

Improving the discovery of Motifs in high-dimensional sequences of varying length

M. Adnan Haider

Master of Science
School of Informatics
University of Edinburgh

2013

Abstract

Acknowledgements

Many thanks to my mummy for the numerous packed lunches; and of course to Igor, my faithful lab assistant.

Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(M. Adnan Haider)

Table of Contents

1	Introduction	1
2	Datasets	5
3	DTW-Background	7
4	Improving DTW	10
4.1	Feature Selection	13
4.1.1	Signal Filter	14
4.1.2	Downsampling	18
4.2	Feature extraction	19
4.2.1	Domain-dependent feature extraction	20
4.2.2	Domain-independent feature extraction	23
5	Adaptive DTW	28
5.1	Feature extraction	29
5.2	Adaption of DTW	30
5.3	Experimental results	35
5.3.1	Experimental setup	38
	Bibliography	41

Chapter 1

Introduction

Over the course of the last decade, the mining of time-series data have received considerable attention within the data mining and machine learning community. The term 'time series' denotes a set of observations concurring any activity against different periods of time. The duration of time period may be in the order of microseconds or monthly or even annually depending on the domain. Mathematically, a time series is defined by the values $y_1, y_2 \dots y_n$ at times $t_1, t_2 \dots t_n$ where $y = f(t)$. The time t_i acts as an independent variable to estimate dependent variables y_i . The dimensionality of the series is denoted as \mathbf{n} where ' \mathbf{n} ' denotes the length of the sequence.

Time series analysis is used in many applications ranging from sales forecasting, budgetary analysis, stock market analysis and many more. One particular domain where the application of time series analysis is currently very popular is *motif* discovery- the problem of efficiently locating frequent/interesting sub-patterns in the data. The knowledge of motifs has been seen to have important applications in various aspects of data mining tasks. For instance:

- The discovery of association rules the reflect information of 'primitive shapes[1].
- The clustering of data into meaningful subgroups. Clustering is one of the most frequently used data mining tasks. It involves an unsupervised process for partitioning a dataset into meaningful groups. Such algorithms need to be specified on the initial seed of points and the number of cluster of groups. Motifs could potentially be used to address both problems. In addition, seeding the algorithm with motifs rather than random points could speed up convergence [17].

- The identification of important sub-patterns in DNA and gene sequences[11]

In the analysis of speech data, motifs also play a very important role. Recent research have shown that detecting and isolating motifs in speech utterances is equivalent to extracting frequent spoken words or linguistic entities spoken by the speaker(s) [3,5]. These methodologies are based on understanding the underlying structure of the observed data and operate on the acoustic signal directly(i.e there is no intermediate recognition stage to map the audio signal to a symbolic representation). This allows the word acquisition process to be unsupervised which is completely a different approach to the current speech recognition systems that are built using a supervised training methodology employing manually transcribed speech to model the underlying speech process.

To identify and extract motifs from time series data, various clustering algorithms have been proposed. The most widely used and popular approaches include the use of:

1. Dynamic time warping algorithm(DTW) [2,3,4,5,6,11] that clusters similar sequences separated by time shifts or temporal dynamics
2. Single value decomposition(SVD)[12]. The entire time series data set can be approximated by a low-rank approximation matrix achieved through transforming the data into an orthogonal feature space whose dimensionality is much lower than the original data sequences.

But unfortunately, directly applying these clustering algorithms to the features i.e the individual time series points may not lead to appealing results. Although the DTW algorithm is immune towards patterns shifted in time or distorted in size/shape, the time complexity of computing the DTW distance of two series is quadratic and is dependent on the dimensionality of the sequences or in other words the length of the sequences. To address this issue, linear-time constrained versions of DTW (Itakura parallelogram[19], Sakoe-Chiba band[18]) are used to constraint the size of the search space but the use of such constraints impact the accuracy of the algorithm[20].

The SVD too, also suffers from similar problems. For data sets where the dimensionality of the data is much higher than the sample size, the computational cost associated with the factorisation of the matrix is quite large. Apart from the computational cost, one of the main constraints of applying SVD is the requirement for all samples to share the same dimension which in the context of time series data means sequences must share the same length. This constraint greatly reduces the type of time series domains

to which SVD can be applied to extract latent factors that denote motifs. The speech corpus is an example of one such domain. Data sets comprised of speech utterances are a good example where recorded utterances do not share the same dimensionality (i.e the same length) and signals that are acoustically similar may be a contracted/expanded version of each other.

The discovery of motifs in high dimensional time series data(i.e long sequences) that vary in length is still a difficult problem to work with. To address the drawbacks of DTW and SVD, there has been some recent work conducted to improve these algorithms. In the paper “ *Fast time series classification using numerosity reduction*”, the authors address the drawbacks of DTW in handling high dimensional sequences. They propose an adaptive approach that initially uses a strict window constraint to reduce the search space of DTW but then gradually increase size of the window by discarding samples from the training set. Although this methodology improves the time complexity of the dynamic time warping algorithm by heuristically discarding regions in the input space, the methodology is more tailored to smart data selection rather than improving the algorithm itself. In the case of SVD, for high dimensional time series sequences which vary in length, the data matrix suffers in being incomplete. Carelessly addressing only the relatively few known entries is highly prone to over fitting. Earlier works [21] relied on imputation to fill in missing ratings and make the rating matrix dense. However, imputation can be very expensive as it significantly increases the amount of data. In addition, the data may be considerably distorted due to inaccurate imputation.

The goal of this project is to improve the performance of these algorithms in handling time series sequences that have high dimensionality and vary in length . To be precise, in the first half of the project, I will be investigating data mining and machine learning methods to improve the speed of the DTW algorithm **without** degrading the accuracy. And in the second half I will be ...

For this project, I will be using 3 time series data sets (details in chapter 2):

- TIGITS
- INLINESKATE
- CINC_ECG_TORSO

For a majority portion of the analysis, the TIDIGITS corpus will serve as the primary

dataset used to investigate the performances of different models. The reason being the TIGITS corpus consists of long time series sequences that vary in length . Since each time sequence corresponds to a speech utterance spoken by a speaker, as result of environment, context and speaker differences the length of the time series sequences will not be the same. In comparison, the sequences with in each UCR data set share the same dimensionality i.e length. Furthermore, the length of the time series sequences on average is much higher in the TIDIGITS corpus than sequences of any data set in the UCR time series database. Thus the TIDIGITS data set is an ideal choice to investigate the performance of different models in my project.

The dissertation is organised as follows: Chapter 2 gives a description of the 3 time-series datasets used for this project. Chapter 4 provides a detailed back ground description of the DTW algorithm. Chapter 3 and 4 investigates methods to improve the performance of the DTW algorithm in terms of both accuracy and speed. Chapter4...

Chapter 2

Datasets

The primary dataset that I have used for this project is the 'TIGITS' corpus. (I need to give more description here)

For Training : The entire training data is used –To contain the computational complexity, I am using samples from production 'a'

For the training set : To reduce the average mean time , I am using half of the training data set by choosing samples from one production: I have chosen:

225 samples from the boy category

234 samples from the girl category

495 samples from the men category

513 samples from the women category

Note : the size of the training set is half of the original training set but contains examples of all classes [1-9]

For the test set : Due to the high computational complexity, I am using only 1/3 of the test set I have chosen 162 random samples from boys

162 random samples from girls

326 random samples from men

326 random samples from women

Apart from the TIGITS, I have used two datasets from the UCR database:

The description of the data sets used for the next set of experiments are as follows:

1. CinC_ECG_torso

- Length of the time series:1639
- Size of test set:1380
- Size of training set:40
- Number of classes:4

2. InLineSkate

- Length of the time series:1882
- Size of test set:550
- Size of training set:100
- Number of classes:7

Chapter 3

DTW-Background

The Dynamic Time Warping algorithm measures the similarity between sequences varying in both time and speed. Formally, the problem formulation of the algorithm is stated as follows: Given two time series X , and Y , of lengths $|X|$ and $|Y|$,

$$X = x_1, x_2 \dots x_{|X|} \quad (3.1)$$

$$Y = y_1, y_2 \dots y_{|Y|} \quad (3.2)$$

construct a warping path W

$$W = w_1, w_2 \dots w_k \text{ where } \max(|X|, |Y|) \leq k \leq |X| + |Y|$$

- Here k denotes the length of the warping path and the m th element of the warping path is $w_l = (n_l, m_l) \in [1 : N] \times [1 : M]$ for $l \in [1 : k]$ where n_l is an index from the time series X and m_l is an index from the time series Y .

To properly understand the mechanism of the DTW algorithm, the definition of some key terminologies must first be stated:

1. Warping path: An (N, M) -warping path (or simply referred to as warping path if N and M are clear from the context) is a sequence $w = (w_1, \dots, w_k)$ with $w_l = (n_l, m_l) \in [1 : N] \times [1 : M]$ for $l \in [1 : k]$ satisfying the following three conditions.
 - (a) Boundary condition: $p_1 = (1, 1)$ and $p_k = (N, M)$. The boundary condition enforces that the first elements of X and Y as well as the last elements of X and Y to be aligned with each other. In other words, the alignment refers

to the entire sequences X and Y .

- (b) Monotonicity condition requires that the path will not turn back on itself, both the i and j indexes either stay the same or increase, they never decrease.
- (c) Step-size condition: $p_{l+1} - p_l \in \{(1,0), (0,1), (1,1)\}$ for $l \in [1:k-1]$. The step size condition expresses a kind of continuity condition: no element in X and Y can be omitted and there are no replications in the alignment

Intuitively speaking, the (N,M) warping path $p = (p_1, \dots, p_k)$ defines an alignment between two sequences $X = (x_1, x_2, \dots, x_N)$ and $Y = (y_1, y_2, \dots, y_M)$ by assigning the element x_i of X to the element y_j of Y .

2. Optimum Warping Path :

The optimal warp path corresponds to the minimum-distance warp path, where the distance of a warp path W is given as

$$Dist(W) = \sum_{i=1}^K dist(X, Y)_{|(w_i)}$$

$dist(X, Y)_{|(w_i)}$ represents the distance computed using an appropriate cost function between the time series points of x_{ni} of sequence X and y_{mi} of sequence Y .

$$dist(X, Y)_{|(w_i)} = dist(x_{ni}, y_{mi})$$

The goal of the DTW algorithm is to compute the distance of the optimal warping path between two time series sequences. Instead of attempting to solve the entire problem all at once, the DTW algorithm utilises the technique of dynamic programming to find an optimum alignment between two sequences through the computation of local distances between the points in the temporal sequence. A two-dimensional $|X|$ by $|Y|$ cost matrix D , is constructed where the value at $D(i, j)$ is the minimum-distance warp path that can be constructed from the two time series $X = x_1, \dots, x_n$ and $Y = y_1, \dots, y_m$.

$$D(i, j) = Dist(i, j) + \min[D(i-1, j), D(i-1, j-1), D(i, j-1)]$$

The figure below gives an example of the working the algorithm.

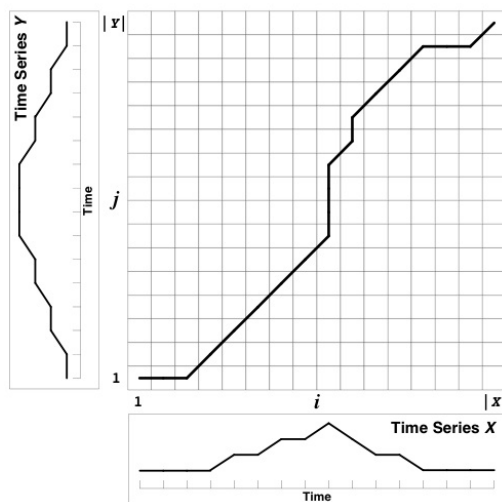


Figure 3.1: A cost matrix with the minimum-distance warp path traced through it.

Chapter 4

Improving DTW

The Dynamic Time Warping(DTW) algorithm is the one of the oldest algorithms that is used to compare and cluster sequences varying in time, length and speed. Given two temporal sequences, the algorithm utilises the technique of dynamic programming to compute the cost of the optimum alignment path between them. The computed cost gives an indication of the degree of similarity. The smaller the cost, the more similar the sequences are. Intuitively speaking, DTW is a clustering algorithm that clusters similar patterns varying in time and speed. The time and computational complexity of this algorithm is $O(n^2)$ where n denotes the length of the sequences that are being compared. Thus for time series domains having high dimensions(long sequences), the time and computational costs incurred by the algorithm are quite high which makes DTW a very unattractive choice for clustering or discovering motifs in high dimensional data sets. Another drawback for working in high-dimensional spaces is the contrast between the distances of nearest and furthest points. The distances between such points become increasingly smaller as the dimensionality increases. This makes it difficult to construct meaningful cluster groups in such spaces.

To address the issue of the curse of dimensionality, DTW algorithms employ a window constraint to reduce the search space. The most commonly used are Sakoe-Chuba Band[18] and the Itakura window constraint[19]. Figure[] gives an illustration on the nature of these window constraints. These constraints determine the allowable shapes that a warping path can take by restricting the DTW to find an optimal warping path only through the constrained window. As the dimensionality(length) of the sequences increases, the size of the window is adjusted accordingly. Rigid window constraints

impose a more rigid alignment that prevent an overly temporal skew between two sequences, by keeping frames of one sequence from getting too far from the other. The vast majority of the data mining researchers use a Sakoe-Chiba Band with a 10% width for the global constraint[23] to constraint the time complexity of DTW to a minimum. For clustering data sets such as speech utterances, the effect produced by such global constraints is highly undesirable. If we consider two utterances of a word spoken at different time frames, the patterns can have an overly temporal askew between them as result of the different contexts in which the words are spoken and/or as a result of different speakers speaking the same word. Thus it is necessary to explore alternative techniques to window constraints that can reduce the time complexity of the DTW algorithm to a minimum without decreasing accuracy.

Before investigating methods to improve the DTW algorithm itself, it is highly necessary to first understand the nature of the data sequences that the DTW is presented with. By achieving a thorough understanding of the data, we can achieve dimensionality reduction by isolating and identifying smaller set of new(current) features that are more relevant for the problem in hand. In this chapter, I investigate data-driven preprocessing techniques that use the domain knowledge to understand the underlying intrinsic structure of the lower-dimensional space on which the data lives . In other words, I am interested to observe whether the performance of the DTW algorithm will improve if we replace the time series sequence with a smaller sequence of vectors that captures knowledge of the domain and the trends of the time series sequences.

There are presently two groups of preprocessing techniques commonly used to address this issue:

- Feature Selection
- Feature Extraction

Feature selection techniques involve selecting only a subset of attributes from the original data. With respect to the time series data, the process refers to sub-sampling the sequence. One of the most popular approaches to feature selection is the exploratory data analysis(EDA). EDA is an approach to data analysis that postpones the usual assumptions about what kind of model the data follows with the more direct approach of allowing the data itself to reveal its underlying structure and models. The particular techniques employed in EDA are often quite simple, consisting of various techniques of:

1. Plotting the raw data such as data traces, histograms, histograms, probability plots, lag plots, block plots, and Youden plots.
2. Plotting simple statistics such as mean plots, standard deviation plots, box plots, and main effects plots of the raw data.
3. Positioning such plots so as to maximise our natural pattern-recognition abilities, such as using multiple plots per page.

Feature extraction processes on the other hand are concerned with the range of techniques that apply an appropriate functional mapping to the original attributes to extract new features. The intuition behind feature extraction is that the data vectors $\{x_n\}$ typically lie close to a non-linear manifold whose intrinsic dimensionality is smaller than that of the input space as a result of strong correlations between the input features. Hence by using appropriate functional mapping, we obtain a smaller set of features that capture the intrinsic correlation between the input features. By doing so, we move from working in high dimensional spaces to working in low dimensional spaces. The choice of appropriate functional mapping can also improve the clustering of data. For example, let's consider figure 4.1: The left-hand plot represents the locations of two dimensional data points in the original input space. The colours red and blue denote the classes to which the data points belong to. To cluster the data with respect to their classes, it will be ideal if we can partition the input space into disjoint regions where points belonging to the same class occupy the same region. This is achieved by mapping the points to a feature space spanned by two gaussian basis functions (shown on the right). Now, we can partition the feature space into two disjoint regions, one of each cluster.

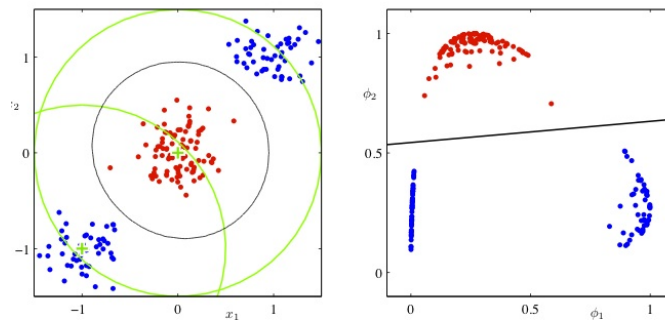


Figure 4.1: The figure on the right corresponds to location of the data points in the feature space spanned by gaussian basis functions $\phi_1(x)$ and $\phi_2(x)$

In the rest of this chapter, I explore a range of feature selection and extraction methods

and investigate whether their application can improve the performance of the DTW algorithm in terms of both accuracy and time complexity.

4.1 Feature Selection

The computational and time complexity associated with the DTW algorithm is governed by the dimensionality of the time series. To get a feel of the data, I employed exploratory data analysis on the isolated word utterances belonging to the test and training data sets that I constructed from the TIDIGITS corpus. The aim here to identify and isolate redundant features from the time series data. To get an idea about the structure of the data, I have studied the plots of the time series sequences along with listening to the individual samples. Figure 4.2 gives the plot of raw signal corresponding to the word '8' by a speaker from the *boy* category. From the visual and auditory analysis, I have made the following observations:

- Long durations of silence occupy the beginning and end of each utterance. These durations of silence segments are considerably long compared to the interesting regions in the acoustic signal that actually contain information about the spoken digit. Removing these silence segments do not only reduce the dimensionality of the time series but also result in minimal loss of information.
- Through listening to numerous samples, I have discovered that the recordings are highly distorted when played in *matlab* even when the data is scaled so that the sound is played as loud as possible without clipping. The distorted signal fails to provide any type of auditory clue about category of the speaker i.e whether the speaker belongs to { boy,girl, men,women} and the signal must be played multiple times for the word to be correctly identified.

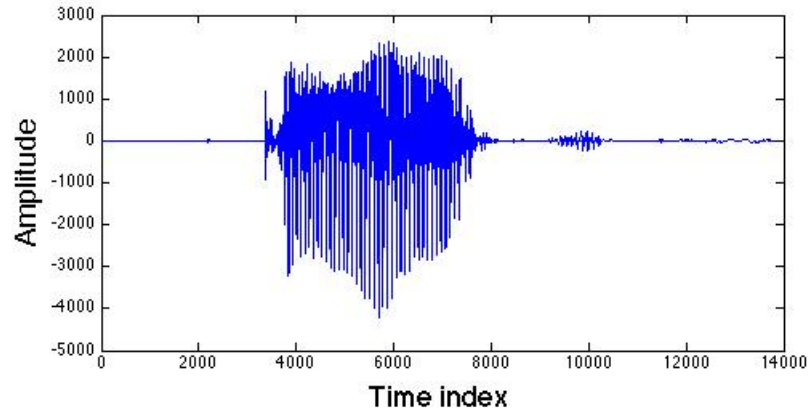


Figure 4.2: 'Raw 'signal

4.1.1 Signal Filter

Thus to remove these redundant attributes from the time series sequence, I have constructed the following algorithm: 'silencefilter' that performs feature selection by removing segments of silence. An outline of the algorithm is as follows:

Algorithm 1 SignalFilter

```

1: procedure SILENCEFILTER(signal) ▷ raw signal
2:   threshold = 0
3:   maxAmplitude = max(rawSignal)
4:   Adapt the threshold based on the value taken by the maximum amplitude
5:   output ← removeSilence(rawSignal, threshold)
6:   return output
7: end procedure
  
```

- The algorithm removes all samples in the times series sequence whose magnitude is less than the threshold. The threshold used is an adaptive parameter. By using the information of the signal's maximum amplitude the algorithm sets the threshold accordingly. It raises the threshold for signals corresponding to speakers having a loud and deep voice and lowers the threshold for signals corresponding to speakers having gentle and low voice.

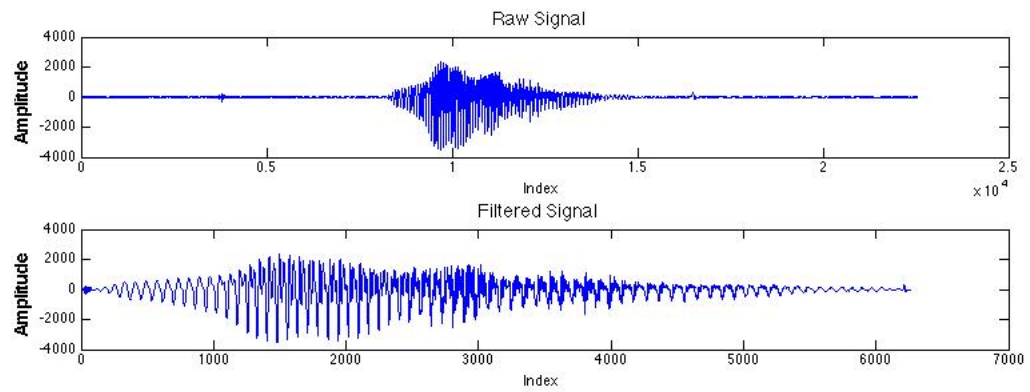


Figure 4.3: shows the raw acoustic signal corresponding to the utterances of the digit '8' alongside with the version that has its dimensionality reduced by the filter discussed above.

From figure 4.3, it can be observed that the filter preserves the interesting patterns associated with the utterance while succeeding in reducing the dimensionality of the data. To investigate the effect of introducing this prior feature selection step on the performance of the DTW algorithm, I conducted the following experiment:

- Objective : Performance comparison between DTW equipped with a feature selection step against the based line DTW
- Dataset used: TIDIGITS Test-set size : 976 samples

category	sample size
boy	162
girl	162
men	326
women	326

Training data set size: 1467

category	sample size
boy	225
girl	234
men	495
women	513

- An outline of DTW used algorithm used for this experiment is given below.

Algorithm 2 Value-Based DTW

```

1: procedure VALUE-BASED(seq1, seq2)                                ▷ two raw sequences
2:    $w = \max(\lceil 0.1 * \max(n, m) \rceil, \text{abs}(n-m))$                 ▷ Window constraint
3:   for  $i=1$ : to  $\text{length}(\text{seq1})$  do                                ▷ Initialise the DTW cost matrix
4:      $\text{DTW}(i, 0) = \infty$ 
5:   end for
6:   for  $i=1$  to  $\text{length}(\text{seq2})$  do
7:      $\text{DTW}(0, i) = \infty$ 
8:   end for
9:   for  $i=2$  to  $\text{length}(\text{seq1})$  do
10:    for  $j=\max(2, i-w)$  to  $\min(\text{length}(\text{seq2}), i+w)$  do    ▷  $\text{cost}(a, b) \equiv \text{euclid}(a, b)$ 
11:       $\text{DTW}(i, j) = \text{cost}(\text{seq1}(i), \text{seq2}(j)) + \min\{ \text{DTW}(i-1, j) + \text{DTW}(i, j-1) + \text{DTW}(i-1, j-1) \}$ 
12:    end for
13:  end for
14:  return  $\text{result} = \frac{\text{DTW}(n, m)}{nm}$                                 ▷  $n=\text{length}(\text{seq1}), m=\text{length}(\text{seq2})$ 
15: end procedure

```

Note : The DTW algorithm is subjected to an adaptive window constraint. The focus of my research here is to improve the accuracy of the DTW algorithm while reducing the time and computational cost to a **minimum**. Even after applying the feature selection process, from initial experiments I have found the dimensionality of the time series sequences is still very high. Thus for these experiments, I have employed the Sakoe-Chuba band that is adaptive in size : $w = \max(\lceil 0.1 * \max(n, m) \rceil, \text{abs}(n-m))$. The lower bound of the window size is set to 10% the size of the longest sequence because its is the standard size that the vast majority of the data mining researchers[23] use to keep the time complexity of DTW to a minimum.

- Results:

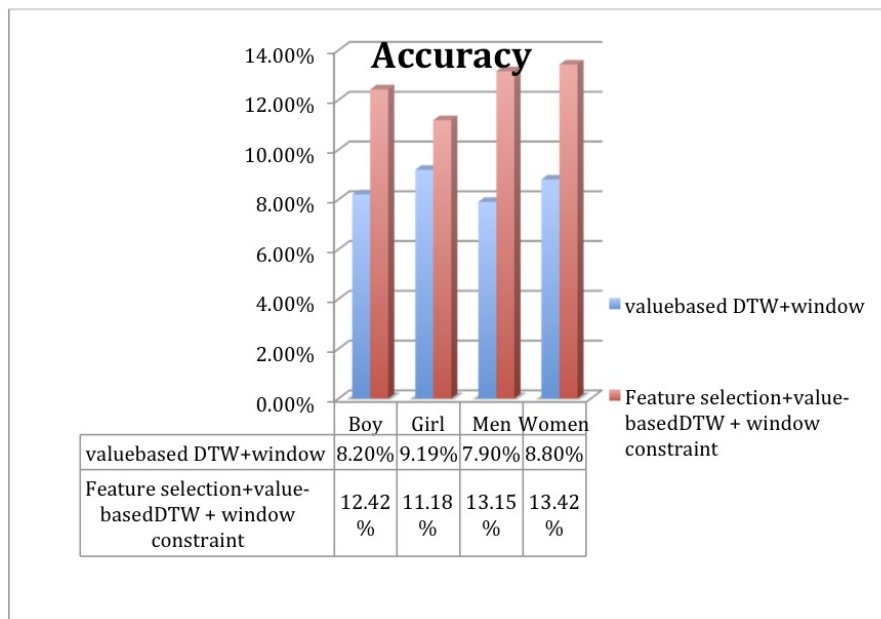


Figure 4.4

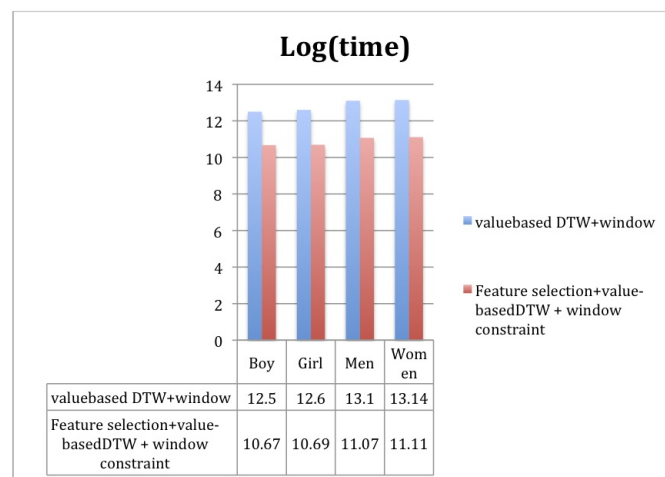


Figure 4.5

Observations:

From the results above, it can be seen that the DTW algorithm achieves very poor accuracy. The reason for this lack of poor recall may be attributed to one of these or a combination of the following three factors:

- raw values- the value of a data point in a time series sequence is not a complete picture of the data point in relation to the rest of the sequence.

- window constraint- the optimum warping path exists outside the boundaries of the Sakoe-Chuba bands[3,6].
- not using MFCC values- The data set comprises of speech utterances. It is a widely known fact that for speech data, the MFCC feature vectors captures the information of phones that make up a word and thus can be used to differentiate between utterances of different lexical identities. (details of MFCC to follow)[3,4,5,6,7].

Employing the prior feature selection process allows DTW subjected to global window constraint to improve **both** its accuracy and time complexity.

Reasons:

- The DTW algorithm, due to the high dimensionality of the time series sequences is subjected to a window constraint that forces the algorithm to operate on the diagonal region of the DTW cost matrix. Removing these redundant features increases the accuracy because these features primarily represent segments of silence and since all utterances contain silence segments, taking these silences into account degrades the performance as they bring in an unwanted notion of similarity in dissimilar patterns.
- The size of the DTW cost matrix is $O(mn)$. Achieving dimensionality reduction through feature selection reduces the size of the cost matrix and thus decreases the computational cost.

From this observation, it can be concluded that using raw values does degrade the performance of the DTW. The experiments to follow later in this chapter will attempt to investigate the degree of the contributions of each factors towards the poor recall performance of the DTW algorithm.

4.1.2 Downsampling

From further exploratory data analysis, I have observed that if I down-sample the utterances by $\frac{1}{2}$ which in other words means decreasing the sampling frequency by half, the resultant sampled signal is much clearer to understand. Sub-sampling the utterances by half involves removing every other sample from the

time series. From the observation of figure 4.6, it can be seen that this technique keeps the global trend of the signal intact but results in the loss of local information. Furthermore through listening the sampled signals, I have discovered that losing some **local information** actually cleans the signal in a manner that allows the listener to identify the speaker's category and the utterance's class with ease.

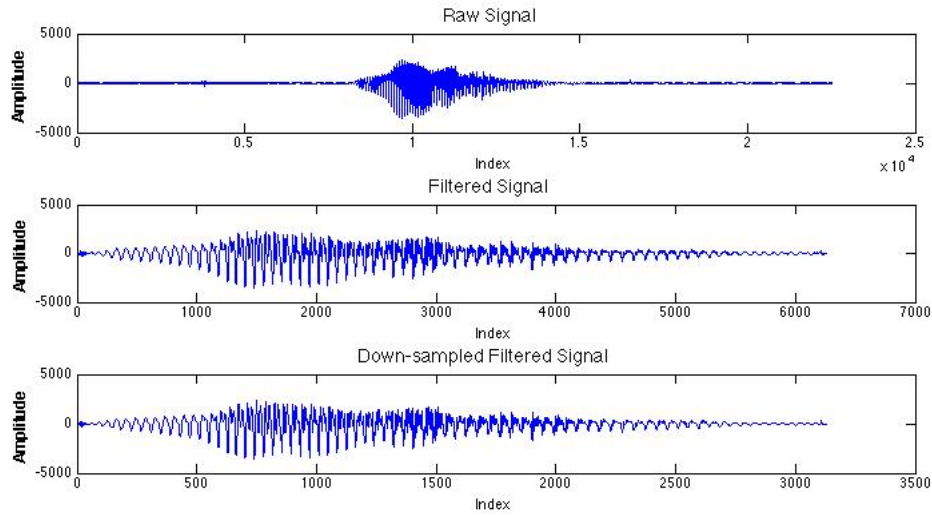


Figure 4.6: shows the raw acoustic signal of the digit '8' (top figure), the silence removed version of the signal(middle) and the silence removed and down sampled version of the acoustic signal (bottom)

To investigate whether further reducing the dimensionality of the time sequences can improve the performance of the DTW, I conducted a third experiment using the same data set and the sample DTW algorithm discussed in 4.1.1. The results found are as follows:

4.2 Feature extraction

From the analysis conducted so far, it can be concluded that heuristically selecting only significant attributes from the time series sequences does improve the accuracy and the speed of the DTW. However, as we have seen, the accuracy of the algorithm is very low. In this section, I investigate the influence of employing domain-dependent and domain dependent feature extraction methodologies on the speed and accuracy of the DTW algorithm. There are two motivations behind conducting this analysis:

- The primary motivation is to investigate whether the low accuracy of DTW is a result of not using features that incorporates information about the domain and the trends of the sequence. The features that we have considered so far are the raw values indexed by time.
- The main focus is to improve the speed of the DTW algorithm without degrading accuracy. By using appropriate functional mapping, we can map the data to lower dimensional feature space that can captures the intrinsic qualities of the data. This not achieves dimensionality reduction of the time series sequences but also the potential to boost accuracy.

The motivation behind this approach is to investigate to what degree we can improve the performance of the standard algorithm across different domains without making changes to the algorithm itself.

4.2.1 Domain-dependent feature extraction

The main data set that I am working with is the TIGITS corpus which is composed of speech utterances. DTW formally is a domain independent algorithm. It will interesting to investigate to what degree is the performance of the algorithm effected if we model the information of the domain into the DTW algorithm. Since at the moment, i am investigating techniques that improve the performance of the DTW algorithm on high dimensional data without making changes to the algorithm itself, in this section, I investigate domain-dependent feature extraction methods that embed the knowledge of the domain in the feature extraction phase.

For speech, the most commonly used features are the MFCC features-mel cepstrum ceptral coefficients. This feature representation is based on the idea of the cepstrum. For human speech, a speech waveform is created when a glottal source waveform of a particular frequency is passed through the vocal tract which because of its shape has a particular filtering characteristic. The exact position of the vocal tract is in fact the key attribute in providing useful information about phones(units of sounds). Cepstrum provides a useful way to separate the information of the vocal tract from the glottal source.

A sketch of the MFCC feature extraction is given below:

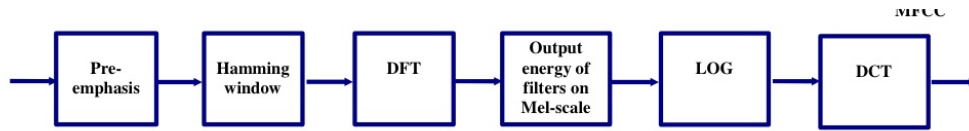


Figure 4.7: MFCC feature extraction

- i Pre-emphasis: boosts the energy of the signal at high frequencies to improve phone detection
- ii Windowing: partitions the time series sequence into frames using a hamming window
- iii DFT: extracts spectral information at different frequency bands
- iv Mel scale : transforming to mel scale improves recognition performance by allowing the model to take into account the property of human hearing
- v Log : makes the feature less sensitive to variations in input such as power variations on the speakers mouth.
- vi Cepstrum : separate the information of the vocal tract from the glottal source. The first 12 cepstral values from spectrum of the log of the spectrum are used

Through the windowing process, the MFCC features extraction achieves dimensionality reduction. Each sequence is segmented into frames of length 20 to 30 ms which are then through appropriate functional mapping are converted into sequences of MFCC feature vectors. Since the result sequence of vectors is much smaller than the length of the original sequence, the size of the DTW cost matrix is much smaller than before, This in turn lowers the time and computation cost incurred by the algorithm. One of the focal points of research for this project is to investigate alternative measures to using global window constraints that minimises the time and a computational cost incurred by the DTW to minimum without decreasing accuracy when working in high dimensional spaces. So the question that now lies is whether we can achieve greater reduction in dimensionality. The feature selection procedure that I discussed in the previous section reduces the size of the sequences by removing segments of silence followed the renaming segments by 1/2. As a prior step to MFCC feature extraction, if we use the first half of this feature selection process(i.e silence removal) and then apply MFCC feature extraction, we achieve a dimensionality reduction which is

greater than using either of the processes alone.

To test this idea I ran two versions of DTW on the TIGITS test set. The first version is equipped with a two step preprocessing stage: removal of silence followed by MFCC feature extraction while the 2nd version only employs MFCC feature extraction as pre-processing step. For this experiment, I had to impose a window constraint on the later version of DTW to contain the time and computational cost. Although MFCC feature extraction does achieve dimensional reduction. From doing initial tests, I have observed that the time taken by the algorithm to compare any two sequences is still significantly high. To therefore reduce the computational time, I was forced to use the same window constraint that I employed for the previous experiments. A summary of the results of all experiments is given by the figures below:

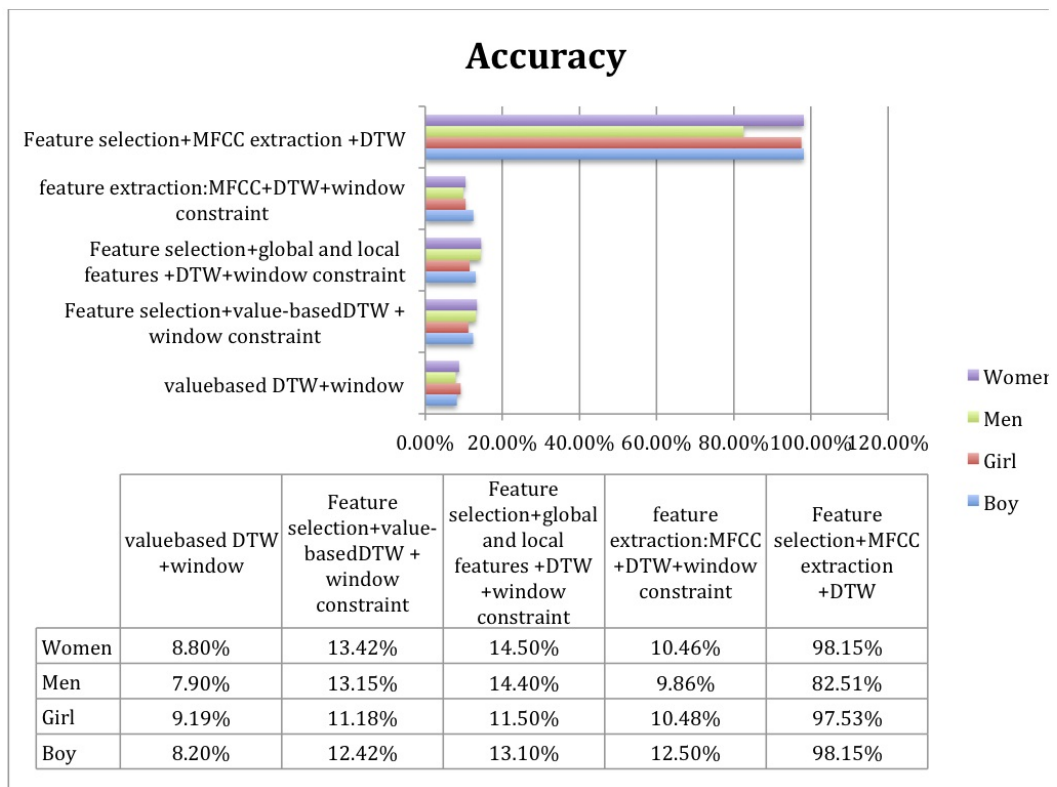


Figure 4.8: MFCC feature extraction

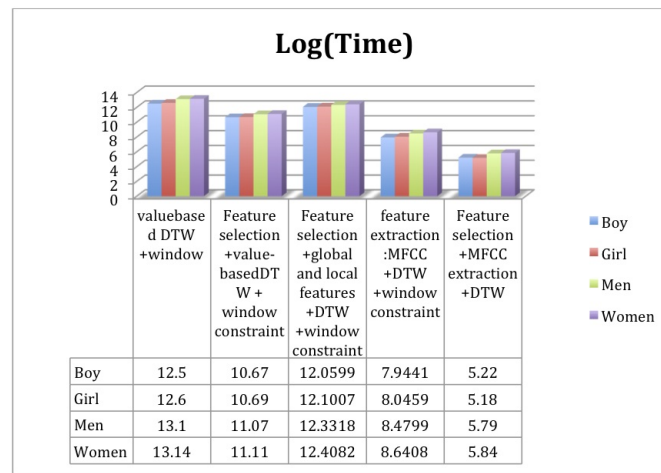


Figure 4.9: MFCC feature extraction

Observations:

- It can be observed that adapting DTW to incorporate a pre-processing stage that involves removal of redundant features through silence removal and employing a feature extraction mapping that integrates domains knowledge in the feature extraction process achieves almost perfect accuracy and incurs the lowest computational time. To be precise, adapting DTW to be domain-dependent allows the algorithm to achieve near perfect accuracy while incurring the minimum computational cost.
- From analysing the time complexity associated with each of the two versions, it can be seen that partitioning the sequences into frames actually leads to greater reduction in dimensionality of the time series than removing redundant features.

To summarise, from the observation of the results gathered from the experiments it can be concluded that employing preprocessing steps that include the domain knowledge greatly improves the accuracy and time complexity of the DTW algorithm when high dimensional spaces and in some domain such the TIGITS corpus eliminates the necessity of impose global window constraints to achieve minimum computational cost.

4.2.2 Domain-independent feature extraction

The fundamental problem of baseline (value-based) DTW is that the numerical value of a data point in a time series sequence is not a complete picture of the data point in relation to the rest of the sequence. The context such as the position of the points in relation to their neighbours is ignored. To fix this issue, an alternative form of DTW known as *Derivative* DTW is proposed but it fails to achieve better performance across all domains as it ignores to take into account the common sub-patterns between two sequences (mainly global trends). Ideally we need to use features that contain information about the overall shapes of the sequences plus the local trend around the points. This allows the DTW to build a complete picture of the data point in relation to the rest of the sequence and hence achieve a better optimal alignment between the two sequences.

For feature extraction, the methodology that I have used for this setup is based on Xie and Wiltgen's paper[]. In their paper, the authors highlight a domain-independent feature extraction process where each point in the time series sequence is replaced by a 4 dimensional vector. In this vector, the first two features correspond to information regarding the local trends around a point and the last two features reflect the position of that point in the global shape of the sequence. From the experiments conducted on the UCR data sets, it has been observed that embedding DTW with this feature extraction process yields greater accuracy across all datasets.

Definition of local feature given in [] is as follows:

$$f_{\text{local}}(r_i) = (r_i - r_{i-1}, r_i - r_{i+1})$$

The extraction of global features is constrained by two factors: the features that reflect information about global trends and the features must be in the same scaling order as the local features. Being in the same scale allows them to be combined with local features. In [] the authors used the following method to extract global features from the time series sequence:

$$f_{\text{global}}(r_i) = (r_i - \sum_{k=1}^{i-1} \frac{r_k}{i-1}, r_i - \sum_{k=i+1}^M \frac{r_k}{M-i})$$

Note : The local and global features have no definition for the first and last points in a sequence.

When working with high dimensional time series data, the main drawback of employing this feature extraction method is that it does not offer the advantage of dimensionality reduction. The dimensionality of the feature space is just two dimensions less than the dimensionality of the original data. The DTW algorithm combined with this feature extraction process therefore suffers from the curse of dimensionality as before. To tackle this issue, as a prior step to the feature extraction process, I applied the feature selection process that I have discussed in the previous section to remove redundant features.

The length of the resultant sequence of the vectors is still large. To address the issue of high computational cost, I have used the same window constraint applied the DTW algorithm equipped the two preprocessing stages of feature selection and extraction on the test data set that I had constructed from the TIGITS corpus. A summary of the results are given below:

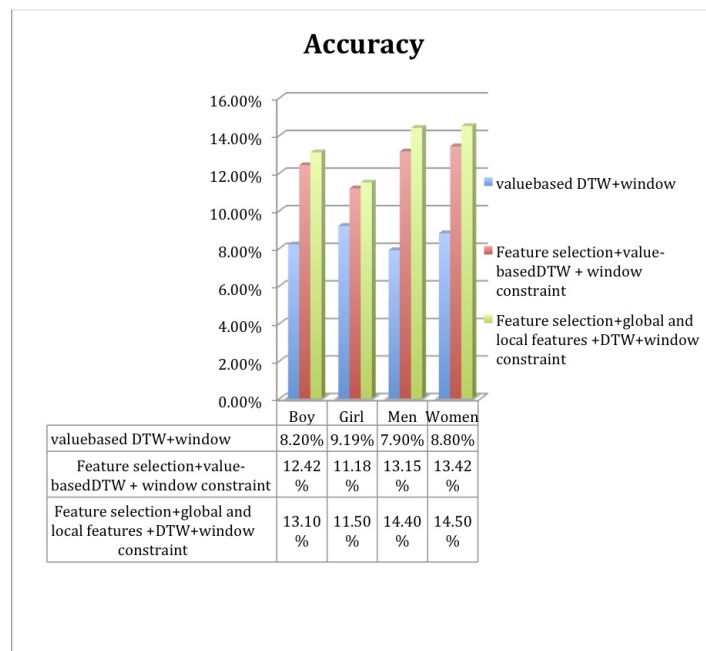


Figure 4.10

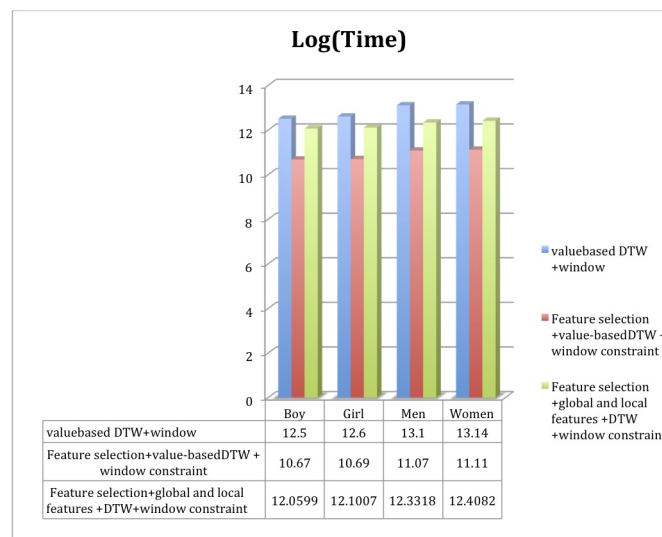


Figure 4.11

Observation:

- Equipping DTW with a preprocessing stage that involves feature selection and extraction of local and global features result in a greater improvement in the accuracy of the algorithm.

Explanation:

- * The algorithm now warps the time axis to match the local and global trends between points in the sequences instead of just their values. Since similar patterns will share the same trends, adding the feature extraction phase therefore improves the accuracy.
- The computational cost incurred by the algorithm is higher than the previous version that uses only the feature selection process.

Explanation:

- * The cost of applying the euclidean metric on vectors $>$ cost of applying the euclidean metric on points. Since the euclidean metric is applied mn times. The overall computational cost increases.

The problem of working with a single data set is that if the model design is iterated many times using a limited size data set, then some over-fitting to the validation data can occur. To ensure that the models have not over-fitted to the test set, I ran the two versions of DTW :one that uses the preprocessing step of

feature extraction while the other just uses the raw values for computing similarity, on the InlineSkate and Cinc_Ecg_Torso time series datasets[]. The results found are as follows:

Note: Both versions of DTW are equipped with the window constraint $w = \max(\lceil 0.1 * \max(n,m) \rceil, \text{abs}(n-m))$ to reduce the time and computational cost to a minimum.

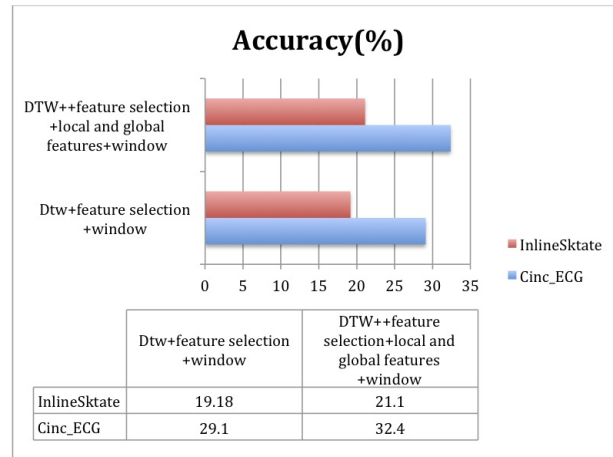


Figure 4.12

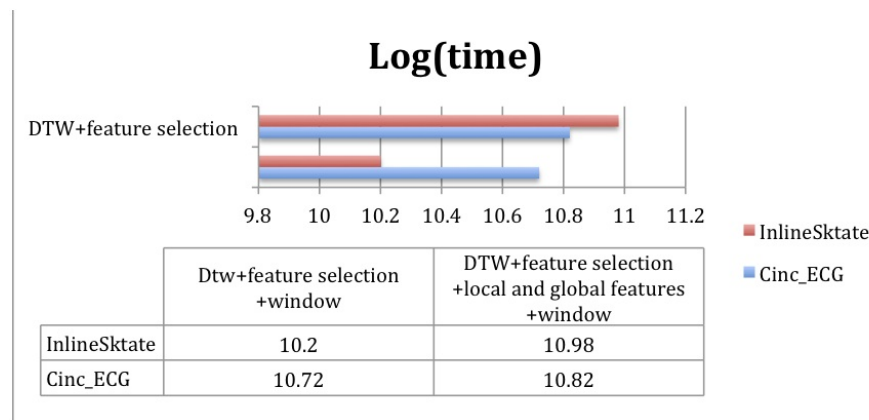


Figure 4.13

The differences between the two versions of DTW in terms of accuracy and time complexity are consistent with the observations made in the previous experiment with the TIDIGITS data set. This gives valid supports to the analysis of the tests that I have conducted so far

Chapter 5

Adaptive DTW

In the previous chapter, I have investigated techniques on different pre-processing strategies that can improve the performance of the general DTW algorithm in working with high dimensional time series datasets. These techniques that I am investigating so far primarily focus on the improving the quality of the data rather than the algorithm itself. The results found from the experiments so far, have shown that understanding the intrinsic properties of the data and factoring in the domain information can not only improve the performance of the DTW but may even discard the need for a the global windowing constraint(as we have seen for the model that augments feature selection and MFCC feature extraction) to reduce the time and computational complexity. The DTW algorithm is a memory based algorithm that employs a similarity metric to compare a sequence with all sequences in the training in an iterative manner. Since the whole training set is used during the testing phase, the computational complexity makes the algorithm very attractive to use. The computational cost of a DTW algorithm is (mn) where m and n denote the length of the two time series sequences currently being compared. Using longer sequences increases the size of the DTW cost matrix hence resulting into a greater number of computations. The domain-dependent pre-processing methodology doesn't guarantee the dropping of the global window constraint that is used to reduce the search space when tackling high dimensional data. As we have seen from the experiments, when working with high-dimensional time-series data, the accuracy of the DTW algorithm using a window constraint suffers greatly even if it's equipped with domain dependent/independent features. Hence from a scientific point of view, it is of

great interest to research methods to improve the DTW algorithm so it can constraint the time and computational complexity associated with high-dimensional data without degrading the accuracy by too much. In this chapter, I investigate an unsupervised methodology that:

- incorporates information about local and global trends in the feature extraction process
- employs an adaptive DTW that tackles the issue of the large time and computational complexity by moving from working on time series sequences to sequences of segmented time-slices. To counter the tradeoff in the decrease in accuracy, the algorithm is equipped with a kernel function(self-proposed) that is designed to measure the similarity of sub-sequences more accurately than standard euclidean metric by being invariant toward time-dilation and scale.

5.1 Feature extraction

For feature extraction, the methodology that I have used for this setup is based on Xie and Wiltgen's paper[] that I have already discussed in the previous section. Each point in the time series sequence is replaced by a 4 dimensional vector where the first two features correspond to information regarding the local trends around a point and the last two features reflect the position of that point in the global shape of the sequence. Definition of local feature as given in [3.2.1] :

$$f_{\text{local}}(r_i) = (r_i - r_{i-1}, r_i - r_{i+1})$$

Definition of global feature:

$$f_{\text{global}}(r_i) = (r_i - \sum_{k=1}^{i-1} \frac{r_k}{i-1}, r_i - \sum_{k=i+1}^M \frac{r_k}{M-i})$$

5.2 Adaption of DTW

The feature extraction methodology discussed above maps the time series sequence to a time series sequence of vectors whose length is $\|X_n\| - 2$. (where $\|X_n\|$ denotes the length of the original time series sequence). The DTW augmented with these features will still suffer from large time and computational complexity if the dimensionality of the data is high. In the MFCC feature extraction process, the time series sequence is first segmented into series of frames of length 20ms. Through appropriate functional mapping, each frame is then mapped to a vector. Because the length of the resultant sequence of vectors is much smaller than the length of the original time series, the size of the DTW cost matrix is reduced resulting in lower time and computational cost associated with each comparison.

Similar to the MFCC extraction process, the time series of 4d vectors extracted in the feature extraction stage are segmented using windows of width 5ms. The original time series is reduced to series of matrices where the length of the new series is 5 times smaller than before. Now if we adapt the cost function of DTW to work on series of matrices rather than series of vectors we can achieve a large improvement in the time and computational cost associated in the testing phase without imposing a **window** constraint.

The problem now can be shifted to finding an appropriate kernel that can be used to compute the similarity between matrices composed of feature vectors. Ideally, we want a metric that takes into account the variation of speed and time when comparing two similar subsequences. We will want to compare the global and local properties associated with a point in one subsequence with the global and local properties of points at different regions in the second sub-sequence illustrated by figure 2. Using a euclidean metric in this scenario is inappropriate. The euclidean metric in this context is identical to linear time warping where the two subsequences will be matched based on a linear match of the two temporal dimensions. In our context, we need a kernel that computes the similarity between two sub-sequences by warping the time axis.

The motivation behind the kernel that I propose for aiding DTW to tackle high-dimensional data comes from the polynomial kernel.

Let x and z be two dimensional vectors. Consider the simple polynomial kernel

of degree 2 : $k(x, z) = (x^T z)^2$. This kernel can be expressed as :

$$\begin{aligned}
 k(x, z) &= (x^T z)^2 \\
 &= (x_1 z_1 + x_2 z_2)^2 \\
 &= x_1^2 z_1^2 + 2x_1 z_1 x_2 z_2 + x_2^2 z_2^2 \\
 &= (x_1^2, 2x_1 x_2, x_2^2)(z_1^2, 2z_1 z_2, z_2^2)^T \\
 &= \phi(x)^T \phi(z)
 \end{aligned}$$

The 2nd order polynomial kernel is equivalent to a corresponding feature mapping ϕ that contains terms of order 2. Now, if we generalise this notion then $k(x, z) = (x^T z)^M$ contains all monomials of order M. If we imagine x and z to be two images, then the polynomial kernel represents a particular weighted sum of all possible products of M pixels in the first image with M pixels in the second image.

Using this as motivation I propose the following kernel:

$$k(x, z) = \left\langle \sum_{i=1}^n x_i, \sum_{j=1}^n z_j \right\rangle$$

where n denotes the length of the window and x_i and z_j represents the 4-dimensional features indexed by the points in two sub-sequences.

To motivate the reasoning behind the construction of this particular kernel let's consider the following signals:

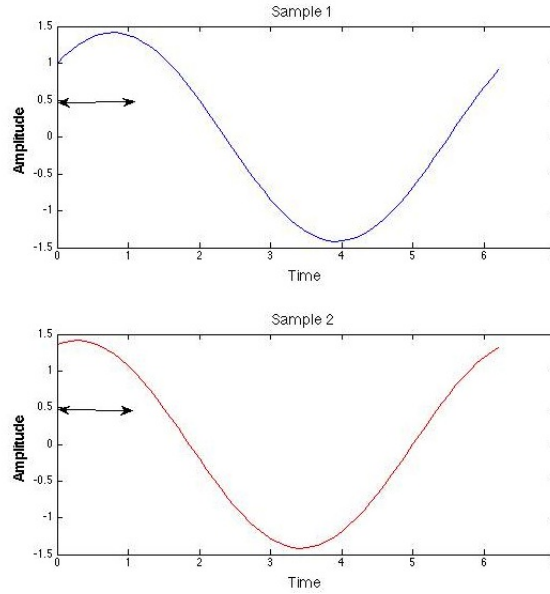


Figure 5.1: Two signals separated by translation

The signal denoted by the ‘red’ color is a ‘slower’ version of the signal denoted by the ‘blue’ color. In the above example, if we are comparing the similarity between the time slices spanned by the arrows, an ideal kernel must be invariant to the time offsets of the signals and thus should consider all possible pairings between the vectors in the subsequences. Intuitively speaking, the kernel must behave like a DTW algorithm..

For time slices of width n , the kernel metric can be expanded and expressed as :

$$\begin{aligned}
 k(x, z) &= \left\langle \sum_{i=1}^n x_i, \sum_{j=1}^n z_j \right\rangle \\
 &= \langle (x_1 + x_2 + x_3 + \dots), (z_1 + z_2 + z_3 + \dots) \rangle \\
 &= \langle x_1, z_1 \rangle + \langle x_1, z_2 \rangle + \langle x_1, z_3 \rangle + \dots + \langle x_2, z_1 \rangle + \langle x_2, z_2 \rangle + \langle x_2, z_3 \rangle + \dots
 \end{aligned}$$

From above expression, we can see that the proposed kernel corresponds to a sum of all possible dot products of pairs belonging to the set $\{(x_i, z_j) | x_i \in \text{seq1}, z_j \in \text{seq2}\}$. Similar to the polynomial kernel, the proposed kernel allows us to match all possible pairs of vectors belonging to the two sub-sequences given by the matrices. It is easy to check that this proposed kernel is in fact a valid kernel:

- $K(x, z) = K(z, x) \Rightarrow$ the function is symmetric.

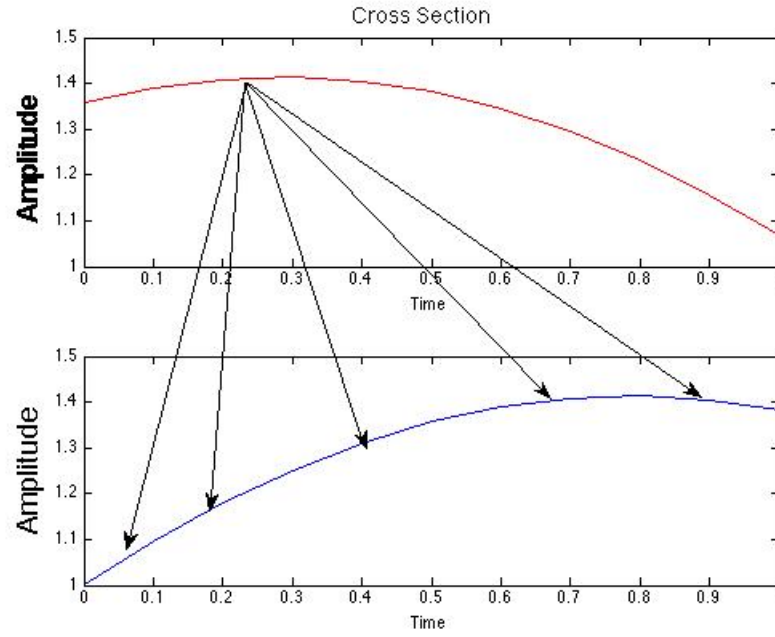


Figure 5.2: Two identical subsequences varying in time

- The kernel satisfies Mercer's theorem : $K(x,z) = \phi(x)^T \phi(z)$ where the feature mapping corresponds to a finite summation of vectors $\phi(y) = \sum_{i=1}^n y_i$.

Augmenting the kernel to the DTW algorithm allows DTW to work on high-dimensional time sequences with using a window constraint. However the accuracy and computational cost of the DTW is now dependent on the size of the time slices used to segment the original sequences:

- The accuracy of DTW increases as the width of the windows decrease. Using subsequence allows the similarity measure to be dominated by the dot products of points whose local and global features are most alike. However using smaller windows achieve lesser dimensionality reduction. Thus the time and computational complexity suffers.

To use this kernel as an appropriate cost function in the DTW algorithm, we need a functional mapping that:

1. constraints the codomain to be in the range from 0 to ∞ .
2. ensures larger values given by the function signify great degree of dissimilarity and smaller values signify a high degree of similitude.

An ideal cost function that make use of dot product sis the *arc-cosine*. Hence I

embedded the kernel function in the cosine distance:

$$\theta = \frac{\langle X, Z \rangle}{|X||Z|}$$

where $X = \sum_{i=1}^n x_i$ and $Z = \sum_{j=1}^n z_j$

A formal outline of the algorithm is as follows:

Algorithm 3 Adapted DTW

```

1: procedure VALUE-BASED(seq1, seq2)           ▷ two sequences of feature vectors
2:   seq_1 ← segment(seq1, n)   ▷ Segment the sequences using a window of size n
3:   seq_2 ← segment(seq2, n)
4:   for i=1: length(seq_1) do                 ▷ Initialise the DTW cost matrix
5:     DTW(i, 0) = ∞
6:   end for
7:   for i=1 to length(seq_2) do
8:     DTW(0, i) = ∞
9:   end for
10:  for i=2 to length(seq_1) do
11:    for j=max(2, i-w) to min(length(seq_2), i+w) do
12:      DTW(i, j) =  $\theta = \frac{\langle X, Z \rangle}{|X||Z|} + \min\{ \text{DTW}(i-1, j) + \text{DTW}(i, j-1) + \text{DTW}(i-1, j-1) \}$ 
13:    end for                                ▷  $X = \sum_{i=1}^n x_i$  and  $Z = \sum_{j=1}^n z_j$ 
14:  end for
15:  return result =  $\frac{\text{DTW}(n, m)}{nm}$            ▷ n=length(seq1), m=length(seq2)
16: end procedure

```

5.3 Experimental results

The changes that I have introduced, in the previous section to the ‘Dynamic Time Warping’ algorithm is aimed to improve the algorithm’s performance in handling high dimensional time series data. In this section, I investigate the performance of my proposed algorithm by running it on the test data set that I have constructed from TIDIGITS test corpus and the time complexities and accuracies against the methodologies that I have investigated so far:

- Approach 1
 - * Apply feature selection process to remove segments of silence and down sample the remaining segment to improve the quality.
 - * Apply value-based DTW(which we denote as baseline) using the most constrained window size and a euclidean metric.
- Approach 2
 - * Apply no feature selection process
 - * Perform feature extraction by extracting MFCC features
 - * Apply DTW using the most constrained window size and a euclidean metric.
- Approach 3
 - * Apply feature selection process that only removes segments of silence
 - * Extract MFCC features
 - * Apply DTW augmented with the euclidean metric.
- Approach 4
 - * Apply feature selection process to remove segment of utterance and down sample the remaining segment to improve the quality.
 - * Apply the feature extraction process discussed in [3.2.1] to extract local and global features.
 - * Apply DTW using the most constrained window size and a euclidean metric.

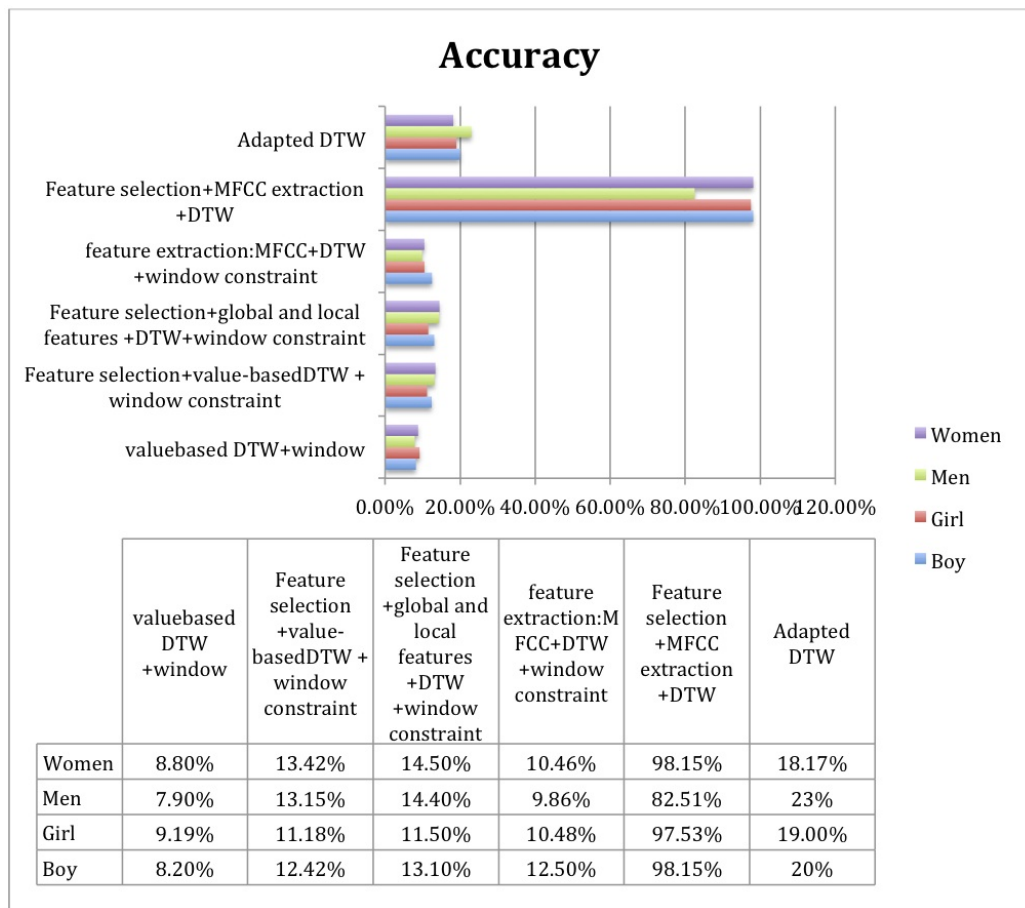


Figure 5.3: Accuracy

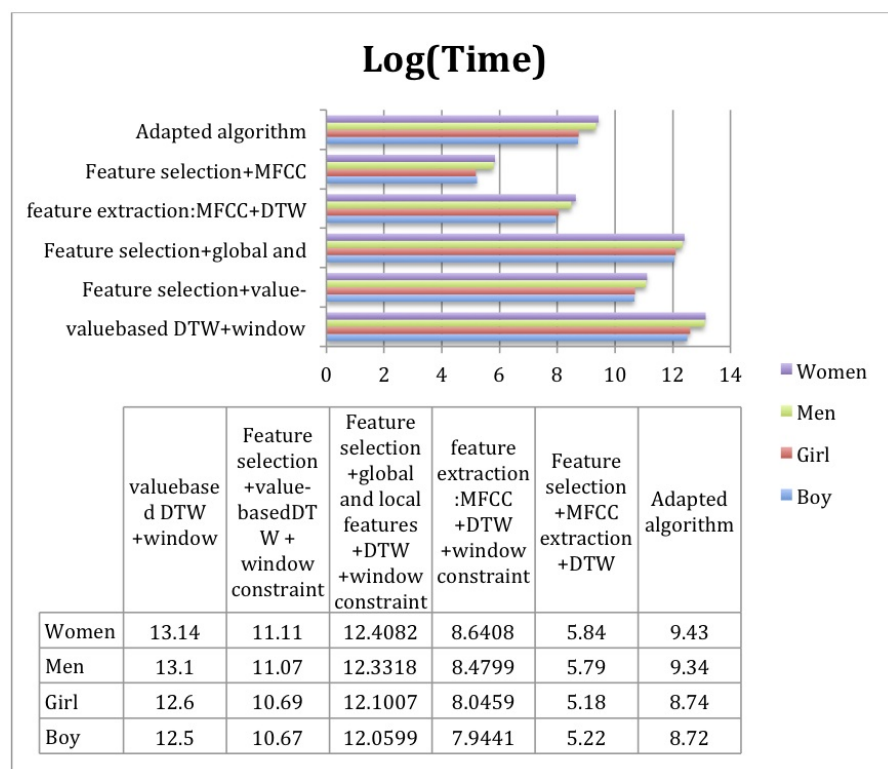


Figure 5.4: Time complexity

There are quite number of interesting observations that can made from the graphs and the tables given by figures [4.3,4.4].

- The proposed algorithm achieves better accuracy for test samples belonging to all categories than any of algorithms that employ the rigid window constraint of $w = \max(\lceil 0.1 * \max(n,m) \rceil, \text{abs}(n-m))$. The most interesting result is that the new algorithm incurs a lower computational cost than any of these DTW algorithms.
- From examining the improvements in accuracy across all categories, it can be concluded that for the TIGITS dataset, the new algorithm is invariant to variations introduced in the acoustic signals by different speakers .
- The methodology behind my proposed approach and that of approach 4 includes the same pre-processing stage. Th input to the DTW algorithm is constructed by using the domain dependent feature selection process mentioned in section followed by a domain independent feature extraction mapping (i.e the local and global features defined in section). Both approaches differ however, in their use of a cost function and a window constraint. The proposed approach doesn't subject DTW to any window constraints and utilised the kernel function that I constructed in the cost metric. Approach 4 on the other hand, employs the euclidean metric an subjects DTW to the window constraint of $w = \max(\lceil 0.1 * \max(n,m) \rceil, \text{abs}(n-m))$ The proposed DTW segments sequences into frames and employ a cost function on the frames. This reduces the dimensionality of the time series sequences and allows the algorithm to achieve a time and computational cost than is lower than the cost incurred by any of the investigated adaptations of the DTW algorithm that employs window constraints.

To confirm that the performance of the new algorithm is not tailored for this particular time-series data set, I have applied the tested the algorithm on the two largest time series data sets in UCR database.

Unlike the speech utterances, the time series sequences within each data set share the same length.

5.3.1 Experimental setup

The focus now is to check how well/ bad is the performance of the new DTW algorithm against DTW algorithms using window constraints when applied to datasets that belong to other application domains . The domain -dependent pre-processing policies are dropped for this set of experiments as these procedures were specifically designed for speech data. Thus in this section, I compare my proposed approach against:

- Approach 1
- Approach 4

As a I mentioned, the domain-dependent feature selection process of silence removal and subsampling is dropped from all approaches. However, the feature extraction process that involves extracting local and global features is kept since this procedure is domain independent.

One of the factors that I have also investigated in these experiments is the size of the time slices used in the algorithm that I proposed. For the TIDIGITS data set, I have used window slices of 50 ms. Reducing the size of the windows should principally increase both accuracy and time-complexity . The core kernel used by the new algorithm is based on the function:

$$k(x, z) = < \sum_{i=1}^n x_i, \sum_{j=1}^n z_j >$$

$k(x, z)$ represents the sum of all possible dot-products . Having a smaller window allows the sum to be dominated by dot products of vectors that are most similar. However smaller window frames results in longer time series sequence of frames. This in turn increases the time and computational complexity incurred by the DTW algorithm.

Figure [4.5] and figure[4.6] shows the accuracies and log(time) of the algorithms for the two datasets:

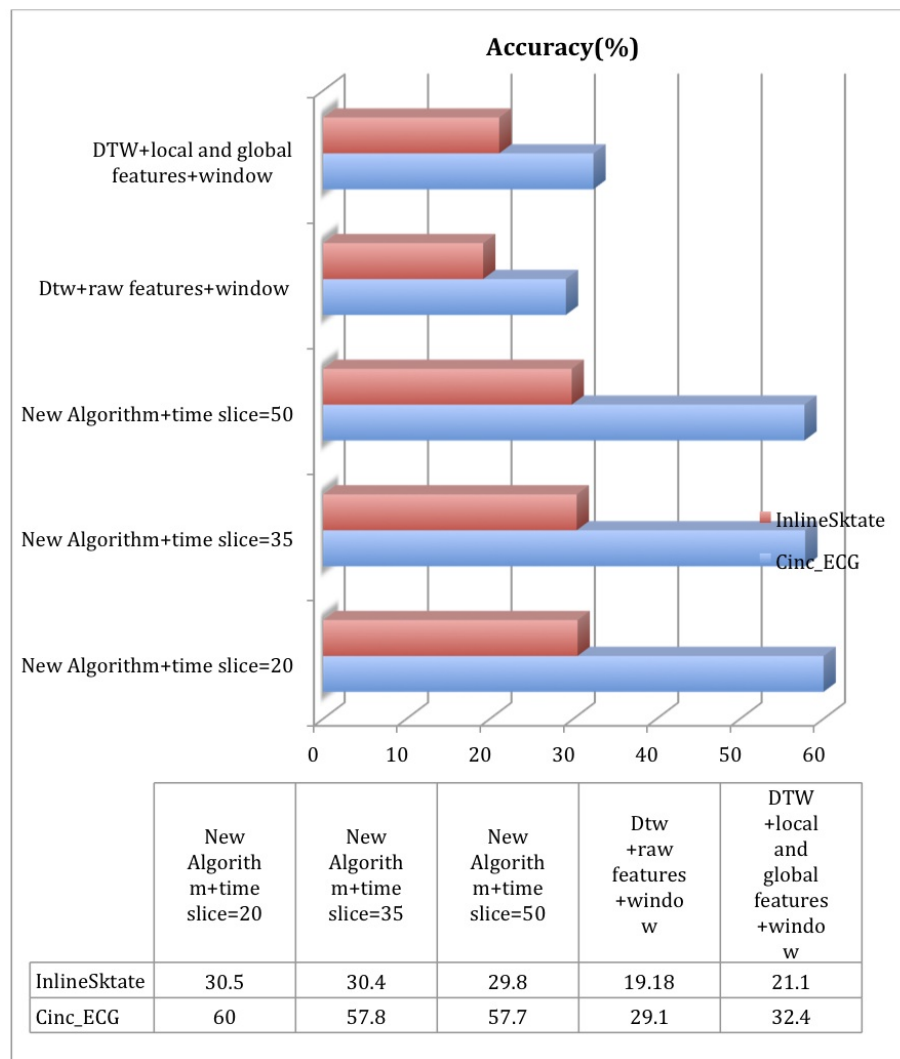


Figure 5.5: Accuracy

Observation:

- The new algorithm indeed achieves better accuracy than any versions of the window constrained DTW algorithm employing domain-independent feature extraction
- Comparatively, the performance of the new algorithm does improve if smaller window slices are employed to partition the time series sequence of vectors.

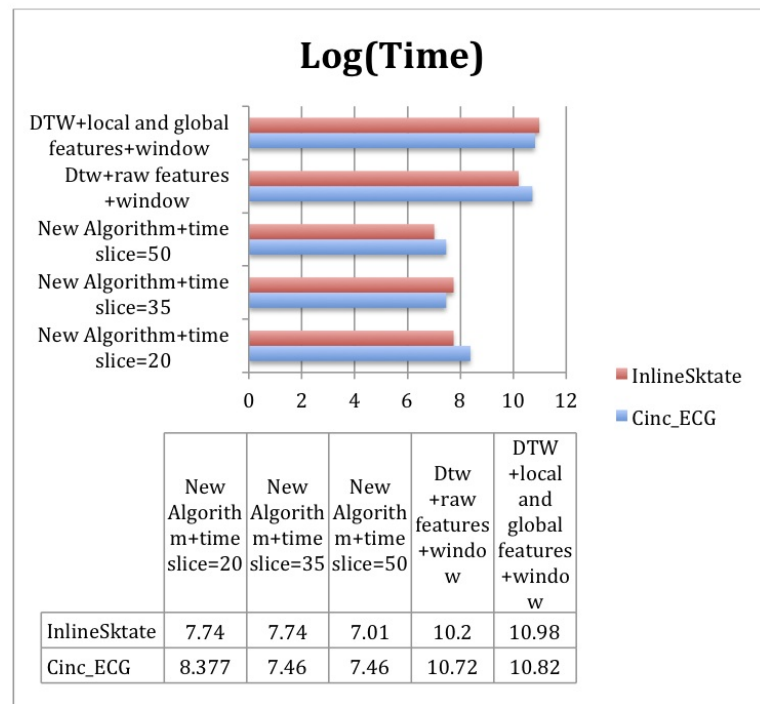


Figure 5.6: Time complexity

Observation:

- The new algorithm incurs less time and computational cost than any versions of the window constrained DTW algorithm employing domain-independent feature extraction
- Comparatively, the time complexity of the new algorithm increases when smaller window slices are used to partition the sequence

Bibliography

- [1] Das, G., Lin, K., Mannila, H., Renganathan, G. Smyth, P. (1998). “*Rule discovery from time series*”. In proceedings of the 4th Int’l Conference on Knowledge Discovery and Data Mining. New York, NY, Aug 27-31. pp 16-22.
- [2] Ying Xie, Bryan Witgen “*Adaptive Feature Based Dynamic Time Warping*”, International Journal of Computer Science And Network Security, January 2010.
- [3] Alex S .Park James R.Glass *Unsupervised Pattern Discovery in Speech*, IEEE Transactions On Audio Speech And Language Processing, VOL 16, January 2008.
- [4] Mathias De Wachter, Mike Matton, Kris Demuynck, Patrick Wambacq “*Template Based Continuous Speech Recognition*” IEEE Transactions On Audio And Speech Processing ,2007.
- [5] Yaodang Zhang and James R.Glass “*Towards Multi-Speaker Unsupervised Speech Pattern Discovery*” in Proc 2009.
- [6] Yaodang Zhang and James R.Glass ” *Unsupervised spoken keyword spotting via segmental DTW on Gaussian Posterior-grams*” in Proc 2009.
- [7] Mathias De Wachter, Mike Matton, Kris Demuynck, Patrick Wambacq ”A *Locally Weighted Distance Measure For Example Based Speech Recognition*” ICASSP 2004.
- [8] Michael A .Carlin, Samuel Thomas, Aren Jansen, Hyek Hermansky “*Rapid Evaluation of Spoken Term Discovery* ”, INTERSPEECH 2011: 821-824.

- [9] Hui Zhang, Tu Bao ho, Yang Zhang, Mao-Song-Lin “*Unsupervised Feature Extraction for Time Series Clustering Using Orthogonal Wavelet Transform*” Informatica 30(2006) 305-319.
- [10] S.Mallet “*A Wavelet Tour of Signal Processing*” Academic Press ,San Diego ,second edition,1999.
- [11]] A. Brazma, I. Jonassen, I. Eidhammer, and D. Gilbert, “*Approaches to the automatic discovery of patterns in biosequences* ” J. Comput. Biol.,vol. 5, no. 2, pp. 279305, 1998.
- [12] F Korn, H. Jagadish and C.Faloutsos. “*Efficiently supporting ad hoc queries in large datasets of time sequences*”. In Proceedings of the ATM of the ACM SIGMOID International Conference of Management Of Dat, pages 289-300.
- [13] Chun-Lin, Liu “*A Tutorial of the Wavelet Transform*” February 23, 2010.
- [14] Josif Grabocka, Erind Bedalli and Lars Schmidt-Thiem “*Efficient Classification of Long Time-Series*”, Information Systems and Machine Learning Lab.ICT Innovations 2012, pages 47-57.
- [15] Jessica Lin Eamonn Keogh Stefano Lonardi Pranav Patel “*Finding Motifs in Time Series*”, CiteSeer 2002.
- [16] Lee A.Barford, R. Shane Fazzio, DavidR. Smith Instruments and Photonics Laboratory “*An Introduction to Wavelets*” September, 1992.
- [17] Fayyad, U., Reina, C. . Bradley. P (1998). “*Initialisation of iterative refinement clustering algorithms*”. In Proceedings of the 4th International Conference on Knowledge Discovery and Data Mining. New York, NY, Aug 27-31. pp 194-198
- [18] Sakoe, H. chiba, S. (1978). “*Dynamic programming algorithm optimization fro spoken word* ” Trans. Acoustics, Speech, and Signal Proc., Vol. ASSP-26. pp. 43-49.
- [19] Itakura, F. (1975). “*Minimum prediction residual principle applied to speech recognition*”. IEE Speech, and Signal Proc., Vol. ASSP-23, pp. 52-72.

- [20] Fu, A.W., Keogh, E., Lau, L.Y.H., Ratanamahatana, C.A. (2005). “*Scaling and Time Warping in Time Series Querying*”. VLDB 05, pp. 649-660.
- [21] Xiaopeng Xi, Eamonn Keogh, Christian Shelton, Li Wei “*Fast Time Series Classification Using Numerosity Reduction*” ICML '06 Proceedings of the 23rd international conference on Machine learning Pages 1033-1040
- [22] D. Kim and B. Yum, “*Collaborative Filtering Based on Iterative Principal Component Analysis*, Expert Systems with Applications 28 (2005), 823830.
- [23] Chotirat Ann Ratanamahatana , Eamonn Keogh “*Three Myths about Dynamic Time Warping Data*”, Proceedings of SIAM International Conference on Data Mining (2005).
- [24] Rabiner, L., Rosenberg, A. Levinson, S. (1978). “*Considerations in dynamic time warping algorithms for discrete word recognition*”. IEEE Trans. Acoustics Speech, and Signal Proc., Vol. ASSP-26, pp. 575-582.