]}{c_{20}^*} \\ c_2=\dfrac{-q[e^{2aL}\left(\cos 2aL+\sin 2aL\right)+e^{3aL}\left(\sin aL-\cos aL\right)+e^{aL}\left(\cos aL-3\sin aL\right)-1]}{c_{21}^*} \\ c_3=\dfrac{-qe^{aL}[\cos aL-\sin aL+e^{aL}\left(\cos 2aL+\sin 2aL-2\right)-e^{2aL}\left(\cos aL+\sin aL\right)+e^{3aL}]}{c_{21}^*} \\ c_4=\dfrac{-qe^{aL}[\cos aL+\sin aL+e^{aL}\left(\cos 2aL-\sin 2aL\right)-e^{2aL}\left(\cos aL+3\sin aL\right)+e^{3aL}]}{c_{21}^*} \end{cases}$$

where

$$c_{20}^*=k\left(e^{4aL}-6e^{2aL}+4\cos^2 aLe^{2aL}+1\right),$$

$$c_{21}^*=k\left(e^{4aL}-4e^{2aL}+2\cos 2aLe^{2aL}+1\right).$$

2）Bco2

$$v(x,EI,k,q)=e^{ax}\left(c_1\cos ax+c_2\sin ax\right)+e^{-ax}\left(c_3\cos ax+c_4\sin ax\right)+q/k$$

where $a=\dfrac{\sqrt{2}}{2}\sqrt[4]{k/EL}$.

The values of coefficient are as follows

$$\begin{cases} c_1=-q\left(\cos aLe^{aL}+1\right)/c_{22}^* \\ c_2=-q\sin aLe^{aL}/c_{22}^* \\ c_3=-qe^{aL}\left(\cos aL+e^{aL}\right)/c_{22}^* \\ c_4=-q\sin aLe^{aL}/c_{22}^* \end{cases}$$

where

$$c_{22}^*=k\left(e^{2aL}+2\cos aLe^{aL}+1\right)$$

3）Bco3

$$v(x,EI,k,q)=e^{ax}\left(c_1\cos ax+c_2\sin ax\right)+e^{-ax}\left(c_3\cos ax+c_4\sin ax\right)+q/k$$

where $a=\dfrac{\sqrt{2}}{2}\sqrt[4]{k/EL}$.

The values of coefficient are as follows

$$\begin{cases} c_1=-q\left[e^{2aL}\left(\cos 2aL-\sin 2aL+2\right)+1\right]/c_{23}^* \\ c_2=-q\left[e^{2aL}\left(\cos 2aL+\sin 2aL\right)-1\right]/c_{23}^* \\ c_3=-qe^{2aL}\left[e^{2aL}+\sqrt{2}\sin\left(2aL+\dfrac{\pi}{4}\right)+2\right]/c_{23}^* \\ c_4=-qe^{2aL}\left[e^{2aL}-\sqrt{2}\cos\left(2aL+\dfrac{\pi}{4}\right)\right]/c_{23}^* \end{cases}$$

where $c_{23}^*=k\left(e^{4aL}+4e^{2aL}+2e^{2aL}\cos 2aL+1\right)$

4）Bco4

$$v(x,EI,k,q)=e^{ax}\left(c_1\cos ax+c_2\sin ax\right)+e^{-ax}\left(c_3\cos ax+c_4\sin ax\right)+q/k$$

where $a=\dfrac{\sqrt{2}}{2}\sqrt[4]{k/EL}$,

The values of coefficient are as follows

$$\begin{cases} c_1=-q\left[e^{2aL}\sin 2aL-e^{2aL}+1\right]/c_{24}^* \\ c_2=qe^{2aL}\left(\cos 2aL-1\right)/c_{24}^* \\ c_3=-qe^{2aL}\left(\sin 2aL-e^{2aL}+1\right)/c_{24}^* \\ c_4=qe^{2aL}\left(\cos 2aL-1\right)/c_{24}^* \end{cases},$$

where

$$c_{24}^*=k\left(2e^{2aL}\sin 2aL-e^{4aL}+1\right)$$

3 The solution of PDE3: $EI\dfrac{d^4v(x)}{dx^4}+N\dfrac{d^2v(x)}{dx^2}=q$

1）Bco1

$$v(x,EI,N,q)=c_1+c_2x+c_3\cos bx+c_4\sin bx+qx^2/N$$

where $b=\sqrt{N/EI}$

The values of coefficient are as follows

$$\begin{cases} c_1=\dfrac{-Lq\left(bL-2\sin bL-bL\cos bL\right)}{Nb\left(2\cos bL-bL\sin bL-2\right)} \\ c_2=\dfrac{-Lq}{N} \\ c_3=\dfrac{Lq\left(bL-2\sin bL-bL\cos bL\right)}{k\left(2\cos bL-bL\sin bL-2\right)} \\ c_4=\dfrac{Lq}{Nb} \end{cases}$$

2）Bco2

$$v(x,EI,N,q)=c_1+c_2x+c_3\cos bx+c_4\sin bx+qx^2/N$$

where $b=\sqrt{N/EI}$

The values of coefficient are as follows

$$\begin{cases} c_1=\dfrac{-2qEL}{N^2} \\ c_2=\dfrac{-Lq}{N} \\ c_3=\dfrac{2qEL}{N^2} \\ c_4=\dfrac{-2qEL\left(\cos bL-1\right)}{N^2\sin bL} \end{cases}$$

3）Bco3

$$v(x,EI,N,q)=c_1+c_2x+c_3\cos bx+c_4\sin bx+qx^2/N$$

where $b=\sqrt{N/EI}$

The values of coefficient are as follows

$$\begin{cases} c_1 = \dfrac{-2qEL\cos bL}{N^2} \\ c_2 = \dfrac{-2q\sin bL}{Nb} \\ c_3 = \dfrac{2qEL\cos bL}{N^2} \\ c_4 = \dfrac{2q\sin bL}{Nb} \end{cases}$$

4 The solution of PDE4: $EI\dfrac{d^4v(x)}{dx^4} + kv(x) + N\dfrac{d^2v(x)}{dx^2} = q$

1）When $N^2 < 4kEI$,

$$v(x,EI,k,N,q) = e^{a_1x}(c_1\cos b_1x + c_2\sin b_1x) + e^{-a_1x}(c_3\cos b_1x + c_4\sin b_1x) + q/k$$

where

$$a_1 = \frac{1}{2}\sqrt{2\sqrt{k/EI} - N/EI}\ ,$$

$$b_1 = \frac{1}{2}\sqrt{2\sqrt{k/EI} + N/EI}\ .$$

① Bco1

$$\begin{cases} c_1 = \left.\begin{array}{l} [-qN/EI\,(1 - e^{a_1L}\cos b_1L + e^{3a_1L}\cos b_1L - e^{2a_1L}\cos 2b_1L) \\ -2q\sqrt{k/EI}\,(1 - 2e^{2a_1L} - e^{a_1L}\cos b_1L + e^{3a_1L}\cos b_1L + e^{2a_1L}\cos 2b_1L) \\ -4a_1b_1q(e^{a_1L}\sin b_1L + e^{3a_1L}\sin b_1L - e^{2a_1L}\sin 2b_1L)] \end{array}\right/ c_{40}^* \\ c_2 = \left.\begin{array}{l} [-qN/EI\,(e^{a_1L}\sin b_1L + e^{3a_1L}\sin b_1L - e^{2a_1L}\cos 2b_1L) \\ -2q\sqrt{k/EI}\,(e^{3a_1L}\sin b_1L - 3e^{a_1L}\sin b_1L + e^{2a_1L}\sin 2b_1L) \\ -4a_1b_1q(e^{a_1L}\cos b_1L - e^{3a_1L}\cos b_1L - e^{2a_1L}\cos 2b_1L)] \end{array}\right/ c_{40}^* \\ c_3 = \left.\begin{array}{l} [-qNe^{a_1L}/EI\,(\cos b_1L + e^{3a_1L} - e^{2a_1L}\cos b_1L - e^{a_1L}\cos 2b_1L) \\ -2qe^{a_1L}\sqrt{k/EI}\,(\cos b_1L - 2e^{2a_1L} + e^{3a_1L} - e^{2a_1L}\cos b_1L + e^{a_1L}\cos 2b_1L) \\ -4a_1b_1qe^{a_1L}(e^{a_1L}\sin 2b_1L + e^{2a_1L}\sin b_1L - \sin 2b_1L)] \end{array}\right/ c_{40}^* \\ c_4 = \left.\begin{array}{l} [-qNe^{a_1L}/EI\,(\sin b_1L + e^{a_1L}\sin 2b_1L - 3e^{2a_1L}\sin b_1L) \\ -2qe^{a_1L}\sqrt{k/EI}\,(\sin b_1L - e^{2a_1L}\sin 2b_1L + e^{2a_1L}\sin 2b_1L) \\ -4a_1b_1qe^{a_1L}(\cos b_1L - e^{2a_1L}\cos b_1L - e^{a_1L}\cos 2b_1L)] \end{array}\right/ c_{40}^* \end{cases}$$

where

$$\begin{aligned} c_{40}^* = k[2\sqrt{k/EI}\,(1 + e^{4a_1L} - 4e^{2a_1L} + 2e^{2a_1L}\cos 2b_1L) \\ + N/EI(1 + e^{4a_1L} - 2e^{2a_1L}\cos 2b_1L)] \end{aligned}$$

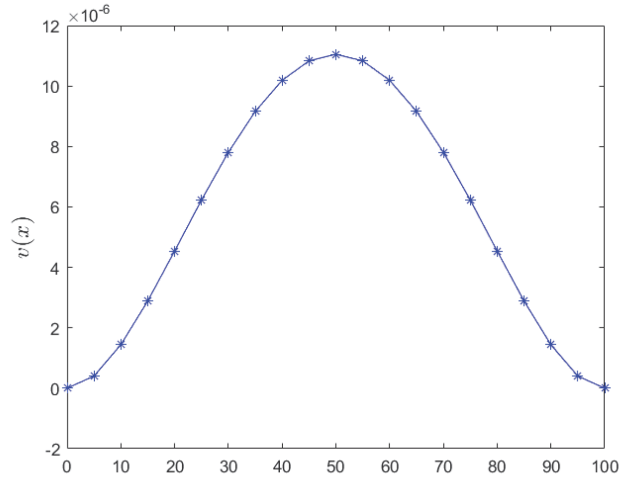
② Bco2 and Bco3

The solutions have the same form, but the coefficients of the solutions are different. The deflection curves $v(x)$ varying with the length of SFT for several boundary conditions can be given in Figure 2. The values of other parameters are as follows:

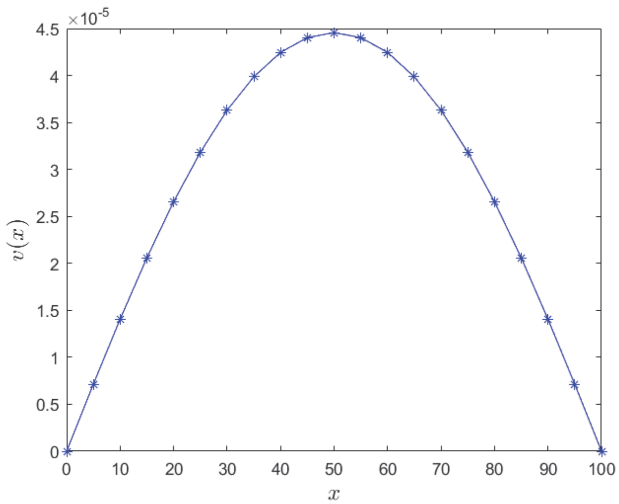
EI=$2.22\times10^{13}\,\mathrm{N\cdot m^2}$,

$q = 1000\,\mathrm{N/m}$,

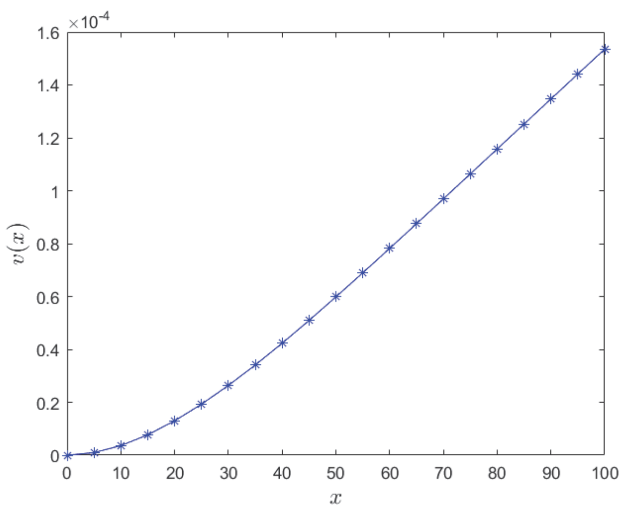
$k = 7\times10^6\,\mathrm{N/m^2}$, $N = 2\times10^8\,\mathrm{N}$ 。



(a)Boc1



(b)Boc2



(c)Boc3

Fig. 2 PDE4(tube is anchored and has axial force)deflection curve under different boundary conditions

2）When $N^2 \geq 4kEI$

$$v(x,EI,N,q) = c_1\cos a_2x + c_2\sin a_2x + c_3\cos b_2x + c_4\sin b_2x + q/k$$

where

$$a_2=\sqrt{\frac{1}{2EI}\left(N-\sqrt{N^2-4kEI}\right)},$$

$$b_2=\sqrt{\frac{1}{2EI}\left(N+\sqrt{N^2-4kEI}\right)}.$$

5 The solution of Equation (1)

Set $v(x,t)=\phi(t)Y(t)$, then

$$\frac{\phi^{(4)}(x)}{\phi(x)}+\frac{N}{EI}\frac{\phi''(x)}{\phi(x)}+\frac{k}{EI}=-\frac{m}{EI}\frac{Y''(t)}{Y(t)}=a^4$$

and

$$\begin{cases}Y''(t)+w^2Y(t)=0\\ \phi^{(4)}(x)+g^2\phi''(x)-b^4\phi(x)=0\end{cases}$$

where $w^2=\frac{a^4EI}{m}$, $b^4=a^4-\frac{k}{EI}$, $g^2=\frac{N}{EI}$.

For the second equation, the solution is

$$\phi(x)=c_1\cos\delta x+c_2\sin\delta x+c_3\cosh\varepsilon x+c_4\sinh\varepsilon x$$

where

$$\delta=\sqrt{\sqrt{b^4+\frac{g^4}{4}}+\frac{g^2}{2}},$$

$$\varepsilon=\sqrt{\sqrt{b^4+\frac{g^4}{4}}-\frac{g^2}{2}}$$

Then we get the solution of ω as follows

$$\omega_n=\sqrt{(\varepsilon L)_n^4\frac{EI}{mL^4}+(\varepsilon L)_n^2\frac{EI}{mL^2}+\frac{k}{m}}$$

or

$$\omega_n=\sqrt{(\delta L)_n^4\frac{EI}{mL^4}-(\delta L)_n^2\frac{EI}{mL^2}+\frac{k}{m}}$$

where the values of $(\varepsilon L)_n$ and $(\delta L)_n$ are related to the boundary condition.

1）Bco1: we need to satisfy

$$-2\delta\varepsilon(\cos\delta L\cosh\varepsilon L-1)-(\delta^2-\varepsilon^2)\sin\delta L\sin\varepsilon L=0$$

The solution of equation (1) is does not exist。

2）Bco2: we need to satisfy

$$-(\delta^2+\varepsilon^2)^2\sin\delta L\sinh\varepsilon L=0$$

Then

$$\sin\delta L=0$$

Then

$$\delta_n=\frac{n\pi}{L},n=1,2,\cdots$$

Then

$$\omega_n=\sqrt{\left(\frac{n\pi}{L}\right)^4\frac{EI}{m}-\left(\frac{n\pi}{L}\right)^2\frac{EI}{m}+\frac{k}{m}}$$

3）Bco3, we need to satisfy

$$\delta\varepsilon^5+\delta^5\varepsilon+2\delta^3\varepsilon^3\cosh\varepsilon L\cos\varepsilon L+\delta^2\varepsilon^4\sin\varepsilon L\sinh\delta L-\delta^4\varepsilon^2\sin\varepsilon L\sinh\delta L=0$$

The solution of equation (1) does not exist.

4）Bco4, we need to satisfy

$$-(\delta^2+\varepsilon^2)^2\delta^2\varepsilon^2(\delta\cos\delta L\sinh\varepsilon L-\varepsilon\sin\delta L\cosh\varepsilon L)=0$$

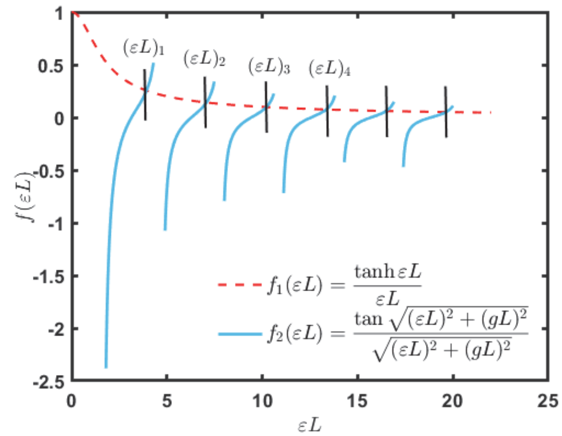


Fig.3 solutions of $\tan\delta L/\delta=\tanh\varepsilon L/\varepsilon$

Then $\frac{\tan\delta L}{\delta}=\frac{\tanh\varepsilon L}{\varepsilon}$, and

$$(\varepsilon L)_1=3.93,(\varepsilon L)_2=7.08,(\varepsilon L)_3=10.22\cdots,$$

as shown in Figure 3.

Just substitute $\omega_n=\sqrt{(\varepsilon L)_n^4\frac{EI}{mL^4}+(\varepsilon L)_n^2\frac{N}{mL^2}+\frac{k}{m}}$ into equation (1).

SUMMARY

In this paper, the analytical solutions of deflection and natural vibration frequency of SFT are studied systematically. Equations with solutions including PDE1 (with Bco1,2,3), PDE2 (with Bco 1,2,3,4), PDE3 (with Bco1,2,3), PDE4 (with Bco1,2,3), and equations without solutions including equation (1) with Bco1 and Bco3 and PDE1, PDE3, PDE4 with Bco4 are studied. The physical significance of unsolved equations is discussed. For equation (1) with the boundary condition Bco1 and Bco3, corresponding frequency solutions cannot be obtained. This is because that when the boundary conditions in Bco1 and Bco3 are substituted into the general solution, the determinant of the coefficient matrix of the linear system cannot be equal to 0, namely the boundary conditions Bco1 or Bco3 may be self-contradictory and the solution under simultaneous constraints cannot be obtained. For PDE1, PDE3 and PDE4, there is no solution when Bco4 is specified, because the boundary condition does not match the equation. Under the constraints of PDE1, PDE3 and PDE4, the boundary Bco4 does not exist.

Other problems worthy of further study include: the deflection of SFT under horizontal load, the natural vibration frequency of the structure, the torsional angle generated by the torsional force along the pipe body and the torsional natural vibration frequency. Through the analytical solutions of these problems, we can further study the advantages and disadvantages of the linear curve of the SFT and the torsion of the main structure of the SFT, which is less studied at present.

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